Dynamic adoption of Central Bank Digital Currency in a stochastic game

Daniele Marazzina
Department of Mathematics
Politecnico di Milano, Italy.

Joint paper with
Barucci, E. and **Brachetta, M.**

Bayes Business School - March 22^{nd}, 2023

Agenda

1. Overview and main takeways

Central Bank Digital Currency (CBDC)

Central Bank Digital Currency (CBDC) is a new form of money that exists only in digital form. Instead of printing money, the central bank issues widely accessible digital coins so that digital transactions and transfers become simple.

- is like banknotes, legal tender feature, but it is only digital
- is not like electronic payments/cryptoassets (privately managed).
- may share some features with cash (anonymity)
- may be cost effective.

1. Overview and main takeways

Is going to be adopted by citizens?
We provide a model on its adoption and some key features: remuneration, subsidies and anonymity.

Takeaways:

1. Adoption of CBDC is high and it depends on remuneration and technology productivity.
2. Remuneration effectiveness depends on transaction costs: if they are low it is better to remunerate CBDC as reserves, otherwise it is better to provide a subsidy with no remuneration.
3. Competition with an anonymous stablecoin is in favor of CBDC, multiple equilibria arise.

2. Motivation

Two facts:

- Citizens use commercial or private money instead of central bank (CB) money.
- Surge of interests on cryptoassets (not only Bitcoin but also stablecoins).

Main issue: CB money is not anymore central, risks for citizens (bank runs, monetary sovereignty).

Is the CBDC the answer?

Maybe, but..... drawbacks: substitution between deposits and CBDC. Financial intermediation at risk.

Goal: CBDC should have success but not too much.

Research question:

- How to design CBDC? Remuneration, anonymity, constraints. Should the CBDC look like cash or like commercial money?

3. Model formulation

We consider three types of agents:

- an infinite number of potential users differentiated by their preferences for digital payments, $i \in [0, 1]$;
- the CB issuing a CBDC on a platform;
- a developer (either a financial intermediary or a miner/validator) providing work for the platform.

We need to model:

- technology
- interest rates: CB deposit facility; commercial deposits; CBDC rate.
- the objective functions of the agents.

Technology

We follow Cong et al. RFS (2021) on Bitcoin.

Given a reference probability space, we define the platform **productivity** A:

$$dA_t = L_t A_t(\mu\, dt + \sigma_A\, dW_t^A), \qquad A_0 > 0. \tag{3.1}$$

where

- L is the **labor force** by developers or financial intermediaries,
- W^A a Wiener process.

Interest rates

European Central Bank (ECB), and many other CBs, have decided not to introduce a new policy rate for CBDC. Its remuneration will be linked to the Deposit Facility Rate (remuneration of reserves by the ECB).
The deposit facility rate r_t^{DF} is modeled as a two-states Markov chain M:

$$r_t^{DF} \doteq r^{DF}(M_t) = \begin{cases} \underline{r} & \text{if } M_t = 0, \\ \overline{r} & \text{if } M_t = 1, \end{cases} \tag{3.2}$$

with $0 \leq \underline{r} < \overline{r}$ and switching probability λ.
The interest rate of commercial deposits $\widehat{r}_t$ is modeled as the deposit facility rate plus a mean reverting spread:

$$d\widehat{r}_t = dr_t^{DF} + dr_t, \qquad \bar{r}_0 > 0, \tag{3.3}$$

where r_t is modeled as a CIR process:

$$dr_t = \bar{a}(\bar{b} - r_t)\,dt + \sigma_r \sqrt{r_t}\,dW_t^r, \qquad r_0 > 0. \tag{3.4}$$

Agents

We consider a continuum of agents indexed by $i \in [0, 1]$.

- These agents are differentiated according to their transaction needs u_i: the higher is u_i the more the agent is willing to transact on the platform using CBDC rather than through payment systems that build on bank deposits, i.e., credit or debit cards.
- We may interpret u_i as the preference by the agent for anonymity (CBDC is likely to be more anonymous than standard electronic payments) and for a safer tool, being central bank's money.

