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ABSTRACT

In this paper we generalize the existing approximate factor model analysis by specifying vector autoregressive moving average (VARMA) dynamics for latent factors. We show that when factors are obtained as linear combinations of observable series their dynamic process is generally a VARMA. Moreover, this generalization can be motivated by the usual arguments of parsimony, invertibility and marginalization issues in which VARMA models outperform the VAR representations. We apply our approach in two pseudo-out-of-sample forecasting exercises using an U.S. monthly balanced panel and a Canadian monthly panel taken from Boivin, Giannoni and Stevanović (2009, 2008) respectively. We find that considering VARMA representations for factors helps in predicting several key macroeconomic aggregates relatively to standard factor based forecasting models.

Key words: factor analysis, VARMA process, forecasting.

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1. **Introduction**

As information technology improves and we move forward in time, the availability of economic and finance time series grows in both time and cross-section size directions. A simple visit to web sites of statistical agencies in developed countries gives us access to thousands of economic indicators while in high-frequency data finance literature researchers deal with panels where the time period is measured in seconds! But this huge amount of information causes curse of dimensionality problem when standard time series tools are used. Since most of these series are relatively highly correlated, at least within some categories, there is a need for a method that will not only reduce the dimension but also use the reduced system in a meaningful way. The most popular method in the literature is factor model analysis, or what is called today "large dimensional approximate factor model" which is an extension of classical factor model by allowing some limited cross-section and time correlation among idiosyncratic components (as long as a small number of largest eigenvalues of the covariance matrix of common components diverge when the number of series tends to infinity, while the remaining eigenvalues as well as the eigenvalues of the covariance matrix of specific components are bounded) and where the "large dimensional" stands for both time and cross-section size asymptotics. While the factor models (approximate static and dynamic) were introduced in finance and macroeconomic literature by Chamberlain and Rothschild (1983), and Sargent and Sims (1977) and Geweke (1977) respectively, the literature on large dimensional factor models started with Stock and Watson (2002a) and Forni, Hallin, Lippi and Reichlin (2000). Further theoretical advances were given by Bai (2003), Bai and Ng (2002), Forni et al. (2004, 2005), among others. In applied work, these models were used in forecasting macroeconomic aggregates (Banarjee, Masten and Massimilano (2006), Stock and Watson (2002b), Forni et al. (2005)), in structural macroeconomic analysis (Bernanke, Boivin and Eliasz (2005) and Favero, Marcellino and Neglia (2005)), in nowcasting or economic monitoring (Aruoba, Diebold and Scotti (2008) and Giannone, Reichlin and Small (2008)), in weak instrument literature (Bai and Ng (2008) and Kapetanious and Marcellino (2008)), and in estimation of dynamic stochastic general equilibrium models (Boivin and Giannoni (2006)).

The specification of factors' dynamics have not been studied in the literature per se and in most of cases it is assumed that factors follow a finite order vector autoregressive (VAR) process. The importance of how well the factors' process is specified depends on the technique used to estimate the factor model and on the research goal. In two-step method developed by Stock and Watson (2002a), the factors' process does not matter for approximation of factors, but this could be an issue if we use a likelihood-based technique which relies on the completely parameterized process. Doz, Giannone, and Reichlin (2006) do a Monte Carlo study where they compare principal components (PC) method, estimation of dynamic factor model parameters using PC estimates then a single pass of the Kalman filter, and maximum likelihood (PC for starting values, then use EM algorithm to convergence), relative to how well those methods approximate true factors. The conclusion is that all three methods behave relatively similarly. Once the factors are approximated (or estimated), the specification of their process could be important if we use it to obtain results of interest. This is the case in forecasting economic indicators since we can use forecasting models that are just projections on factors' space or the forecasting models that rely on the factor structure where the forecasts of factors are necessary. Boivin and Ng (2005) compare combinations of factor estimation methods and forecasting equation specifications from the perspective of forecast MSE. In particular, they compare projection-based models (as in Stock and Watson (2002b)) and those based on factor structure where the forecast of common component is obtained either using VAR process or in nonparametric way (as in Forni et al. (2000)). They conclude that projection-based method which use PC estimates generally works best.

In this paper we generalize the existing approximate factor model analysis by specifying vector autoregressive moving average (VARMA) dynamics for latent factors. We show that when factors are obtained as linear combinations of observable series their dynamic process is generally a VARMA. Moreover, this generalization can be motivated by the usual arguments of parsimony, invertibility and marginalization issues in which VARMA models outperform the VAR representations (see Lütkepohl (1987) for details). Hence, assuming finite order VAR structure can be miss-leading in finite sample if the MA component of the true process is important or we do not use the true number of factors. However, VARMA modeling has not been used a lot in the literature because of the identification and estimation issues. The general VARMA representation is not identified so one must find an identified form (usually an echelon form), and the estimation requires nonlinear techniques (maximum likelihood or nonlinear least squares) which becomes very difficult with the number of series. To overcome these issues we use identified VARMA forms and generalized least squares method proposed in Dufour and Pelletier (2008).

Once we have argued that factors' dynamics should be modeled as VARMA, the objective of this paper is to see if allowing for VARMA dynamics in factors can help in forecasting exercise in terms of forecast MSE. To do so, we estimate the factors as principal components of the contemporaneous covariance matrix of observables and then fit four different identified VARMA forms on these PCs. We apply our approach in two pseudo-out-of-sample forecasting exercises using an U.S. monthly balanced panel and a Canadian monthly panel taken from Boivin, Giannoni and Stevanović (2009, 2008) respectively. We find that considering VARMA representations for factors helps in predicting several key macroeconomic aggregates relatively to standard factor-based forecasting models, both using projection models and those based on VAR factors' structure. Finally, we perform a series of Monte Carlo simulations in which VARMA specifications help a lot especially in small sample cases.

In the next section we present some results on linear transformations of vector stochastic processes. The Section 3 studies identification issues of VARMA models, while the link between VARMA and factors representations is studied in Section 4. The dynamic factor model with VARMA process for factors is proposed in Section 5. The estimation of factor models and VARMA processes is discussed in Section 6. The forecasting models that we use in simulations and empirical application are presented in section 7. Monte Carlo simulation results and empirical results are presented in Section 8 and Section 9 respectively.

2. Linear transformations of vector stochastic processes

In this section we resume results on linear transformations of vector stochastic processes studied in Lütkepohl (1987, chapter 4) and provide similar results for two special cases of interest. Exploring the features of such transformed processes is important since in practice many data are obtained by temporal and spatial aggregations and/or transformed by linear filtering techniques before being used. Moreover, in macroeconomics we are often interested in modeling and representing the economy by specifying a multivariate stochastic process on a small number of economic indicators (usually already individually linearly transformed). Hence, we work on a marginalized process that can be seen as a linear transformation of the original process of economic indicators. Finally, if we are interested in dimension-reduction methods such as principal components we end up with variables constructed as linear transformations of observable series. Thus, it is important to take into account the information on the nature of the original process when modeling its linear transformations.

The most important result concerns linear transformations of a zero mean N-dimensional, stationary, nondeterministic stochastic process. Suppose a such process X_t that has MA representation

$$X_t = \sum_{i=0}^{\infty} \Phi_i u_{t-i} = \Phi(L) u_t, \quad \Phi_0 = I_K, \tag{2.1}$$

where u_t is white noise with $E(X_t) = 0$, $E(u_t u_t') = \Sigma_u$, $E(X_t X_t') = \Sigma_X$, $E(X_t X_{t+h}') = \Gamma_X(h)$, and $\det(\Phi(z)) \neq 0$ for $|z| < 1$. We are interested in linear transformation F_t of X_t,

$$F_t = C X_t, \tag{2.2}$$

where C is an $(K \times N)$ matrix of rank K that is fixed over time. Then, given the nature of the process of X_t we have that

1. F_t is also stationary, nondeterministic and has zero mean. Thus, it has an MA representation

$$F_t = \sum_{i=0}^{\infty} \Psi_i v_{t-i} = \Psi(L) v_t, \quad \Psi_0 = I_K, \tag{2.3}$$

   where v_t is K-dimensional white noise with $E(v_t v_t') = \Sigma_v$.

2. If Σ_u is nonsingular and C is of full rank M, then $\det(\Psi(z)) \neq 0$ for $|z| < 1$.

This result considers a very general case where X_t is a vector stochastic process with an MA representation. If it is invertible, the cases of finite or infinite VAR processes are then covered.

However, in practice only a finite number of parameters can be estimated in such multivariate time series models. In following we discuss the linear transformations of finite order vector stochastic processes, and especially of widely used VAR(MA) models.

Suppose X_t an N-dimensional finite order MA(q) process,

$$X_t = u_t + M_1 u_{t-1} + \ldots + M_q u_{t-q} = M(L) u_t \tag{2.4}$$

with $\det(M(z)) \neq 0$ for $|z| < 1$ and nonsingular white noise noise covariance matrix Σ_u. Let C be a $(K \times N)$ matrix of full rank K. Then, it can be shown that $F_t = C X_t$ has an invertible MA(q*) representation

$$F_t = v_t + N_1 v_{t-1} + \ldots + N_q^* v_{t-q^*} = N(L)v_t \tag{2.5}$$

with $\det(N(z)) \neq 0$ for $|z| < 1$ where v_t is K-dimensional white noise with nonsingular matrix Σ_v, the N_i are $(K \times K)$ coefficient matrices and $q^* \leq q$.

It is worth noting that some conditions in previous result can be relaxed. Nonsingular covariance matrix Σ_u and C of full rank are not necessary so there may be exact linear dependencies among components of X_t and F_t. This more general case is proved in Lütkepohl (1984). Another remark concerns $q*$. It is easy to construct examples where $q* < q$ by properly choosing C and $M(L)$. Hence, finding lower bound for $q*$ will imply finding restrictions on C and $M(L)$.

Using previous results it can be shown that the VARMA class of models is closed with respect to linear transformation while this is not necessary the case with VAR models. More precisely, let X_t be an N-dimensional, stable, invertible VARMA(p,q) process

$$\Phi(L)X_t = \Theta(L)u_t. \tag{2.6}$$

and let C be an $(K \times N)$ matrix of rank K<N. Then, as stated in Corollary 11.1.1 in Lütkepohl (2005), $F_t = CX_t$ has a VARMA(p^*,q^*) representation with $p^* \leq Np$, and $q^* \leq (N-1)p + q$. Hence, a linear transformation of a finite order VARMA process still has a finite VARMA representation but with possibly higher autoregressive and moving average orders [1].

Since the most used time series model to approximate the process of economic time series is a finite order VAR, it is important to note that this class of models is not closed with respect to linear transformations. Given that many macroeconomic indicators are in fact obtained as temporal and/or cross-sectional aggregations of sectoral (micro) indicators and are sometimes filtered using linear techniques, this is an argument in favor of allowing for VARMA specification. In following corollary we show that a linear transformation, the one that reduces the dimension of the original VAR process, will typically have a VARMA representation.

Corollary 2.1 LINEAR TRANSFORMATION OF VAR(p) PROCESS.

Let X_t be an N-dimensional, stable VAR(p) process and let C be an $(K \times N)$ matrix of rank K. Then, $F_t = CX_t$ has a VARMA(p^*,q^*) representation with

$$p^* \leq Np$$

and

$$q^* \leq (N-1)p.$$

Moreover, we can get tighter bounds if $K > 1$:

$$p^* \leq (N-K+1)p$$

and

$$q^* \leq (N-K)p.$$

[1] Corollary 11.1.2 in Lütkepohl (2005) gives tighter bounds for these orders when $K > 1$.

PROOF. Without loss of generality, write the process X_t as centered VAR(p)

$$\Phi(L)X_t = u_t. \tag{2.7}$$

Premultiplying (2.7) by the adjoint $\Phi(L)^*$ of $\Phi(L)$ and using $|\Phi(L)| = \Phi(L)^*\Phi(L)$ gives

$$|\Phi(L)|X_t = \Phi(L)^* u_t. \tag{2.8}$$

Since $|\Phi(z)^*| \neq 0$ for $|z| \leq 1$, (2.8) is a stable and invertible VARMA representation of X_t. Premultiplying (2.8) by C yields

$$|\Phi(L)|F_t = C\Phi(L)^* u_t. \tag{2.9}$$

The AR operator $|\Phi(L)|$ is a polynomial in L of degree Np (or less) and the elements of $\Phi(L)^*$ are all polynomials of degree $(N-1)p$. Thus, the right-hand side of (2.9) is a linearly transformed finite order MA process which, by Proposition 2.1, has an MA(q^*) representation with $q^* \leq (N-1)p$. To obtain tighter upper bounds, see the proof of Corollary 11.1.2 in Lütkepohl (2005). □

Finally, it is interesting to note that a nonsingular transformation of X_t preserves its VARMA representation, so the VAR class is closed under linear transformations that keep the same dimension of the original process.

Corollary 2.2 NONSINGULAR LINEAR TRANSFORMATION OF VARMA(p,q) PROCESS.
Let X_t be an N-dimensional, stable VARMA(p,q) process and let C be an $(N \times N)$ invertible matrix. Then, $F_t = CX_t$ preserves the VARMA(p,q) representation.

PROOF. Suppose X_t an N-dimensional, stable and invertible VARMA(p,q) process

$$\Phi(L)X_t = \Theta(L)u_t$$

where $\Phi(L) = (I - \Phi_1 L - \ldots - \Phi_p L^p)$ and $\Theta(L) = (I - \Theta_1 L - \ldots - \Theta_q L^q)$, and let C be an $(N \times N)$ nonsingular matrix. Premultiply by C both sides of last equation to get

$$C\Phi(L)C^{-1}CX_t = C\Theta(L)C^{-1}Cu_t.$$

Define $\bar{\Phi}(L) = (I - C\Phi_1 C^{-1} L - \ldots - C\Phi_p C^{-1} L^p)$, $\bar{\Theta}(L) = (I - C\Theta_1 C^{-1} L - \ldots - C\Theta_q C^{-1} L^q)$, $F_t = CX_t$, and $v_t = Cu_t$. Then, we get a VARMA(p,q) representation for linearly transformed process F_t

$$\bar{\Phi}(L)X_t = \bar{\Theta}(L)u_t$$

. □

3. Identification issues in VARMA representations

In previous section we showed that linear transformations of a VARMA (or VAR) process yield also a VARMA representation for the transformed process but with possibly different degrees of AR and MA polynomials. Now, we are interested in identification issues concerning the VARMA representation of F_t. The identification problem arises at two levels. First, one must find an identified VARMA representation in terms of AR and MA parameters. If we are interested in forecasting this is enough. But if we want to do structural analysis, for example study the dynamic impact of a structural shock using impulse responses, we need to identify the structural form of an identified VARMA representation. This gives the second class of identification issues.

3.1. Finding an identified VARMA representation

Before going further, let us briefly present the typical identification problem in VARMA models and several identified representations. The identification problem arises since a VARMA representation of X_t is not unique. If X_t has a VARMA representation (2.6), the corresponding MA representation is

$$X_t = \Psi(L) u_t \tag{3.1}$$

with $\Psi(L) = \Phi(L)^{-1}\Theta(L)$. Then we say that two VARMA representations are equivalent if $\Phi(L)^{-1}\Theta(L)$ results in the same operator $\Psi(L)$, i.e., they have the same autocovariance structure. Thus, to ensure uniqueness of a VARMA representation, we must impose restrictions on the AR and MA operators such that for a given $\Psi(L)$ there is one and only one set of operators $\Phi(L)$ and $\Theta(L)$ that generates the infinite MA representation in (3.1).

There are many ways to identify the process in (2.6) and in following we state five identified VARMA representations: the echelon form and the final equation form that are well known in the literature, and three representations proposed by Dufour and Pelletier (2008).

Definition 3.1 (Echelon form) *The VARMA representation in (2.6) is said to be in echelon form if the AR and MA operators $\Phi(L) = [\phi_{ij}(L)]_{i,j=1,\dots,N}$ and $\Theta(L) = [\theta_{ij}(L)]_{i,j=1,\dots,N}$ satisfy the following conditions: all operators $\phi_{ij}(L)$ and $\theta_{ij}(L)$ in the i-th row of $\Phi(L)$ and $\Theta(L)$ have the same degree p_i with the form*

$$\begin{aligned}
\phi_{ii}(L) &= 1 - \sum_{m=1}^{p_i} \phi_{ii,m} L^m, \quad for \quad i = 1, \dots, N \\
\phi_{ij}(L) &= - \sum_{m=p_i-p_{ij}+1}^{p_i} \phi_{ij,m} L^m, \quad for \quad j \neq i \\
\theta_{ij}(L) &= \sum_{m=0}^{p_i} \theta_{ij,m} L^m, \quad for \quad i,j = 1, \dots, N, \quad with \quad \Theta_0 = \Phi_0.
\end{aligned}$$

Further, in the VAR operator $\phi_{ij}(L)$,

$$p_{ij} = \begin{cases} \min(p_i + 1, p_j) & for \quad i \geq j, \\ \min(p_i, p_j) & for \quad i < j, \end{cases} \qquad i, j = 1, \ldots, N.$$

i.e., p_{ij} specifies the number of free coefficients in the operator $\phi_{ij}(L)$ for $j \neq i$. The row orders $(p_1, \ldots, p_N)$ are the Kronecker indices and their sum $\sum_{i=1}^{N} p_i$ is the McMillan degree. For the VARMA orders we have in general $p = q = \max(p_1, \ldots, p_N)$.

Definition 3.2 (Final AR equation form (FAR)) *The VARMA representation in (2.6) is said to be in final AR equation form if $\Phi(L) = \phi(L)I_N$, where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ is a scalar polynomial with $\phi_p \neq 0$.*

Definition 3.3 (Final MA equation form (FMA)) *The VARMA representation in (2.6) is said to be in final MA equation form if $\Theta(L) = \theta(L)I_N$, where $\theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$ is a scalar polynomial with $\theta_q \neq 0$.*

Definition 3.4 (Diagonal MA equation form (DMA)) *The VARMA representation in (2.6) is said to be in diagonal MA equation form if $\Theta(L) = \mathrm{diag}[\theta_{ii}(L)] = I_N - \Theta_1 L - \cdots - \Theta_q L^q$ where $\theta_{ii}(L) = 1 - \theta_{ii,1}L - \cdots - \theta_{ii,q_i}L^{q_i}$, $\theta_{ii,q_i} \neq 0$, and $q = \max_{1 \leq i \leq N}(q_i)$.*

Definition 3.5 (Diagonal AR equation form (DAR)) *The VARMA representation in (2.6) is said to be in diagonal AR equation form if $\Phi(L) = \mathrm{diag}[\phi_{ii}(L)] = I_N - \Phi_1 L - \cdots - \Phi_p L^p$ where $\phi_{ii}(L) = 1 - \phi_{ii,1}L - \cdots - \phi_{ii,p_i}L^{p_i}$, $\phi_{ii,p_i} \neq 0$, and $p = \max_{1 \leq i \leq N}(p_i)$.*

An interesting fact from results on linear aggregations VARMA processes in Section 2 is that the aggregated process F_t can always have an identified VARMA representation in final AR equation form. But this representation may not be attractive for several reasons. First, it is quite far from usual VAR models by excluding lagged values of other variables in each equation. Moreover, the AR coefficients are the same in all equations which will require a polynomial of very high order. Second, the interaction between the different variables is modeled through the MA part of the model, which may be very complex in structural analysis.

