Health Status Mortality Modeling Based on A Multiple-state Markov Ageing Model

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Outline

Motivation

Why consider heterogeneity in mortality modelling

- High uncertainty in mortality development.
  - Systematic mortality risk (longevity risk)
  - Mortality heterogeneity
- Key to the fair pricing of mortality-linked products.
- Increasing attention has been paid on mortality heterogeneity.

Why use health status to identify heterogeneity

- Directly linked to the mortality compared to health risk factors or socio-economy status.
- Health care costs are significant to both individuals and the government.

Literature Review I

1. **Stochastic mortality models** – systematic mortality risk
   - Discrete time stochastic mortality models
     - Including Lee-Carter models (Lee and Carter, 1992) and CBD models (Cairns et al., 2006).
     - Time-series models popular in modeling mortality trend.
     - Not compatible with the valuation of mortality-linked products.
   - Continuous time stochastic mortality models
     - Affine term structure model (ATSM) (Duffie and Kan, 1996; Blackburn and Sherris, 2013).
     - Satisfy important requirements for applications (Schrager, 2006) and proved to be appropriate in fitting historical mortality data (Blackburn and Sherris, 2013).
     - No clear link between model and human ageing process (Liu and Lin, 2012).

Literature Review II

2. **Mortality models with heterogeneity**
   - Observed Heterogeneity Models
     - Cox proportional hazards model (Cox, 1972).
     - Generalized Linear Mixed Models (Meyricke and Sherris, 2013).
     - Limited by high data demand.
   - Unobserved Heterogeneity Models – model heterogeneity mortality from standard mortality
     - Frailty models (Vaupel et al., 1979; Manton et al., 1986; Su and Sherris, 2012).
     - Markov ageing models (MAMs) (Le Bras, 1976; Lin and Liu, 2007; Su and Sherris, 2012; Liu and Lin, 2012; Sherris and Zhou, 2014).

Research Aims I

Markov Ageing Models (MAMs)

- Deterministic MAMs (Le Bras, 1976; Lin and Liu, 2007; Su and Sherris, 2012)
- Stochastic MAMs
  - subordinate time change process + no time trend in intensities (Liu and Lin, 2012)
  - Subordinate time change process + time trend in intensities (Sherris and Zhou, 2014)

Affine Term Structure Models (ATSMs)

- Single Cohort Mortality Models (Dahl, 2004; Biffs, 2005; Dahl and Moller, 2006; Schrager, 2006)
- Multi-cohort Mortality Models (Blackburn and Sherris, 2013; Jevtic et al., 2013; Xu et al., 2015)

Gap

Observable Health Status

- More direct relationship with mortality than socio-economic status (Sherris and Zhou, 2014)
- Determinant of survival probabilities (Hurd et al. 2001)
- Influencing factors for annuities purchase (Turra and Mitchell, 2004)

Extending the framework of Sherris and Zhou (2014) by modeling mortality intensities in MAMs with stochastic affine processes rather than simple parametric functions

Research Aims II

- Develop a multiple state mortality model with heterogeneity
  - Stochastic mortality intensities following affine type processes.
  - Observable health status as heterogeneity factors.
- Calibrate to (Australian) cohort mortality data and cross sectional health data
- Capturing uncertainty of mortality dynamics in both the aggregated level and health status levels.
- Projecting of health distribution development.

Markov Ageing Models (MAMs)

1. Ageing process is modeled in terms of changes in physiological functions.
2. Physiological age:
   - a *relative health index* representing the degree of ageing;
   - a range of physiological ages to represent heterogeneity;
   - higher physiological ages can be viewed as worse health status with higher mortality rates.
3. Phase-type distribution for the time until death.

Deterministic MAMs – Lin and Liu(2007), Su and Sherris(2012)

1. Model based on 'physiological age'.
2. n transient states and 1 absorbing state (death).
3. Transition rates matrix:

$$\Lambda = \begin{pmatrix} -(\lambda_1 + q_1) & \lambda_1 & 0 & \dots & 0 \\ 0 & -(\lambda_2 + q_2) & \lambda_2 & \dots & 0 \\ 0 & 0 & -(\lambda_3 + q_3) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -q_n \end{pmatrix},$$

   - λ_i will be constant after finite development periods;
   - q_i is a function of state i and has no time trend (also include additional constant parameter to capture the hump ages) .
4. Phase-type distribution: $S(t) = \alpha \exp(\Lambda t)e$, α stands for initial distribution.

Stochastic MAMs – Liu and Lin(2013), Sherris and Zhou(2014)

1. Small number of states (5 transient states) - facilitate the incorporation of health data.
2. Similar underlying multi-state model.
3. Subordinate Gamma time process γ_t to capture the systematic risk.
4. Sherris and Zhou (2014) make the matrix time-inhomogeneous by taking into account time trend in transition intensity functions:
    - $q_i(t) = a \times e^{bt} + c_i$,
    - $\lambda_i(t) = m_i \times (t-1) + n_i$.