We denote by F the cross-sectional cumulative distribution function of u_i, $i \in [0, 1]$, assuming that $F\colon [0, +\infty) \to [0, 1]$ is continuous and invertible, that is

$$u_i = F^{-1}(i) \quad \forall i \in [0, 1].$$

The agents' problem

At each instant of time t, each agent $i \in [0,1]$ has to decide whether to use commercial money or CBDC.
As in Cong et al. (2021), the instantaneous payoff for agent i from holding $x_{i,t} \geq 0$ CBDC is

$$d\nu_{i,t} = \underbrace{x_{i,t}^{1-\alpha}(N_t^{\gamma} A_t u_i)^{\alpha} dt}_{\text{utility}} + \underbrace{(v_t - \varphi)dt}_{\text{net fixed cost}} - \underbrace{x_{i,t}(\widehat{r}_t - r_t^{CBDC})\, dt}_{\text{opportunity cost}} - \underbrace{kx_{i,t}dt}_{\text{prop. cost}}, \tag{3.5}$$

where

- N_t:number of platform users
- u_i: preference for digital payments, distributed according to F (continuous and invertible).
- v_t: money subsidy from the CB (conditional upon adoption)
- $\varphi > 0$: fixed cost for the user
- $k \geq 0$: proportional transaction cost
- $\widehat{r}_t$: interest rate on bank deposit
- r_t^{CBDC}: interest rate on CBDC.

$$d\nu_{i,t} = \underbrace{x_{i,t}^{1-\alpha}(N_t^{\gamma} A_t u_i)^{\alpha} dt}_{\text{utility}} + \underbrace{(v_t - \varphi) dt}_{\text{net fixed cost}} - \underbrace{x_{i,t}(\widehat{r}_t - r_t^{CBDC})\, dt}_{\text{opportunity cost}} - \underbrace{k x_{i,t} dt}_{\text{prop. cost}},$$

- utility is affected by a network effect (the number of agents adopting CBDC), technology productivity and agent's preference for digital payments.
- there is a fixed cost for adopting the CBDC, the CB can subsidize it.
- there is a proportional transaction fee.
- adopting the CBDC you loose the remuneration on deposit but you get the one on CBDC

The total payoff is

$$\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\, dy_{i,t}\right],$$

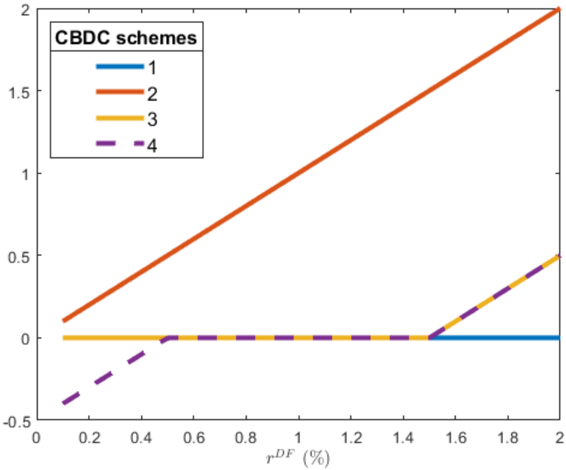
where $\rho > 0$ is the intertemporal discounting factor and

$$dy_{i,t} = \max\{0, \max_{x_{i,t}} d\nu_{i,t}\}.$$

CBDC remuneration

We compare four schemes for the CBDC remuneration as considered by ECB:

1. $r^{CBDC} = 0$ (like cash)
2. $r^{CBDC} = r^{DF}$ (like banks' reserves)
3. $r^{CBDC} = [r^{DF} - k_2]^+, \ \underline{r} < k_2 < \bar{r}$ (negative spread with respect to deposits)
4. $r^{CBDC}(t) = -[r^{DF}(t) - k_1]^- + [r^{DF}(t) - k_2]^+, \ \ \underline{r} < k_1, \ k_2 < \bar{r}$. (negative spread with respect to deposits plus negative interest rates)



Schemes 2 and 3 allow to control the disintermediation effect.
Scheme 4 allows negative interest rates.

