Socio-economic Differentiation in Experienced Mortality Modelling and its pricing implications

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The demographic data are scaled to a certain degree and sensitive information is censored

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Motivation

Socio-economic differentiation in mortality

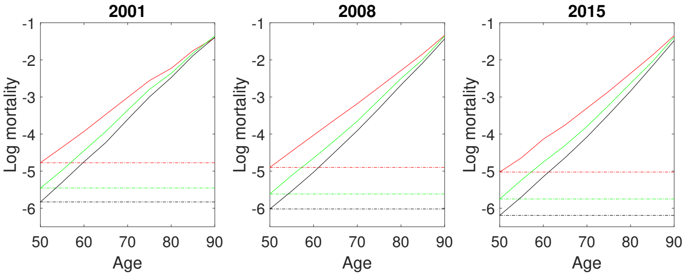


Figure: Logarithm central mortality rates by IMD decile (red: poor, green: middle, black: wealthy), England (Data:`https://goo.gl/AJGrq6`)

- Higher socio-economic profiles (sub-populations) enjoy lower mortality rates in the national mortality

Motivation

- Is socio-economic differentiation exist also in **Insured mortality**?
  - If so, how this differentiation moves in "leve" and "trend"?
- How **Uncertainty** is defined for differentiated mortality?
- Business motivation behind Differentiation: flexible and fair price.
- What is the **pricing implications** of such socio-economic differentiation?
  - Poor people compensate for rich people when "*one price*".
  - Undesirable wealth transfer from the low-income profiles to high-income profiles.

Existing models for socio-economic mortality differentiation

- **Classical actuarial approach**: treat socioeconomic attributes as traditional differentiator (e.g., gender, age) in the life table
  - Divide the portfolio into subpopulations specific to gender, age, period, and socioeconomic attributes....
  - Cells of these life tables are usually sparse and difficult to make useful statistical inference. Special treatments are needed for continuous socioeconomic attributes.
- Gschlössl et al. [2011] suggest regression analysis as an appropriate tool to estimate mortality differentials

Existing models for socio-economic mortality differentiation

- **Regression analysis:** treat socioeconomic attributes as independent variables to explain the individual death/alive response.
  - Proportional hazard model, e.g., Cox [1992]
  - Survival analysis, e.g., Richards [2008]
  - Poisson regression, e.g.,Gschlössl et al. [2011]
  - Logistic regression, e.g., Madrigal et al. [2011]
- **National mortality** rates are overlooked in Differentiated experienced mortality, in these regression analyses.
- **Current (Pension) business** especially in The Netherlands are built in connection with National Mortality (via the so-called **Experience Factor**).

Existing Models for Socio-economic Mortality differentiation

- **Regression analysis and mortality experience:** modelling the shifts from national mortality foreces in term of socioeconomic differences
  - For example, van Berkum et al. [2018] employ Poisson generalized additive model to demonstrate the outstanding mortality risk factors in a pension fund.
  - Bridging Plat [2009] (Experienced mortality modelling) and Gschlössl et al. [2011] (Poisson regression analysis).
  - Results: **Salary** info as one of the most significant differentiators.

Existing Models for Socio-economic Mortality differentiation

- Little knowledge on how socio-economic mortality differentiation evolves over time in portfolio.
  - Portfolio: Socio-economic differentiation only in "*level*"? Or also in "*trend*"?
- **Examining how differentiation evolve over time** is crucial for **pricing implications**
  - (1) Limitation of the data (2) The business need of the flexible & fair pricing. (3) Regulatory concerns.

Existing pricing

- Mortality Differentiation is connected to **long-term life/pension liabilities** and evolves the *Uncertainty, risk-margins and SCR estimation* in **Multi-price structure**
- Salahnejhad and Pelsser [2016] implemented two Riks-margin valuations based on the **Conditional Scenario Generation**
  - **EIOPA risk-margin Price**: The aggregate risk-margins along Best-estimate (by regulators)
  - **Time-consistent Price**: Backward iteration of the one-period operator
  - Both include Repetitive **Conditional Pricing Operators** with high load of calculations
  - extra developments in Dhaene et al. [2017]
- Numerical Method: **Regression-based methods** to Price with **Conditional Operators** see Longstaff and Schwartz [2001]

This paper

- Mortality modelling on an important risk factor, i.e., salary information,
- Relatively better data quality across time in industry.
- We build **Stochastic Differentiated Experienced Mortality** model by extending Plat [2009].
- Easy to integrate in current Business setting (easy to add more variables).
- We render pricing implications on longevity derivatives when we take into account salary differentiation over time, comparing to No-differentiation.
    - Special case of taking mortality differentiation into an index like SCR, price etc.