With respect to these concerns, a more interesting representation is the diagonal MA form. It is easy to specify contrary to echelon form since we don't need to deal with rules for the orders of the off-diagonal elements in the AR and MA operators. From the practitioners' point of view, it is very appealing since it can be seen as a simple extension of the VAR model. Simply adding lags of u_{it} to equation i is a natural extension of the VAR model which could give a more parsimonious representation. It also has the advantage of putting the simple structure on the MA polynomials, the part which complicates the estimation, rather than the AR part as in the final AR equation form.

4. Link between VARMA and factor representations

In this section we study the relations between VARMA process of X_t and its factor representations. In Theorem 4.1, we postulate a factor model and show the implication of the process of factors on the process of X_t [2].

Theorem 4.1 IMPLICATION OF FACTORS' PROCESS ON THE PROCESS OF X_t.

1. *Consider the following factor model:*

$$X_t = \Lambda F_t + u_t \tag{4.1}$$

$$[I - \Phi(L)L]F_t = a_t \tag{4.2}$$

where X_t is an $N\times 1$ vector of series, Λ is an $N \times K$ matrix of factor loadings, F_t is a $K\times 1$ vector of factors, u_t and a_t are two uncorrelated white noises with covariance matrices Σ_u and Σ_a respectively, and $\Phi(z) = [\Phi_1 z - \ldots - \Phi_p z^p]$. The innovations u_t can be such that (4.1) is an exact or approximate factor model. Then, X_t follows a VARMA(p,p) process

$$[I - A(L)L]X_t = [I - A(L)L]u_t + \Lambda a_t \tag{4.3}$$

where $A(L) = \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda'$.

2. *If F_t follows a VARMA(p,q) process*

$$[I - \Phi(L)L]F_t = [I - \Theta(L)L]a_t \tag{4.4}$$

then X_t has VARMA(p,max(p,q)) representation

$$[I - A(L)L]X_t = [I - A(L)L]u_t + \Lambda[I - \Theta(L)L]a_t \tag{4.5}$$

PROOF. Premultiply (4.1) by $(\Lambda'\Lambda)^{-1}\Lambda'$, assuming that $(\Lambda'\Lambda)$ is nonsingular, and go back one period to get

$$F_{t-1} = (\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} - (\Lambda'\Lambda)^{-1}\Lambda' u_{t-1}.$$

Then, replacing for F_{t-1} in (4.2) yields

$$F_t = \Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} + a_t - \Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1}.$$

Finally, replace for F_t in (4.1) to obtain

$$X_t = \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} + u_t - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1} + \Lambda a_t,$$

which is exactly (4.3). This is a VARMA(p,p) process since u_t and a_t are two uncorrelated white noises.

[2] We will briefly present classical and modern factor analysis in the next section. Here, we focus only on the relationship between VARMA and factor representations of a vector stochastic process.

To obtain (4.5), we follow the same steps except that we use (4.4) instead of (4.2), which yields

$$X_t = \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} + u_t - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1} + \Lambda a_t - \Lambda\Theta(L)a_{t-1}.$$

The VAR part is still of order p, while the MA part is of order max(p,q) since the autocovariances of $\nu_t \equiv u_t - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1} + \Lambda a_t - \Lambda\Theta(L)a_{t-1}$ go to zero for $h > max(p, q)$. □

An interesting implication of the Theorem 4.1 is that X_t will never have a finite order VAR representation.

Corollary 4.1 . *If X_t has a factor representation as in (4.1), it cannot have a finite order VAR representation.*

PROOF. From (4.3) we can see that the only way to cancel MA part of the process of X_t is to set $A(L)$ to zero, but this cancels the AR part also. Moreover, setting $A(L)$ to zero means canceling VAR dynamics of factors ($\Phi(L) = 0$) and/or eliminating the factor structure ($\Lambda = 0$). When factors has VARMA dynamics, we see from (4.5) that the situation is the same except that we have to cancel $\Theta(L)$. □

However, if VARMA representation of X_t is invertible, it has a VAR(∞) that in practice can be approximated by finite order VAR.

The next question is what are the implications of the process of X_t on the factors' representation? In other words, what are the implications of underlying structure of X_t on the representation of latent factors when the latter are calculated as linear transformations of X_t? We will enumerate all cases studied in Section 2, i.e. MA, MA(q), VARMA(p,q) and VAR(p) representation for X_t. The results are summarized in Theorem 4.2.

Theorem 4.2 IMPLICATION OF X_t'S REPRESENTATION ON THE PROCESS OF F_t.

Suppose that factors are computed as linear combinations of elements of X_t that has a factor representation as in (4.1). Then the following results hold:

(i) *If X_t has an MA representation as in (2.1), F_t has an MA representation.*

(ii) *If X_t has a finite MA(q) representation as in (2.4), F_t follows a finite MA(q^*) process with $q \le q^*$.*

(iii) *If X_t follows VARMA(p,q) process as in (2.6), F_t has VARMA(p^*,q^*) representation with $p^* \le Np$ and $q^* \le q + (N-1)p$ (or $p^* \le (N-K+1)p$ and $q^* \le q + (N-K)p$).*

(iv) *If X_t follows VAR(p) process as in (2.7), F_t has VARMA(p^*,q^*) representation with $p^* \le Np$ and $q^* \le (N-1)p$.*

PROOF. We have that K-dimensional process F_t is a linear transformation of X_t, i.e. $F_t = CX_t$ where C is $K \times N$. Then we obtain the results using Propositions and Corollaries from Section 2.
For (i), if X_t has an MA representation as in (2.1), then by Proposition 4.1 in Lütkepohl (1987) the K-dimensional process $F_t = CX_t$ has an MA representation.
For (ii), if X_t has an MA(q^*) representation as in (2.4), then by Proposition 4.2 in Lütkepohl (1987) the K-dimensional process $F_t = CX_t$ has an MA(q^*) representation.
For (iii) when X_t follows VARMA(p,q) process as in (2.6), then by Corollaries 11.1.1 and 11.1.2 in Lütkepohl (2005) $F_t = CX_t$ has VARMA(p^*,q^*) representation with appropriate bounds for AR and MA orders.
Finally for (iv), if X_t follows VAR(p) process as in (2.7), then by Corollary 2.1 $F_t = CX_t$ follows VARMA(p^*,q^*) process. □

As in Section 2, the invertibility characteristics of X_t still hold for F_t if Σ_u is nonsingular and C is of full rank K. The invertibility condition is important in practice. From Theorem 4.2 we see that if X_t is invertible (and stable if VARMA or VAR) and if F_t inherits these characteristics, then it has an infinite VAR representation that can be approximated in practice by a finite order VAR.

Another interesting result concerns stability of factor loadings over time and the implication on the factors' dynamics. If factor loadings Λ are time varying, then the MA coefficients of F_t's process are also time varying when these are estimated as principal components of X_t. Moreover, if F_t has VAR(∞) representation and is approximated by finite order VAR, their parameters are also time varying.

To resume, we recall arguments in favor of VARMA modeling of factors' process.

(i) Whenever the joint process of series in X_t is VAR or VARMA, if the factors are estimated as principal components they follow a VARMA process. Since most of series in X_t are usually linearly transformed before estimation (seasonal adjustments, temporal aggregations, contemporaneous aggregations, standardization), and following results in the Section 2, the transformed process from which factors are extracted has a VARMA representation. Moreover, we showed in Theorem 4.1 that factor structure on X_t implies a VARMA representation on X_t.

(ii) VARMA representations are usually more parsimonious and could produce more relevant statistical inference. As it was found in Dufour and Pelletier (2008) the introduction of MA operator allows for the reduction of the required AR order so we can get more precise estimates and thus more precise structural analysis using impulse responses since the coefficients of impulse-responses functions are nonlinear combinations of AR and MA estimates. Moreover, in terms of forecasting power, VARMA models present theoretical advantages over VAR representation (see Lütkepohl (1987)).

(iii) Finally, note that imposing a VARMA process on factors can be viewed from two perspectives. First, if one uses factor analysis as dimension-reduction method to model dynamics of large time series data sets, then implications of the underlying process of X_t on F_t should be considered, and using Theorem 4.2 we see that VARMA is a natural process for factors. On the other side, if we suppose that the true representation of the world is factor representation, i.e. there exists a small number of structural shocks that generate observable series, considering a VARMA process on factors instead of VAR is an interesting generalization motivated by the usual arguments of parsimony, invertibility and marginalization issues. Moreover, if we underestimate the number of factors, even if F_t has VAR representation, the subvector of F_t will follow a VARMA process.

We have considered factor model in static form without loss of generality since it is always possible to write dynamic factor model in static form. This more general case is treated in the following section where we introduce dynamic factor model with VARMA process for factors.

5. Factor models and VARMA processes

In this section we propose a dynamic factor model (DFM) with VARMA processes for idiosyncratic components and factors (that will sometimes be labeled as DFM-VARMA for notational simplicity). As discussed above, this generalization can be motivated by parsimony, invertibility, marginalization and linear transformations (seasonal adjustments, temporal and spatial aggregations) issues.

Following the notation in Stock and Watson (2005), the DFM where factors and specific components have VARMA(p_f,q_f) and ARMA($p_{x,i}$,$q_{x,i}$) representations respectively, can be written as

$$\begin{aligned} X_{it} &= \tilde{\lambda}_i(L) f_t + u_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T && (5.1) \\ u_{it} &= \delta_i(L) u_{i,t-1} + \nu_{it} - \gamma_i(L)\nu_{it-1} && (5.2) \\ f_t &= \Gamma(L) f_{t-1} + \Theta(L)\eta_t && (5.3) \end{aligned}$$

where $\tilde{\lambda}_i(L)$ is a lag polynomial, $\delta_i(L)$ is a $p_{x,i}$-degree lag polynomial, $\gamma_i(L)$ is a $q_{x,i}$-degree lag polynomial, $\Gamma(L) = [\Gamma_1 L + \ldots + \Gamma_{p_f} L^{p_f}]$, $\Theta(L) = [I - \Theta_1 L - \ldots - \Theta_{q_f} L^{q_f}]$, ν_{it} is an n-dimensional white noise uncorrelated with q-dimensional white noise process η_t.
The exact DFM is obtained if the following assumption is satisfied:

$$E(u_{it}u_{js}) = 0 \quad \forall i, j, t, s, \quad i \neq j.$$

If the cross-correlation of errors is weak (tends to zero when N tends to infinity), we obtain approximate DFM [3].

Substracting $\delta_i(L)u_{it-1} - \gamma_i(L)\nu_{it-1}$ from both sides of (5.1) gives a DFM with serially uncorrelated idiosyncratic errors:

$$X_{it} = \lambda_i(L) f_t + \delta_i(L) X_{it-1} + \nu_{it} - \gamma_i(L)\nu_{it-1}, \tag{5.4}$$

where $\lambda_i(L) = (1 - \delta_i(L)L)\tilde{\lambda}_i(L)$.

[3] See Bai and Ng (2008) for an overview of classical and modern factor analysis literature, and distinction between exact and approximate factor models

Then, we can write the DFM in the following form:

$$X_t = \lambda(L)f_t + D(L)X_{t-1} + \nu_t - \gamma(L)\nu_{t-1} \tag{5.5}$$

$$f_t = \Gamma(L)f_{t-1} + \Theta(L)\eta_t \tag{5.6}$$

where

$$\lambda(L) = \begin{bmatrix} \lambda_1(L) \\ \vdots \\ \lambda_n(L) \end{bmatrix}, D(L) = \begin{bmatrix} \delta_1(L) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_n(L) \end{bmatrix},$$

$$\gamma(L) = \begin{bmatrix} \gamma_1(L) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_n(L) \end{bmatrix}, \nu_t = \begin{bmatrix} \nu_{1t} \\ \vdots \\ \nu_{nt} \end{bmatrix}.$$

It is worth noting that if $\Theta(L) = I$ and $\gamma_i(L) = 0 \quad \forall i$ we obtain the case treated in Stock and Watson (2005).

To obtain DFM in static form, suppose that $\tilde{\lambda}(L)$ has finite degree p-1, and let $F_t = [f_t' \quad f_{t-1}' \ldots f_{t-p+1}']'$. Let the dimension of F_t be k, where $q \leq k \leq qp$. Then, we can write

$$X_t = \Lambda F_t + u_t \tag{5.7}$$

$$u_t = D(L)u_{t-1} + \nu_t - \gamma(L)\nu_{t-1} \tag{5.8}$$

$$F_t = \Phi(L)F_{t-1} + G\Theta(L)\eta_t \tag{5.9}$$

where Λ is a $n \times k$ matrix, the i^{th} row consists of the coefficients of $\tilde{\lambda}_i(L)$, $\Phi(L)$ contains coefficients of $\Gamma(L)$ and zeros, and G is $k \times q$ matrix that loads (structural) shocks η_t to static factors (consists of 1's and 0's).

Again, if $\Theta(L) = I$ and $\gamma(L) = 0$ we obtain the factor model in static form that was used to forecast time series (Stock and Watson (2002b, 2008), Boivin and Ng (2005)) and to study the impact of monetary policy shocks using impulse-responses functions in factor-augmented VAR (FAVAR) model (Bernanke, Boivin and Eliasz (2005), Boivin, Giannoni and Stevanović (2008)).

6. **Estimation**

Several estimation methods of factor models and VARMA processes (separately) have been proposed in the literature, and we resume some of them in this section.

6.1. **Estimation of factor models**

Here we recall main estimation methods of dynamic (static) factor models with $\Theta(L) = I$ and $\gamma_i(L) = 0 \quad \forall i$.

The unknown coefficients in (5.1)-(5.3) (or in its static form (5.7)-(5.9)) can be estimated by Gaussian maximum likelihood using the Kalman filter (or by Quasi ML), see Engle and Watson (1981), Stock and Watson (1989), Sargent (1989). This method is computationally burdensome when N is very large, but also the misspecification becomes very likely. However, there are some recent improvements: Kalman filter speedup by Jungbacker and Koopman (2008), using principal components as very good starting values then a single pass of the Kalman filter by Giannone, Reichlin, and Sala (2004), principal components for starting values then use EM algorithm to convergence by Doz, Giannone, and Reichlin (2006).

An alternative to simultaneous equations likelihood-based estimation is two-step principal components (PC) procedure where factors are approximated in the first step and then the dynamic process of factors is estimated in the second step. The main result is that factors can be approximated by Principal Components Analysis (PCA) estimator. Stock and Watson (2002a) prove consistency of these estimators in approximate factor model when both cross-section and time sizes, N, T, go to infinity, and without restrictions on N/T. Moreover, they justify using $\hat{F}_t$ as regressor without adjustment. Bai and Ng (2006) improve those results by showing that PCA estimators are $\sqrt{T}$ consistent and asymptotically normal if $\sqrt{T}/N \rightarrow 0$. Except when T/N goes to zero, inference should take into account the effect of generated regressors. In this approach, and especially in case where $\delta_i(L) = 0 \quad \forall i$, the dynamic process of static factors doesn't affect the approximation of factors. In case where idiosyncratic errors are serially correlated, Stock and Watson (2005) suggest an iterative PC procedure (EM algorithm).

Since PCA estimator is motivated by least squares problem, if there is heteroskedasticity (or cross-correlation or serial correlation) in error term of equation (5.7), we could do better by using GLS (or WLS), which gives Generalized Principal Component (GPC) estimator. The infeasible WLS estimator of F and Λ from (5.7) solves the following least squares problem

$$\min_{F_1,\ldots F_T,\Lambda} \sum_{t=1}^{T} (X_t - \Lambda F_t)' \Sigma_u^{-1} (X_t - \Lambda F_t)$$

where $\Sigma_u = E[u_t u_t']$. The solution is $\hat{\Lambda}$ = first r eigenvectors of $\Sigma_u^{-1/2} \Sigma_X \Sigma_u^{-1/2}\,'$, where Σ_X is the covariance matrix of X_t, and $\hat{F}_t = \hat{\Lambda}' X_t$. Since Σ_u is unobservable, the feasible WLS consists of finding $\hat{\Sigma}_u$. For example, Boivin and Ng (2006) use $\hat{\Sigma}_e = \mathrm{diag}(\hat{\Sigma}_u)$ which accords from exact DFM restrictions.

Another approach is to use dynamic principal components (DPC). However DPC analysis produce two-sided estimates of the factors (depend on past and future information) and thus these estimates are not suitable for forecasting or for structural VAR analysis in which information set timing assumptions are used to identify shocks.

Since there exist several ways to estimate DFM and to approximate factors, the natural question is which estimator to use? Theoretical results ranking is MLE, PC, GPC. Given the parameters, the Kalman filter estimator of F_t is the optimal estimator if the errors are Gaussian. For non Gaussian errors, the Kalman filter estimator is the MMSE estimator, but this doesn't take parameter estimation error into account. The simulation evidence doesn't give a clear answer. Choi (2007) compares PC, infeasible GPC, and feasible GPC in a Monte Carlo study. He finds efficiency gains for feasible

GPC in some cases, but the estimation of Σ_u hurts performance relative to infeasible GPC. Doz, Giannone, and Reichlin (2006) do a Monte Carlo study where they compare PC, estimation of DFM parameters using PC estimates then a single pass of the Kalman filter, and ML (PC for starting values, then use EM algorithm to convergence), relative to how well those methods approximate true factors. The conclusion is that all three methods behave relatively similarly. Boivin and Ng (2005) compare combinations of factor estimation methods and forecasting equation specifications, from the perspective of forecast MSE, and conclude that PCA estimator combined with unrestricted forecasting model generally works best[4].