The developer and the CB

The optimal quantity of CBDC held by the i-th agent at time $t \geq 0$ is denoted by $x^*_{i,t}$.
Then the gross CBDC demand (volume) is

$$V_t = \int_0^1 x^*_{i,t}\, di.$$

The financial intermediary chooses $L = \{L_t\}_{t \geq 0}$ to maximize

$$\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\,(\underbrace{kV_t}_{\text{revenues}} - \underbrace{(L_t + \theta L_t^2)}_{\text{costs}})\, dt\right].$$

The intermediary earns proportional transaction fees, exerts labor force with a quadratic cost.

The CB selects the subsidy $v = \{v_t\}_{t\geq 0}$ to minimize

$$\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(\underbrace{\omega\left(\frac{V_t}{A_t} - \overline{V}\right)^2}_{\text{target deviation}} + \underbrace{N_t v_t}_{\text{total subsidy}}\right) dt\right],$$

The target is provided by volume compared to the technology.
The CB minimizes the quadratic costs and the amount of subsidies.

4. Solving the model

Given the strategies of CB and intermediary, we can solve the agent's problem. There is a threshold $\overline{u}_t$ determining the adoption of the technology.

$$\begin{cases} x^*_{i,t} = \left(\frac{\widehat{r}_t - r_t^{CBDC} + k}{1-\alpha}\right)^{-\frac{1}{\alpha}} N_t^{\gamma} A_t u_i, \quad \text{if } u_i > \bar{u}_t \\ \bar{u}_t = \left(\frac{\widehat{r}_t - r_t^{CBDC} + k}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}} \frac{\varphi - v_t}{\alpha N_t^{\gamma} A_t}, \\ N_t = 1 - F(\bar{u}_t), \\ V_t = \left(\frac{\widehat{r}_t - r_t^{CBDC} + k}{1-\alpha}\right)^{-\frac{1}{\alpha}} N_t^{\gamma} A_t \int_{\bar{u}_t}^{+\infty} u \, dF(u). \end{cases} \tag{4.1}$$

Then we can evaluate the optimal response functions of CB (v^*) and developer (L^*) through optimal control stochastic techniques yielding an equilibrium.

The aggregate demand of CBDC and the number of adopters can be written as a function of the interest rate spread r_t, technology productivity and policy rate: $V_t = V(A_t, r_t, M_t)$, $N_t = N(A_t, r_t, M_t)$.

The value function for the developer is

$$W(a, r, m) = \max_{L \in \chi_t} \mathbb{E}\left[\int_t^{+\infty} e^{-\rho s}(kV_s - L_s - \theta L_s^2)\, ds \middle| A_t = a, r_t = r, M_t = m\right], \tag{4.2}$$

for all $(a, r, m) \in (0, +\infty)^2 \times \{0, 1\}$.

Exploiting the notation

$$W(a,r,0) = W^0(a,r) \quad \text{and} \quad W(a,r,1) = W^1(a,r)\ \forall (a,r) \in (0,+\infty)^2,$$

according to the Hamilton-Jacobi-Bellman (HJB) approach, we look for two suitable functions W^0 and W^1 satisfying the following differential equations:

$$\begin{cases} \rho W^0(a,r) = \mathcal{L}W^0(a,r) + kV(a,r) + \lambda(W^1(a,r) - W^0(a,r)) \\ + \max_{L\geq 0}\{\mu a L \frac{\partial W^0}{\partial a}(a,r) + \frac{\sigma_A^2}{2} a^2 L^2 \frac{\partial^2 W^0}{\partial a^2}(a,r) - L - \theta L^2\}, \\ \rho W^1(a,r) = \mathcal{L}W^1(a,r) + kV(a,r) + \lambda(W^0(a,r) - W^1(a,r)) \\ + \max_{L\geq 0}\{\mu a L \frac{\partial W^1}{\partial a}(a,r) + \frac{\sigma_A^2}{2} a^2 L^2 \frac{\partial^2 W^1}{\partial a^2}(a,r) - L - \theta L^2\}, \end{cases} \tag{4.3}$$

where $\mathcal{L}$ is the differential operator:

$$\mathcal{L}f(a,r) = a(b-r)\frac{\partial f}{\partial r}(a,r) + \frac{\sigma_r^2}{2} r \frac{\partial^2 f}{\partial r^2}(a,r) \tag{4.4}$$

A similar approach can be applied to the optimization problem of the CB

5. Model calibration

We calibrate interest rate processes from market data.

