StMoMo: An R Package for **St**ochastic **Mo**rtality **Mo**delling

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Agenda

- Motivation and Literature Review
- Generalised Age-Period-Cohort mortality models
- **StMoMo** package
- Conclusions

StMoMo: **St**ochastic **Mo**rtality **Mo**delling

Who is MoMo?

StMoMo: Stochastic Mortality Modelling

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Who is MoMo?

Stochastic **Mo**rtality **Mo**delling — **StMoMo**: **St**ochastic **Mo**rtality **Mo**delling

Who is MoMo?

Advances in mortality modelling

- **Lee-Carter model** (Lee and Carter 1992)
  - Add more bilinear age-period components (Renshaw and Haberman 2003)
  - Add a cohort effect (Renshaw and Haberman 2006)
- Two factor **CBD model** (Cairns, Blake, and Dowd 2006)
  - Add cohort effect, quadratic age term (Cairns et al. 2009)
  - Combine with features of the Lee-Carter (Plat 2009)
- **Many more models** proposed in the literature (e.g. Aro and Pennanen (2011), O'Hare and Li (2012), Börger, Fleischer, and Kuksin (2013), Alai and Sherris (2014))

Mortality modelling in R

- **Demography** (Hyndman 2014)
  - Lee-Carter model and several of its variants
- **ilc** (Butt, Haberman, and Shang 2014)
  - Lee-Carter with cohorts and Lee-Carter under a Poisson framework
- **Lifemetrics** (`http://www.macs.hw.ac.uk/~andrewc/lifemetrics/`)
  - CBD and extensions
  - Lee-Carter with cohorts and Lee-Carter under a Poisson framework

Limitation of existing R packages

- Not easily expandable to include new models
- Limited forecasting and simulation capabilities
- Limited tools for goodness-of-fit analysis
- Do not allow for parameter uncertainty

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- **StMoMo** seeks to overcome these limitations

StMoMo: An R package for **St**ochastic **Mo**rtality **Mo**delling

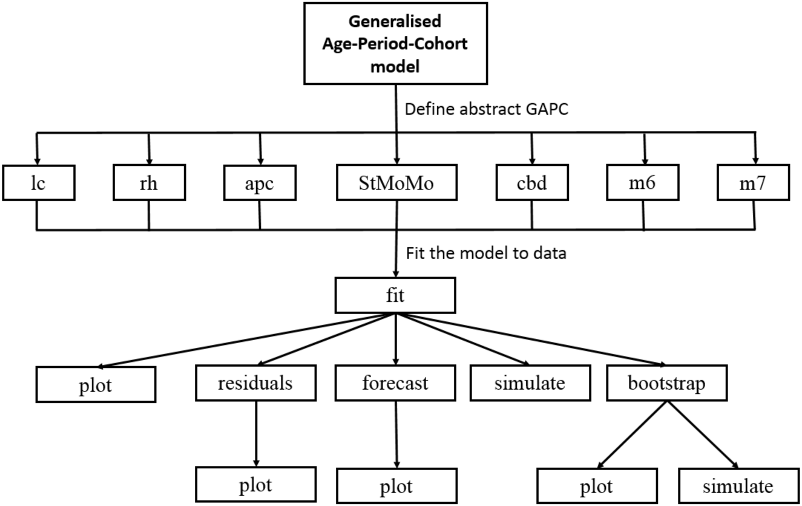
- On CRAN:
  http://cran.r-project.org/web/packages/StMoMo/
- Development version on Github:
  https://github.com/amvillegas/StMoMo
- To install the stable version on R CRAN:

```
install.packages("StMoMo")
```

- To load within R:

```
library(StMoMo)
```

Overview of the structure of **StMoMo**



Generalised Age-Period-Cohort stochastic mortality models

StMoMo is based on the unifying framework of the family of Generalised Age-Period-Cohort stochastic mortality models

- General Age-Period-Cohort model structure (Hunt and Blake 2015)
- Generalised (non-)linear model (Currie 2014)