6.2. Estimation of VARMA models

Standard estimation methods for VARMA models are maximum likelihood and nonlinear least squares. But these methods require nonlinear optimization which may not be feasible when the number of parameters is relatively large. Dufour and Pelletier (2008) consider a generalization of the regression-based estimation method proposed by Hannan and Rissanen (1982).
Consider a K-dimensional zero mean process Y_t generated by the VARMA(p,q) model:

$$A(L)Y_t = B(L)U_t \tag{6.10}$$

where $A(L) = I_K - A_1L - \cdots - A_pL^p$, $B(L) = I_K - B_1L - \cdots - B_qL^q$, and U_t is a sequence of uncorrelated random variables. Assume det$[A(z)] \neq 0$ for $|z| \leq 1$ and det$[B(z)] \neq 0$ for $|z| \leq 1$ so the process Y_t is stable and invertible. Split the whole vector of VARMA parameters, γ, in two parts γ_1 (the parameters for the AR part) and γ_2 (MA part): $\gamma = [\gamma_1 \quad \gamma_2]'$. For VARMA in diagonal MA equation form, we have

$$\gamma_1 = [a_{1\bullet,1}, \ldots, a_{1\bullet,p}, \ldots, a_{K\bullet,1}, \ldots, a_{K\bullet,p}], \tag{6.11}$$
$$\gamma_2 = [b_{11,1}, \ldots, b_{11,q_1}, \ldots, b_{KK,1}, \ldots, b_{KK,q_K}]. \tag{6.12}$$

The estimation method involves three steps.
Step 1. Estimate a VAR(n_T) to approximate the VARMA(p,q) and recuperate the residuals defined as:

$$\hat{U}_t = Y_t - \sum_{l=1}^{n_T} \hat{\Pi}_l^{n_T} Y_{t-l}, \quad T > 2Kn_T. \tag{6.13}$$

Step 2. With the residuals from step 1, compute an estimate of the covariance matrix of U_t, $\hat{\Sigma}_U = \frac{1}{T}\sum_{t=n_t+1}^{T} \hat{U}_t\hat{U}_t'$, and estimate by GLS the following multivariate regression,

$$A(L)Y_t = [B(L) - I_K]\hat{U}_t + e_t,$$

to get estimates $\tilde{A}(L)$ and $\tilde{B}(L)$. The estimator is

[4] This unrestricted model will be discussed in Section 7.

$$\hat{\gamma} = [\sum_{t=l}^{T} \hat{Z}'_{t-1} \hat{\Sigma}_U^{-1} \hat{Z}_{t-1}]^{-1} [\sum_{t=l}^{T} \hat{Z}'_{t-1} \hat{\Sigma}_U^{-1} Y_t] \tag{6.14}$$

with $l = n_T + \max(p, q) + 1$. Setting

$$\begin{aligned}
\mathbf{Y}_{t-1}(p) &= [y_{1,t-1}, \ldots, y_{K,t-1}, \ldots, y_{1,t-p}, \ldots, y_{K,t-p}], \\
\hat{\mathbf{U}}_{t-1} &= [\hat{u}_{1,t-1}, \ldots, \hat{u}_{K,t-1}, \ldots, \hat{u}_{1,t-q}, \ldots, \hat{u}_{K,t-q}], \\
\hat{\mathbf{u}}_{k,t-1} &= [\hat{u}_{k,t-1}, \ldots, \hat{u}_{k,t-q_k}],
\end{aligned}$$

the matrix $\hat{Z}_{t-1}$ is:

$$\hat{Z}_{t-1} = \begin{bmatrix} \mathbf{Y}_{t-1}(p) & \cdots & 0 & \hat{\mathbf{u}}_{1,t-1} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{Y}_{t-1}(p) & 0 & \cdots & \hat{\mathbf{u}}_{K,t-1} \end{bmatrix}.$$

Step 3. Using the second step estimates, form new residuals

$$\tilde{U}_t = Y_t - \sum_{i=1}^{p} \tilde{A}_i Y_{t-i} + \sum_{j=1}^{q} \tilde{B}_j \tilde{U}_{t-j}$$

initiating with $\tilde{U}_t = 0$, $t \leq \max(p, q)$, and define

$$X_t = \sum_{j=1}^{q} \tilde{B}_j X_{t-j} + Y_t,$$

$$W_t = \sum_{j=1}^{q} \tilde{B}_j W_{t-j} + \tilde{U}_t,$$

initiating with $X_t = W_t = 0$ for $t \leq \max(p, q)$. Compute a new estimate of Σ_U, $\hat{\Sigma}_U = \frac{1}{T} \sum_{t=\max(p,q)+1}^{T} \tilde{U}_t \tilde{U}'_t$. Then, regress by GLS $\tilde{U}_t + X_t - W_t$ on $\tilde{V}_{t-1}$ with

$$\tilde{V}_t = \sum_{j=1}^{q} \tilde{B}_j \tilde{V}_{t-j} + \tilde{Z}_t$$

where $\tilde{Z}_t$ is just like $\hat{Z}_t$ from step 2 except it is computed with $\tilde{U}_t$ instead of $\hat{U}_t$ to obtain regression coefficient $\hat{A}_i$ and $\hat{B}_j$:

$$\hat{\gamma} = \left[\sum_{t=\max(p,q)+1}^{T} \tilde{V}'_{t-1} \tilde{\Sigma}_U^{-1} \tilde{V}_{t-1} \right]^{-1} \left[\sum_{t=\max(p,q)+1}^{T} \tilde{V}'_{t-1} \tilde{\Sigma}_U^{-1} \left[\tilde{U}_t + X_t - W_t \right] \right]. \tag{6.15}$$

The consistency and asymptotic normality of above estimators are derived in DP (2008).

In previous steps the orders of the AR and MA operators were supposed known. In practice they are usually estimated by statistical methods or suggested by theory. Dufour and Pelletier (2008) propose an information criteria to be applied in the second step of estimation procedure above. For all $p_i \leq P$ and $q_i \leq Q$ compute

$$\log(\det\tilde{\Sigma}_U) + \dim(\gamma)\frac{(\log T)^{1+\delta}}{T}, \quad \delta > 0. \tag{6.16}$$

Choose $\hat{p}_i$ and $\hat{q}_i$ as the set which minimizes the information criteria (6.16). The properties of estimators $\hat{p}_i$ and $\hat{q}_i$ are given in the paper.

6.3. **Estimation of factor models *and* VARMA processes**

In our model we need to estimate both factors (and loadings) and their dynamic process. As in the existing literature, we can estimate the system (5.1)-(5.3) (or in its static form (5.7)-(5.9)) simultaneously by assuming some distributions for errors. This method is already difficult with respect to numerical optimization with factors having a simple VAR structure. Hence, adding MA part to factors' process should not help, or should make it worse, since estimating a simple VARMA process is not easy. However, it would be interesting to see if considering VARMA processes can help in approximation of true factors, and this is a part of ongoing research project.

Instead of likelihood-based approach we use two-step estimation. In the first step $\hat{F}_t$ are computed as first K principal components of the contemporaneous covariance matrix of X_t. Then, in the second step we estimate the VARMA representation (5.9) using $\hat{F}_t$ [5].

7. **Applications in macroeconomics**

The factor models have been widely used in many sciences and domains. In particular, the macroeconomic applications can be separated in several categories: forecasting (and nowcasting) of macroeconomic aggregates, structural analysis where shocks with meaningful economic interpretation have been identified, testing the implications of DFM structure and in estimation of structural macroeconomic models. In this paper we consider the forecasting exercise to see if allowing for VARMA dynamics in estimated factors can help in forecasting some macroeconomic indicators of interest.

7.1. **Forecasting time series**

Consider a simplified version of static model (5.7)-(5.9)) assuming that F_t is scalar

$$X_{it} = \lambda_i F_t + u_{it} \tag{7.1}$$

$$u_{it} = \delta_i u_{it-1} + \nu_{it} - \gamma_i \nu_{it-1} \tag{7.2}$$

$$F_t = \phi F_{t-1} + \eta_t - \theta\eta_{t-1} \tag{7.3}$$

[5] Another approach would be to adapt the iterative algorithm used in Stock and Watson (2005) to consider the ARMA dynamics of idiosyncratic component.

Then, after replacing for two last equations and rearranging, we get the forecast of X_{T+1} based on informational set at T

$$X_{iT+1|T} = \delta_i X_{iT} + \lambda_i(\phi - \delta_i)F_T - \lambda_i \theta \eta_T - \gamma_i \nu_{iT}. \tag{7.4}$$

Now, we can summarize several implications of (7.4). As discussed in Boivin and Ng (2005), in standard factor model where idiosyncratic components and factor follow autoregressive processes, if $\lambda_i \neq 0$, i.e. factor structure with respect to variable i exists, and $\phi \neq \delta_i$, meaning that common and specific components do not have the same dynamics, considering F_t in predicting X_{it} should perform better than AR forecast in terms of mean squared error (MSE).

Allowing for MA parts in dynamics of common and specific components generalizes this finding. Suppose again $\lambda_i \neq 0$. If MA coefficients are not zero, $\theta \neq 0$ and $\gamma_i \neq 0$, ignoring moving average structure will produce higher forecast errors even if $\phi = \delta_i$.

It is important to note that forecasting performance of X_{it}, given a factor structure, is affected by the choice of estimation method to get factors and by the choice of forecasting equation. Boivin and Ng (2005) address these issues by considering static and dynamic factors approximations with three types of forecasting equations: unrestricted (where X_{iT+h} is forecasted using X_{iT}, F_T and their lags), direct (where dynamic process of factors is estimated and used to first forecast F_{T+h} and then get X_{iT+h}), and nonparametric (no parametric assumptions are made about the dynamics of factors nor their relations to observables). Their simulation results show that the unrestricted forecast equation using static factors generally does best in terms of relative MSE to autoregressive alternative. Moreover, it seems that these findings are mainly caused by the choice of forecasting equation.

In our approach, allowing for MA structure should help in forecasting X_{it} if the process of factors is well approximated. If the MA part of the VARMA process of factors is weak, i.e. coefficients of $[I - \Theta(L)]^{-1}$ go rapidly to zero, the finite VAR approximation should perform as well as if VARMA representation is estimated. In practice, due to estimation error, it will be not surprising to see that simpler method perform better. However, if coefficients of $[I - \Theta(L)]^{-1}$ vanishes slowly, or in more extreme case if VARMA representation is not invertible, estimating parsimonious VARMA process should outperform a long VAR approximation.

7.1.1. **Pseudo-out-of-sample forecasting exercise**

To evaluate if allowing for VARMA structure in approximate factor model analysis can help in predicting some macroeconomic aggregates we conduct a pseudo-out-of-sample forecasting exercise in which we compare our DFM-VARMA approach to some standard factor-based forecasting techniques used in the literature. Following Boivin and Ng (2005), the forecasting equations that we consider are divided in two categories: those that do not consider the dynamic process of factors (called “Unrestricted”in Boivin and Ng (2005) and “Diffusion index”, or DI and DI-AR, in Stock and Watson (2002)), and those that first predict the common and specific components using their dynamic processes and then form the forecast of the series of interest. Moreover, in the latter we distinguish between sequential and direct techniques to obtain forecasts.

In this exercise we estimate factors with principal components analysis from the contemporaneous covariance matrix of observables X_t so only the second type of forecasting equations can be affected by allowing for VARMA process for factors. We compare the results for four identified VARMA forms labeled "Diag MA", "Diag AR", "Final MA"and "Final AR". Also, we only model the factors' process as VARMA while for the idiosyncratic component we assume an AR(p) process.

Let us resume more formally the factor-based forecasting equations before presenting the simulation and empirical results:

- First type:
$$X_{i,T+h|T} = \alpha^h + \sum_{j=1}^{m} \beta_{ij}^h F_{T-j+1} + \sum_{j=1}^{p} \rho_{ij}^h X_{i,T-j+1}$$

  - Unrestricted: $m \geq 1, p \geq 0$
  - DI: $m = 1, p = 0$
  - DI-AR: $m = 1$

- Second type:
$$X_{i,T+h|T} = \lambda_i' F_{T+h|T} + u_{i,T+h|T}$$
  where $u_{i,T+h|T}$ is forecasted sequentially or directly using AR(p) process while the factors' dynamics is approximated by VAR(p) (giving Sequential and Direct forecasts as in Boivin and Ng (2005)) or by one of four identified VARMA forms in sequential way

  - Sequential: $F_{T+h|T} = \hat{\Phi}(L) F_{T+h-1|T}$
  - Direct: $F_{T+h|T} = \hat{\Phi}^h(L) F_T$
  - VARMA: $F_{T+h|T} = \hat{\Phi}(L) F_{T+h-1|T} + \hat{\Theta}(L) \eta_{T+h-1|T}$ where VARMA contains four different models (Diag MA, Diag AR, Final MA, Final AR) defined by corresponding lag polynomials $\Phi(L)$ and $\Theta(L)$.

The benchmark forecasting model used to compare previous equations is standard AR(p) that was used in other studies (Stock and Watson (2002b), Boivin and Ng (2005)). However, given the factor structure for observable series, the finite order autoregressive process is only an approximation of the process of X_{it}. After the Theorem 4.1 we have that the implied process for each element of X_t is a finite order ARMA. Hence, if the MA part for a particular series of interest is very important and close to non-invertibility region, we need a very long autoregressive model to approximate the true process and this can affect the forecasting performance. In our forecasting exercise we also include ARMA modeling as an alternative to see if it outperforms the autoregressive specifications and especially how does it performs relative to factor-based models.

8. Monte Carlo simulations

To illustrate the performance of our approach we run several Monte Carlo simulation exercises where we compare forecasting performance of second type models: DFM-VARMA (in four identified forms) to DFM-VAR models. Here, we present results where data are simulated using the static factor model with MA(1) dynamics for factors and where the idiosyncratic component is simulated as in Boivin and Ng (2005) and as in Onatski (2009) to capture "weak factor structure" characteristics:

$$X_{it} = \lambda_i F_t + u_{it} \tag{8.1}$$

$$F_t = \eta_t - B\eta_{t-1} \tag{8.2}$$

where $i = 1, \ldots, N$, $t = 1, \ldots, T$, $\eta_t \sim N(0, 1)$, and the generating process of u_{it} will be specified in each simulation exercise.

Here, we present two different simulation exercises. In the first series of simulations we vary the time and cross-section dimensions, the nature of idiosyncratic components and the importance of MA component in factors' dynamics. In the second exercise, we fix the time and cross-section dimensions to 100 each, simulate idiosyncratic component as in Boivin and Ng (2005) and vary the number of factors.

8.1. **Simulation exercise 1**

- Time dimension: $T \in \{50, 100, 600\}$.
- Cross-section dimension: $N = \{50, 100, 130\}$.
- Number of factors: $K \in \{2, 4\}$
- Idiosyncratic component dynamics:

$$\begin{aligned} u_{it} &= \rho_N u_{i-1,t} + \xi_{it} \\ \xi_{it} &= \rho_T \xi_{i,t-1} + \epsilon_{it} \\ \epsilon_{it} &\sim N(0,1) \end{aligned}$$

  where the cross-section dependance (CSD) is controlled by parameter $\rho_N \in \{0.1, 0.5, 0.9\}$, and the time dependance (TD) is controlled by parameter $\rho_T \in \{0.1, 0.9\}$.

- VARMA orders: estimated as in Dufour and Pelletier (2008).
- AR order for idiosyncratic component: 1.
  - Case 1:
    - K = 2

$$B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}$$

    - VAR order: 6.
  - Case 2:

- K = 4

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- VAR order: 4.

The results from this simulation exercise are presented in Tables 1 and 2 for Cases 1 and 2 respectively. The numbers represent MSE of four DFM-VARMA identified forms over the MSE of DFM-VAR Direct or Iterative forecasting models.

In Case 1 and for situations where the time dimension is small, i.e. T=50, the DFM-VARMA models outperform DFM-VAR Direct model especially at long horizons. The huge improvement at horizons 24 and 36 is due to small sample size. Compared to DFM-VAR Iterative model, the DFM-VARMA forms still produce better forecasts in terms of MSE, but the improvement is less important than compared to multi-step-ahead forecasting VAR-based model. However, we can see that MA forms outperform DFM-VAR Iterative model up to 20% at short and long horizons. When the time size becomes more important, T=100 and T=600, the improvement of VARMA-based models is moderate but they still produce better forecasts especially at longer horizons when compared to Direct VAR model and at shorter horizons when compared to Iterative VAR model. The picture is quite similar in results from simulation specification in Case 2. However, in this case and for most of time and cross-section sizes and specifications of idiosyncratic components, the DFM-VARMA gives better results than in Case 1.

Another interesting result is that DFM-VARMA models seem to perform better in situation of weaker factor structure, that is in cases where the cross-section size is smaller, N=50 compared to N=100, and for a fixed nature of idiosyncratic component correlation structure. Finally, when time and cross-section sizes are comparable to what we have in data (T=600, N=130), the DFM-VARMA models perform better in Case 2 than in Case 1, due to very persistent MA part in factor dynamics in Case 2.

Table 1: Results from simulation exercise 1, case 1

	$\rho_T = 0.9, \rho_N = 0.5$								$\rho_T = 0.9, \rho_N = 0.1$							
	RELATIVE MSE (TO VAR(6) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(6) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.0078	1.1405	0.9235	1.3858	1.0061	1.0945	0.9084	1.4722	1.0203	1.0656	0.8897	1.2688	0.9977	1.0303	0.9026	1.3464
2	1.0199	1.0852	0.9483	1.3189	1.0302	1.0762	0.9383	1.3660	0.9689	1.0113	0.8982	1.1708	1.0013	1.0406	0.9038	1.1735
4	0.8872	0.9459	0.8350	1.0746	0.9338	1.0242	0.8745	1.1542	0.9142	0.9508	0.8616	1.0391	0.9032	0.9166	0.8461	1.0029
6	0.8122	0.9181	0.7635	0.9536	0.8514	0.9375	0.7954	1.0010	0.8420	0.8656	0.7851	0.9213	0.8841	0.8798	0.8054	0.9182
12	0.6311	0.8392	0.6072	0.7198	0.6857	0.9278	0.6533	0.8036	0.6401	0.7487	0.6235	0.7038	0.7042	0.8089	0.6850	0.7582
18	0.4913	0.7186	0.4754	0.5339	0.5181	0.8285	0.4955	0.5744	0.5208	0.6774	0.5133	0.5609	0.5469	0.6970	0.5296	0.5742
24	0.3762	0.6192	0.3706	0.4237	0.3846	0.7215	0.3788	0.4291	0.4095	0.5979	0.4124	0.4322	0.4380	0.5724	0.4282	0.4499
36	0.1394	0.2429	0.1369	0.1480	0.1445	0.3006	0.1422	0.1560	0.1417	0.2169	0.1402	0.1447	0.1453	0.2152	0.1424	0.1535
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.0761	1.1170	1.0004	1.6656	1.0130	1.0126	1.0093	1.0070	1.0622	1.0751	0.9990	1.3846	0.9978	0.9980	0.9984	0.9927
2	1.0865	1.1495	1.0193	1.5676	0.9962	0.9956	0.9952	0.9951	1.0578	1.0368	0.9913	1.2818	0.9935	0.9951	0.9935	0.9933
4	1.0537	1.0890	1.0038	1.4432	0.9945	0.9950	0.9947	0.9947	1.0254	1.0088	0.9729	1.2141	0.9890	0.9894	0.9891	0.9891
6	1.0168	1.0392	0.9686	1.3060	0.9945	0.9954	0.9946	0.9946	1.0058	0.9720	0.9477	1.1812	0.9892	0.9892	0.9892	0.9892
12	0.9183	0.9915	0.8960	1.2573	0.9871	0.9883	0.9873	0.9873	0.9480	0.9163	0.8819	1.0303	0.9919	0.9918	0.9919	0.9919
18	0.8886	0.9848	0.8552	1.1123	0.9831	0.9880	0.9832	0.9832	0.9371	0.9068	0.8823	1.0173	0.9784	0.9784	0.9784	0.9784
24	0.8643	0.9706	0.8198	1.1203	0.9831	0.9830	0.9828	0.9828	0.9441	0.8755	0.8626	1.0214	0.9807	0.9807	0.9807	0.9807
36	0.8078	0.9754	0.7956	1.0742	0.9863	0.9846	0.9847	0.9847	0.8591	0.8376	0.8013	0.9264	0.9796	0.9796	0.9796	0.9796