Deposit facility rate
We consider ECB official data from January 1999 to August 2022.
We set the facility rate at 0 in state 0 and the rate in state 1 is determined minimizing the average square distance between the observed value and the closest of the two level.
λ is estimated as the reciprocal of the average length of the periods between two consecutive jumps in the two levels version.

Deposit rate
We take the EONIA as a proxy for the deposit rate. We calibrate the process r_t from the spread between EONIA and the deposit facility rate, with daily data from January 1999 to December 2021.
We apply a log-likelihood procedure.

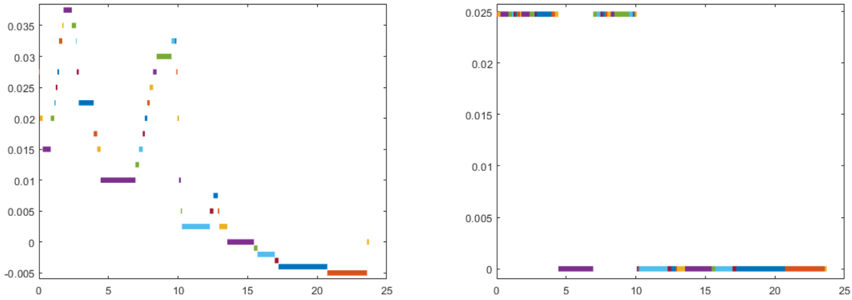


Figure: Observed deposit facility rate (left), two state estimate (right) .

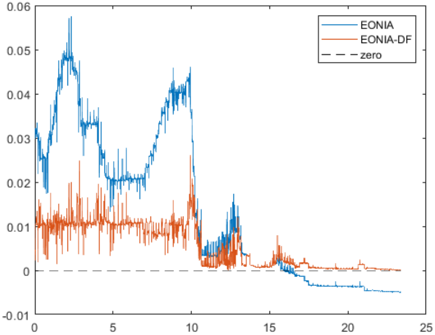


Figure: Spread between EONIA and deposit facility rate.

Baseline parameters

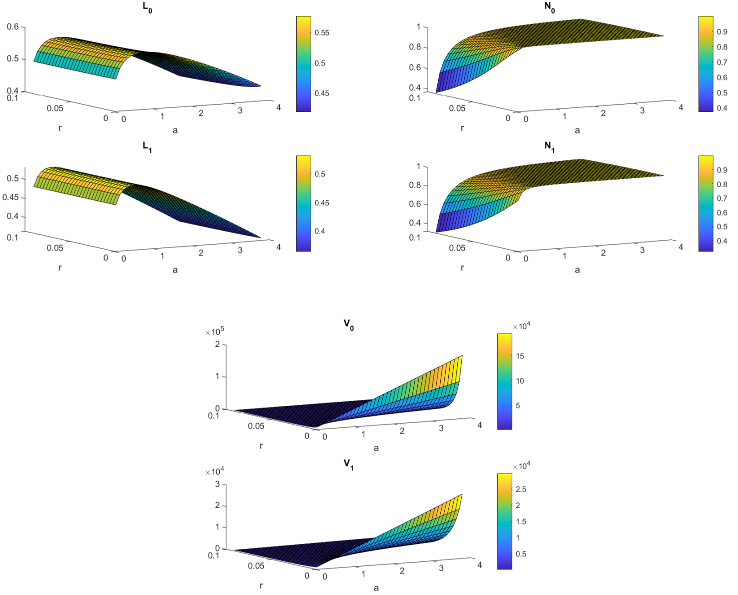
The parameters of the interest rate processes are:

- Parameters of process (3.4): $\bar{a} = 4.4157$; $\bar{b} = 0.0053$; $\sigma_r = 0.2011$;
- Deposit facility parameters: $\underline{r} = 0$, $\bar{r} = 2.475\%$, $\lambda = 0.1686$;