General Age-Period-Cohort model structure

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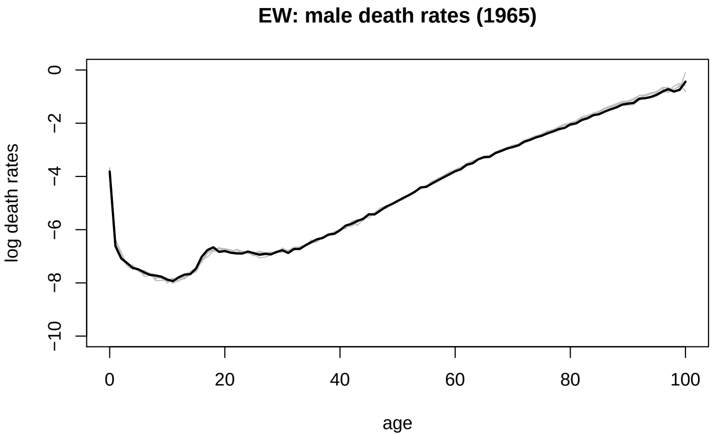

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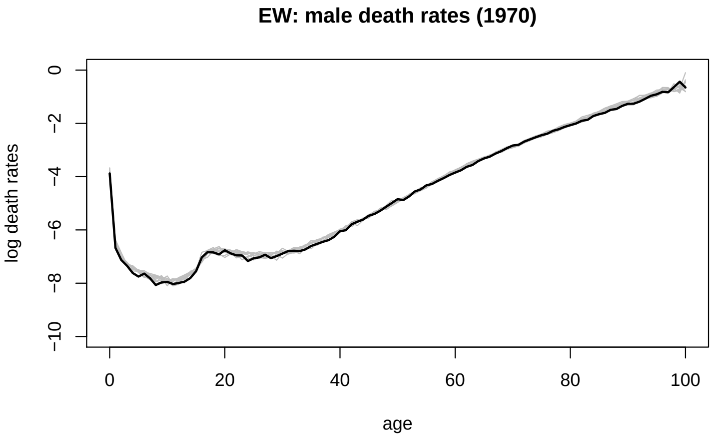

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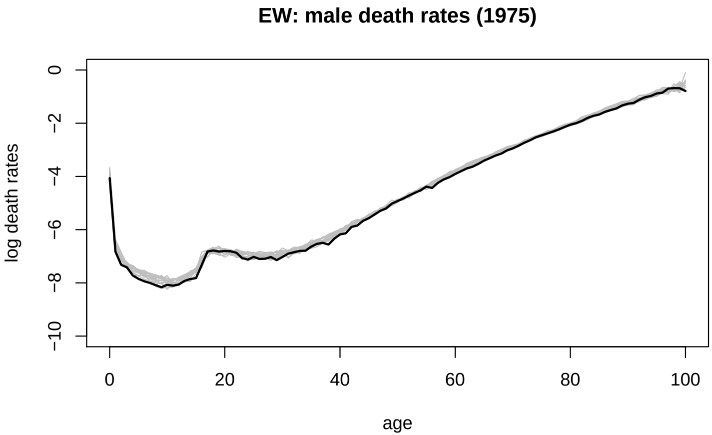

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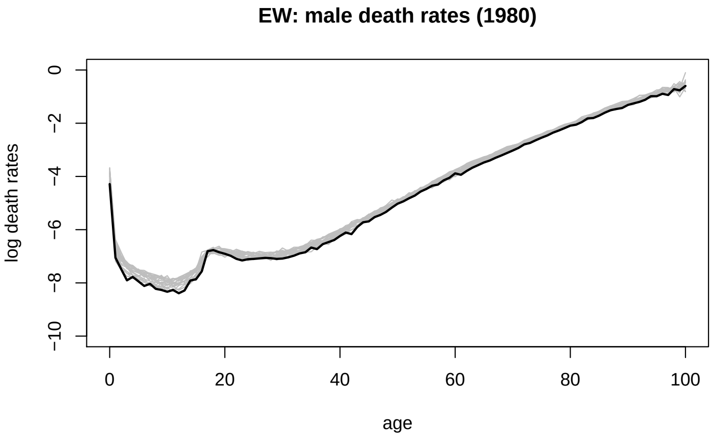