Model Definitions I

1. 4 level health status and one absorbing death state.
2. Time-inhomogeneous transition intensity matrix $\Lambda(t)$:

$$\begin{pmatrix} -(\lambda_{1,t}+\mu_{1,t}) & \lambda_{1,t} & 0 & 0 \\ 0 & -(\lambda_{2,t}+\mu_{2,t}) & \lambda_{2,t} & 0 \\ 0 & 0 & -(\lambda_{3,t}+\mu_{3,t}) & \lambda_{3,t} \\ 0 & 0 & 0 & -\mu_{4,t} \end{pmatrix}$$

3. The time until death will then follows a phase-type distribution with representation $(\pi_0, \Lambda(t))$, where π_0 is the initial health distribution.

Model Definitions II

4. Health transition intensity: $\lambda_{i,t} = a_i + b \cdot e^{c \cdot t}$
   - a_i – status dependent health transition intensity;
   - $b \cdot e^{c \cdot t}$ – ageing trend of health transition intensities for each status.
5. Mortality intensity: $\mu_{i,t}$
   - Instantaneous mortality intensity: $\mu_i(t) = X(t) + Y_i(t)$
     - $X(t)$ – population development factor; $Y_i(t)$ – health status adjusting factor. (Non-mean reverting stochastic processes)
   - Average force of moratliy:

$$\bar{\mu}_i(t,T) = -\frac{B(t,T)}{T-t}X(t) - \frac{B_i(t,T)}{T-s}Y_i(t) - \frac{C(t,T)}{T-t} - \frac{C_i(t,T)}{T-t}$$

     - $\mu_{i,t}$ will be calculated by combing factor and factor loadings

Model Definitions III

6. Phase-type properties (Lin and Liu, 2007; Sherris and Zhou, 2014):
   - The survival probability in t years' time:

$$S(t) = \pi_0 \exp\left(\sum_{s=1}^{t} \Lambda(s)\right) e,$$

     where e is the column vector of ones.
   - The probability for an individual alive at time t is in state i:

$$\pi_i(t) = P(J_t = i \mid T > t) = \frac{P_i(t)}{S(t)},$$

     where

$$P_i(t) = P(J_t = i, T > t) = [\pi_0 e^{\sum_{s=1}^{t} \Lambda(s)}]_i,$$

     $\pi(t) = [\pi_1(t), \pi_2(t), \cdots, \pi_4(t)]$ represent the health distribution at time t.

Data Source

The model was calibrated to mortality and health data for Australian population (male and female combined).

- **Human Mortality Database(HMD)**: one year death rates and life tables (1921-2013), cohort death rates can be derived from this.
- **WHO mortality database**: number of deaths from each health condition + corresponding population in each 5 year interval from age 5 to 84, up to year 2015.
- **National Health Survey**: prevalence of long-term conditions: 10 year interval from age 15 to 75, across year 2007-08, 2011-12 and 2014-15.

Health Status Classification

1. **Severity Index**

$$\textit{Index value} = \frac{\textit{yearly death rate for each ICD chapter}}{\textit{yearly prevalence in that chapter}}$$

2. **Health Status Classification**
   - H1: ICD chapter 4,5,7,8,12,13,14,16
   - H2: ICD chapter 3,6,9,10,11
   - H3: ICD chapter 1,15
   - H4: ICD chapter 2

Note: We use International Classification of Diseases (ICD) to define the health conditions.

Mortality Intensities I

1. **Aggregated Mortality intensities**
   - Estimation Method – Kalman Filter Algorithm
     - $\mu_t^0 = X(t)$, $dX(t) = aX(t)dt + \sigma dW(t)$
     - Measurement equation:

$$\begin{pmatrix} \bar{\mu}_0(t,t+1) \\ \bar{\mu}_0(t,t+2) \\ \cdots \\ \bar{\mu}_0(t,t+n) \end{pmatrix} = \begin{pmatrix} -\dfrac{1-e^{\alpha}}{\alpha} \\ -\dfrac{1-e^{2\alpha}}{2\alpha} \\ \cdots \\ -\dfrac{1-e^{n\alpha}}{n\alpha} \end{pmatrix} X(t) - \begin{pmatrix} C(1) \\ \dfrac{C(2)}{2} \\ \cdots \\ \dfrac{C(n)}{n} \end{pmatrix} + \begin{pmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \cdots \\ \varepsilon_n(t) \end{pmatrix},$$

     - State transition equation:

$$X_t = \Phi X_{t-1} + \eta_t, \quad \eta_t \sim N(0, Q),$$

where $\Phi = e^{\alpha}$ and $Q = -\dfrac{\sigma^2}{2\alpha}(1 - e^{2\alpha})$.