	RELATIVE MSE (TO VAR(6) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(6) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.0078	1.1405	0.9235	1.3858	1.0061	1.0945	0.9084	1.4722	1.0203	1.0656	0.8897	1.2688	0.9977	1.0303	0.9026	1.3464
2	1.0126	1.0774	0.9415	1.3095	1.0234	1.0691	0.9321	1.3570	0.9805	1.0234	0.9090	1.1848	1.0082	1.0477	0.9100	1.1815
4	0.9744	1.0388	0.9170	1.1801	1.0087	1.1063	0.9446	1.2467	0.9902	1.0298	0.9331	1.1254	0.9881	1.0028	0.9257	1.0972
6	0.9771	1.1044	0.9185	1.1472	1.0052	1.1068	0.9390	1.1818	1.0000	1.0281	0.9325	1.0943	1.0380	1.0330	0.9456	1.0781
12	0.9149	1.2165	0.8802	1.0434	0.9656	1.3067	0.9200	1.1317	0.9214	1.0777	0.8975	1.0130	0.9626	1.1057	0.9363	1.0364
18	0.8988	1.3147	0.8699	0.9768	0.9436	1.5088	0.9023	1.0460	0.9115	1.1854	0.8983	0.9816	0.9521	1.2136	0.9221	0.9997
24	0.8625	1.4195	0.8497	0.9714	0.9041	1.6962	0.8905	1.0088	0.8556	1.2493	0.8616	0.9029	0.9393	1.2276	0.9183	0.9649
36	0.8121	1.4144	0.7971	0.8619	0.8697	1.8094	0.8561	0.9391	0.8176	1.2517	0.8088	0.8351	0.8566	1.2680	0.8391	0.9044
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.0761	1.1170	1.0004	1.6656	1.0130	1.0126	1.0093	1.0070	1.0622	1.0751	0.9990	1.3846	0.9978	0.9980	0.9984	0.9927
2	1.0487	1.1095	0.9838	1.5129	0.9959	0.9953	0.9949	0.9949	1.0489	1.0280	0.9829	1.2710	0.9948	0.9965	0.9949	0.9946
4	1.0446	1.0797	0.9952	1.4309	0.9986	0.9990	0.9987	0.9987	1.0364	1.0197	0.9833	1.2272	0.9931	0.9934	0.9932	0.9932
6	1.0440	1.0670	0.9945	1.3409	0.9992	1.0002	0.9993	0.9994	1.0547	1.0192	0.9937	1.2386	0.9981	0.9981	0.9981	0.9981
12	1.0432	1.1264	1.0179	1.4283	1.0002	1.0015	1.0004	1.0004	1.0749	1.0390	0.9999	1.1682	1.0001	1.0001	1.0001	1.0001
18	1.0651	1.1804	1.0251	1.3332	1.0001	1.0051	1.0002	1.0002	1.1030	1.0674	1.0385	1.1973	1.0000	0.9999	1.0000	1.0000
24	1.0808	1.2137	1.0252	1.4010	1.0004	1.0003	1.0000	1.0000	1.1481	1.0647	1.0490	1.2422	1.0000	1.0000	1.0000	1.0000
36	1.0764	1.2997	1.0602	1.4313	1.0016	0.9999	1.0000	1.0000	1.1326	1.1042	1.0564	1.2213	0.9999	0.9999	0.9999	0.9999

Table 1: Results from simulation exercise 1, case 1, continued

	$\rho_T = 0.1, \rho_N = 0.9$								$\rho_T = 0.1, \rho_N = 0.1$							
	RELATIVE MSE (TO VAR(6) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(6) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.8978	0.9108	0.8924	0.9362	0.9329	0.8880	0.8585	0.9208	0.9439	0.8761	0.7969	0.9618	0.9289	0.8919	0.8155	0.9675
2	0.8522	0.8716	0.8606	0.9168	0.8289	0.8194	0.8228	0.8642	0.8029	0.7863	0.7764	0.8459	0.7888	0.7900	0.7736	0.8569
4	0.8381	0.8420	0.8524	0.8601	0.8195	0.8213	0.8187	0.8238	0.7894	0.7542	0.7533	0.7742	0.7513	0.7533	0.7525	0.7687
6	0.8213	0.8227	0.8225	0.8210	0.7852	0.7806	0.7799	0.7795	0.7580	0.7420	0.7409	0.7438	0.7477	0.7488	0.7446	0.7458
12	0.7923	0.7906	0.7905	0.7907	0.7630	0.7569	0.7567	0.7568	0.6773	0.6751	0.6751	0.6754	0.6575	0.6604	0.6560	0.6613
18	0.6803	0.6770	0.6771	0.6772	0.6582	0.6576	0.6577	0.6577	0.5757	0.5700	0.5701	0.5761	0.5741	0.5753	0.5704	0.5728
24	0.5367	0.5363	0.5364	0.5364	0.4865	0.4864	0.4863	0.4862	0.4106	0.4074	0.4073	0.4084	0.4329	0.4303	0.4304	0.4317
36	0.0946	0.0956	0.0944	0.0944	0.0801	0.0799	0.0799	0.0800	0.0726	0.0721	0.0721	0.0721	0.0719	0.0722	0.0721	0.0721
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9680	0.9676	0.9560	0.9515	0.9931	0.9995	0.9926	0.9921	0.9702	0.9672	0.9290	0.9316	0.9838	0.9874	0.9868	0.9840
2	0.9332	0.9304	0.9306	0.9310	0.9881	0.9929	0.9882	0.9878	0.8998	0.9053	0.8985	0.8993	0.9816	0.9904	0.9811	0.9811
4	0.9338	0.9261	0.9257	0.9257	0.9882	0.9895	0.9893	0.9894	0.9095	0.9003	0.9000	0.8997	0.9891	0.9894	0.9891	0.9891
6	0.9467	0.9350	0.9351	0.9351	0.9831	0.9831	0.9830	0.9830	0.8806	0.8771	0.8767	0.8767	0.9821	0.9822	0.9821	0.9821
12	0.9358	0.9359	0.9359	0.9359	0.9825	0.9825	0.9825	0.9825	0.8855	0.8841	0.8839	0.8839	0.9778	0.9778	0.9778	0.9778
18	0.9297	0.9298	0.9297	0.9297	0.9874	0.9873	0.9873	0.9873	0.8725	0.8704	0.8702	0.8702	0.9852	0.9852	0.9852	0.9852
24	0.9140	0.9142	0.9143	0.9143	0.9887	0.9886	0.9886	0.9886	0.8711	0.8707	0.8709	0.8709	0.9815	0.9815	0.9815	0.9815
36	0.9044	0.9047	0.9043	0.9043	0.9929	0.9930	0.9930	0.9930	0.8183	0.8185	0.8183	0.8183	0.9790	0.9790	0.9790	0.9790

	RELATIVE MSE (TO VAR(6) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(6) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.8978	0.9108	0.8924	0.9362	0.9329	0.8880	0.8585	0.9208	0.9439	0.8761	0.7969	0.9618	0.9289	0.8919	0.8155	0.9675
2	0.8879	0.9082	0.8966	0.9552	0.8644	0.8545	0.8580	0.9012	0.8438	0.8264	0.8159	0.8890	0.8259	0.8271	0.8099	0.8971
4	0.9270	0.9313	0.9427	0.9513	0.9119	0.9139	0.9110	0.9167	0.9067	0.8664	0.8653	0.8893	0.8629	0.8652	0.8643	0.8828
6	0.9552	0.9567	0.9565	0.9547	0.9441	0.9386	0.9378	0.9373	0.9249	0.9055	0.9040	0.9076	0.9174	0.9188	0.9135	0.9151
12	0.9872	0.9850	0.9849	0.9852	0.9840	0.9761	0.9758	0.9759	0.9791	0.9759	0.9758	0.9763	0.9783	0.9826	0.9760	0.9839
18	0.9957	0.9910	0.9910	0.9911	0.9934	0.9925	0.9926	0.9926	0.9937	0.9838	0.9840	0.9944	0.9994	1.0015	0.9928	0.9971
24	0.9973	0.9966	0.9967	0.9968	0.9911	0.9909	0.9908	0.9905	0.9866	0.9788	0.9786	0.9812	1.0005	0.9944	0.9946	0.9977
36	0.9979	1.0082	0.9965	0.9966	0.9925	0.9896	0.9896	0.9908	0.9902	0.9836	0.9835	0.9836	0.9919	0.9958	0.9946	0.9944
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9680	0.9676	0.9560	0.9515	0.9931	0.9995	0.9926	0.9921	0.9702	0.9672	0.9290	0.9316	0.9838	0.9874	0.9868	0.9840
2	0.9508	0.9479	0.9481	0.9485	0.9901	0.9949	0.9903	0.9898	0.9096	0.9152	0.9083	0.9091	0.9847	0.9935	0.9842	0.9842
4	0.9755	0.9675	0.9671	0.9671	0.9936	0.9948	0.9947	0.9948	0.9557	0.9461	0.9458	0.9455	0.9949	0.9953	0.9950	0.9950
6	0.9990	0.9867	0.9867	0.9867	0.9974	0.9974	0.9973	0.9973	0.9717	0.9679	0.9674	0.9674	0.9971	0.9972	0.9971	0.9971
12	0.9987	0.9987	0.9987	0.9987	1.0002	1.0002	1.0002	1.0002	0.9995	0.9979	0.9977	0.9977	1.0000	1.0000	1.0000	1.0000
18	1.0001	1.0002	1.0001	1.0001	1.0001	1.0000	1.0000	1.0000	1.0025	1.0001	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
24	0.9995	0.9997	0.9998	0.9998	1.0000	0.9999	0.9999	0.9999	0.9998	0.9993	0.9996	0.9996	1.0001	1.0001	1.0001	1.0001
36	1.0003	1.0007	1.0002	1.0002	0.9998	1.0000	1.0000	1.0000	0.9997	0.9999	0.9996	0.9996	1.0000	1.0000	1.0000	1.0000

Table 2: Results from simulation exercise 1, case 2

	$\rho_T = 0.9, \rho_N = 0.5$								$\rho_T = 0.9, \rho_N = 0.1$							
	RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.8168	0.8338	0.7830	1.0004	0.8211	0.8364	0.8081	1.0849	0.8024	0.8060	0.7948	0.9410	0.8482	0.8043	0.8569	0.9851
2	0.7715	0.7747	0.7444	0.7682	0.7379	0.7581	0.7481	0.7793	0.7213	0.7259	0.7188	0.7389	0.7198	0.7139	0.7166	0.7150
4	0.6972	0.7636	0.6885	0.7148	0.6981	0.7450	0.6974	0.7262	0.7059	0.7161	0.6937	0.7051	0.6773	0.6853	0.6833	0.6975
6	0.6538	0.7624	0.6533	0.6698	0.6733	0.7685	0.6621	0.6729	0.6507	0.6855	0.6536	0.6589	0.6163	0.6507	0.6318	0.6394
12	0.5453	0.7673	0.5343	0.5335	0.5497	0.7743	0.5494	0.5466	0.5597	0.6293	0.5507	0.5509	0.5290	0.6072	0.5395	0.5395
18	0.4128	0.6560	0.4091	0.4046	0.4379	0.7022	0.4381	0.4331	0.4203	0.5301	0.4205	0.4190	0.4431	0.5177	0.4435	0.4434
24	0.3114	0.5082	0.3001	0.2976	0.3007	0.5244	0.2998	0.2955	0.2956	0.3807	0.3015	0.2995	0.3025	0.3673	0.2975	0.2945
36	0.1427	0.2401	0.1432	0.1412	0.1446	0.2674	0.1454	0.1421	0.1389	0.1853	0.1332	0.1318	0.1403	0.1817	0.1416	0.1407
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9727	0.9732	0.9352	1.1557	0.9965	0.9946	0.9957	0.9979	0.9810	0.9378	0.9130	1.0321	0.9863	0.9899	0.9861	0.9882
2	0.9055	0.9078	0.8919	1.0099	0.9792	0.9800	0.9776	0.9801	0.8919	0.8915	0.8767	0.9066	0.9816	0.9926	0.9822	0.9826
4	0.9020	0.9038	0.8942	0.9336	0.9793	0.9847	0.9796	0.9797	0.8858	0.8770	0.8756	0.8916	0.9769	0.9821	0.9769	0.9770
6	0.8990	0.9078	0.8898	0.9155	0.9782	0.9782	0.9782	0.9782	0.8762	0.8694	0.8697	0.8712	0.9753	0.9755	0.9753	0.9753
12	0.8367	0.9146	0.8277	0.8523	0.9748	0.9749	0.9748	0.9748	0.8448	0.8384	0.8347	0.8370	0.9668	0.9668	0.9668	0.9668
18	0.8115	0.9658	0.7972	0.7983	0.9771	0.9771	0.9771	0.9771	0.8227	0.8171	0.8022	0.8114	0.9738	0.9738	0.9738	0.9738
24	0.7895	1.0204	0.7764	0.7984	0.9791	0.9791	0.9791	0.9791	0.7964	0.7994	0.7781	0.7835	0.9732	0.9732	0.9732	0.9732
36	0.7170	0.9889	0.7026	0.7137	0.9721	0.9721	0.9721	0.9721	0.7362	0.7713	0.7215	0.7249	0.9709	0.9709	0.9709	0.9709

	RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.8168	0.8338	0.7830	1.0004	0.8211	0.8364	0.8081	1.0849	0.8024	0.8060	0.7948	0.9410	0.8482	0.8043	0.8569	0.9851
2	0.8286	0.8320	0.7994	0.8251	0.7797	0.8011	0.7905	0.8235	0.7833	0.7884	0.7807	0.8025	0.7672	0.7610	0.7639	0.7622
4	0.8697	0.9526	0.8589	0.8917	0.8609	0.9187	0.8600	0.8955	0.8698	0.8824	0.8548	0.8688	0.8359	0.8457	0.8432	0.8607
6	0.8943	1.0429	0.8937	0.9163	0.9173	1.0471	0.9021	0.9168	0.8961	0.9440	0.9001	0.9073	0.8810	0.9302	0.9031	0.9141
12	0.9307	1.3097	0.9119	0.9106	0.9372	1.3199	0.9367	0.9318	0.9712	1.0919	0.9556	0.9560	0.9384	1.0770	0.9569	0.9570
18	0.9047	1.4379	0.8967	0.8869	0.9440	1.5138	0.9446	0.9337	0.9320	1.1754	0.9323	0.9292	0.9607	1.1225	0.9614	0.9612
24	0.9289	1.5163	0.8952	0.8877	0.9200	1.6044	0.9174	0.9040	0.9305	1.1987	0.9492	0.9430	0.9554	1.1603	0.9398	0.9302
36	0.8715	1.4667	0.8748	0.8623	0.8821	1.6315	0.8874	0.8671	0.9142	1.2191	0.8767	0.8671	0.9301	1.2043	0.9384	0.9327
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9727	0.9732	0.9352	1.1557	0.9965	0.9946	0.9957	0.9979	0.9810	0.9378	0.9130	1.0321	0.9863	0.9899	0.9861	0.9882
2	0.9165	0.9188	0.9027	1.0221	0.9820	0.9828	0.9804	0.9829	0.9077	0.9074	0.8923	0.9227	0.9830	0.9941	0.9836	0.9840
4	0.9632	0.9651	0.9549	0.9970	0.9884	0.9938	0.9887	0.9888	0.9564	0.9469	0.9454	0.9626	0.9911	0.9964	0.9911	0.9912
6	1.0000	1.0099	0.9898	1.0184	0.9992	0.9993	0.9992	0.9992	0.9935	0.9858	0.9861	0.9878	1.0002	1.0004	1.0002	1.0002
12	1.0057	1.0994	0.9950	1.0245	1.0000	1.0000	1.0000	1.0000	1.0110	1.0033	0.9988	1.0016	1.0001	1.0001	1.0001	1.0001
18	1.0058	1.1971	0.9881	0.9894	1.0000	1.0000	1.0000	1.0000	1.0217	1.0147	0.9962	1.0076	1.0000	1.0000	1.0000	1.0000
24	1.0060	1.3002	0.9893	1.0174	1.0000	1.0000	1.0000	1.0000	1.0175	1.0214	0.9941	1.0010	1.0000	1.0000	1.0000	1.0000
36	1.0081	1.3903	0.9878	1.0034	1.0000	1.0000	1.0000	1.0000	1.0153	1.0636	0.9950	0.9997	1.0000	1.0000	1.0000	1.0000

Table 2: Results from simulation exercise 1, case 2

	$\rho_T = 0.1, \rho_N = 0.9$								$\rho_T = 0.1, \rho_N = 0.1$							
	RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9207	0.8569	0.9599	0.9455	0.9556	0.8779	0.9330	0.9213	0.9759	0.8416	0.9349	0.9087	0.9368	0.8434	0.9225	0.8822
2	0.6689	0.6830	0.6899	0.6822	0.6670	0.6853	0.6780	0.6627	0.6416	0.6599	0.6531	0.6360	0.6411	0.6687	0.6575	0.6271
4	0.6246	0.6309	0.6261	0.6250	0.6442	0.6652	0.6245	0.6214	0.5737	0.5921	0.5650	0.5602	0.6129	0.6269	0.6215	0.6087
6	0.6142	0.6198	0.6241	0.6240	0.6262	0.6153	0.6106	0.6100	0.5872	0.6063	0.6060	0.6033	0.5694	0.5743	0.5574	0.5562
12	0.5207	0.5402	0.5464	0.5464	0.5372	0.5398	0.5420	0.5418	0.4849	0.4933	0.4871	0.4871	0.5170	0.5173	0.5219	0.5221
18	0.4600	0.4555	0.4419	0.4419	0.4442	0.4465	0.4464	0.4464	0.4073	0.4019	0.3921	0.3921	0.3917	0.3982	0.3957	0.3957
24	0.2904	0.2895	0.2903	0.2903	0.2847	0.2872	0.2817	0.2817	0.2554	0.2597	0.2494	0.2494	0.2696	0.2706	0.2624	0.2624
36	0.1062	0.1074	0.1092	0.1092	0.1021	0.1033	0.0987	0.0987	0.0969	0.0969	0.0967	0.0967	0.0965	0.0994	0.0971	0.0971
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.1330	1.0075	0.9066	1.2783	0.9592	0.9935	0.9641	0.9572	1.5484	0.9748	0.8213	2.1043	0.9237	0.9960	0.9028	0.9907
2	0.8541	0.8709	0.8453	0.8923	0.9719	0.9951	0.9724	0.9727	0.8596	0.8784	0.8165	0.9017	0.9677	1.0016	0.9682	0.9685
4	0.8501	0.8490	0.8428	0.8509	0.9742	0.9914	0.9742	0.9742	0.8287	0.8205	0.8020	0.8184	0.9734	1.0058	0.9733	0.9735
6	0.8452	0.8432	0.8425	0.8425	0.9721	0.9790	0.9721	0.9721	0.8310	0.8218	0.8178	0.8186	0.9699	1.0212	0.9699	0.9696
12	0.8331	0.8326	0.8328	0.8331	0.9761	0.9766	0.9760	0.9760	0.8086	0.7958	0.7957	0.7969	0.9653	0.9949	0.9653	0.9655
18	0.8056	0.8093	0.8052	0.8049	0.9713	0.9829	0.9712	0.9712	0.7650	0.7590	0.7590	0.7579	0.9661	1.0172	0.9661	0.9661
24	0.8044	0.8034	0.8036	0.8038	0.9683	0.9745	0.9682	0.9682	0.7630	0.7624	0.7623	0.7629	0.9721	0.9918	0.9721	0.9718
36	0.7643	0.7610	0.7610	0.7613	0.9680	0.9698	0.9679	0.9679	0.7210	0.7188	0.7188	0.7191	0.9658	0.9918	0.9658	0.9657

	RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9207	0.8569	0.9599	0.9455	0.9556	0.8779	0.9330	0.9213	0.9759	0.8416	0.9349	0.9087	0.9368	0.8434	0.9225	0.8822
2	0.7223	0.7375	0.7449	0.7366	0.7213	0.7410	0.7331	0.7167	0.6860	0.7055	0.6983	0.6800	0.6809	0.7102	0.6983	0.6660
4	0.8030	0.8111	0.8050	0.8036	0.8032	0.8294	0.7787	0.7748	0.7679	0.7925	0.7563	0.7498	0.7922	0.8103	0.8032	0.7867
6	0.8952	0.9035	0.9097	0.9096	0.9050	0.8892	0.8825	0.8816	0.8384	0.8657	0.8653	0.8615	0.8590	0.8663	0.8409	0.8391
12	0.9339	0.9690	0.9801	0.9801	0.9716	0.9763	0.9803	0.9799	0.9487	0.9651	0.9529	0.9530	0.9696	0.9702	0.9787	0.9791
18	1.0024	0.9927	0.9629	0.9629	0.9849	0.9900	0.9898	0.9898	0.9950	0.9820	0.9580	0.9579	0.9649	0.9810	0.9748	0.9748
24	0.9944	0.9912	0.9940	0.9940	0.9974	1.0062	0.9868	0.9868	0.9803	0.9967	0.9572	0.9572	0.9890	0.9929	0.9627	0.9627
36	0.9870	0.9976	1.0146	1.0146	0.9876	0.9999	0.9546	0.9546	0.9963	0.9965	0.9948	0.9948	0.9796	1.0089	0.9850	0.9850
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.1330	1.0075	0.9066	1.2783	0.9592	0.9935	0.9641	0.9572	1.5484	0.9748	0.8213	2.1043	0.9237	0.9960	0.9028	0.9907
2	0.8822	0.8995	0.8731	0.9216	0.9692	0.9923	0.9697	0.9700	0.8804	0.8997	0.8363	0.9235	0.9610	0.9946	0.9614	0.9618
4	0.9293	0.9281	0.9213	0.9302	0.9814	0.9987	0.9814	0.9814	0.9292	0.9200	0.8992	0.9176	0.9745	1.0069	0.9744	0.9746
6	0.9798	0.9775	0.9767	0.9768	0.9963	1.0034	0.9962	0.9962	0.9759	0.9651	0.9603	0.9613	0.9904	1.0428	0.9904	0.9901
12	0.9978	0.9972	0.9973	0.9977	1.0000	1.0005	0.9999	0.9999	1.0133	0.9972	0.9971	0.9986	0.9996	1.0303	0.9996	0.9998
18	1.0004	1.0049	0.9999	0.9995	1.0001	1.0120	1.0000	1.0000	1.0081	1.0003	1.0003	0.9988	0.9999	1.0528	0.9999	0.9999
24	1.0011	0.9998	1.0001	1.0003	1.0001	1.0065	1.0000	1.0000	1.0012	1.0004	1.0003	1.0011	1.0000	1.0203	1.0000	0.9997
36	1.0040	0.9998	0.9997	1.0002	1.0001	1.0020	1.0000	1.0000	1.0029	0.9998	0.9999	1.0003	1.0000	1.0269	1.0000	0.9998

8.2. **Simulation exercise 2**

- Time dimension: $T = 100$.
- Cross-section dimension: $N = 100$.
- Number of factors: $K \in \{3, 4, 6\}$
- Idiosyncratic component dynamics: $u_{it} = \kappa\nu_{it}$, $\nu_{it} \sim N(0, \sigma^2_{\nu_i})$ such that the common component explains a fraction ϑ of the variance of X_t. Here, ϑ is set to 0.5 while for the first series in panel X_t, the one that is forecasted, we have $var(\lambda_1 F_t)/var(X_{1t}) = 0.75$.
- MA coefficients matrices:
  - K = 3

$$B = \begin{bmatrix} 0.2350 & 0 & 0 \\ 0 & 0.2317 & 0 \\ 0 & 0 & 0.5776 \end{bmatrix}$$

  - K = 4

$$B = \begin{bmatrix} 0.3365 & 0 & 0 & 0 \\ 0 & 0.2420 & 0 & 0 \\ 0 & 0 & 0.0610 & 0 \\ 0 & 0 & 0 & 0.4735 \end{bmatrix}$$

  - K = 6

$$B = \begin{bmatrix} 0.1558 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4827 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4525 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5320 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6604 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2763 \end{bmatrix}$$

- VAR order: 4.
- VARMA orders: estimated as in Dufour and Pelletier (2008).
- AR order for idiosyncratic component: 1.

The results from this simulation exercise are presented in Table 3. We can conclude that DFM-VARMA models performance improves with the number of factors which is a consequence of parsimony principle. Moreover, in comparison with Direct DFM-VAR forecasting model the VARMA based models are better especially in long horizons, with exception for very short horizon in the case of 6 factors model.

Table 3: Results from simulation exercise 2

	RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS											
	K=3				K=4				K=6			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9638	0.9643	0.9285	0.9330	0.9194	0.9182	0.8866	0.8927	0.7282	0.6615	0.6905	0.6907
2	0.9085	0.9174	0.9076	0.9133	0.8792	0.8901	0.8805	0.8866	0.8261	0.8615	0.8244	0.8385
4	0.8971	0.8966	0.8965	0.8961	0.8764	0.8775	0.8764	0.8769	0.8030	0.8030	0.8010	0.8072
6	0.9038	0.9037	0.9035	0.9036	0.8548	0.8549	0.8548	0.8549	0.9182	0.9180	0.9182	0.9204
12	0.8808	0.8807	0.8807	0.8807	0.8416	0.8418	0.8418	0.8418	0.7983	0.7997	0.7983	0.7983
18	0.8831	0.8831	0.8831	0.8831	0.8455	0.8454	0.8454	0.8454	0.9393	0.9383	0.9393	0.9393
24	0.8757	0.8756	0.8756	0.8756	0.8425	0.8425	0.8425	0.8425	0.7287	0.7286	0.7287	0.7287
36	0.8344	0.8343	0.8343	0.8343	0.7930	0.7932	0.7932	0.7932	0.5466	0.5466	0.5466	0.5466
	RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS											
	K=3				K=4				K=6			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9638	0.9643	0.9285	0.9330	0.9194	0.9182	0.8866	0.8927	0.7282	0.6615	0.6905	0.6907
2	0.9197	0.9288	0.9188	0.9246	0.9092	0.9205	0.9106	0.9168	0.9296	0.9695	0.9277	0.9435
4	0.9685	0.9680	0.9679	0.9675	0.9562	0.9574	0.9562	0.9568	0.9406	0.9406	0.9383	0.9456
6	0.9927	0.9926	0.9925	0.9926	0.9851	0.9852	0.9850	0.9852	0.9467	0.9466	0.9467	0.9490
12	1.0001	1.0000	1.0000	1.0001	1.0002	1.0005	1.0005	1.0005	0.9803	0.9820	0.9803	0.9802
18	0.9997	0.9996	0.9996	0.9996	1.0038	1.0037	1.0037	1.0037	0.9957	0.9947	0.9957	0.9957
24	1.0009	1.0008	1.0008	1.0008	1.0010	1.0009	1.0009	1.0009	0.9978	0.9977	0.9978	0.9978
36	0.9998	0.9997	0.9997	0.9997	0.9993	0.9995	0.9995	0.9995	0.9986	0.9986	0.9986	0.9986

9. **Empirical application**

We conduct the same out-of-sample forecasting exercise for two different sets of data. In the first exercise we use a balanced monthly panel from Boivin, Giannoni and Stevanović (2009) (essentially the upgraded version of the data used in Stock and Watson (2002b)) containing 128 monthly US economic and financial indicators spanning from 1959 to 2008. The second application deals with a Canadian balanced monthly panel from Boivin, Giannoni and Stevanović (2008) containing 332 series observed from 1981 to 2008. The series are initially transformed to induce stationarity. The detailed description of the series and their transformations are in the appendix. The main results are presented in following sections.

9.1. **Main empirical results with US data**

The Relative MSE (relative to benchmark AR(p) model) results are presented in Table 3. The pseudo-out-of-sample evaluation period is 1988M01-2008M12. In the forecasting models Unrestricted, DI, and DI-AR, the number of factors, their number of lags and the number of lags of X_{it} are estimated using a Bayesian information criteria, and these can vary over the whole evaluation period. In the case of forecasting models based on the factors' process, the number of factors is fixed to 4.

Table 3: RMSE relative to Direct AR(p) forecasts

Industrial production: total										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	0.8706	**0.8457**	0.8958	0.9443	0.9443	0.8971	0.9019	0.9132	0.8985	0.9700
2	1.0490	0.9938	1.0106	1.0157	1.0665	**0.9074**	0.9202	0.9112	0.9123	1.0026
4	1.1934	1.0411	1.0527	1.0711	1.2214	**0.8947**	0.9906	0.8970	0.9481	0.9710
6	1.1496	1.0238	1.0245	1.1743	1.3528	0.9248	1.0494	**0.9202**	0.9847	0.9918
12	1.2486	1.0445	1.0389	1.0933	1.3682	1.0008	1.2215	1.0075	1.0371	**0.9713**
18	1.0507	1.0048	1.0207	1.0662	1.2508	1.0511	1.5098	1.0615	1.1206	**0.9910**
24	1.0393	1.0628	1.0748	1.0128	1.0863	0.9858	1.7920	0.9959	1.1061	**0.9604**
36	1.0092	1.0906	1.1437	1.2364	1.0421	0.9855	3.0304	0.9883	1.1795	**0.9826**
48	1.0147	1.1110	1.1212	1.1063	1.0355	0.9921	5.5321	0.9922	1.1681	**0.9856**
Civilian labor force: employed. total										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	0.8264	0.8832	0.8451	0.8202	0.8202	**0.8004**	0.8075	0.8027	0.8008	1.0496
2	0.9407	0.9391	0.9381	0.9477	0.9591	0.8931	**0.8805**	0.8961	0.8852	1.0422
4	0.9766	0.9739	0.9937	1.0204	1.0551	0.9213	0.8997	0.9200	**0.8991**	0.9993
6	1.0776	1.0799	1.0937	1.0714	1.1550	0.9667	0.9526	0.9636	**0.9455**	1.0032
12	1.0741	1.0742	1.0722	1.0137	1.1654	0.9718	0.9912	0.9704	0.9558	**0.9507**
18	1.0471	1.0488	1.0472	0.9735	1.1391	1.0073	1.1386	1.0096	1.0391	**0.9721**
24	1.0237	1.0580	1.0268	**0.9641**	1.1002	1.0154	1.2806	1.0177	1.0856	0.9893
36	0.9573	0.9099	0.9703	0.9507	0.9477	0.9070	1.5452	0.9043	1.0098	**0.8957**
48	**0.9227**	0.9236	0.9250	0.9576	0.9989	0.9652	2.4022	0.9624	1.0482	0.9550
Consumer price index: all items										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	0.8806	**0.8700**	**0.8700**	0.9228	0.9228	0.9144	0.9432	0.8856	0.9072	1.0143
2	0.9866	0.9942	0.9942	0.9612	0.9730	0.9309	0.9427	0.9274	**0.9170**	0.9856
4	1.0656	1.0732	1.0732	1.0398	1.0170	1.0007	1.0665	0.9895	**0.9792**	1.0129
6	1.1343	1.1334	1.1334	1.0349	1.0101	0.9946	1.0752	0.9939	**0.9928**	1.0364
12	1.1173	1.1279	1.1279	1.0821	**0.9513**	0.9572	1.1958	0.9553	1.0408	1.0297
18	1.0311	1.0379	1.0379	1.0430	0.9654	**0.8894**	1.1021	0.8909	0.9673	0.9391
24	0.9644	1.0712	1.0712	0.9510	0.9980	0.8819	1.1851	**0.8791**	0.9713	0.8805
36	0.7645	**0.7627**	**0.7627**	0.9870	0.9470	0.8329	1.4591	0.8385	0.9126	0.8619
48	0.8663	0.8488	0.8488	0.9361	0.9536	**0.8292**	2.2640	0.8335	0.8864	0.8511
Federal funds rate										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	**0.8375**	7.9025	1.1331	2.5170	2.5170	1.7198	3.1430	1.6070	2.1519	0.9304
2	**0.5932**	2.3977	0.8468	1.2457	1.3039	1.0068	2.0381	0.9462	1.3972	0.8919
4	**0.5156**	1.0252	0.6709	0.7663	0.8053	0.7065	1.5720	0.6562	0.9504	0.9087
6	**0.5576**	0.7488	0.6719	0.7309	0.6908	0.6189	1.4871	0.5742	0.8662	0.9433
12	0.5888	0.5916	0.6790	0.6416	0.6479	0.5757	1.5622	**0.5325**	0.8013	1.0424
18	0.6245	0.6387	0.6407	0.6404	0.6792	0.5591	1.8109	**0.5141**	0.7447	0.8674
24	0.5747	0.6950	0.6169	0.6505	0.6864	0.5449	2.0802	**0.5106**	0.6765	0.7937
36	0.5795	0.5411	**0.3621**	0.7218	0.7389	0.5420	3.0782	0.5312	0.6158	0.7088
48	**0.4743**	0.5066	0.5829	0.8742	0.7607	0.5065	4.6229	0.5088	0.5114	0.6457

We can see from the Table 3 that allowing VARMA structure for factors in second-type forecasting equations improves the forecasts of some key macroeconomic indicators across several horizons. In the case of Industrial production, Stock and Watson's diffusion index model performs the best for very short horizon of one month, while diagonal MA and final MA VARMA forms outperform other methods for horizons 2, 4 and 6 months. Finally, the ARMA model produce the smallest

RMSE for the long-term forecasts which is an interesting result for practitioners since the long-term forecasts are necessary for budget and fiscal planning. In the case of Employment three identified VARMA forms outperform all other standard factor-based models for short and mid-term horizons while ARMA still produce the smallest RMSE in long-term forecasting except for 2 and 4 years horizons where Direct and Unrestricted models produce best results.

On the nominal side, diffusion index model produces the best forecasts of CPI at horizon 1, the model where factors dynamics are modeled in final AR VARMA form works the best for horizons 2, 4 and 6. In long-term forecasts VARMA-based models perform the best for horizons 18, 24 and 48 months, while Sequential and Diffusion index models dominate for horizons 12 and 48.

Finally, the Unrestricted model is clearly the best suited to forecast the federal funds rate at short and mid-term horizons while VARMA-based model in Final MA form performs very well for horizons 12, 18 and 24 months.

Another interesting question that we addressed when presenting forecasting models is to compare forecasts obtained by ARMA model to factor-based predictions. This is motivated by the fact that if factor structure holds, each observable series has an ARMA representation. Hence, forecasting an observable series, for which factor structure holds, using a factor-based equation or an appropriated ARMA model should give the same forecasts in theory. However in practice there are estimation uncertainty and model misspecification that can make theoretically same models produce completely different forecasts. In Table 4 we present mean squared errors of all factor-based equations relative to ARMA model forecasts. The results in bold characters represent cases where ARMA model outperform the factor-based alternative in terms of MSE.

Table 4: RMSE relative to ARMA(p,q) forecasts

Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
Industrial production: total									
1	0.8975	0.8719	0.9235	0.9735	0.9735	0.9248	0.9298	0.9414	0.9263
2	**1.0463**	0.9912	**1.0080**	**1.0131**	**1.0637**	0.9050	0.9178	0.9088	0.9099
4	**1.2290**	**1.0722**	**1.0841**	**1.1031**	**1.2579**	0.9214	**1.0202**	0.9238	0.9764
6	**1.1591**	**1.0323**	**1.0330**	**1.1840**	**1.3640**	0.9324	**1.0581**	0.9278	0.9928
12	**1.2855**	**1.0754**	**1.0696**	**1.1256**	**1.4086**	**1.0304**	**1.2576**	**1.0373**	**1.0677**
18	**1.0602**	**1.0139**	**1.0300**	**1.0759**	**1.2622**	**1.0606**	**1.5235**	**1.0711**	**1.1308**
24	**1.0822**	**1.1066**	**1.1191**	**1.0546**	**1.1311**	**1.0264**	**1.8659**	**1.0370**	**1.1517**
36	**1.0271**	**1.1099**	**1.1640**	**1.2583**	**1.0606**	**1.0030**	**3.0841**	**1.0058**	**1.2004**
48	**1.0295**	**1.1272**	**1.1376**	**1.1225**	**1.0506**	**1.0066**	**5.6129**	**1.0067**	**1.1852**
Civilian labor force: employed. total									
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
1	0.7873	0.8415	0.8052	0.7814	0.7814	0.7626	0.7693	0.7648	0.7630
2	0.9026	0.9011	0.9001	0.9093	0.9203	0.8569	0.8448	0.8598	0.8494
4	0.9773	0.9746	0.9944	**1.0211**	**1.0558**	0.9219	0.9003	0.9206	0.8997
6	**1.0742**	**1.0765**	**1.0902**	**1.0680**	**1.1513**	0.9636	0.9496	0.9605	0.9425
12	**1.1298**	**1.1299**	**1.1278**	**1.0663**	**1.2258**	**1.0222**	**1.0426**	**1.0207**	**1.0054**
18	**1.0772**	**1.0789**	**1.0773**	**1.0014**	**1.1718**	**1.0362**	**1.1713**	**1.0386**	**1.0689**
24	**1.0348**	**1.0694**	**1.0379**	0.9745	**1.1121**	**1.0264**	**1.2945**	**1.0287**	**1.0973**
36	**1.0688**	**1.0159**	**1.0833**	**1.0614**	**1.0581**	**1.0126**	**1.7251**	**1.0096**	**1.1274**
48	0.9662	0.9671	0.9686	**1.0027**	**1.0460**	**1.0107**	**2.5154**	**1.0077**	**1.0976**
Consumer price index: all items									
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
1	0.8682	0.8577	0.8577	0.9098	0.9098	0.9015	0.9299	0.8731	0.8944
2	**1.0010**	**1.0087**	**1.0087**	0.9752	0.9872	0.9445	0.9565	0.9409	0.9304
4	**1.0520**	**1.0595**	**1.0595**	**1.0266**	**1.0040**	0.9880	**1.0529**	0.9769	0.9667
6	**1.0945**	**1.0936**	**1.0936**	0.9986	0.9746	0.9597	**1.0374**	0.9590	0.9579
12	**1.0851**	**1.0954**	**1.0954**	**1.0509**	0.9239	0.9296	**1.1613**	0.9277	**1.0108**
18	**1.0980**	**1.1052**	**1.1052**	**1.1106**	**1.0280**	0.9471	**1.1736**	0.9487	**1.0300**
24	**1.0953**	**1.2166**	**1.2166**	**1.0801**	**1.1334**	**1.0016**	**1.3459**	0.9984	**1.1031**
36	0.8870	0.8849	0.8849	**1.1451**	**1.0987**	0.9664	**1.6929**	0.9729	**1.0588**
48	**1.0179**	0.9973	0.9973	**1.0999**	**1.1204**	0.9743	**2.6601**	0.9793	**1.0415**
Federal funds rate									
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
1	0.9002	**8.4937**	**1.2179**	**2.7053**	**2.7053**	**1.8485**	**3.3781**	**1.7272**	**2.3129**
2	0.6651	**2.6883**	0.9494	**1.3967**	**1.4619**	**1.1288**	**2.2851**	**1.0609**	**1.5665**
4	0.5674	**1.1282**	0.7383	0.8433	0.8862	0.7775	**1.7299**	0.7221	**1.0459**
6	0.5911	0.7938	0.7123	0.7748	0.7323	0.6561	**1.5765**	0.6087	0.9183
12	0.5649	0.5675	0.6514	0.6155	0.6215	0.5523	**1.4987**	0.5108	0.7687
18	0.7200	0.7363	0.7386	0.7383	0.7830	0.6446	**2.0877**	0.5927	0.8585
24	0.7241	0.8756	0.7772	0.8196	0.8648	0.6865	**2.6209**	0.6433	0.8523
36	0.8176	0.7634	0.5109	**1.0183**	**1.0425**	0.7647	**4.3428**	0.7494	0.8688
48	0.7346	0.7846	0.9027	**1.3539**	**1.1781**	0.7844	**7.1595**	0.7880	0.7920

Results in Table 4 suggest several interesting points. In case of Industrial production, all factor-based models do better than ARMA at short term horizon of one month. For longer horizons, ARMA does much better than Unrestricted and Sequential models, and its performance improve with horizons, while the improvement is more moderate with respect to DI, DI-AR and Direct models. Compared to DFM-VARMA forecasts, ARMA model does better only after horizon 12.