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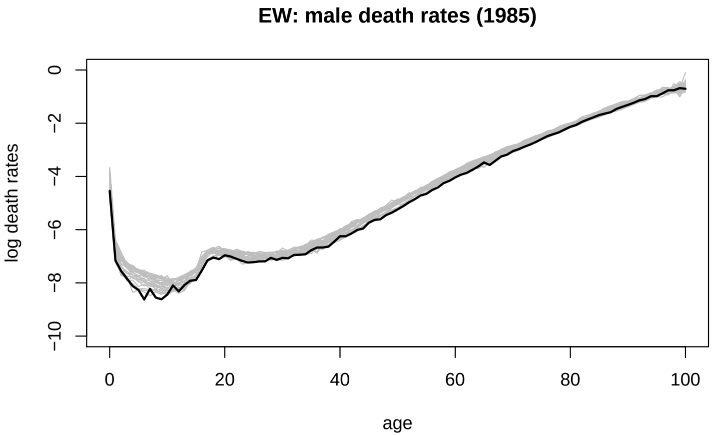

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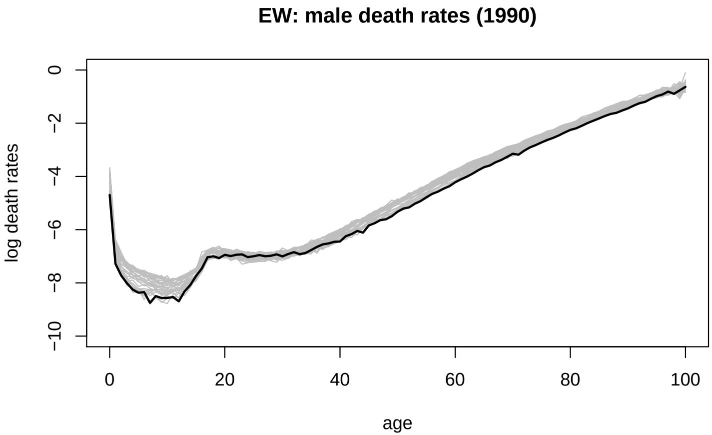

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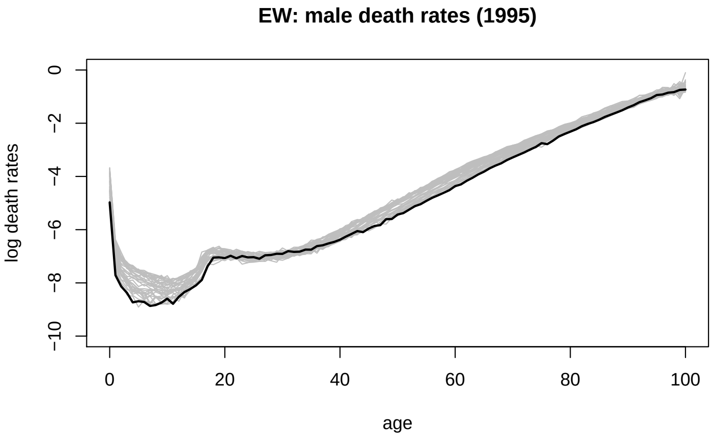

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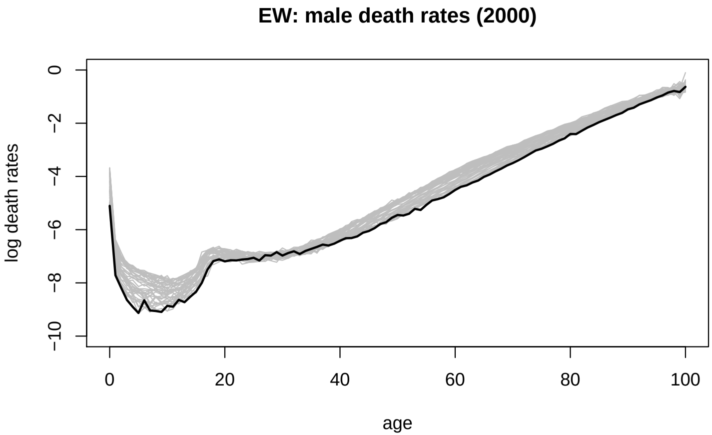