Mortality Intensities II

- Estimation Results

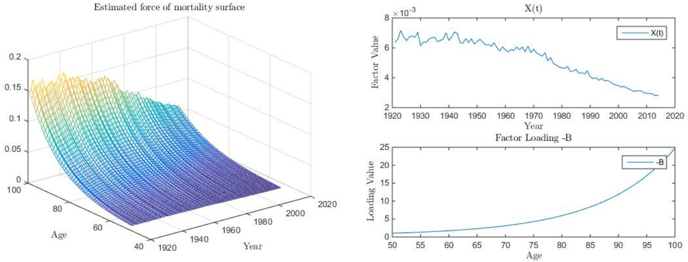


Figure: $\bar{\alpha} = 0.0937$, $\bar{\sigma} = 3.6046e - 4$

Mortality Intensities III

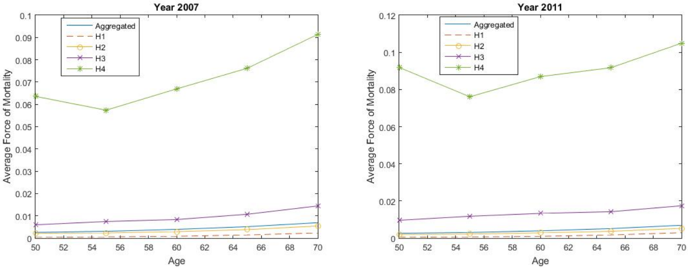
2. **Status Dependent Mortality intensities**
   - Estimation Method
     - $\mu_i(t) = X(t) + Y_i(t)$
     - $dY_i(t) = \alpha_i Y_i(t)dt + \sigma_i dW^i(t)$
     - Minimizing the calibration error:

$$\theta_i^* = argmin_{\theta_i} \sqrt{\sum_{\tau=1}^{n} (\mu_i(\tau) - \bar{\mu}_i(\tau))^2},$$

where $\theta_i = (\alpha_i, \sigma_i)$, $\mu_i(\tau)$ from true data and $\bar{\mu}_i(\tau)$ from affine model

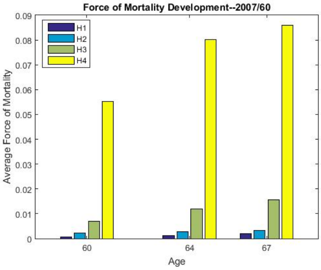
Mortality Intensities IV

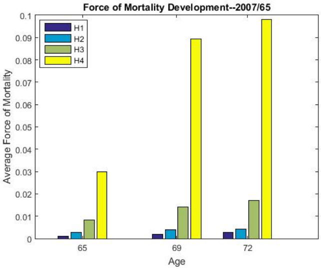
- Data Analysis
  - Period Force of Mortality – 2007/2011



Mortality Intensities V

- Cohort Force of Mortality – 2007/60,2007/65





Health Transition Intensities I

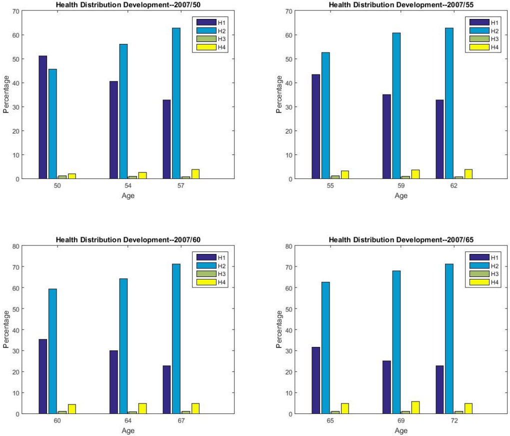
1. Estimation Method
   - $\lambda_{i,t} = a_i + b \cdot e^{c \cdot t}, i = 1, 2, 3$
   - Getting health distribution from prevalence data
   - Minimizing the calibration error:

$$\beta^* = argmin_\beta \sqrt{\sum_{\tau=1}^{n} (\pi(\tau) - \bar{\pi}(\tau))^2}$$

where $\pi(t) = [\pi_1(t), \pi_2(t), \ldots, \pi_4(t)]$ and $\beta = (a_i, b, c)$

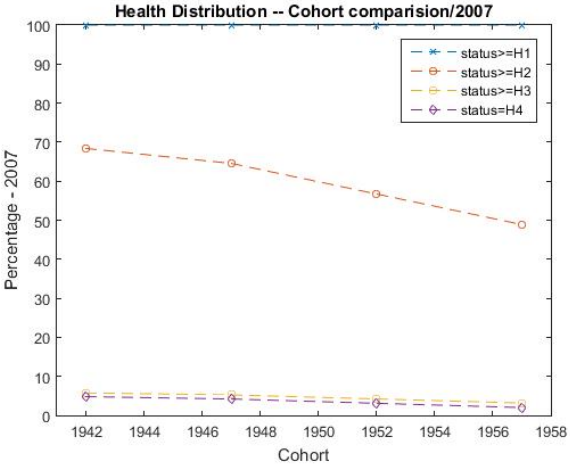
Health Transition Intensities II

2. Data Analysis
   - Health Distribution – Age Trend



Health Transition Intensities III

- Health Distribution – Cohorts Comparison



Summary

1. Focusing on establishing a multiple state mortality model considering health heterogeneity.
2. Aims to better capturing the trend and uncertainty of mortality development by involving the ATSMs into MAMs.
3. Working on fitting the model by combing the health and status dependent mortality data.
4. Future work: link to retirement product design and retirement planning.

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