As fa as the other parameters are concerned, we set:

- productivity parameters: $\mu = 0.25$; $\sigma_A = 0.75$;
- Instantaneous payoff and expected value of future revenues parameters: $\gamma = 0.1$; $\alpha = 0.4$; $\varphi = 1$; $k = 0.01$; $\theta = 0.04$;
- Intertemporal discounting factor: $\rho = 0.035$;
- F is lognormal, with mean 0.2 and standard deviation 1.5;
- target $\bar{V} = 2000$ and $\omega = 0.5$;
- in the third and fourth remuneration scheme we set $k_1 = 0.005$, $k_2 = 0.015$.

Figure: $r^{CBDC} = [r^{DF} - k_2]^+$ & No Central Bank: Optimal control L, optimal N and optimal total amount V.



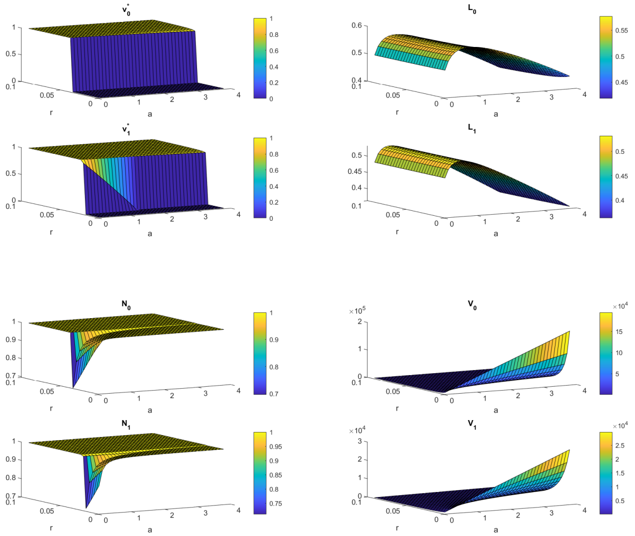
Participation rate increases as the productivity goes up because of the higher utility benefit
Participation rate decreases as the deposit interest rate increases because the opportunity cost of detaining CBDC becomes higher.

The same pattern is observed for CBDC volume.

The adoption rate is very high, for a large set of the state space we reach 100%. An interest rate higher than 5% together with a low productivity is needed to observe a low adoption rate.

As far as the job effort is concerned, we observe that it is increasing and then decreasing in A.

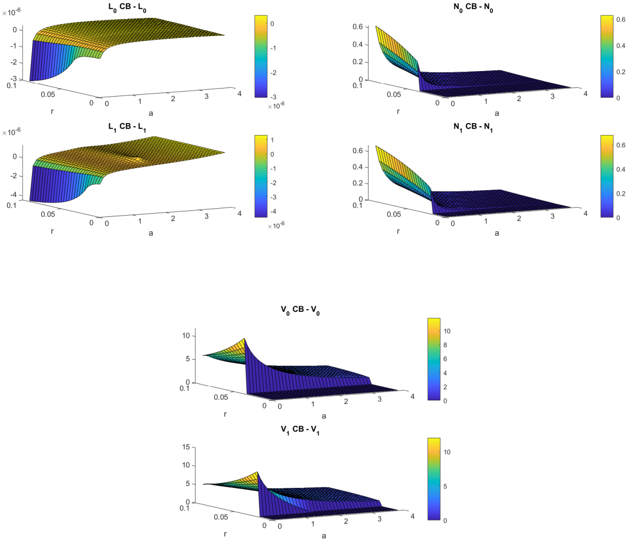
Figure: $r^{CBDC} = [r^{DF} - k_2]^+$ & Central Bank: Optimal v, optimal control L, optimal N and optimal total amount V.



With CB's intervention:

- v^* is high for high interest rates, null for low ones. The CB's subsidy is always less or equal than the adoption fee $\varphi = 1$.
- The pattern of volume and job effort with respect to A and r is similar to what is observed with no CB's intervention.
- A different pattern emerges for the participation rate: a U-shape pattern with respect to the interest rate is observed for low productivity.