General Age-Period-Cohort model structure

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$$\log \mu_{xt} \quad =$$

General Age-Period-Cohort model structure

$$\log \mu_{xt} \quad = \quad \alpha_x$$

General Age-Period-Cohort model structure

$$\log \mu_{xt} = \alpha_x + \kappa_t$$

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General Age-Period-Cohort model structure

$$\log \mu_{xt} \quad = \quad \alpha_x \quad + \quad \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t{}^{(i)}$$

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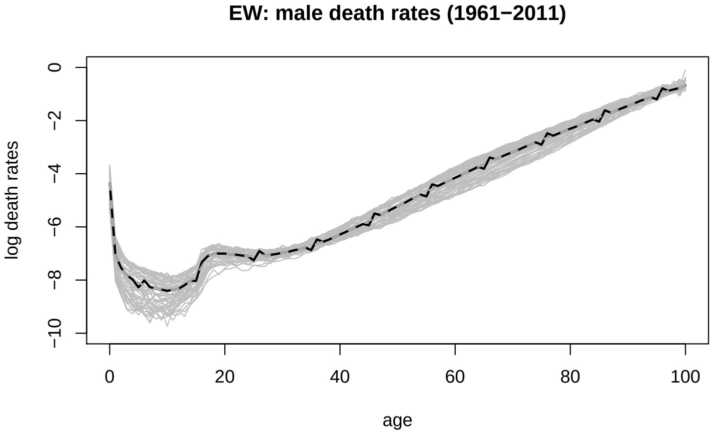

$$\log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t{}^{(i)} + \gamma_{t-x}$$

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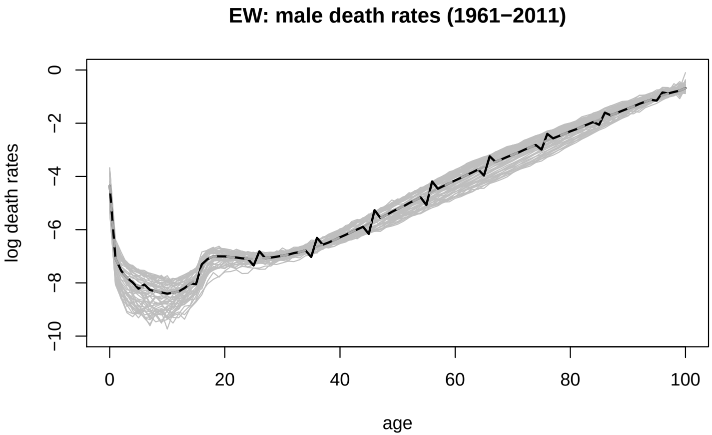

$$\log \mu_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} {\kappa_t}^{(i)} + \beta_x^{(0)} \gamma_{t-x}$$

Generalised Age-Period-Cohort stochastic mortality models

1. **Random Component**:

$$D_{xt} \sim \mathrm{Poisson}(E^c_{xt}\mu_{xt}) \quad \text{or} \quad D_{xt} \sim \mathrm{Binomial}(E^0_{xt}, q_{xt})$$

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$$\eta_{xt} \quad = \quad \alpha_x \quad + \quad \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} \quad + \quad \beta_x^{(0)} \gamma_{t-x}$$

- *Lee-Carter type*: $\beta_x^{(i)}$, non-parametric
- *CBD type*: $\beta_x^{(i)} \equiv f^{(i)}(x)$, pre-specified parametric function

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3. **Link Function**:

$$g\left(\mathbb{E}\left(\frac{D_{xt}}{E_{xt}}\right)\right) = \eta_{xt}$$

- log-Poisson: $\eta_{xt} = \log \mu_{xt}$
- logit-Binomial: $\eta_{xt} = \text{logit}\, q_{xt}$

