

The Period–Cohort Annuity Valuation Gap under Mortality Model Uncertainty

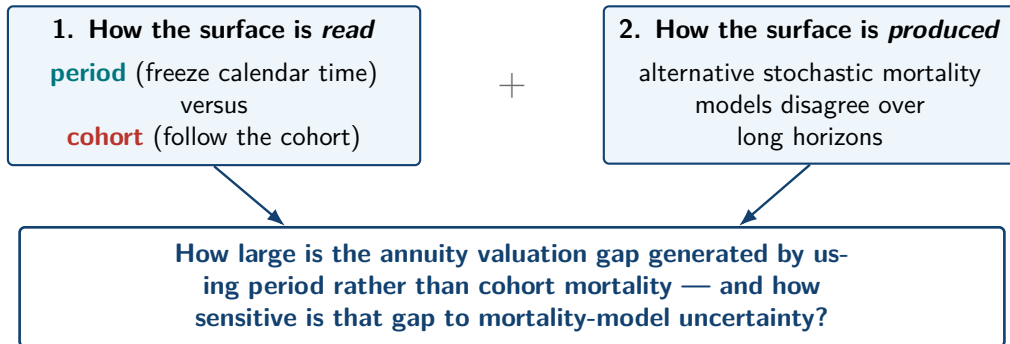
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The research problem

An annuity liability is a discounted stream of payments weighted by **survival probabilities** — and those probabilities are not observed. They are *constructed*, in two separate steps.



two interacting sources of valuation uncertainty

The analysis is divided into two complementary parts.

Part I — Demography.

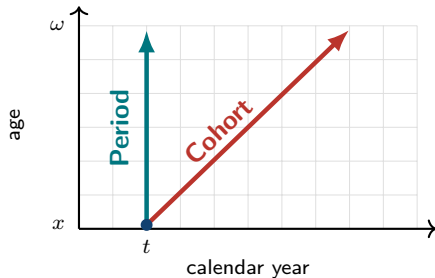
The first part focuses on **mortality forecasting**. Several stochastic mortality models are considered and combined through alternative ensemble rules, with particular attention to a **Shapley-based approach**.

Part II — Finance.

The second part studies the **financial implications** of the projected mortality dynamics. The analysis compares period- and cohort-based annuity valuations and assesses the role of **mortality-model uncertainty**.

PART I — DEMOGRAPHY—Period and cohort mortality

Let $m_{x,t}$ the central death rate at age x in calendar year t , and $p_{x,t}$ the corresponding one-year survival probability.



Period — freeze calendar time

$${}_k p_x^P(t) = \exp\left(-\sum_{h=0}^{k-1} m_{x+h,t}\right) \quad (1)$$

Cohort — follow the cohort

$${}_k p_x^C(t) = \exp\left(-\sum_{h=0}^{k-1} m_{x+h,t+h}\right) \quad (2)$$

If mortality improves over time, cohort survival is typically higher (Wilmoth, 2005; Ayuso et al., 2021).

Mortality Models and Model Uncertainty

Mortality projections provide the demographic basis for the period-cohort valuation problem.

We consider a set of stochastic mortality forecasting models,

$$\mathcal{M} = \{M_1, \dots, M_J\}.$$

For each model $j \in \mathcal{M}$, let

$$\widehat{m}_{x,t}^{(j)}$$

denote the forecast of the central death rate at age x and calendar year t .

The forecasting stage therefore produces a collection of projected mortality surfaces,

$$\left\{ \widehat{m}_{x,t}^{(j)} \right\}_{j \in \mathcal{M}}.$$

Rather than selecting a single specification *a priori*, we account for **mortality-model uncertainty** through forecast combination.

Forecast Combination

For each $j \in \mathcal{M}$, a rolling procedure generates one-step-ahead mortality forecasts on a validation grid $\mathcal{G}_V$.

Forecast accuracy is measured on the mortality-rate scale:

$$L_j = \frac{1}{|\mathcal{G}_V|} \sum_{(x,t) \in \mathcal{G}_V} \left(\hat{m}_{x,t}^{(j)} - m_{x,t} \right)^2.$$

The validation results are used to retain a subset of models $\mathcal{M}_B \subseteq \mathcal{M}$ and to construct a convex combination,

$$\hat{m}_{x,t}^{ens} = \sum_{j \in \mathcal{M}_B} \omega_j \hat{m}_{x,t}^{(j)}, \quad \sum_{j \in \mathcal{M}_B} \omega_j = 1, \quad \omega_j \geq 0.$$

The different combination rules differ in the way the weights ω_j are assigned.