Data

- Individual-level panel data traces from 2002 to 2016.
- The salary classes are characterized as:
  - MSC1: $modalSal_{i,t} < 1$,
  - MSC2: $1 < modalSal_{i,t} < 2$,
  - MSC3: $2 < modalSal_{i,t}$.
- $modalSal_{i,t}$ is ratio of the yearly salary of individual i at year t over the national modal salary at year t.
- We focus on the male records from age 33 to 77 with about 99.8% salary information coverage.
- Total number of observations is up to 660,000 individuals with 15 years observations.

Regression analysis

$$D_{i,t} \sim \beta_{0,t} + \beta_{1,t} \times age_{i,t} + \sum_g \beta_{g,t} \times s^g_{i,t} + \epsilon_{i,t} \quad (1)$$

$D_{i,t}$ is the death indicator of the pensioner i at the year t. $s^g_{i,t}$ is the salary level indicator.

Regression analysis: logistics regression

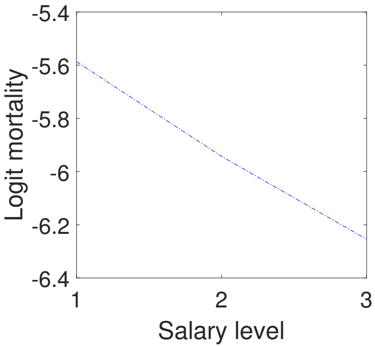


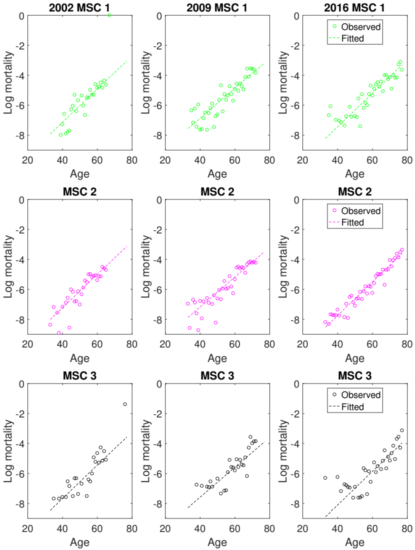
Figure: The logit of the salary-specific overall central death rates

Regression analysis: logistics regression

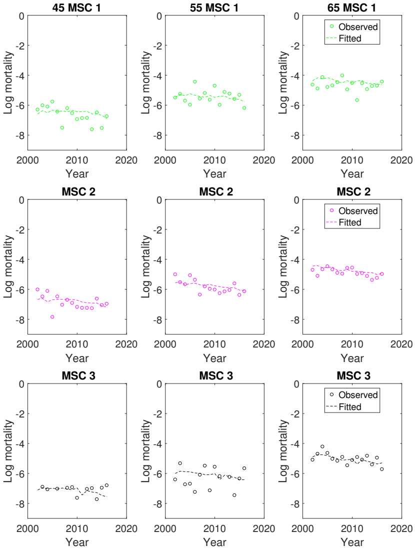
- According to the rather logarithm linear relation, logistic regression works properly for salary differentiation.
- Logistic regression is easier to apply in practice.

$$logit(q^g_{x,t}) \sim \beta_{0,t} + \beta_{1,t} * x + \beta_{g,t} * s^g_{i,t} \quad (2)$$

Results across ages



Results across years



Experience Mortality Modelling: Portfolio

Plat model (with slight modification) for differentiation level g

$$log(a^g_{x,t}) = (\frac{x-d}{w-d})f^g_t + \phi^g_{x,t} \tag{3}$$

- $a^g_{x,t} = \frac{\sum_{s=t}^{s=t+k} \hat{m}^g_{x,s} E^g_{x,s}}{\sum_{s=t}^{s=t+k} m^{pop}_{x,s} E^g_{x,s}}$,
- $\hat{m}^g_{x,s}$ is obtained from logistic regression.
- $\phi^g_{x,t}$ follows a multivariate normal distribution containing all the subgroups and the time varying components of the national population.