Figure: Differences between the values with and without CB.



In the no intervention region (low interest rate), no difference between the two models is observed in job effort, volume and participation rate.

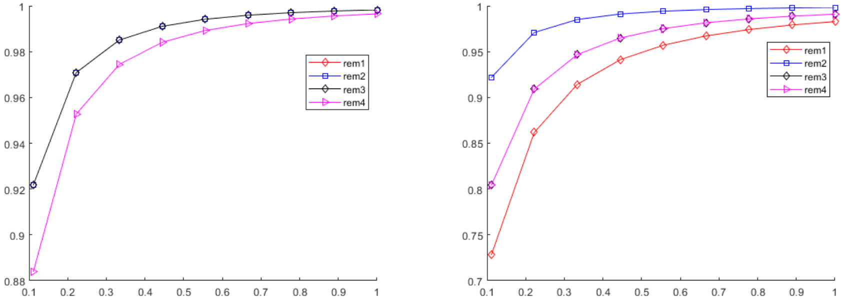
For high productivity and low interest rate, the effect of CB's intervention is almost null.

The CB drastically impacts on N when productivity is low and the interest rate increases, pushing towards full participation. For any level of productivity, the delta in the participation rate due to the intervention of the CB is increasing in the interest rate.

For a low productivity the delta in volume is not monotonic but bell-shaped: as interest rate goes up, the CB starts to subsidize consumers, an increase in volume is observed but then the effect is decreasing.

The effect on job effort goes in the opposite direction: the CB's intervention induces a smaller job effort mostly when the productivity is low and the interest rate is high.

Figure: No Central Bank: Optimal N as a function of a for the different remuneration schemes, for state 0 (left) and state 1 (right). $r = 1\%$.



The productivity rate A affects the adoption rate N for the four different remuneration schemes in the two states.

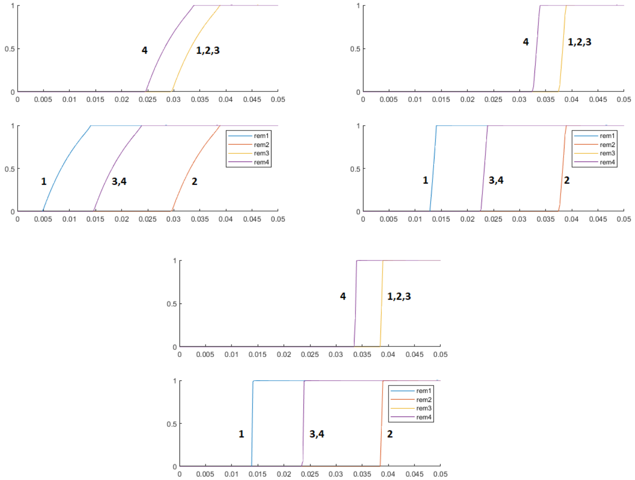
In state 0 (low interest rate), the first three remuneration schemes render a null remuneration and therefore the adoption rates coincide, for the fourth remuneration scheme we have a negative remuneration of the CBDC and therefore the adoption rate of CBDC is lower than for the other schemes.

In state 1 (high interest rate): the second scheme yields a remuneration that coincides with r^{DF}, remuneration of CBDC for scheme 3 and 4 is 0.975%, according to the first scheme, remuneration is null. As expected, the lower is the remuneration of CBDC the lower is the adoption rate.

Remuneration significantly affects the adoption rate (between 10 and 20%) for low productivity, the effect is decreasing as productivity increases.

To boost adoption, CBDC should be remunerated as reserves.

Figure: Central Bank v_t^* as a function of r: $a = 0.01$ (upper left), $a = 0.1$ (upper right), $a = 0.5$ (lower). State 0 (upper part of the figures), state 1 (lower part).



To further investigate the effect associated with the intervention of the CB, we plot v^* as a function of r for different values of the productivity ($a = 0.01,\ 0.1,\ 0.5$), for the four remuneration schemes in the two states (0 and 1).