Shapley (1953) ensemble I

For a given model $j \in \mathcal{M}_B$, consider any coalition $S \subseteq \mathcal{M}_B \setminus \{j\}$.

Define the equally weighted mortality forecast and its validation loss

$$\hat{m}_{x,t}^{(S)} = \frac{1}{|S|} \sum_{k \in S} \hat{m}_{x,t}^{(k)}, \quad L(S) = \frac{1}{|\mathcal{G}_V|} \sum_{(x,t) \in \mathcal{G}_V} \left(\hat{m}_{x,t}^{(S)} - m_{x,t} \right)^2$$

the value of a coalition is $v(S) = -L(S)$.

Adding model j produces the marginal contribution

$$\Delta_j(S) = v(S \cup \{j\}) - v(S).$$

The Shapley value averages this contribution over *all possible coalitions*:

$$\phi_j = \sum_{S \subseteq \mathcal{M}_B \setminus \{j\}} \frac{|S|!(|\mathcal{M}_B| - |S| - 1)!}{|\mathcal{M}_B|!} \Delta_j(S).$$

Shapley (1953); Bimonte et al. (2025, 2026)

Part I — Demography

Shapley ensemble II

The Shapley values measure model contributions, but they are neither sign-constrained nor normalised.

We first standardise them:

$$z_j = \frac{\phi_j - \bar{\phi}}{s_\phi}.$$

They are then mapped into convex weights through

$$w_j^{SH} = \frac{\exp(z_j)}{\sum_{k \in \mathcal{M}_B} \exp(z_k)}, \quad w_j^{SH} > 0, \quad \sum_{j \in \mathcal{M}_B} w_j^{SH} = 1.$$

The resulting mortality forecast is

$$\hat{m}_{x,t}^{SH} = \sum_{j \in \mathcal{M}_B} w_j^{SH} \hat{m}_{x,t}^{(j)}.$$

Combination is performed on mortality rates, before annuity valuation.

Equal-weight benchmark

$$w_j^{EW} = \frac{1}{|\mathcal{M}_B|}.$$

All retained models receive the same weight.

Performance-based benchmark

Let

$$\xi_j = \frac{L_j}{\max_{k \in \mathcal{M}_B} L_k}, \quad \xi_j \in [0, 1].$$

The weights are defined as

$$w_j^{PB} = \frac{\exp(-\xi_j)}{\sum_{k \in \mathcal{M}_B} \exp(-\xi_k)}.$$

Models with lower validation loss receive larger weights.

Data

- Human Mortality Database — **Italy**
- **male, female, total** populations
- ages 60, ..., 100

Sample split

- validation: **2000–2010**
- test: **2011–2019**
- re-estimation for projection:
1960–2019

Expanding rolling-origin design: at each forecast origin, models are estimated using only the information available up to that point.

Model selection is **population-specific**.

Candidate models

LC	Lee–Carter
APC	age–period–cohort
RH	Renshaw–Haberman
CBD	Cairns–Blake–Dowd
M6	CBD with cohort effect
PLAT	Plat
HU	Hyndman–Ullah functional
CPSPLINE	constrained P-spline

Model references: Lee and Carter (1992); Renshaw and Haberman (2006); Plat (2009); Camarda (2019).

Selected models and combination weights

For each population, only the five models with the lowest L_j values are retained for the ensemble construction.

Validation-based ensemble weights on the selected mortality models

Model	Male			Female			Total		
	Shapley	Perf.	Equal	Shapley	Perf.	Equal	Shapley	Perf.	Equal
HU	0.450	0.237	0.200	0.318	0.242	0.200	0.347	0.238	0.200
CPSPLINE	0.229	0.217	0.200	0.230	0.223	0.200	0.223	0.223	0.200
LC	0.196	0.206	0.200	0.264	0.233	0.200	0.271	0.230	0.200
PLAT	0.033	0.149	0.200	0.161	0.195	0.200	0.131	0.197	0.200
RH	0.093	0.190	0.200	—	—	—	—	—	—
APC	—	—	—	0.027	0.108	0.200	—	—	—
CBD	—	—	—	—	—	—	0.028	0.111	0.200