Generalised Age-Period-Cohort stochastic mortality models

4. **Set of parameter constraints**:
    - Most mortality models are only identifiable up to a transformation
    - Need parameters constraints to ensure identifiability
    - **Constraint function** v mapping an arbitrary vector of parameters

$$\theta := \left(\alpha_x, \beta_x^{(1)}, ..., \beta_x^{(N)}, \kappa_t^{(1)}, ..., \kappa_t^{(N)}, \beta_x^{(0)}, \gamma_{t-x}\right)$$

into a vector of transformed parameters

$$v(\theta) = \tilde{\theta} = \left(\tilde{\alpha}_x, \tilde{\beta}_x^{(1)}, ..., \tilde{\beta}_x^{(N)}, \tilde{\kappa}_t^{(1)}, ..., \tilde{\kappa}_t^{(N)}, \tilde{\beta}_x^{(0)}, \tilde{\gamma}_{t-x}\right)$$

satisfying the model constraints with no effect on the predictor η_{xt} (i.e. θ and $\tilde{\theta}$ result in the same η_{xt})

GAPC stochastic mortality models with **StMoMo**

GAPC model are constructed using the function

```
StMoMo(link, staticAgeFun, periodAgeFun,
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  - `periodAgeFun`: list of length N defining $\beta_x^{(i)}$, $i = 1, \ldots, N$.
  - `cohortAgeFun`: defines parameter $\beta_x^{(0)}$
- `constFun`: Implementation of **constraint function** $v(\theta) = \tilde{\theta}$ which defines the **set of parameter constraints**

$$\boxed{\eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x}}$$

GAPC stochastic mortality models with **StMoMo**

Model	Predictor (η_{xt})
LC	$\alpha_x + \beta_x^{(1)}\kappa_t^{(1)}$
CBD	$\kappa_t^{(1)} + (x-\bar{x})\kappa_t^{(2)}$
APC	$\alpha_x + \kappa_t^{(1)} + \gamma_{t-x}$
M7	$\kappa_t^{(1)} + (x-\bar{x})\kappa_t^{(2)} + \left((x-\bar{x})^2 - \hat{\sigma}_x^2\right)\kappa_t^{(3)} + \gamma_{t-x}$

- For consistency, all under a log-Poisson setting:

$$D_{xt} \sim \mathrm{Poisson}(E_{xt}^c \mu_{xt})$$

$$\log \mu_{xt} = \eta_{xt}$$

Lee-Carter model (Lee and Carter 1992)

Predictor: $\eta_{xt} = \alpha_x + \beta_x^{(1)} \kappa_t^{(1)}$

Constraints: $\sum_x \beta_x^{(1)} = 1, \qquad \sum_t \kappa_t^{(1)} = 0$

$v(\theta) = \tilde{\theta}$**:** $\left(\alpha_x, \beta_x^{(1)}, \kappa_t^{(1)}\right) \rightarrow \left(\alpha_x + c_1 \beta_x^{(1)}, \frac{1}{c_2} \beta_x^{(1)}, c_2(\kappa_t^{(1)} - c_1)\right)$

with $\quad c_1 = \frac{1}{n} \sum_t \kappa_t^{(1)} \quad c_2 = \sum_x \beta_x^{(1)}$

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$$v(\theta) = \tilde{\theta}\text{:} \quad \left(\alpha_x, \beta_x^{(1)}, \kappa_t^{(1)}\right) \to \left(\alpha_x + c_1\beta_x^{(1)}, \frac{1}{c_2}\beta_x^{(1)}, c_2(\kappa_t^{(1)} - c_1)\right)$$

$$\text{with} \quad c_1 = \frac{1}{n}\sum_t \kappa_t^{(1)} \quad c_2 = \sum_x \beta_x^{(1)}$$

```
#Define constraint function
constLC <- function(ax, bx, kt, b0x, gc, wxt, ages){
c1 <- mean(kt[1, ], na.rm = TRUE)
c2 <- sum(bx[, 1], na.rm = TRUE)
list(ax = ax + c1 * bx[, 1], bx[, 1] = bx[, 1] / c2,
     kt[1,] = c2 * (kt[1, ] - c1))}
#Define model
LC <- StMoMo(link = "log", staticAgeFun = TRUE,
             periodAgeFun = "NP", constFun = constLC)
```

CBD model (Cairns, Blake, and Dowd 2006)

Predictor: $\eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)}$

Constraints: No constraints necessary

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Constraints: No constraints necessary

```
#B2: x - \bar{x}
f2 <- function(x, ages) x - mean(ages)
#Define model
CBD <- StMoMo(link = "log", staticAgeFun = FALSE,
              periodAgeFun = c("1", f2))
```

Model definition: Predefined functions for common models

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```
LC <- lc()
CBD <- cbd(link = "log")
APC <- apc()
M7 <- m7(link = "log")
```