For Employment, the conclusion is quite similar relative to DFM-VARMA, while the first-type models perform better than ARMA for horizons 1, 2, 4, and 48.

In case of Consumer price index, again all factor-based models perform better at horizon 1 but ARMA seems to be a better choice for most of horizons relatively to first-type models. Moreover, we can find an DFM-VARMA representation that outperforms ARMA model at all horizons. Finally, the picture is quite different in case of Federal funds rate where the Unrestricted forecasts outperform completely ARMA model and the latter beats all other factor-based models at short horizons of one and two months only.

Overall, based on these results we can say that ARMA model is a very good alternative for standard factor-based models in forecasting key macroeconomic indicators, especially for long-term horizons in case of real-activity variables. This is not very surprising since ARMA model are very parsimonious. However, is outperformed in most of cases by our DFM-VARMA specifications.

We just discussed the forecasting performance of all alternative models and concluded that second-type forecasting models with VARMA structure perform generally the best in this particular exercise. But it is also of interest to see more directly how approximating factors' process by a VARMA process compares to usual forecasting model where the factors' dynamics is fitted by VAR process. In Table 5 we present MSE values of VARMA-based forecasting models relative to Direct and Sequential second-type model. The numbers in bold character present cases where VARMA-based model performs better that the VAR alternative.

Table 5: MSE of VARMA-based models relative to VAR-based forecasting factor model

	Industrial production: total							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9500**	**0.9551**	**0.9671**	**0.9515**	**0.9500**	**0.9551**	**0.9671**	**0.9515**
2	**0.8934**	**0.9060**	**0.8971**	**0.8982**	**0.8508**	**0.8628**	**0.8544**	**0.8554**
4	**0.8353**	**0.9248**	**0.8375**	**0.8852**	**0.7325**	**0.8110**	**0.7344**	**0.7762**
6	**0.7875**	**0.8936**	**0.7836**	**0.8385**	**0.6836**	**0.7757**	**0.6802**	**0.7279**
12	**0.9154**	1.1173	**0.9215**	**0.9486**	**0.7315**	**0.8928**	**0.7364**	**0.7580**
18	**0.9858**	1.4161	**0.9956**	1.0510	**0.8403**	1.2071	**0.8487**	**0.8959**
24	**0.9733**	1.7694	**0.9833**	1.0921	**0.9075**	1.6496	**0.9168**	1.0182
36	**0.7971**	2.4510	**0.7993**	**0.9540**	**0.9457**	2.9080	**0.9484**	1.1318
48	**0.8968**	5.0005	**0.8969**	1.0559	**0.9581**	5.3424	**0.9582**	1.1281
	Civilian labor force: employed. total							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9759**	**0.9845**	**0.9787**	**0.9763**	**0.9759**	**0.9845**	**0.9787**	**0.9763**
2	**0.9424**	**0.9291**	**0.9456**	**0.9341**	**0.9312**	**0.9180**	**0.9343**	**0.9229**
4	**0.9029**	**0.8817**	**0.9016**	**0.8811**	**0.8732**	**0.8527**	**0.8720**	**0.8521**
6	**0.9023**	**0.8891**	**0.8994**	**0.8825**	**0.8370**	**0.8248**	**0.8343**	**0.8186**
12	**0.9587**	**0.9778**	**0.9573**	**0.9429**	**0.8339**	**0.8505**	**0.8327**	**0.8201**
18	1.0347	1.1696	1.0371	1.0674	**0.8843**	**0.9996**	**0.8863**	**0.9122**
24	1.0532	1.3283	1.0556	1.1260	**0.9229**	1.1640	**0.9250**	**0.9867**
36	**0.9540**	1.6253	**0.9512**	1.0622	**0.9571**	1.6305	**0.9542**	1.0655
48	1.0079	2.5086	1.0050	1.0946	**0.9663**	2.4048	**0.9635**	1.0494
	Consumer price index: all items							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9909**	1.0221	**0.9597**	0.9831	**0.9909**	1.0221	**0.9597**	**0.9831**
2	**0.9685**	**0.9808**	**0.9648**	0.9540	**0.9567**	**0.9689**	**0.9531**	**0.9424**
4	**0.9624**	1.0257	**0.9516**	0.9417	**0.9840**	1.0487	**0.9730**	**0.9628**
6	**0.9611**	1.0389	**0.9604**	0.9593	**0.9847**	1.0644	**0.9840**	**0.9829**
12	**0.8846**	1.1051	**0.8828**	0.9618	1.0062	1.2570	1.0042	1.0941
18	**0.8527**	1.0567	**0.8542**	0.9274	**0.9213**	1.1416	**0.9228**	1.0020
24	**0.9273**	1.2462	**0.9244**	1.0213	**0.8837**	1.1875	**0.8809**	**0.9732**
36	**0.8439**	1.4783	**0.8495**	0.9246	**0.8795**	1.5408	**0.8854**	**0.9637**
48	**0.8858**	2.4185	**0.8904**	0.9469	**0.8695**	2.3742	**0.8741**	**0.9295**
	Federal funds rate							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.6833**	1.2487	**0.6385**	**0.8549**	**0.6833**	1.2487	**0.6385**	**0.8549**
2	**0.8082**	1.6361	**0.7596**	1.1216	**0.7721**	1.5631	**0.7257**	1.0716
4	**0.9220**	2.0514	**0.8563**	1.2402	**0.8773**	1.9521	**0.8149**	1.1802
6	**0.8468**	2.0346	**0.7856**	1.1851	**0.8959**	2.1527	**0.8312**	1.2539
12	**0.8973**	2.4349	**0.8300**	1.2489	**0.8886**	2.4112	**0.8219**	1.2368
18	**0.8730**	2.8278	**0.8028**	1.1629	**0.8232**	2.6662	**0.7569**	1.0964
24	**0.8377**	3.1978	**0.7849**	1.0400	**0.7939**	3.0306	**0.7439**	**0.9856**
36	**0.7509**	4.2646	**0.7359**	**0.8531**	**0.7335**	4.1659	**0.7189**	**0.8334**
48	**0.5794**	5.2881	**0.5820**	**0.5850**	**0.6658**	6.0772	**0.6689**	**0.6723**

The most of results in Table 5 are in bold character implying that DFM-VARMA models outperform the standard DFM-VAR specification for most of horizons and identified VARMA forms.

This is especially the case for Industrial production where all VARMA forms produce smaller MSE than VAR-based forecasts and the improvements seem to be more important with mid-term horizons and relative to Direct model. In the case of Civilian labor force, considering VARMA helps again in predicting but the gain is less important and VAR-based model with multi-step forecasts perform even better for long-term horizons, while the iterative VAR-based model is outperformed by DFM-VARMA models in MA forms.

On the nominal side, the DFM-VARMA models in both MA forms seem to perform better that the VAR-based alternatives in predicting CPI, and this improvement rises with forecast horizons. Finally, the same VARMA forms perform quite well relatively to both Direct and Sequential models in case of Federal Funds Rate, and again the gain rises with forecast horizon.

9.2. **Number of factors in second-type forecasting models**

The results in Table 1 are obtained by fixing the number of factors in the second-type forecasting equations at 4 for all evaluation periods. In contrast, the number of factors can vary over time and is estimated in the first-type forecasting equations. There exists a list of criteria to estimate the number of static factors but their success in practice is mitigated.

Another way to vary the number of factors in forecasting equations of the second type is to set it equal to one of the estimates in the first-type models, but the question remains: to which one? Generally, the estimated number of factors included in "Unrestricted" model is smaller than in "DI" since in the former the information contained in lags of dependent variable is important enough to force the information criterion to pick a smaller number of factors.

On the other hand, when forecasting factors alone we use information contained in the lags of each factor in the case of VAR, and/or innovations in the case of identified VARMA forms. Hence, with respect to parsimony principle, a smaller number of factors would be more appropriate if the dynamic relations between factors are important (higher number of lags) while we should include more factors if there is no persistent relations between them and then explain more of the variance in observable series. In our robustness analysis we did the same forecasting exercise as in previous section but where we the number of factors used in second-type models was the same as in one of the first-type model. The overall forecasting performance results from these exercises produce quite similar picture to the one in Tables 1 and 3. This is not surprising with respect to VARMA-based models since the VARMA models are closed to marginalization.

9.3. **Main empirical results with Canadian data**

In application with US data we had a situation where the time size of informational panel is much bigger than the cross-section size, i.e. 600 time periods versus 128 series. Using Canadian data from Boivin, Giannoni and Stevanović (2008) we perform the same pseudo-out-of-sample forecasting exercise as in previous sections. In this data set there 332 economic indicators measured from 1981 to 2008, which gives a time size of 334. The evaluation period is 1998-2008.

Table 6: RMSE relative to Direct AR(p) forecasts

Employment										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	1.0221	1.0165	1.0920	0.9854	0.9854	**0.9410**	0.9854	0.9601	1.0362	1.0151
2	0.9874	0.9751	0.9457	0.9998	0.9920	**0.9059**	0.9920	0.9236	1.0597	1.0092
4	1.0604	1.0865	1.1204	0.9783	0.9399	0.9298	0.9399	**0.9221**	1.0503	1.0060
6	1.1928	1.1408	1.1667	1.1130	0.9760	0.9641	0.9760	**0.9286**	1.0615	1.0011
12	**0.9822**	1.1197	1.2073	1.0402	0.9914	1.0194	0.9914	0.9938	1.0889	1.0760
18	1.2135	1.5923	1.6208	1.3230	**0.9792**	1.0282	1.0740	0.9845	1.1923	1.1054
24	1.3133	1.9476	1.9595	1.1989	**0.9803**	1.0290	1.0022	0.9819	1.1401	1.0937
36	1.7336	2.1289	2.2198	1.5687	0.9201	0.9395	0.9441	**0.9190**	1.0639	1.0442
48	1.7698	1.5115	1.2833	1.7333	0.9788	0.9734	0.9905	**0.9608**	1.0926	1.0829
Consumer price index: all items										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	0.8779	**0.8501**	0.8567	0.9146	0.9146	0.8563	0.9130	0.8647	0.9512	0.8811
2	0.9028	**0.8720**	0.8790	0.9946	0.9804	0.8895	0.9804	0.9040	0.9798	0.9226
4	0.9139	0.9082	0.9000	0.9737	0.9328	0.8826	0.9328	**0.8816**	0.9430	0.9069
6	0.8800	0.8701	0.8811	0.9307	0.8853	0.8403	0.8853	**0.8399**	0.8900	0.9062
12	0.9921	1.0585	1.0140	1.0178	0.9845	0.9318	0.9845	**0.9070**	1.0255	1.0207
18	1.0114	1.0143	**1.0083**	1.0362	1.0138	1.0504	1.0847	1.0130	1.0368	1.1184
24	0.9810	1.0563	1.0743	0.9671	0.9460	0.9655	0.9938	**0.9508**	1.0340	1.0804
36	**0.9844**	1.1165	1.1126	1.0140	1.0179	1.0325	1.0309	1.0160	1.1187	1.1287
48	**0.9919**	1.3307	1.3174	1.0908	1.0550	1.0415	1.0318	1.0554	1.1554	1.1832
Producer price index: all manufacturing										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	1.0079	1.0035	1.0094	1.0097	1.0097	0.9985	1.0070	1.0175	1.0443	**0.9931**
2	1.0088	0.9732	0.9835	1.0317	1.0077	0.9852	1.0077	0.9874	1.0499	**0.9729**
4	0.9841	1.0255	1.0280	1.0115	0.9810	0.9986	0.9810	0.9852	1.0483	**0.9803**
6	0.9759	1.0083	1.0103	0.9885	0.9701	0.9830	0.9701	0.9781	0.9958	**0.9580**
12	1.0246	1.0274	1.0294	1.0183	1.0142	**0.9916**	1.0142	0.9942	0.9973	1.0123
18	**0.9740**	0.9998	1.0026	0.9905	0.9828	0.9789	0.9837	0.9815	0.9894	0.9842
24	**0.9927**	1.0204	1.0230	1.0159	1.0027	0.9956	1.0018	0.9981	0.9984	1.0040
36	1.0363	1.0763	1.0947	0.9850	0.9831	0.9790	0.9814	0.9804	**0.9755**	0.9842
48	**0.9890**	1.0761	1.0632	0.9927	1.0108	1.0032	1.0050	1.0110	0.9969	1.0143

From the results in Table 6 we obtain quite similar conclusions as in the case of US data set that VARMA-based forecasting models perform better for the most of horizons. Hence, modeling factors' dynamics as VARMA processes can help in forecasting economic series even in the case where the cross-section and time size are of same magnitude.

In Table 7 we redo the same exercise as in Table 4, but for Canadian application.

Table 7: MSE of VARMA-based models relative to VAR-based forecasting factor model

	Employment							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9500**	**0.9551**	**0.9671**	**0.9515**	**0.9500**	**0.9551**	**0.9671**	**0.9515**
2	**0.8934**	**0.9060**	**0.8971**	**0.8982**	**0.8508**	**0.8628**	**0.8544**	**0.8554**
4	**0.8353**	**0.9248**	**0.8375**	**0.8852**	**0.7325**	**0.8110**	**0.7344**	**0.7762**
6	**0.7875**	**0.8936**	**0.7836**	**0.8385**	**0.6836**	**0.7757**	**0.6802**	**0.7279**
12	**0.9154**	1.1173	**0.9215**	**0.9486**	**0.7315**	**0.8928**	**0.7364**	**0.7580**
18	**0.9858**	1.4161	**0.9956**	1.0510	**0.8403**	1.2071	**0.8487**	**0.8959**
24	**0.9733**	1.7694	**0.9833**	1.0921	**0.9075**	1.6496	**0.9168**	1.0182
36	**0.7971**	2.4510	**0.7993**	**0.9540**	**0.9457**	2.9080	**0.9484**	1.1318
48	**0.8968**	5.0005	**0.8969**	1.0559	**0.9581**	5.3424	**0.9582**	1.1281
	Consumer price index: all items							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9759**	**0.9845**	**0.9787**	**0.9763**	**0.9759**	**0.9845**	**0.9787**	**0.9763**
2	**0.9424**	**0.9291**	**0.9456**	**0.9341**	**0.9312**	**0.9180**	**0.9343**	**0.9229**
4	**0.9029**	**0.8817**	**0.9016**	**0.8811**	**0.8732**	**0.8527**	**0.8720**	**0.8521**
6	**0.9023**	**0.8891**	**0.8994**	**0.8825**	**0.8370**	**0.8248**	**0.8343**	**0.8186**
12	**0.9587**	**0.9778**	**0.9573**	**0.9429**	**0.8339**	**0.8505**	**0.8327**	**0.8201**
18	1.0347	1.1696	1.0371	1.0674	**0.8843**	**0.9996**	**0.8863**	**0.9122**
24	1.0532	1.3283	1.0556	1.1260	**0.9229**	1.1640	**0.9250**	**0.9867**
36	**0.9540**	1.6253	**0.9512**	1.0622	**0.9571**	1.6305	**0.9542**	1.0655
48	1.0079	2.5086	1.0050	1.0946	**0.9663**	2.4048	**0.9635**	1.0494
	Producer price index: all manufacturing							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9909**	1.0221	**0.9597**	**0.9831**	**0.9909**	1.0221	**0.9597**	**0.9831**
2	**0.9685**	**0.9808**	**0.9648**	**0.9540**	**0.9567**	**0.9689**	**0.9531**	**0.9424**
4	**0.9624**	1.0257	**0.9516**	**0.9417**	**0.9840**	1.0487	**0.9730**	**0.9628**
6	**0.9611**	1.0389	**0.9604**	**0.9593**	**0.9847**	1.0644	**0.9840**	**0.9829**
12	**0.8846**	1.1051	**0.8828**	**0.9618**	1.0062	1.2570	1.0042	1.0941
18	**0.8527**	1.0567	**0.8542**	**0.9274**	**0.9213**	1.1416	**0.9228**	1.0020
24	**0.9273**	1.2462	**0.9244**	1.0213	**0.8837**	1.1875	**0.8809**	**0.9732**
36	**0.8439**	1.4783	**0.8495**	**0.9246**	**0.8795**	1.5408	**0.8854**	**0.9637**
48	**0.8858**	2.4185	**0.8904**	**0.9469**	**0.8695**	2.3742	**0.8741**	**0.9295**

As in the case of US data, we find that DFM-VARMA models generally outperform VAR-based factor forecasting models, and two MA forms seem to be the best choices.

10. **Conclusion**

In this paper we proposed a dynamic factor model were factors have VARMA dynamics (DFM-VARMA) which can be seen as a generalization of the existing approximate factor model studied by Stock and Watson (2005). We first emphasized results on linear transformations of vector stochastic processes and then showed in Theorem 4.2 that when factors are obtained as linear combinations of observable series their dynamic process is generally a VARMA and not a VAR that is usually assumed in the literature. Moreover, this generalization can be motivated by the usual arguments of parsimony, invertibility and marginalization issues in which VARMA models outperform the VAR representations (see Lütkepohl (1987) for details). Hence, assuming finite order VAR structure can be miss-leading in finite sample if the MA component of the true process is important or we do not use the true number of factors.

We also showed in Theorem 4.1 that a factor structure, where factors have VAR or VARMA representation, implies a VARMA representation on observable series. Hence, each variable in the observable data set have ARMA dynamics meaning that in a forecasting performance comparison exercise the ARMA model should be considered as alternative to factor-based forecasting models.

In order to illustrate the performance of our approach we performed a series of Monte Carlo simulations and found that VARMA specifications help a lot especially in small sample cases where the best improvement were at longer horizons, but also in cases where the sample sizes are comparable to our empirical exercise

To demonstrate the importance of modeling factors dynamics as VARMA processes in real world we conducted a pseudo-out-of-sample forecasting exercise using Canadian and US large data sets and find that DFM-VARMA models help in predicting several key macroeconomic aggregates relatively to standard factor-based forecasting models, both using projection models and those based on VAR factors' structure. Moreover, we find that univariate ARMA model is also a very good alternative, especially at long forecasting horizons.