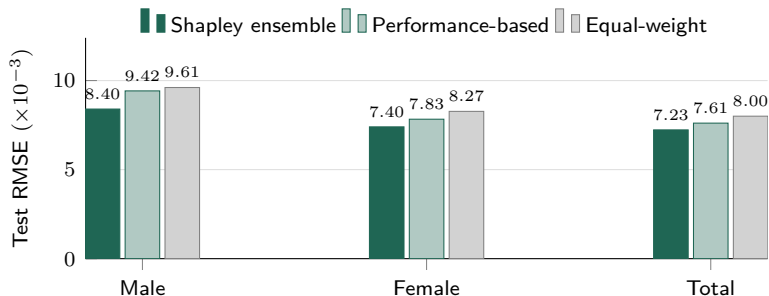
HU leads everywhere

Largest Shapley weight in all three populations.

Selection $\neq$ contribution

APC (female) and CBD (total) are selected, but receive only 2.7% and 2.8% Shapley weight.

Out-of-sample forecast accuracy

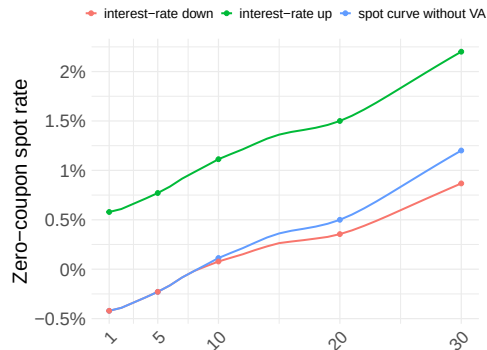


The Shapley ensemble attains the **lowest test RMSE** in all three populations.

Combination weights are estimated on the validation sample and held fixed over the test period.

Discounting: the EIOPA risk-free term structure

PART II — FINANCE ■ from mortality forecasts to annuity values



EIOPA EUR risk-free term structure.

- EIOPA **EUR basic/up/down** term structures
- reference date: **31 December 2019**
- **no volatility adjustment**
- common discount curve for period and cohort valuation

If $y(k)$ denotes the continuously compounded zero-coupon rate at maturity k ,

$$d_k = \exp(-k y(k)).$$

Contract and discounting are held **fixed**; **only the survival probabilities change** between the two valuations.

Annuity liabilities and valuation gap

For a given financial scenario, define

Period liability

$$V_t^P = B_t \sum_{k=1}^{\omega-x_r} d_k {}_k p_{x_r}^P(t)$$

Cohort liability

$$V_t^C = B_t \sum_{k=1}^{\omega-x_r} d_k {}_k p_{x_r}^C(t)$$

B_t denotes the annual annuity benefit at valuation year t .

$$\text{RelGap}_t = \frac{V_t^C - V_t^P}{V_t^P}, \quad c_t = \frac{V_t^P}{V_t^C}.$$

- $\text{RelGap}_t > 0 \implies$ **period valuation understates the cohort-consistent liability.**
- c_t is the benefit adjustment factor that makes the cohort-consistent liability equal to the period-based liability.

$x_r = 67$, $\omega = 100$, valuation years 2020–2050, and $B_t = 1$. For 2050, cohort valuation requires mortality projections up to 2083.

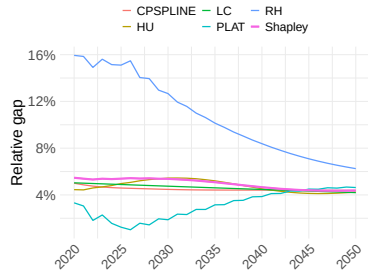
Shapley mortality ensemble — baseline EIOPA curve

Male					Female					Total			
Year	V^P	V^C	Rel. gap %	c_t	V^P	V^C	Rel. gap %	c_t	V^P	V^C	Rel. gap %	c_t	
2020	16.7	17.6	5.47	0.948	19.3	20.3	4.77	0.954	18.1	19.1	5.18	0.951	
2025	17.2	18.1	5.39	0.949	19.7	20.5	4.17	0.960	18.5	19.4	4.89	0.953	
2030	17.5	18.5	5.37	0.949	20.1	20.9	4.03	0.961	18.9	19.8	4.78	0.954	
2040	18.4	19.2	4.66	0.955	20.7	21.6	4.28	0.959	19.6	20.5	4.65	0.956	
2050	19.1	19.9	4.38	0.958	21.3	22.3	4.36	0.958	20.3	21.2	4.61	0.956	