Model definition: Predefined functions for common models

```
LC <- lc()
CBD <- cbd(link = "log")
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M7 <- m7(link = "log")
```

```
## Poisson model with predictor:
log m[x,t] = a[x] + b1[x] k1[t]

## Poisson model with predictor:
log m[x,t] = k1[t] + f2[x] k2[t]

## Poisson model with predictor:
log m[x,t] = a[x] + k1[t] + g[t-x]

## Poisson model with predictor:
log m[x,t] = k1[t] + f2[x] k2[t] +
f3[x] k3[t] + g[t-x]
```

Model fitting: Data

Sample data for England & Wales males aged 0-100 for the period 1961-2011

```
Dxt <- EWMaleData$Dxt
Ext <- EWMaleData$Ext
ages <- EWMaleData$ages   #0-100
years <- EWMaleData$years #1961-2011
```

Model fitting: Data

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```

```
Dxt
```

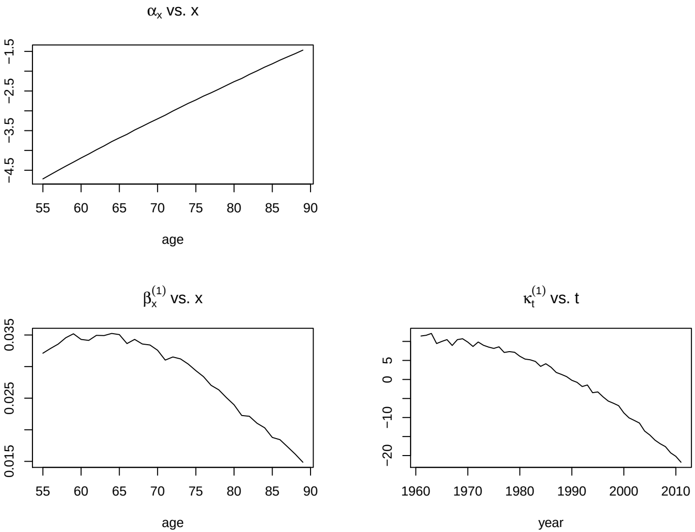
```
##    1961  1962  1963  1964 1965 1966 1967 1968 1969
## 0 9988 10573 10401 10011 9518 9357 8673 8705 8331
## 1  665   598   665   588  571  616  549  552  567
## 2  398   353   378   354  354  389  374  381  381
## 3  249   259   261   254  292  301  281  316  275
```

Model fitting

```r
#Ages for fitting
ages.fit <- 55:89
#Fit other models
LCfit <- fit(LC, Dxt = Dxt, Ext = Ext, ages = ages,
        years = years, ages.fit = ages.fit)
APCfit <- fit(APC, Dxt = Dxt, Ext = Ext, ages = ages,
        years = years, ages.fit = ages.fit)
CBDfit <- fit(CBD, Dxt = Dxt, Ext = Ext, ages = ages,
        years = years, ages.fit = ages.fit)
M7fit <- fit(M7, Dxt = Dxt, Ext = Ext, ages = ages,
        years = years, ages.fit = ages.fit)
```

Parameter estimates

```r
plot(LCfit)
```



Goodness-of-fit: Residuals

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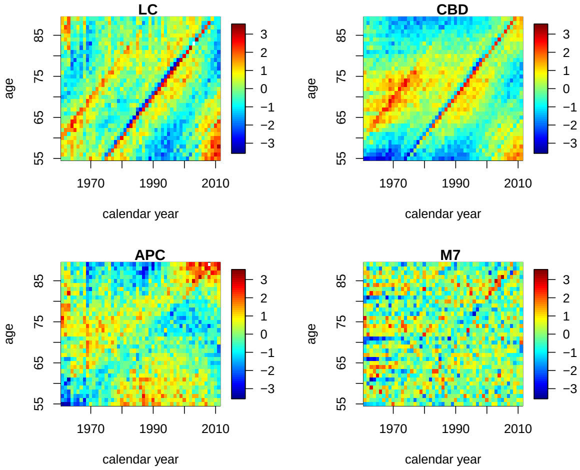
```
#Compute residuals
LCres <- residuals(LCfit)
CBDres <- residuals(CBDfit)
APCres <- residuals(APCfit)
M7res <- residuals(M7fit)
```