Appendix

The data used in our empirical application are presented in this appendix. US data are taken from Boivin, Giannoni and Stevanovic (2009), while the Canadian data are from Boivin, Giannoni and Stevanovic (2008). The transformation codes (labeled T-Code) are: 1 - no transformation; 2 - first difference; 4 - logarithm; 5 - first difference of logarithm.

US Data

No.	Series Code	T-Code	Series Description
			Real output and income
1	IPS10	5	INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX
2	IPS11	5	INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL
3	IPS12	5	INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS
4	IPS13	5	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
5	IPS14	5	INDUSTRIAL PRODUCTION INDEX - AUTOMOTIVE PRODUCTS
6	IPS18	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
7	IPS25	5	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
8	IPS29	5	INDUSTRIAL PRODUCTION INDEX - DEFENSE AND SPACE EQUIPMENT
9	IPS299	5	INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS
10	IPS306	5	INDUSTRIAL PRODUCTION INDEX - FUELS
11	IPS32	5	INDUSTRIAL PRODUCTION INDEX - MATERIALS
12	IPS34	5	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
13	IPS38	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
14	IPS43	5	INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)
15	PMP	1	NAPM PRODUCTION INDEX (PERCENT)
16	PMI	1	PURCHASING MANAGERS' INDEX (SA)
17	UTL11	1	CAPACITY UTILIZATION - MANUFACTURING (SIC)
18	YPR	5	PERS INCOME CH 2000 $,SA-US
19	YPDR	5	DISP PERS INCOME,BILLIONS OF CH (2000) $,SAAR-US
20	YP@V00C	5	PERS INCOME LESS TRSF PMT CH 2000 $,SA-US
21	SAVPER	2	PERS SAVING,BILLIONS OF $,SAAR-US
22	SAVPRATE	1	PERS SAVING AS PERCENTAGE OF DISP PERS INCOME,PERCENT,SAAR-US
			Employment and hours
23	LHEL	5	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
24	LHELX	4	EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF
25	LHEM	5	CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)
26	LHNAG	5	CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)
27	LHTUR	1	UNEMPLOYMENT RATE: BOTH SEXES, 16-19 YEARS (%,SA)
28	LHU14	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
29	LHU15	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
30	LHU26	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
31	LHU27	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)
32	LHU5	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
33	LHU680	1	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
34	LHUEM	5	CIVILIAN LABOR FORCE: UNEMPLOYED, TOTAL (THOUS.,SA)
35	AHPCON	1	AVG HR EARNINGS OF PROD WKRS: CONSTRUCTION ($,SA)
36	AHPMF	1	AVG HR EARNINGS OF PROD WKRS: MANUFACTURING ($,SA)
37	PMEMP	1	NAPM EMPLOYMENT INDEX (PERCENT)
38	CES002	5	EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE
39	CES003	5	EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING
40	CES004	5	EMPLOYEES ON NONFARM PAYROLLS - NATURAL RESOURCES AND MINING
41	CES011	5	EMPLOYEES ON NONFARM PAYROLLS - CONSTRUCTION
42	CES015	5	EMPLOYEES ON NONFARM PAYROLLS - MANUFACTURING
43	CES017	5	EMPLOYEES ON NONFARM PAYROLLS - DURABLE GOODS
44	CES033	5	EMPLOYEES ON NONFARM PAYROLLS - NONDURABLE GOODS
45	CES046	5	EMPLOYEES ON NONFARM PAYROLLS - SERVICE-PROVIDING
46	CES048	5	EMPLOYEES ON NONFARM PAYROLLS - TRADE, TRANSPORTATION, AND UTILITIES
47	CES049	5	EMPLOYEES ON NONFARM PAYROLLS - WHOLESALE TRADE
48	CES053	5	EMPLOYEES ON NONFARM PAYROLLS - RETAIL TRADE
49	CES088	5	EMPLOYEES ON NONFARM PAYROLLS - FINANCIAL ACTIVITIES
50	CES140	5	EMPLOYEES ON NONFARM PAYROLLS - GOVERNMENT
51	CES151	1	AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - GOODS-PRODUCING
52	CES153	1	AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - CONSTRUCTION
53	CES154	1	AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACTURING
54	CES155	1	AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACT. OVERTIME HOURS
55	CES156	1	AVG WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - DURABLE GOODS
56	CES275	5	AVG HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - GOODS-PRODUCING
57	CES277	5	AVG HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - CONSTRUCTION
58	CES278	5	AVG HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACTURING
			Real Consumption
59	JQCR	5	REAL PERSONAL CONS EXP QUANTITY INDEX (200=100), SAAR
60	JQCNR	5	REAL PERSONAL CONS EXP-NONDURABLE GOODS QUANTITY INDEX (200=100), SAAR
61	JQCDR	5	REAL PERSONAL CONS EXP-DURABLE GOODS QUANTITY INDEX (200=100), SAAR
62	JQCSVR	5	REAL PERSONAL CONS EXP-SERVICES QUANTITY INDEX (200=100), SAAR
			Real inventories and orders
63	MOCMQ	5	NEW ORDERS (NET) - CONSUMER GOODS and MATERIALS, 1996 DOLLARS (BCI)
64	MSONDQ	5	NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1996 DOLLARS (BCI)
65	PMDEL	1	NAPM VENDOR DELIVERIES INDEX (PERCENT)
66	PMNO	1	NAPM NEW ORDERS INDEX (PERCENT)
67	PMNV	1	NAPM INVENTORIES INDEX (PERCENT)

			Housing starts
68	XMTOSA	4	RESIDENTIAL CONSTRUCTION PRIVATE HOUSING UNITS STARTED: TOTAL UNITS (THOUS.,SAAR)
69	HUSTSZ	4	HOUSING STARTS: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)
70	HSFR	4	HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA
71	HSMW	4	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
72	HSNE	4	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
73	HSSOU	4	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
74	HSWST	4	HOUSING STARTS:WEST (THOUS.U.)S.A.
			Exchange rates
75	EXRCAN	5	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)
76	EXRUK	5	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
77	EXRUS	5	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
			Price indexes
78	PMCP	1	NAPM COMMODITY PRICES INDEX (PERCENT)
79	PW561	5	PRODUCER PRICE INDEX: CRUDE PETROLEUM (82=100,NSA)
80	PWCMSA	5	PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)
81	PWFCSA	5	PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)
82	PWFSA	5	PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
83	PWIMSA	5	PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)
84	PUNEW	5	CPI-U: ALL ITEMS (82-84=100,SA)
85	PUS	5	CPI-U: SERVICES (82-84=100,SA)
86	PUXF	5	CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)
87	PUXHS	5	CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)
88	PUXM	5	CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100,SA)
89	PUXX	5	CPI-U: ALL ITEMS LESS FOOD AND ENERGY (82-84=100,SA)
90	PUC	5	CPI-U: COMMODITIES (82-84=100,SA)
91	PUCD	5	CPI-U: DURABLES (82-84=100,SA)
92	PU83	5	CPI-U: APPAREL & UPKEEP (82-84=100,SA)
93	PU84	5	CPI-U: TRANSPORTATION (82-84=100,SA)
94	PU85	5	CPI-U: MEDICAL CARE (82-84=100,SA)
			Stock prices
95	FSDJ	5	COMMON STOCK PRICES: DOW JONES INDUSTRIAL AVERAGE
96	FSDXP	1	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)
97	FSPCOM	5	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
98	FSPIN	5	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
99	FSPXE	1	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)
			Money and credit quantity aggregates
100	FM1	5	MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)
101	FM2	5	MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$,
102	FMFBA	5	MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)
103	FMRNBA	2	DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA)
104	FMRRA	5	DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)
105	CCINRV	5	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)
			Miscellaneous
106	UOMO83	1	COMPOSITE INDEXES LEADING INDEX COMPONENT INDEX OF CONSUMER EXPECTATIONS UNITS: 1966.1=100 NSA, CONFBOARD AND U.MICH.
			Interest rates and bonds
107	FYGM3	1	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)
108	FYGM6	1	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)
109	FYGT1	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
110	FYGT10	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
111	FYGT20	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,20-YR.(% PER ANN,NSA)
112	FYGT3	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,3-YR.(% PER ANN,NSA)
113	FYGT5	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)
114	FYPR	1	PRIME RATE CHG BY BANKS ON SHORT-TERM BUSINESS LOANS(% PER ANN,NSA)
115	FYAAAC	1	BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
116	FYAAAM	1	BOND YIELD: MOODY'S AAA MUNICIPAL (% PER ANNUM)
117	FYAC	1	BOND YIELD: MOODY'S A CORPORATE (% PER ANNUM,NSA)
118	FYAVG	1	BOND YIELD: MOODY'S AVERAGE CORPORATE (% PER ANNUM)
119	FYBAAC	1	BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
120	SFYGM3	1	FYGM3-FYFF
121	SFYGM6	1	FYGM6-FYFF
122	SFYGT1	1	FYGT1-FYFF
123	SFYGT5	1	FYGT5-FYFF
124	SFYGT10	1	FYGT10-FYFF
125	SFYAAAC	1	FYAAAC-FYFF
126	SFYBAAC	1	FYBAAC-FYFF
127	FYFF	1	INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)
128	Bspread10Y	1	FYBAAC-FYGT10

Canadian Data

No.	StatCan no	Code	Series category
			Table 326-0020 Consumer Price Index Canada, Provinces
1	v41690973	**5**	All-items (2002=100)
2	v41690974	**5**	Food (2002=100)
3	v41690993	**5**	Dairy products (2002=100)
4	v41691046	**5**	Food purchased from restaurants (2002=100)
5	v41691051	**5**	Rented accommodation (2002=100)
6	v41691055	**5**	Owned accommodation (2002=100)
7	v41691065	**5**	Natural gas (2002=100)
8	v41691066	**5**	Fuel oil and other fuels (2002=100)
9	v41691108	**5**	Clothing and footwear (2002=100)
10	v41691129	**5**	Private transportation (2002=100)
11	v41691153	**5**	Health and personal care (2002=100)
12	v41691170	**5**	Recreation, education and reading (2002=100)
13	v41692942	**5**	All-items excluding eight of the most volatile components (Bank of Canada definition) (2002=100)
14	v41691232	**5**	All-items excluding food (2002=100)
15	v41691233	**5**	All-items excluding food and energy (2002=100)
16	v41691238	**5**	All-items excluding energy (2002=100)
17	v41691237	**5**	Food and energy (2002=100)
18	v41691239	**5**	Energy (2002=100)
19	v41691219	**5**	Housing (1986 definition) (2002=100)
20	v41691222	**5**	Goods (2002=100)
21	v41691223	**5**	Durable goods (2002=100)
22	v41691225	**5**	Non-durable goods (2002=100)
23	v41691229	**5**	Goods excluding food purchased from stores and energy (2002=100)
24	v41691230	**5**	Services (2002=100)
25	v41691231	**5**	Services excluding shelter services (2002=100)
26	v41691244	**5**	Newfoundland and Labrador; All-items (2002=100)
27	v41691369	**5**	Newfoundland and Labrador; All-items excluding food and energy (2002=100)
28	v41691363	**5**	Newfoundland and Labrador; Goods (2002=100)
29	v41691367	**5**	Newfoundland and Labrador; Services (2002=100)
30	v41691379	**5**	Prince Edward Island; All-items (2002=100)
31	v41691503	**5**	Prince Edward Island; All-items excluding food and energy (2002=100)
32	v41691497	**5**	Prince Edward Island; Goods (2002=100)
33	v41691501	**5**	Prince Edward Island; Services (2002=100)
34	v41691513	**5**	Nova Scotia; All-items (2002=100)
35	v41691638	**5**	Nova Scotia; All-items excluding food and energy (2002=100)
36	v41691632	**5**	Nova Scotia; Goods (2002=100)
37	v41691636	**5**	Nova Scotia; Services (2002=100)
38	v41691648	**5**	New Brunswick; All-items (2002=100)
39	v41691773	**5**	New Brunswick; All-items excluding food and energy (2002=100)
40	v41691767	**5**	New Brunswick; Goods (2002=100)
41	v41691771	**5**	New Brunswick; Services (2002=100)
42	v41691783	**5**	Quebec; All-items (2002=100)
43	v41691909	**5**	Quebec; All-items excluding food and energy (2002=100)
44	v41691903	**5**	Quebec; Goods (2002=100)
45	v41691907	**5**	Quebec; Services (2002=100)
46	v41691919	**5**	Ontario; All-items (2002=100)
47	v41692045	**5**	Ontario; All-items excluding food and energy (2002=100)
48	v41692039	**5**	Ontario; Goods (2002=100)
49	v41692043	**5**	Ontario; Services (2002=100)
50	v41692055	**5**	Manitoba; All-items (2002=100)
51	v41692181	**5**	Manitoba; All-items excluding food and energy (2002=100)
52	v41692175	**5**	Manitoba; Goods (2002=100)
53	v41692179	**5**	Manitoba; Services (2002=100)
54	v41692191	**5**	Saskatchewan; All-items (2002=100)
55	v41692317	**5**	Saskatchewan; All-items excluding food and energy (2002=100)
56	v41692311	**5**	Saskatchewan; Goods (2002=100)
57	v41692315	**5**	Saskatchewan; Services (2002=100)
58	v41692327	**5**	Alberta; All-items (2002=100)
59	v41692452	**5**	Alberta; All-items excluding food and energy (2002=100)
60	v41692446	**5**	Alberta; Goods (2002=100)
61	v41692450	**5**	Alberta; Services (2002=100)
62	v41692462	**5**	British Columbia; All-items (2002=100)
63	v41692588	**5**	British Columbia; All-items excluding food and energy (2002=100)
64	v41692582	**5**	British Columbia; Goods (2002=100)
65	v41692586	**5**	British Columbia; Services (2002=100)
			Table 026-0001 Building permits, residential values and number of units
66	v14098	**1**	Canada; Total dwellings (number of units) [D848383]
67	v41651	**1**	Canada; Total dwellings (dollars - thousands) [D845521]
68	v13824	**1**	Newfoundland and Labrador; Total dwellings (number of units) [D847651]
69	v41560	**1**	Newfoundland and Labrador; Total dwellings (dollars - thousands) [D845363]
70	v13859	**1**	Prince Edward Island; Total dwellings (number of units) [D847658]
71	v41595	**1**	Prince Edward Island; Total dwellings (dollars - thousands) [D845370]
72	v13866	**1**	Nova Scotia; Total dwellings (number of units) [D847665]
73	v41602	**1**	Nova Scotia; Total dwellings (dollars - thousands) [D845377]
74	v13873	**1**	New Brunswick; Total dwellings (number of units) [D847672]
75	v41609	**1**	New Brunswick; Total dwellings (dollars - thousands) [D845384]
76	v13880	**1**	Quebec; Total dwellings (number of units) [D847679]
77	v41616	**1**	Quebec; Total dwellings (dollars - thousands) [D845391]
78	v13887	**1**	Ontario; Total dwellings (number of units) [D847686]
79	v41623	**1**	Ontario; Total dwellings (dollars - thousands) [D845398]
80	v13894	**1**	Manitoba; Total dwellings (number of units) [D847693]

81	v41630	**1**	Manitoba; Total dwellings (dollars - thousands) [D845405]
82	v13901	**1**	Saskatchewan; Total dwellings (number of units) [D847700]
83	v41637	**1**	Saskatchewan; Total dwellings (dollars - thousands) [D845412]
84	v13908	**1**	Alberta; Total dwellings (number of units) [D847707]
85	v41644	**1**	Alberta; Total dwellings (dollars - thousands) [D845419]
86	v13831	**1**	British Columbia; Total dwellings (number of units) [D847714]
87	v41567	**1**	British Columbia; Total dwellings (dollars - thousands) [D845426]
			Table 027-0002 CMHC, housing starts, under constr and completions, SA
88	v730040	**1**	Canada; Total units (units - thousands) [J9001]
89	v729972	**1**	Newfoundland and Labrador; Total units (units - thousands) [J7002]
90	v729973	**1**	Prince Edward Island; Total units (units - thousands) [J7003]
91	v729974	**1**	Nova Scotia; Total units (units - thousands) [J7004]
92	v729975	**1**	New Brunswick; Total units (units - thousands) [J7005]
93	v729976	**1**	Quebec; Total units (units - thousands) [J7006]
94	v729981	**1**	Ontario; Total units (units - thousands) [J7008]
95	v729987	**1**	Manitoba; Total units (units - thousands) [J7011]
96	v729988	**1**	Saskatchewan; Total units (units - thousands) [J7012]
97	v729989	**1**	Alberta; Total units (units - thousands) [J7013]
98	v729990	**1**	British Columbia; Total units (units - thousands) [J7014]
			Table 377-0003 Business leading indicators for Canada
99	v7677	**1**	Average work week, manufacturing; Smoothed (hours) [D100042]
100	v7680	**1**	Housing index; Smoothed (index, 1992=100) [D100043]
101	v7681	**5**	United States composite leading index; Smoothed (index, 1992=100) [D100044]
102	v7682	**5**	Money supply; Smoothed (dollars, 1992 - millions) [D100045]
103	v7683	**5**	New orders, durable goods; Smoothed (dollars, 1992 - millions) [D100046]
104	v7684	**5**	Retail trade, furniture and appliances; Smoothed (dollars, 1992 - millions) [D100047]
105	v7686	**1**	Shipment to inventory ratio, finished products; Smoothed (ratio) [D100049]
106	v7678	**5**	Stock price index, TSE 300; Smoothed (index, 1975=1000) [D100050]
107	v7679	**5**	Business and personal services employment; Smoothed (persons - thousands) [D100051]
108	v7688	**5**	Composite index of 10 indicators; Smoothed (index, 1992=100) [D100053]
			Table 379-0027 GDP at basic prices, by NAICS, Canada, SA, 2002 constant prices
109	v41881478	**5**	All industries [T001] (dollars - millions)
110	v41881480	**5**	Business sector, goods [T003] (dollars - millions)
111	v41881481	**5**	Business sector, services [T004] (dollars - millions)
112	v41881482	**5**	Non-business sector industries [T005] (dollars - millions)
113	v41881485	**5**	Goods-producing industries [T008] (dollars - millions)
114	v41881486	**5**	Service-producing industries [T009] (dollars - millions)
115	v41881487	**5**	Industrial production [T010] (dollars - millions)
116	v41881488	**5**	Non-durable manufacturing industries [T011] (dollars - millions)
117	v41881489	**5**	Durable manufacturing industries [T012] (dollars - millions)
118	v41881494	**5**	Agriculture, forestry, fishing and hunting [11] (dollars - millions)
119	v41881501	**5**	Mining and oil and gas extraction [21] (dollars - millions)
120	v41881524	**5**	Residential building construction [230A] (dollars - millions)
121	v41881525	**5**	Non-residential building construction [230B] (dollars - millions)
122	v41881527	**5**	Manufacturing [31-33] (dollars - millions)
123	v41881555	**5**	Wood product manufacturing [321] (dollars - millions)
124	v41881564	**5**	Paper manufacturing [322] (dollars - millions)
125	v41881602	**5**	Rubber product manufacturing [3262] (dollars - millions)
126	v41881606	**5**	Non-metallic mineral product manufacturing [327] (dollars - millions)
127	v41881637	**5**	Machinery manufacturing [333] (dollars - millions)
128	v41881654	**5**	Electrical equipment, appliance and component manufacturing [335] (dollars - millions)
129	v41881662	**5**	Transportation equipment manufacturing [336] (dollars - millions)
130	v41881663	**5**	Motor vehicle manufacturing [3361] (dollars - millions)
131	v41881674	**5**	Aerospace product and parts manufacturing [3364] (dollars - millions)
132	v41881675	**5**	Railroad rolling stock manufacturing [3365] (dollars - millions)
133	v41881688	**5**	Wholesale trade [41] (dollars - millions)
134	v41881689	**5**	Retail trade [44-45] (dollars - millions)
135	v41881690	**5**	Transportation and warehousing [48-49] (dollars - millions)
136	v41881699	**5**	Pipeline transportation [486] (dollars - millions)
137	v41881724	**5**	Finance, insurance, real estate, rental and leasing and management of companies and enterprises [5A] (dollars - millions)
138	v41881756	**5**	Educational services [61] (dollars - millions)
139	v41881759	**5**	Health care and social assistance [62] (dollars - millions)
140	v41881776	**5**	Federal government public administration [911] (dollars - millions)
141	v41881777	**5**	Defence services [9111] (dollars - millions)
142	v41881779	**5**	Provincial and territorial public administration [912] (dollars - millions)
143	v41881780	**5**	Local, municipal and regional public administration [913] (dollars - millions)
			Tables 329-00(46,38,39) Industrial price indexes, 1997=100
144	v1575728	**5**	Transformer equipment (index, 1997=100) [P5648]
145	v1575754	**5**	Electric motors and generators (index, 1997=100) [P5674]
146	v1575886	**5**	Diesel fuel (index, 1997=100) [P5806]
147	v1575925	**5**	Light fuel oil (index, 1997=100) [P5845]
148	v1575903	**5**	Heavy fuel oil (index, 1997=100) [P5823]
149	v1575934	**5**	Lubricating oils and greases (index, 1997=100) [P5854]
150	v1575958	**5**	Asphalt mixtures and emulsions (index, 1997=100) [P5878]
151	v1575457	**5**	Industrial trucks, tractors and parts (index, 1997=100) [P5329]
152	v1575493	**5**	Parts, air conditioning and refrigeration equipment (index, 1997=100) [P5365]
153	v1575511	**5**	Food products industrial machinery and equipment (index, 1997=100) [P5383]
154	v1575557	**5**	Trucks, chassis, tractors, commercial (index, 1997=100) [P5429]
155	v1575610	**5**	Motor vehicle engine parts (index, 1997=100) [P5482]
156	v3860051	**5**	Motor vehicle brakes (index, 1997=100) [P5512]
157	v3822562	**5**	All manufacturing (index, 1997=100) [P6253]
158	v3825177	**5**	Total excluding food and beverage manufacturing (index, 1997=100) [P6491]