Experience Mortality Modelling: Portfolio

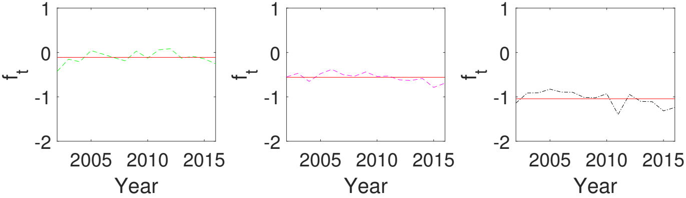


Figure: The time-varying component of the experience factor. Left to right: Salary Class 1, 2, 3

"**Trend Differentiation**" is not not significant for low and high salary classes.

ARIMA(0,0,0) provides a satisfying fit for the time-varying component of experience factor for different salary classes.

$$f_t^g = \delta^g + v_t^g \tag{4}$$

Experience mortality modelling: portfolio bootstrap scheme

To reconcile the uncertainty around the logistic estimation of the experience mortality.

ID	Year	Alive	Age	Modal1	Modal2	Modal3
1	2002	1	50	1	0	0
	2003	1	51	1	0	0
	2004	0	52	0	1	0
2	2002	1	66	1	0	0
	2003	0	67	1	0	0
3	2002	1	45	0	1	0
	2003	1	46	0	1	0
	2004	1	47	0	1	0

Table 1: Panel Data Sample

ID	Year	Alive	Age	Modal1	Modal2	Modal3
1	2002	1	50	1	0	0
	2003	1	51	1	0	0
	2004	0	52	0	1	0
1	2002	1	50	1	0	0
	2003	1	51	1	0	0
	2004	0	52	0	1	0
3	2002	1	45	0	1	0
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	2004	1	47	0	1	0

Table 2: A possible bootstrapped Panel Data Sample

Steps: (1) Re-sample from all **individual panels** in each bootstrap with replacements (2) Re-estimate the logistic regression in each bootstrap. (3) Re-estimate the experience mortality model.

Experience Mortality Modelling: Population

Lee-Carter model Lee and Carter [1992]:

$$\log m_{x,t}^{pop} = a_x + b_x k_t + \varepsilon_{x,t}, \tag{5}$$

$$k_t = d + k_{t-1} + \epsilon_t, \tag{6}$$

Note that ϵ_t follows a multivariate normal distribution alongside with the time varying components of the base and salary differentiated experience factors.

Fitting and Projecting Experienced Mortality

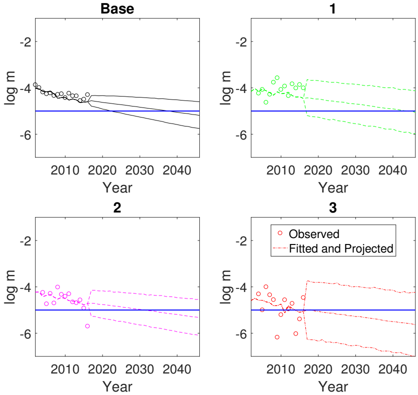

Figure: The observe, fitted, and projections of the portfolio specific mortality for age 67, 95% bootstrapped confidence intervals

CIs $< --- >$ Uncertainty in each salary class and base class.

Uncertainty in Differentiation

Importance of Solvency Capital Requirements (SCR) for **long-dated liabilities** (Typical in Life/Pension).

- **Uncertainty** is a core concern in **Differentiation** specially in long-term life/pension products.
- Uncertainty can be calibrated with respect to the **required SCR** (We are still working to escalate this ...!!)

Let us for now see some **Pricing Implications** instead

- **Unhedgeable risk** involved in mortality/longevity requires apppropriate **Risk-margin (Loading) in Price** on top of the Expected value.
- Risk-margin should make **sufficient buffer capital** to cover the unexpected risk.

Payoff and Expected Value

Consider a Simple Endowment with the payoff 1:

$$G_x(\kappa_T) = f({}_T p_x) = 1 \times N_x(T) \tag{7}$$

with maturity T and starting cohort $N_x(0)$ with age x with underlying mortality trend κ_t.

- $N_x(T) = N_x(0) * {}_T p_x$: Number of survivors at age $x + T$,
- ${}_T p_x$: Projected T-year survival probability random variable.