Goodness-of-fit: Residual heatmaps

```
plot(LCres, type = "colourmap", reslim = c(-3.5, 3.5))
```



Forecasting and simulation

- **Period indexes:** Multivariate random walk with drift

$$\boldsymbol{\kappa}_t = \boldsymbol{\delta} + \boldsymbol{\kappa}_{t-1} + \boldsymbol{\xi}_t^{\kappa}, \qquad \boldsymbol{\kappa}_t = \begin{pmatrix} \kappa_t^{(1)} \\ \vdots \\ \kappa_t^{(N)} \end{pmatrix}, \qquad \boldsymbol{\xi}_t^{\kappa} \sim N(\mathbf{0}, \Sigma),$$

- **Cohort effect:** ARIMA(p, q, d) with drift

$$\Delta^d \gamma_c = \delta_0 + \phi_1 \Delta^d \gamma_{c-1} + \cdots + \phi_p \Delta^d \gamma_{c-p} + \epsilon_c + \delta_1 \epsilon_{c-1} + \cdots + \delta_q \epsilon_{c-q}$$

Forecasting

Model	Model for γ_{t-x}
APC	ARIMA$(1,1,0)$ with drift
M7	ARIMA$(2,0,0)$ with non-zero intercept

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APC	ARIMA$(1, 1, 0)$ with drift
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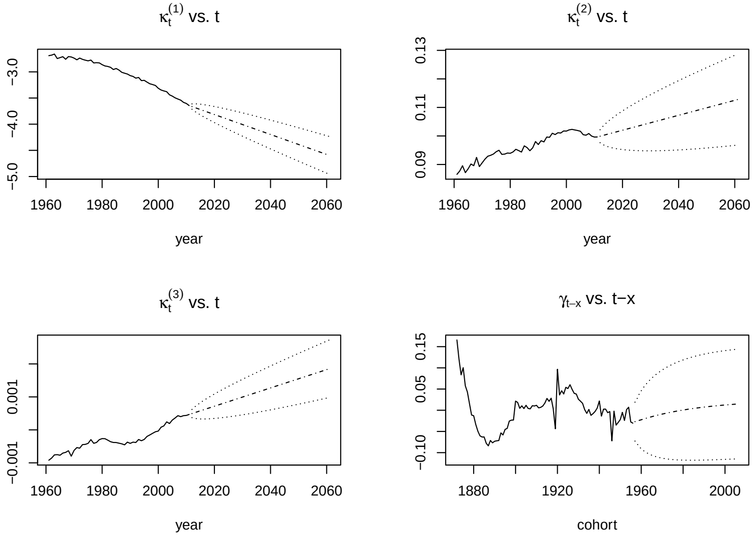
50-year ahead ($h = 50$) central projections: period indexes, cohort index, and one-year death probabilities:

```
LCfor <- forecast(LCfit, h=50)
CBDfor <- forecast(CBDfit, h=50)
APCfor <- forecast(APCfit, h=50, gc.order = c(1,1,0))
M7for <- forecast(M7fit, h=50, gc.order = c(2,0,0))
```

Forecasted period and cohort indexes

```r
plot(M7for, parametricbx = FALSE)
```

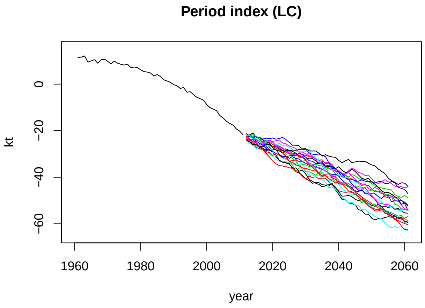


Simulation

```
LCsim <- simulate(LCfit, nsim=500, h=50)
CBDsim <- simulate(CBDfit, nsim=500, h=50)
APCsim <- simulate(APCfit, nsim=500, h=50,
                  gc.order=c(1,1,0))
M7sim <- simulate(M7fit, nsim=500, h=50,
                  gc.order=c(2,0,0))
```

