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NDC Pension Systems under Socioeconomic Longevity Inequality: Financial Sustainability, Actuarial Fairness and Implicit Redistribution

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Motivation

- NDC systems are designed to mimic funded DC schemes in a PAYG setting
- Actuarial equivalence is a central design principle
- Empirical evidence shows a strong **socio-economic gradient in longevity**

$$e_{R_1} < e_{R_2} < \cdots < e_{R_m}$$

Question: what happens to actuarial consistency when survival differs systematically across income groups?

NDC in a nutshell

Notional Defined Contribution (NDC)

- PAYG public pension scheme
- Fixed contribution rate (% of salary)
- Individual notional accounts
- Contributions are virtually capitalised
- At retirement, notional capital is converted into a life annuity

Hybrid structure:

- transversal PAYG logic
- longitudinal DC-like actuarial logic

Research question

In a **steady-state** PAYG NDC system with heterogeneous mortality:
Can we simultaneously satisfy:

- PAYG equilibrium (R1): $C(t) = B(t)$,
- cohort actuarial equilibrium (R2): $PVPC(\tau) = PVFB(\tau)$,
- actuarial fairness across socio-economic classes (R3): $\frac{PVFB_i(\tau)}{PVPC_i(\tau)} = K(\tau) \quad \forall\, i$,
- total attribution of contribution (R4): $C(t) = C^*(t)$,

using **uniform NDC rules**?

If not:

- which constraint must be relaxed,
- and where should redistribution be implemented?

Longevity heterogeneity

Socio-economic gradient of longevity

- Strong empirical link between income and survival
- Higher income → higher life expectancy
- Longevity gaps increase over time

Actuarial issue: regressivity

- Uniform annuitisation uses **average mortality**
- Longer-lived groups receive higher expected lifetime benefits

Ethical requirement for public pensions:

- avoid systematic transfers from short-lived to long-lived groups

General framework

- Heterogeneous population: each individual has a specific salary and a survival pattern
- No perfect ranking between salary and longevity is assumed
- The model allows for possible dependence between salary and post-retirement survival
- For each newly retired individual i: $S(i)$ is salary at retirement and ${}_hp_i$ is the probability of surviving h years after retirement
- Assumptions:
  - career length (N), contribution rate (π), notional interest rate (r), growth rate of new cohorts (η), salary growth rate (γ), and pension indexation rate (λ) are constant
  - same salary distribution at each age

Notation

Retirement cohort size: K^*

Average salary: $\mathbb{E}(S) = \frac{1}{K^*}\sum_{i=1}^{K^*} S(i)$

Annuity factor:

$$A(i) := \sum_{h=1}^{\infty} {}_hp_i \left(\frac{1+\lambda}{(1+\gamma)(1+\eta)}\right)^h$$

Average annuity factor: $\mathbb{E}(A) = \frac{1}{K^*}\sum_{i=1}^{K^*} A(i)$

Growth factors

$$H(r,\gamma) := \sum_{h=0}^{N-1} \left(\frac{1+r}{1+\gamma}\right)^h$$

$$G(\eta) := \sum_{h=0}^{N-1} (1+\eta)^h$$

These terms summarize respectively:

- the accumulation of contributions into notional capital;
- the demographic growth structure of the system.

Standard NDC system with a common annuity factor

Income of the system:

$$In = \pi \cdot K^* \cdot \mathbb{E}(S) \cdot G(\eta) \quad (1)$$

Notional capital at retirement:

$$C(i) = \pi \cdot S(i) \cdot H(r, \gamma)$$

Pension at retirement with common annuity factor $\mathbb{E}(A)$:

$$P(i) = \frac{C(i)}{\mathbb{E}(A)} = \frac{\pi \cdot S(i) \cdot H(r, \gamma)}{\mathbb{E}(A)}$$

Aggregate outcome:

$$Out = \sum_{i=1}^{K^*} P(i) A(i) = \frac{\pi \cdot K^* \cdot H(r,\gamma) \cdot \mathbb{E}(SA)}{\mathbb{E}(A)} \quad (2)$$

Balance decomposition

Using

$$\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y) + \rho_{XY}\,\sigma_X\sigma_Y$$

we obtain

$$Out = \frac{\pi \cdot K^* \cdot H(r,\gamma) \cdot [\mathbb{E}(S)\mathbb{E}(A) + \rho_{SA}\sigma_S\sigma_A]}{\mathbb{E}(A)}$$

Hence,

$$Out = In\frac{H(r,\gamma)}{G(\eta)} + \pi \cdot K^* \cdot H(r,\gamma)\frac{\rho_{SA}\sigma_S\sigma_A}{\mathbb{E}(A)} \quad (3)$$

If salary and $A(i)$ are independent, the second term vanishes.

If also $H(r,\gamma) = G(\eta)$, the system is globally balanced.