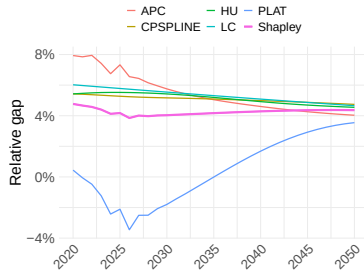
Cohort-consistent liabilities are **systematically higher**:

baseline period-cohort correction $\approx 4\text{--}5.5\%$

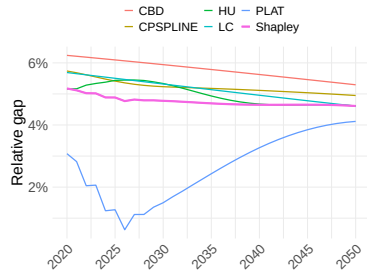
model-implied gaps over time



(a) Male



(b) Female



(c) Total

Alternative mortality models imply **materially different period-cohort valuation gaps**, even under the same EIOPA discount curve.

Robustness to the discounting environment

Mortality projection is held **fixed**; only the EIOPA term structure changes.

Year	Male			Female			Total		
	Down	Spot	Up	Down	Spot	Up	Down	Spot	Up
2020	5.66	5.47	4.92	4.97	4.77	4.25	5.37	5.18	4.63
2030	5.56	5.37	4.84	4.19	4.03	3.61	4.96	4.78	4.28
2050	4.54	4.38	3.92	4.55	4.36	3.89	4.80	4.61	4.12

Relative period-cohort gap (%).

$$\text{RelGap}_t^{\text{down}} > \text{RelGap}_t^{\text{spot}} > \text{RelGap}_t^{\text{up}}$$

Lower discount rates increase the weight of long-dated payments, where the cohort survival advantage is larger.

Mean relative gap (%) — Shapley ensemble

Population	Flat 3%	Baseline EIOPA
Male	4.07	4.96
Female	3.50	4.25
Total	3.90	4.76

$\text{RelGap} > 0$ under both discounting specifications.

The qualitative conclusion is unchanged: the period-based valuation remains lower than the cohort-consistent valuation.

Main conclusions

- 1 The **Shapley ensemble** attains the lowest test RMSE among the combination rules considered, for all three Italian populations.
- 2 **Period** mortality **understates cohort**-consistent annuity liabilities, with a baseline correction of approximately 4–5.5% at age 67.
- 3 The size of the correction depends on **mortality-model uncertainty**, while its economic relevance persists under alternative discounting assumptions.

Future research

- **Joint biometric–financial modelling**: stochastic interest rates instead of deterministic term structures.
- **Multi-population mortality** and richer annuity specifications.

Cohort consistency is not only a demographic modelling issue: it affects the **economic measurement** of long-term annuity liabilities.

- Ayuso, M., Bravo, J. M., Holzmann, R., and Palmer, E. (2021). Addressing the life expectancy gap in pension policy. *Insurance: Mathematics and Economics*, 99:200–221.
- Bacinello, A. R., Millossovich, P., and Chen, A. (2018). The impact of longevity and investment risk on a portfolio of life insurance liabilities. *European Actuarial Journal*, 8(2):257–290.
- Bimonte, G., Russolillo, M., Shang, H. L., and Yang, Y. (2025). Mortality models ensemble via shapley value. *Decisions in Economics and Finance*, 48(2):1131–1159.
- Bimonte, G., Russolillo, M., Yang, Y., and Shang, H. L. (2026). Enhancing mortality forecasting with ensemble learning: A shapley-based approach. *International Journal of Forecasting*.
- Camarda, C. G. (2019). Smooth constrained mortality forecasting. *Demographic Research*, 41:1091–1130.
- Lee, R. D. and Carter, L. R. (1992). Modeling and forecasting US mortality. *Journal of the American Statistical Association*, 87(419):659–671.
- Plat, R. (2009). On stochastic mortality modeling. *Insurance: Mathematics and Economics*, 45(3):393–404.
- Renshaw, A. E. and Haberman, S. (2006). A cohort-based extension to the Lee–Carter model for mortality reduction factors. *Insurance: Mathematics and Economics*, 38(3):556–570.
- Shapley, L. S. (1953). A value for n -person games. In Kuhn, H. W. and Tucker, A. W., editors, *Contributions to the Theory of Games II*, pages 307–317. Princeton University Press.
- Wilmoth, J. R. (2005). On the relationship between period and cohort mortality. *Demographic Research*, 13(11):231–280.