- as productivity increases, the CB starts to subsidize adoption at higher deposit spreads independently of the remuneration scheme.
- the CB starts to subsidize adoption at a lower rate in the high deposit facility rate state (state 1).
- in case of a high deposit facility rate, the deposit spread trigger for the subsidy by the CB is the lowest in case of no remuneration and is the highest in case of a remuneration like the deposit facility rate. In case of a low deposit facility rate (state 0), the lowest trigger is observed for a negative remuneration.
- the intermediate region is wide only for small values of productivity, otherwise the transition in the optimal solution is quick.

We conduct Monte Carlo simulations (T=3 years).

	$k = 0.01$		$k = 0.02$	
Payoff	Mean	Variance	Mean	Variance
		V/A		
1	33150.33	908756499.52	8182.84	30663811.51
2	46914.09	723384955.52	11126.41	17174132.93
3	34280.32	845365910.22	8622.68	25629006.51
4	16138.58	98805386.26	5669.52	6365379.51
		V		
1	5239.38	58701553.76	1300.71	2551996.52
2	7345.90	66596741.42	1753.66	2853766.11
3	5408.86	56723287.53	1367.66	2429229.77
4	2548.44	8730367.49	900.87	848085.56

Table: No Central Bank. Mean and variance considering 1 000 000 Montecarlo simulations with weekly time step, $A_0 = 0.1$, $r_0 = 1\%$.

	$k = 0.01$		$k = 0.02$	
Payoff	Mean	Variance	Mean	Variance
		N		
1	0.88	0.02	0.82	0.02
2	0.94	0.00	0.87	0.01
3	0.91	0.01	0.84	0.02
4	0.88	0.01	0.81	0.02
		L		
1	0.57	0.00	0.57	0.00
2	0.56	0.00	0.56	0.00
3	0.57	0.00	0.56	0.00
4	0.56	0.00	0.56	0.00

Table: No Central Bank. Mean and variance considering 1 000 000 Montecarlo simulations with weekly time step, $A_0 = 0.1$, $r_0 = 1\%$.

The remuneration scheme weakly affects the adoption rate, but it significantly affects transaction volume.

In the no intervention by the CB, the second remuneration scheme is the most effective to favor the adoption of CBDC. The third remuneration scheme ranks second performing slightly better than the no remuneration case. The fourth one performs the worst.

Intervention by the CB doesn't change significantly the adoption rate with the exception provided by the no remuneration (first scheme) in case of a high transaction cost. In this case, the CB significantly subsidizes the adoption of CBDC and the scheme performs the best.

Payoff	$k = 0.01$ Mean	Variance	$k = 0.02$ Mean	Variance
		V/A		
1	33154.11	908525246.86	8205.21	30363039.26
2	46914.17	723378037.23	11126.93	17164072.42
3	34280.82	845333470.81	8626.57	25574824.69
4	16139.16	98788524.70	5674.17	6328558.36
		V		
1	5239.70	58698093.46	1302.60	2547185.60
2	7345.90	66596653.47	1753.69	2853617.13
3	5408.90	56722812.10	1367.98	2428442.51
4	2548.49	8730367.49	901.23	847529.16

Table: With Central Bank. Mean and variance considering 1 000 000 Montecarlo simulations with weekly time step, $A_0 = 0.1$, $r_0 = 1\%$.

	$k = 0.01$		$k = 0.02$	
Payoff	Mean	Variance	Mean	Variance
		N		
1	0.89	0.01	0.88	0.01
2	0.94	0.00	0.87	0.01
3	0.91	0.01	0.85	0.01
4	0.89	0.01	0.82	0.01
		L		
1	0.57	0.00	0.57	0.00
2	0.56	0.00	0.56	0.00
3	0.57	0.00	0.56	0.00
4	0.56	0.00	0.56	0.00
		Cumulative Cost		
1	0.05	0.02	0.32	0.26
2	0.00	0.00	0.01	0.00
3	0.01	0.00	0.05	0.02
4	0.01	0.00	0.07	0.02

Table: With Central Bank. Mean and variance considering 1 000 000 Montecarlo simulations with weekly time step, $A_0 = 0.1$, $r_0 = 1\%$.