Canonical NDC case

If

$$1 + r = (1 + \gamma)(1 + \eta)$$

then

$$H(r, \gamma) = G(\eta)$$

Equation (3) becomes

$$Out = In + \pi \cdot K^* \cdot G(\eta) \frac{\rho_{SA}\, \sigma_S \sigma_A}{\mathbb{E}(A)} \qquad (4)$$

Interpretation:

no dependence ⇒ global balance;

positive dependence ⇒ structural deficit.

Relaxing the constant-salary assumption

For each individual i,

$$S(i,h), \qquad h = 1,\ldots,N$$

with

$$S(i,N) \;=\; S(i)$$

Maintained assumption:

- in steady state, the cross-sectional salary distribution at each career year is the same across cohorts.

Generalized career setting

Income:

$$In = \pi \cdot K^* \sum_{h=1}^{N} (1 + \eta)^{N-h} \mathbb{E}(S_h) \quad (5)$$

where: $\mathbb{E}(S_h) = \frac{1}{K^*} \sum_{i=1}^{K^*} S(i,h)$

Notional capital:

$$C(i) = \pi \sum_{h=1}^{N} S(i,h) \left(\frac{1+r}{1+\gamma}\right)^{N-h} \quad (6)$$

Pension: $P\ (i) = \frac{C(i)}{\mathbb{E}(A)}$

Aggregate outcome:

$$Out = \sum_{i=1}^{K^*} P(i)A(i) = K^* \frac{\mathbb{E}(CA)}{\mathbb{E}(A)} \quad (7)$$

Key implication in the generalized career setting

In this setting we have:

$$Out = K^* \cdot \mathbb{E}(C) + K^* \frac{\rho_{CA}\sigma_C\sigma_A}{\mathbb{E}(A)} \quad (8)$$

Canonical NDC case:

$$1 + r = (1 + \gamma)(1 + \eta)$$

implies

$$\left(\frac{1 + r}{1 + \gamma}\right)^{N-h} = (1 + \eta)^{N-h}$$

so that $K^* \cdot \mathbb{E}(C) = In$

Therefore

$$Out = In + K^* \frac{\rho_{CA}\sigma_C\sigma_A}{\mathbb{E}(A)} \quad (9)$$

Discrete class approximation

General assumptions

- m socio-economic classes
- Class-specific salaries $s_i(x,t)$
- Class-specific survival after retirement ${}_hp_i(x)$

Demographic structure

- total population splitted in m classes: $L_x(t) = \sum_{i=1}^{m} L_x^i(t)$
- survival probability after retirement: ${}_hp_i(x) < {}_hp_j(x)$ for i $< j$

Salary structure

- $s_i(x,t) = s_i(x_0,0) \cdot (1+\gamma)^t \prod_{k=0}^{x-x_0-1}(1+\sigma_i(x_0+k)) \quad x_0 < x < x_r$
- Assumption: $s_i(x,t) < s_j(x,t)$ for i $< j$

Annuity rates

General (uniform) annuity rate

$$A(x_r) = \sum_{h=0}^{\omega - x_r} {}_hp(x_r) \left(\frac{1+\lambda}{1+r}\right)^h = \sum_{h=0}^{\omega - x_r} {}_hp(x_r) \left(\frac{1+\lambda}{(1+\gamma)(1+\eta)}\right)^h = \sum_{h=0}^{\omega - x_r} {}_hp(x_r) \left(\frac{1}{1+j}\right)^h$$

Class specific annuity rate

$$A_i(x_r) = \sum_{h=0}^{\omega - x_r} {}_hp_i(x_r) \left(\frac{1}{1+j}\right)^h$$

Properties

1. $A(x_r) = \sum_{i=1}^{m} w_i A_i(x_r)$ (w_i =proportion of class i in active population)
2. $A_i(x_r) < A_j(x_r)$ for i $< j$

Standard NDC under heterogeneity

Uniform rules (same contribution rate, same annuity divisor)

$$b_i(x_r) = \frac{\sum_{x=x_0}^{x_r-1} \pi \cdot s_i(x) \prod_{k=x}^{x_r-1}(1+r_k)}{A(x_r)}$$

Let us introduce the factor $A = \frac{\sum_{i=1}^{m} w_i \cdot A_i \cdot Q_i}{\sum_{i=1}^{m} w_i \cdot Q_i}$

with $A_i = \frac{A_i(x_r)}{A(x_r)}$ (longevity factor)

and $Q_i = s_i(x_0) \sum_{h=1}^{x_r-x_0}(1+\eta)^{x_r-x_0-h} \prod_{k=0}^{h-1}(1+\sigma_i(x_0+k))$

(salary factor)

We can demonstrate that R1 and R2 hold only if $A = 1$.

When $A > 1$ ($A < 1$) the system exhibits structural losses (gains).

Standard NDC under heterogeneity

In particular, if the classes with high salaries are associated with higher longevity, we generate a structural loss.