Conditional Expected Payoff at time $t < T$:

$$\mathbb{E}\left[f(N_x(T)) \mid N_x(t)\right] \quad , \quad N_x(t) \sim \{\kappa_t \ \& \ a_{x,t}\}$$

EIOPA Risk-margin Price

For a Multi-period valuation to capture the uncertainty in long-term, EIOPA suggests:

$$\Pi_t^{EIOPA}[f(N_x^g(T))] = e^{-r(T-t)} \times \left[h(N_x^g(t)) + \delta \sum_{k=1}^{T-t} \mathbb{V}\mathrm{aR}_q \left[h(N_x^g(t+k)) - h(N_x^g(t)) \;\middle|\; \mathbb{BE}(N_x^g(t+k-1))\right]\right] \tag{8}$$

for each differentiation level $g = \{L, M, H\}$ where

$$h(N_x^g(t+k)) = \mathbb{E}\left[f(N_x^g(T)) \mid N_x^g(t+k)\right]$$

Best-estimate unit payoff at time T given the realizations at time $t+k$,

$\mathbb{BE}(N_x(t+k-1))$ is the best-estimate number of survivors at $t+k-1$ given the initial info at time t.

EIOPA Risk-Margin for Long-term Liabilities

- Measuring the Risk-Margin along the Best-Estimate.

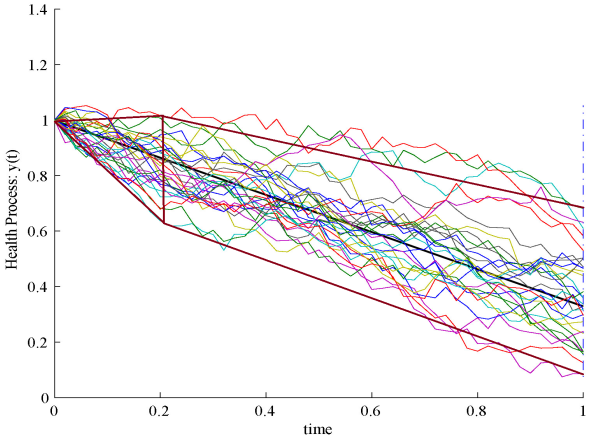

Figure: Simulation of Sample diffusion process for human health over time.

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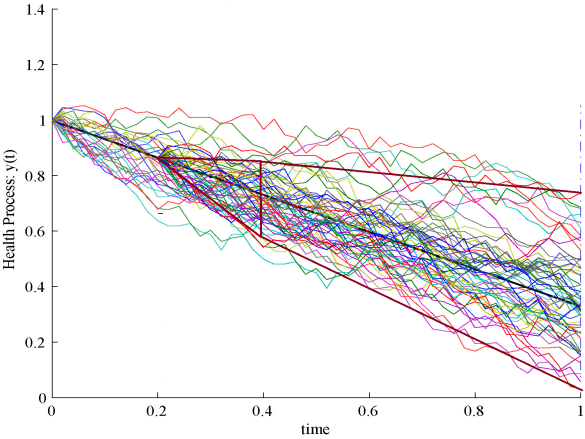

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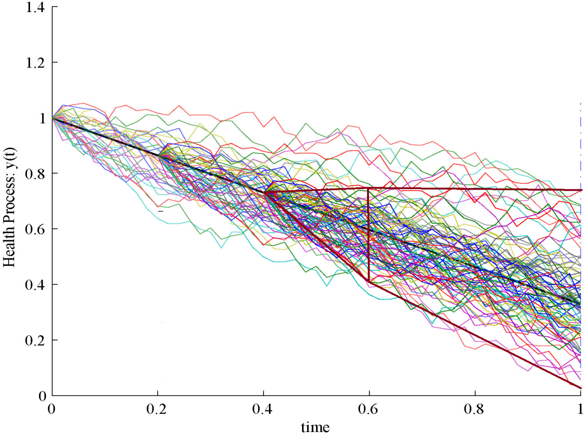