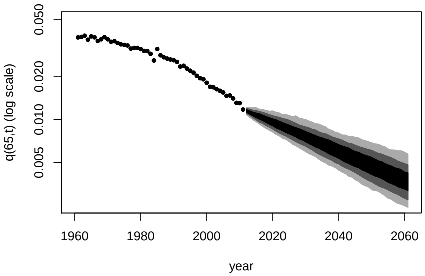
Simulation trajectories

```
#Plot period index trajectories for the LC model
plot(LCfit$years, LCfit$kt[1,],
     xlim=c(1960,2061), ylim=c(-65,15),
     type="l", xlab="year", ylab="kt",
     main="Period index (LC)")
matlines(LCsim$kt.s$years, LCsim$kt.s$sim[1,,1:20],
         type="l", lty=1)
```



Fancharts

```
library(fanplot)
plot(LCfit$years, (Dxt/Ext)["65",], xlim=c(1960,2061),
     ylim=c(0.0025,0.05),  pch =20, log="y",
     xlab="year", ylab="q(65,t) (log scale)")
fan(t(LCsim$rates["65",,]), start=2012,
    probs=c(2.5,10,25,50,75,90,97.5), n.fan=4, ln=NULL,
    fan.col=colorRampPalette(c("black","white")))
```



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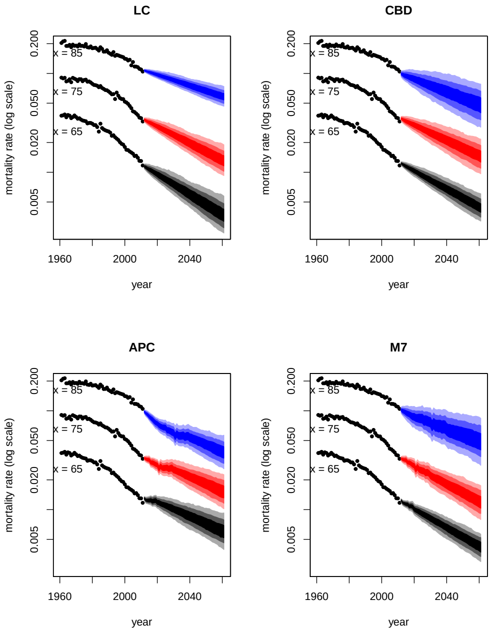
LC
CBD
APC
M7
mortality rate (log scale)
0.005
0.020
0.050
0.200
x = 85
x = 75
x = 65
1960
2000
2040
year

Parameter uncertainty and bootstrapping

StMoMo implements:

- Semiparametric bootstrapping (Brouhns et al., 2005)
- Residuals bootstrapping (Koissi et al., 2006)

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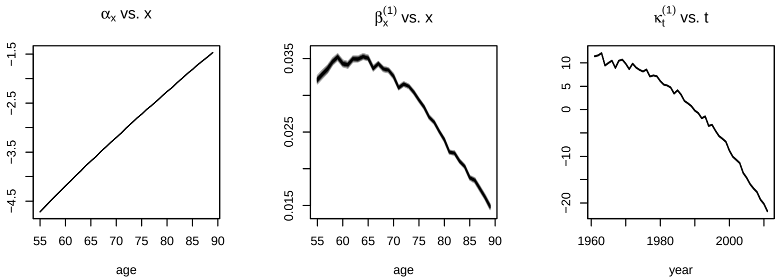
```
LCboot <- bootstrap(LCfit, nBoot=500,
                    type="semiparametric")
plot(LCboot, nCol = 3)
```



Conclusion

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  - Model fitting
  - Analysis of goodness-of-fit
  - Projection and simulations
  - Bootstrapping and parameter uncertainty
- Easy implementation and comparison of a wide range of models making it useful for:
  - Actuaries analysing longevity risk
  - Use in the classroom

Future work

- New models for forecasting time indexes (e.g. VAR models)
- Allow for $\beta_x^{(i)} = f^{(i)}(x; \theta_i)$ (see Hunt and Blake (2014))
- Multipopulation models
- Shiny web app

http://cran.r-project.org/web/packages/StMoMo/
https://github.com/amvillegas/StMoMo

Thank you!

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