159	v3825178	**5**	Food and beverage manufacturing [311, 3121] (index, 1997=100) [P6492]
160	v3825179	**5**	Food and beverage manufacturing excluding alcoholic beverages (index, 1997=100) [P6493]
161	v3825180	**5**	Non-food (including alcoholic beverages) manufacturing (index, 1997=100) [P6494]
162	v3825181	**5**	Basic manufacturing industries [321, 322, 327, 331] (index, 1997=100) [P6495]
163	v3825183	**5**	Primary metal manufacturing excluding precious metals (index, 1997=100) [P6497]
			Table 176-0001 Commodity price index, US$ (index, 82-90=100)
164	v36382	**5**	Total, all commodities (index, 82-90=100) [B3300]
165	v36383	**5**	Total excluding energy (index, 82-90=100) [B3301]
166	v36384	**5**	Energy (index, 82-90=100) [B3302]
167	v36385	**5**	Food (index, 82-90=100) [B3303]
168	v36386	**5**	Industrial materials (index, 82-90=100) [B3304]
			Tables 176-00(46,47), 184-0002 Stock market statistics
169	v37412	**5**	Toronto Stock Exchange, value of shares traded (dollars - millions) [B4213]
170	v37413	**5**	Toronto Stock Exchange, volume of shares traded (shares - millions) [B4214]
171	v37414	**5**	United States common stocks, Dow-Jones industrials, high (index) [B4218]
172	v37415	**5**	United States common stocks, Dow-Jones industrials, low (index) [B4219]
173	v37416	**5**	United States common stocks, Dow-Jones industrials, close (index) [B4220]
174	v37419	**5**	New York Stock Exchange, customers' debit balances (dollars - millions) [B4223]
175	v37420	**5**	New York Stock Exchange, customers' free credit balance (dollars - millions) [B4224]
176	v122620	**5**	Standard and Poor's/Toronto Stock Exchange Composite Index, close (index, 1975=1000) [B4237]
177	v122628	**1**	Toronto Stock Exchange, stock dividend yields (composite), closing quotations (percent) [B4245]
178	v122629	**1**	Toronto Stock Exchange, price earnings ratio, closing quotations (ratio) [B4246]
179	v6384	**5**	Total volume; Value of shares traded (dollars - millions) [D4560]
180	v6385	**5**	Industrials; Value of shares traded (dollars - millions) [D4558]
181	v6386	**5**	Mining and oils; Value of shares traded (dollars - millions) [D4559]
			Table 176-0064 Foreign exchange rates
183	v37426	**1**	United States dollar, noon spot rate, average (dollars) [B3400]
184	v37437	**1**	United States dollar, 90-day forward noon rate (dollars) [B3401]
185	v37452	**1**	Danish krone, noon spot rate, average (dollars) [B3403]
186	v37456	**1**	Japanese yen, noon spot rate, average (dollars) [B3407]
187	v37427	**1**	Norwegian krone, noon spot rate, average (dollars) [B3409]
188	v37428	**1**	Swedish krona, noon spot rate, average (dollars) [B3410]
189	v37429	**1**	Swiss franc, noon spot rate, average (dollars) [B3411]
190	v37430	**1**	United Kingdom pound sterling, noon spot rate, average (dollars) [B3412]
191	v37431	**1**	United Kingdom pound sterling, 90-day forward noon rate (dollars) [B3413]
192	v37432	**1**	United States dollar, closing spot rate (dollars) [B3414]
193	v37433	**1**	United States dollar, highest spot rate (dollars) [B3415]
194	v37434	**1**	United States dollar, lowest spot rate (dollars) [B3416]
195	v37435	**1**	United States dollar, 90-day forward closing rate (dollars) [B3417]
196	v41498903	**1**	Canadian dollar effective exchange rate index (CERI) (1992=100) (dollars)
			Table 176-0043 Interest rates
197	v122550	**1**	Bank rate, last Tuesday or last Thursday (percent) [B14079]
198	v122530	**1**	Bank rate (percent) [B14006]
199	v122495	**1**	Chartered bank administered interest rates - prime business (percent) [B14020]
200	v122505	**1**	Forward premium or discount (-), United States dollar in Canada: 3 month (percent) [B14034]
201	v122509	**1**	Prime corporate paper rate: 1 month (percent) [B14039]
202	v122556	**1**	Prime corporate paper rate: 2 month (percent) [B14084]
203	v122491	**1**	Prime corporate paper rate: 3 month (percent) [B14017]
204	v122504	**1**	Bankers' acceptances: 1 month (percent) [B14033]
205	v122558	**1**	Government of Canada marketable bonds, average yield: 1-3 year (percent) [B14009]
206	v122485	**1**	Government of Canada marketable bonds, average yield: 3-5 year (percent) [B14010]
207	v122486	**1**	Government of Canada marketable bonds, average yield: 5-10 year (percent) [B14011]
208	v122487	**1**	Government of Canada marketable bonds, average yield: over 10 years (percent) [B14013]
209	v122515	**1**	Chartered bank - 5 year personal fixed term (percent) [B14045]
210	v122493	**1**	Chartered bank - non-chequable savings deposits (percent) [B14019]
211	v122541	**1**	Treasury bill auction - average yields: 3 month (percent) [B14007]
212	v122484	**1**	Treasury bill auction - average yields: 3 month, average at values (percent) [B14001]
213	v122552	**1**	Treasury bill auction - average yields: 6 month (percent) [B14008]
214	v122554	**1**	Treasury bills: 2 month (percent) [B14082]
215	v122531	**1**	Treasury bills: 3 month (percent) [B14060]
216	v122499	**1**	Government of Canada marketable bonds, average yield, average of Wednesdays: 1-3 year (percent) [B14028]
217	v122500	**1**	Government of Canada marketable bonds, average yield, average of Wednesdays: 3-5 year (percent) [B14029]
218	v122502	**1**	Government of Canada marketable bonds, average yield, average of Wednesdays: 5-10 year (percent) [B14030]
219	v122501	**1**	Government of Canada marketable bonds, average yield, average of Wednesdays: over 10 years (percent) [B14003]
220	v122497	**1**	Average residential mortgage lending rate: 5 year (percent) [B14024]
221	v122506	**1**	Chartered bank - chequable personal savings deposit rate (percent) [B14035]
222	v122507	**1**	Covered differential: Canada-United States 3 month Treasury bills (percent) [B14036]
223	v122508	**1**	Covered differential: Canada-United States 3 month short-term paper (percent) [B14038]
224	v122510	**1**	First coupon of Canada Savings Bonds (percent) [B14040]
			Table 176-0051 Canada's official international reserves
225	v122396	**5**	Total, Canada's official international reserves (dollars - millions) [B3800]
226	v122397	**5**	Convertible foreign currencies, United States dollars (dollars - millions) [B3801]
227	v122398	**5**	Convertible foreign currencies, other than United States (dollars - millions) [B3802]
228	v122399	**5**	Gold (dollars - millions) [B3803]
229	v122401	**5**	Reserve position in the International Monetary Fund (IMF) (dollars - millions) [B3805]

Table 176-0032 Credit measures

230	v36414	**5**	Total business and household credit; Seasonally adjusted (dollars - millions) [B165]
231	v36415	**5**	Household credit; Seasonally adjusted (dollars - millions) [B166]
232	v36416	**5**	Residential mortgage credit; Seasonally adjusted (dollars - millions) [B167]
233	v36417	**5**	Consumer credit; Seasonally adjusted (dollars - millions) [B168]
234	v36418	**5**	Business credit; Seasonally adjusted (dollars - millions) [B169]
235	v36419	**5**	Other business credit; Seasonally adjusted (dollars - millions) [B170]
236	v36420	**5**	Short-term business credit; Seasonally adjusted (dollars - millions) [B171]

Table 176-0025 Monetary aggregates

237	v37148	**5**	Currency outside banks (dollars - millions) [B1604]
238	v37153	**5**	Canadian dollar assets, total loans (dollars - millions) [B1605]
239	v37154	**5**	General loans (including grain dealers and installment finance companies) (dollars - millions) [B1606]
240	v37107	**5**	Total, major assets (dollars - millions) [B1611]
241	v37111	**5**	Canadian dollar assets, liquid assets (dollars - millions) [B1615]
242	v37112	**5**	Canadian dollar assets, less liquid assets (dollars - millions) [B1616]
243	v37119	**5**	Total personal loans, average of Wednesdays (dollars - millions) [B1622]
244	v37120	**5**	Business loans, average of Wednesdays (dollars - millions) [B1623]
245	v41552793	**5**	Currency outside banks and chartered bank deposits, held by general public (including private sector float) (dollars - millions)
246	v41552795	**5**	M1B (gross) (currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits) (dollars - millions)
247	v41552796	**5**	M2 (gross) (currency outside banks, chartered bank demand and notice deposits, chartered bank personal term deposits, adjustments to M2 (gross) (continuity adjustments and inter-bank demand and notice deposits)) (dollars - millions)
248	v41552797	**5**	Currency outside banks and chartered bank deposits (including private sector float) (dollars - millions)
249	v37130	**5**	Residential mortgages (dollars - millions) [B1632]
250	v41552798	**5**	M2+ (gross) (dollars - millions)
251	v37135	**5**	Chartered bank deposits, personal, term (dollars - millions) [B1637]
252	v37138	**5**	Total, deposits at trust and mortgage loan companies (dollars - millions) [B1639]
253	v37139	**5**	Total, deposits at credit unions and caisses populaires (dollars - millions) [B1640]
254	v37140	**5**	Bankers' acceptances (dollars - millions) [B1641]
255	v37145	**5**	Monetary base (notes and coin in circulation, chartered bank and other Canadian Payments Association members' deposits with the Bank of Canada) (dollars - millions) [B1646]
256	v37146	**5**	Monetary base (notes and coin in circulation, chartered bank and other Canadian Payments Association members' deposits with the Bank of Canada) (excluding required reserves) (dollars - millions) [B1647]
257	v37147	**5**	Canada Savings Bonds and other retail instruments (dollars - millions) [B1648]
258	v41552801	**5**	M2++ (gross) (M2+ (gross), Canada Savings Bonds, non-money market mutual funds) (dollars - millions)
259	v37152	**5**	M1++ (gross) (dollars - millions) [B1652]

Table 282-0087 LFS, SA, Canada and provinces

260	v2062811	**5**	Canada; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
261	v2062815	**1**	Canada; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
262	v2063000	**5**	Newfoundland and Labrador; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
263	v2063004	**1**	Newfoundland and Labrador; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
264	v2063189	**5**	Prince Edward Island; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
265	v2063193	**1**	Prince Edward Island; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
266	v2063378	**5**	Nova Scotia; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
267	v2063382	**1**	Nova Scotia; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
268	v2063567	**5**	New Brunswick; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
269	v2063571	**1**	New Brunswick; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
270	v2063756	**5**	Quebec; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
271	v2063760	**1**	Quebec; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
272	v2063945	**5**	Ontario; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
273	v2063949	**1**	Ontario; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
274	v2064134	**5**	Manitoba; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
275	v2064138	**1**	Manitoba; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
276	v2064323	**5**	Saskatchewan; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
277	v2064327	**1**	Saskatchewan; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
278	v2064512	**5**	Alberta; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
279	v2064516	**1**	Alberta; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)
280	v2064701	**5**	British Columbia; Employment; Both sexes; 15 years and over; Seasonally adjusted (persons - thousands)
281	v2064705	**1**	British Columbia; Unemployment rate; Both sexes; 15 years and over; Seasonally adjusted (rate)

Table 282-0088 Employment by industry

282	v2057603	**5**	Total employed, all industries; Seasonally adjusted (persons - thousands)
283	v2057604	**5**	Goods-producing sector; Seasonally adjusted (persons - thousands)
284	v2057605	**5**	Agriculture [1100-1129, 1151-1152]; Seasonally adjusted (persons - thousands)
285	v2057606	**5**	Forestry, fishing, mining, oil and gas [1131-1133, 1141-1142, 1153, 2100-2131]; Seasonally adjusted (persons - thousands)
286	v2057607	**5**	Utilities [2211-2213]; Seasonally adjusted (persons - thousands)
287	v2057608	**5**	Construction [2361-2389]; Seasonally adjusted (persons - thousands)
288	v2057609	**5**	Manufacturing [3211-3219, 3271-3279, 3311-3399, 3111-3169, 3221-3262]; Seasonally adjusted (persons - thousands)
289	v2057610	**5**	Services-producing sector; Seasonally adjusted (persons - thousands)
290	v2057611	**5**	Trade [4111-4191, 4411-4543]; Seasonally adjusted (persons - thousands)
291	v2057612	**5**	Transportation and warehousing [4811-4931]; Seasonally adjusted (persons - thousands)
292	v2057613	**5**	Finance, insurance, real estate and leasing [5211-5269, 5311-5331]; Seasonally adjusted (persons - thousands)
293	v2057614	**5**	Professional, scientific and technical services [5411-5419]; Seasonally adjusted (persons - thousands)
294	v2057615	**5**	Business, building and other support services [5511-5629]; Seasonally adjusted (persons - thousands)
295	v2057616	**5**	Educational services [6111-6117]; Seasonally adjusted (persons - thousands)
296	v2057617	**5**	Health care and social assistance [6211-6244]; Seasonally adjusted (persons - thousands)
297	v2057618	**5**	Information, culture and recreation [5111-5191, 7111-7139]; Seasonally adjusted (persons - thousands)
298	v2057619	**5**	Accommodation and food services [7211-7224]; Seasonally adjusted (persons - thousands)
299	v2057620	**5**	Other services [8111-8141]; Seasonally adjusted (persons - thousands)
300	v2057621	**5**	Public administration [9110-9191]; Seasonally adjusted (persons - thousands)

			Tables 228-00(01,41) Merchandise imports and exports Canada, SA
301	v183474	**5**	Imports, United States, including Puerto Rico and Virgin Islands (dollars - millions) [D398058]
302	v183475	**5**	Imports, United Kingdom (dollars - millions) [D398059]
303	v183476	**5**	Imports, Other European Economic Community (dollars - millions) [D398060]
304	v183477	**5**	Imports, Japan (dollars - millions) [D398061]
305	v191559	**5**	Exports, United States, including Puerto Rico and Virgin Islands (dollars - millions) [D399518]
306	v191560	**5**	Exports, United Kingdom (dollars - millions) [D399519]
307	v191561	**5**	Exports, Other European Economic Community (dollars - millions) [D399520]
308	v191562	**5**	Exports, Japan (dollars - millions) [D399521]
309	v21386488	**5**	Imports, total of all merchandise (dollars - millions)
310	v21386489	**5**	Imports, Sector 1 Agricultural and fishing products (dollars - millions)
311	v21386492	**5**	Imports, Sector 2 Energy products (dollars - millions)
312	v21386495	**5**	Imports, Sector 3 Forestry products (dollars - millions)
313	v21386496	**5**	Imports, Sector 4 Industrial goods and materials (dollars - millions)
314	v21386500	**5**	Imports, Sector 5 Machinery and equipment (dollars - millions)
315	v21386505	**5**	Imports, Sector 6 Automotive products (dollars - millions)
316	v21386509	**5**	Imports, Sector 7 Other consumer goods (dollars - millions)
317	v21386512	**5**	Imports, Sector 8 Special transactions trade (dollars - millions)
318	v21386514	**5**	Exports, total of all merchandise (dollars - millions)
319	v21386515	**5**	Exports, Sector 1 Agricultural and fishing products (dollars - millions)
320	v21386518	**5**	Exports, Sector 2 Energy products (dollars - millions)
321	v21386522	**5**	Exports, Sector 3 Forestry products (dollars - millions)
322	v21386526	**5**	Exports, Sector 4 Industrial goods and materials (dollars - millions)
323	v21386531	**5**	Exports, Sector 5 Machinery and equipment (dollars - millions)
324	v21386535	**5**	Exports, Sector 6 Automotive products (dollars - millions)
325	v21386539	**5**	Exports, Sector 7 Other consumer goods (dollars - millions)
326	v21386540	**5**	Exports, Sector 8 Special transactions trade (dollars - millions)
			Table 026-0008: Building permits, values by activity sector; Canada
327	v4667	**5**	Total residential and non-residential (dollars - thousands) [D2677]
328	v4668	**5**	Residential (dollars - thousands) [D2681]
329	v4669	**5**	Non-residential (dollars - thousands) [D4898]
330	v4670	**5**	Industrial (dollars - thousands) [D2678]
331	v4671	**5**	Commercial (dollars - thousands) [D2679]
332	v4672	**5**	Institutional and governmental (dollars - thousands) [D2680]

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