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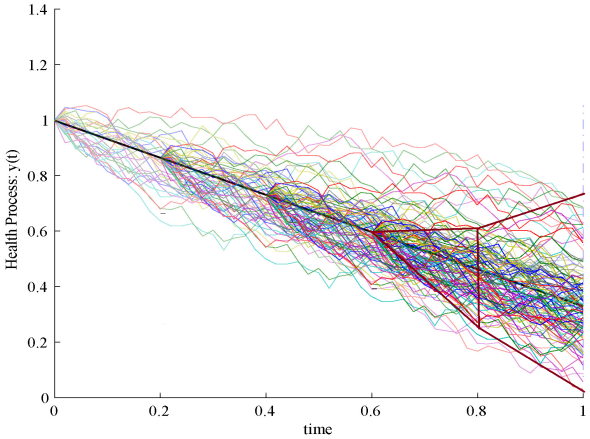

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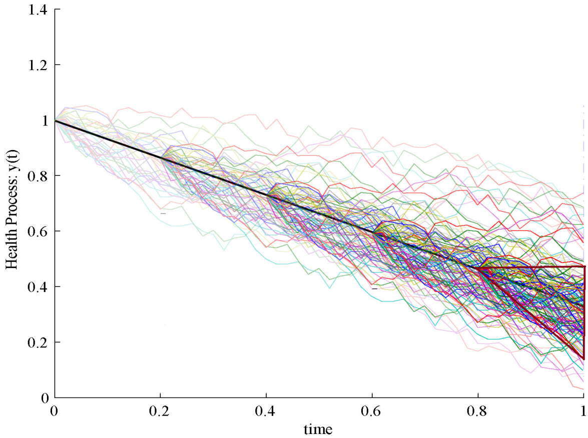

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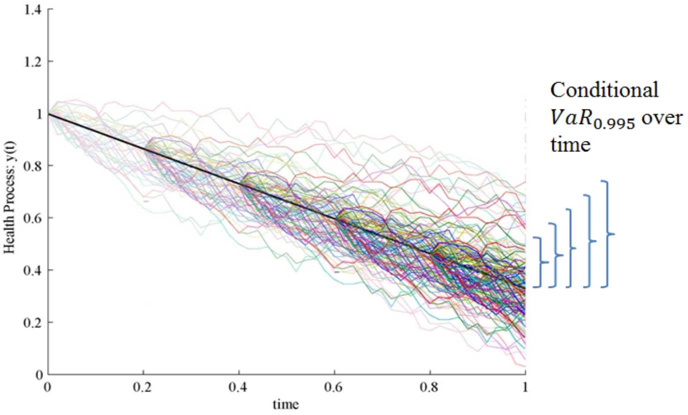

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The idea of Time-consistency

- **What if** in the the mean-time the Best-estimate didn't come true?!
- Core Concept: **Every Future Middle-term State of the Risk Can Initiate A New Market**.

Tomorrow has a New Story!

Yesterday's perception doesn't remain Credible!

- Middle-term dynamics on trends and volatility cause **Uncertainty on Uncertainty**
- **Conditional Scenario Generation** reflects the imagination of the **what if ...** situation.
- Middle-term dynamics should be measured by the **"Middle-term (Re)-Valuation"**.
- **Time-consistency** constructs the price based on the Middle-term (Re)-Valuation

Time-Consistent Risk-margin Price

Take the discrete set $\{0, 1, 2, ..., T-1, T\}$ dividing $[0, T]$,

The **backward iteration of the one-period** cost-of-capital risk-margin price can be represented as below:

$$
\begin{aligned}
&\textbf{(T-1, T)}: && \pi\left(N_x^g(T-1)\right) = \Pi^{CoC}\left[f(N_x^g(T)) \mid N_x^g(T-1)\right] \\
&\textbf{(T-2, T-1)}: && \pi\left(N_x^g(T-2)\right) = \Pi^{CoC}\left[\pi(N_x(T-1)) \mid N_x^g(T-2)\right] \\
&\quad\vdots && \\
&\textbf{(t, t+1)}: && \pi\left(N_x^g(t)\right) = \Pi^{CoC}\left[\pi\left(N_x^g(t+1)\right) \mid N_x^g(t)\right] \\
&\quad\vdots && \\
&\textbf{(0, 1)}: && \pi^{TC}\left(N_x^g(0)\right) = \Pi^{CoC}\left[\pi\left(N_x^g(1)\right) \mid N_x^g(0)\right]
\end{aligned}
\tag{9}
$$