The ranking among the four remuneration schemes is always confirmed considering the transaction volume.

The job effort is insensitive both to the remuneration schemes and to the intervention by the CB.

Remunerating CBDC and subsidizing its adoption are complementary approaches. In case of low transaction costs it is recommended to remunerate the CBDC as reserves, in case of high transaction costs it is better to provide a direct subside and no remuneration.

8. Stablecoin

Holding $x_{i,t}$ units of a stablecoin, with a certain degree of anonymity compared to CBDC, yields the following utility:

$$d\tilde{\nu}_{i,t} = (x_{i,t})^{1-\alpha}(\widetilde{N}_t^{\gamma}\tilde{A}_t \eta u_i)^{\alpha} dt - \tilde{\varphi} dt - x_{i,t}\widehat{r}_t\, dt - \tilde{k} x_{i,t} dt,$$

where $\eta \geq 1$ is the degree of transaction anonymity. When $\eta = 1$ we get indifference for anonymity, while $\eta \to +\infty$ represents total focus on anonymity.

The stable coin is not remunerated, and it is not related to any intervention of the CB. Tradeoff: remuneration+subsidy vs. anonymity

We assume that the single agent will hold $x_{i,t}$ of either CBDC or stablecoin solving the problem

$$dy_{i,t} = \max_{x_{i,t}} \{d\nu_{i,t}, d\tilde{\nu}_{i,t}, 0\}.$$

Payoff	η	state	N	$\widetilde{N}$	V	$\widetilde{V}$	$\partial_t U$	$\partial_t \widetilde{U}$	$\partial_t U + \partial_t \widetilde{U}$
3	1.0	0	0.00	0.92	0	1835	0	29851	29851
			0.92	0.00	1835	0	29851	0	29851
3	1.0	1	0.00	0.71	0	235	0	1103	1103
			0.79	0.00	443	0	3053	0	3053
3	1.5	0	0.00	0.95	0	2766	0	67771	67771
			0.92	0.00	1835	0	29851	0	29851
3	1.5	1	0.00	0.80	0	360	0	2573	2573
			0.79	0.00	443	0	3053	0	3053

Table: Multiple Solutions for $v = 0$

Payoff	η	state	N	$\widetilde{N}$	V	$\widetilde{V}$	$\partial_t U$	$\partial_t \widetilde{U}$	$\partial_t U + \partial_t \widetilde{U}$
3	1.0	0	1.00	0.00	1855	0	30437	0	30437
3	1.0	1	1.00	0.00	458	0	3233	0	3233
3	1.5	0	0.13	0.87	7	2730	91	66285	66376
			0.98	0.02	1342	512	21986	8430	30416
			1.00	0.00	1855	0	30437	0	30437
3	1.5	1	1.00	0.00	458	0	3233	0	3233

Table: Multiple Solutions for $v = 1$

Without CB intervention, there are always two equilibria (independently on η) and the two platforms never coexist.

In case with full CB intervention ($v = 1$) and $\eta = 1$, we only have one solution, with everybody using the CBDC. As η (anonymity of stablecoin) increases, multiple solutions appear, with only the CBDC or where the two platforms co-exist.

Notice that, in all the equilibria with CB's intervention, $N + \widetilde{N} = 1$, that is, all agents exploit one of the two platform. Instead in case of no intervention there is not full adoption of one of the two platforms. We can conclude that the two platforms with a different degree of anonymity coupled with a subsidy by the CB render an equilibrium where both platforms are used with the highest level of satisfaction by citizens.

9. Conclusions

- The CBDC adoption is strongly influenced by the platform productivity (↑) and interest rates (↓).
- The CB plays an active and key role. The subsidy impact depends on market conditions and the remuneration scheme. It is more effective on the participation rate than volume and job effort.
- The second scheme seems more effective without CB intervention.
- The third and fourth schemes seem less effective in terms of adoption and demanded volume, but they are also less expensive.
- The first scheme is the most effective in case of high transaction costs, but brings low demand and the highest CB cost.
- Competition with a stablecoin leads to multiple equilibria, coexistence of both tools of payment is achieved only in case of subsidies.