Π_t^{CoC} The one-year Cost-of-Capital price operator:

$$
\Pi_t^{CoC}\left[N_x^g(t+1) \mid N_x^g(t)\right] =
$$

$$
e^{-r}\left[h(N_x^g(0)) + \delta \mathbb{V}\mathrm{aR}_q\left[h(N_x^g(t+1)) - h(N_x^g(0)) \mid \mathbb{BE}(N_x^g(t))\right]\right] \tag{10}
$$

and $h(N_x^g(t)) = \mathbb{E}\left[f(N_x^g(T)) \mid N_x^g(t)\right]$

Aggregate Risk-margin in Differentiated Prices

- Differentiation creates a **segmented/partitioned portfolio**
- Each differentiation class has **smaller & and more homogeneous groups**.
- Differentiated prices are built based on the dependent structure of the underlying mortality.
- **Differentiated Risk-margins** should cover the **aggregate risk-margin of the base (total) portfolio**:

$$RM(L+M+H) \leq RM(L) + RM(M) + RM(H)$$

Pricing results: 50% quantile

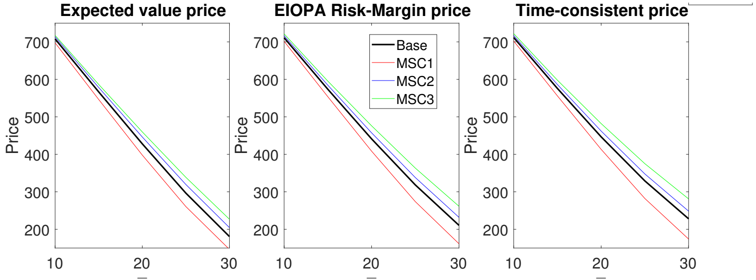

Figure: Longevity bonds for age 40 male

Pricing results: 50% quantile with 95% CIs

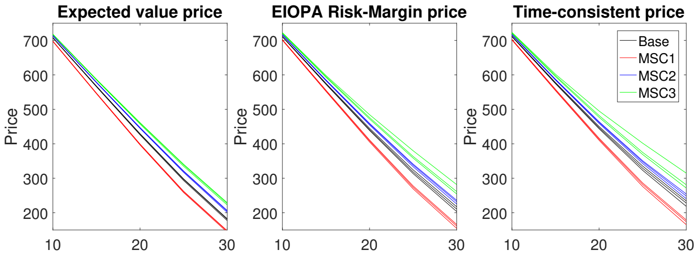

Figure: Longevity bonds for age 40 male

Conclusion

- Our model renders reasonable best estimates alongside with proper confidence intervals.
- Based on our mortality scenarios, we provide three prices of the differentiated longevity bonds for different maturities.
- The pricing results show the price of these bonds are significantly different between the lowest salary group and the highest salary group, comparing to the base group.

Reference

David R Cox. Regression models and life-tables. In *Breakthroughs in statistics*, pages 527–541. Springer, 1992.

J. Dhaene, B. Stassen, K. Barigou, D. Linders, and Z. Chen. Fair valuation of insurance liabilities: Merging actuarial judgement and market-consistency. *Insurance Mathematics and Economics*, 76:14–27, 2017.

Susanne Gschlössl, Pascal Schoenmaekers, and Michel Denuit. Risk classification in life insurance: methodology and case study. *European Actuarial Journal*, 1(1):23–41, 2011.

Ronald D Lee and Lawrence R Carter. Modeling and forecasting us mortality. *Journal of the American statistical association*, 87 (419):659–671, 1992.

F. A. Longstaff and E. S. Schwartz. Valuing american options by simulation: A simple least-squares approach. *The Review of Financial Studies*, 14(1):113–147, 2001.

Ana M Madrigal, Fiona E Matthews, DD Patel, AT Gaches, and

SD Baxter. What longevity predictors should be allowed for when valuing pension scheme liabilities? *British Actuarial Journal*, 16(1):1–38, 2011.

Richard Plat. Stochastic portfolio specific mortality and the quantification of mortality basis risk. *Insurance: Mathematics and Economics*, 45(1):123–132, 2009.

SJ Richards. Applying survival models to pensioner mortality data. *British Actuarial Journal*, 14(2):257–303, 2008.

Ahmad Salahnejhad and Antoon Pelsser. Market-consistent actuarial valuations with application to eiopa risk-margin and time-consistent pricing. 2016.

Frank van Berkum, Katrien Antonio, and Michel Vellekoop. Unraveling relevant risk factors explaining pension fund mortality:. 2018.