Proposition 1:
In a static environment with heterogeneous population and using canonical parameters, classical NDC will no more satisfies criteria R3, but also no more R1 (PAYG Equilibrium) and R2 (Global cohort equilibrium).

Strategy A: dual contribution rates

If we want to stay with an uniform system and restore actuarial PAYG equilibrium, we should pay more contributions without changing the benefits.
We introduce:

- paid rate π
- allocated rate π^*

$$b_i^{(A)}(x_r) = \frac{\sum_{x=x_0}^{x_r-1} \pi^* \cdot s_i(x) \prod_{k=x}^{x_r-1}(1+r_k)}{{}^{A}a(x_r)}$$

where $\pi^* = \frac{\pi}{A}$

The adjustment is actuarially correct but not intuitive.

Proposition 2:
In a static environment with heterogeneous population and using canonical parameters, dual NDC with the link between paid and allocated rates given by the ratio A satisfies criteria R1 and R2 but not R3.

Strategy B: ex-post adjustment

Use class-specific annuity at retirement (Holzman et al., 2019):

$$b_i^{(B)}(x_r) = \frac{\sum_{x=x_0}^{x_r-1} \pi \cdot s_i(x) \prod_{k=x}^{x_r-1}(1+r_k)}{A_i(x_r)}$$

Same accumulation phase as in standard NDC

Adjustment occurs **only at retirement**

Proposition 3:
In a static environment with heterogeneous population and using canonical parameters , NDC with class annuity rate at retirement age satisfies criteria R1, R2, R3, and R4.

Strategy B2: Alternative design based on progressivity

Using a finite number of classes at retirement age could be seen as a direct discrimination….

Switch from a class system to a progressive formula based on salaries (natural parameter in a pension computation)

$$b_i^{(\mathrm{B})}(x_r, t) = \frac{\sum_{x=x_0+1}^{x_r} \pi \cdot s_i\big(x, t-(x_r-x)\big) \cdot (1+r)^{x_r-x}}{A_i(x_r)}$$

$$b_i^{(\mathrm{B2})}(x_r, t) = \frac{\sum_{x=x_0+1}^{x_r} \pi \cdot s_i\big(x, t-(x_r-x)\big) \cdot (1+r)^{x_r-x}}{A(x_r, s_i)}$$

Progressive version of annuity rates

We define the annuity rate as a function of the different slices of salary

$$A(x_r, s_i) = \sum_{c=1}^{i} \alpha_c \frac{s_c(x_r, t) - s_{c-1}(x_r, t)}{s_i(x_r, t)}$$

$$A(x_r, s_1) = \alpha_1 = A_1(x_r)$$

$$A(x_r, s_2) = A_1(x_r)\frac{s_1(x_r, t)}{s_2(x_r, t)} + \alpha_2 \frac{s_2(x_r, t) - s_1(x_r, t)}{s_2(x_r, t)}$$

…

where $\alpha_k = \frac{A_k(x_r)s_k(x_r,t) - A_{k-1}(x_r)s_{k-1}(x_r,t)}{s_k(x_r,t) - s_{k-1}(x_r,t)}$

is the weighted marginal increase of annuity rate

Strategy C: ex-ante adjustment

Anticipate redistribution during accumulation (Holzman et al., 2019) :
Uniform annuity and class-specific allocated rates:

$$b_i^{(C)} = \frac{\sum_{x=x_0+1}^{x_r} \pi_i^* \cdot s_i\big(x, t-(x_r-x)\big) \cdot (1+r)^{x_r-x}}{A(x_r)}$$

With $\pi_i^* = \pi \frac{A(x_r)}{A_i(x_r)}$

Proposition 4:
In a static environment with heterogeneous population and using canonical parameters, NDC with ex-ante adjustment satisfies criteria R1, R2, R3.

We can also develop a progressive version of the allocated rate

Strategy C2: Alternative design based on progressivity

Switch from a class system to a progressive formula based on salaries

$$b_i^{(C)} = \frac{\sum_{x=x_0+1}^{x_r} \pi_i^* \cdot s_i\big(x, t-(x_r-x)\big) \cdot (1+r)^{x_r-x}}{A(x_r)}$$

$$b_i^{(C2)} = \frac{\sum_{x=x_0+1}^{x_r} \sum_{c=1}^{i} \pi^*(s_c)\big[s_c\big(x, t-(x_r-x)\big) - s_{c-1}\big(x, t-(x_r-x)\big)\big] \cdot (1+r)^{x_r-x}}{A(x_r)}$$

where

$$\pi^*(s_i) = \frac{\pi \cdot A(x_r)\left[\frac{\sum_{x=x_0+1}^{x_r} s_i\big(x, \mathrm{t}-(x_r-x)\big) \cdot (1+r)^{x_r-x}}{A^i(x_r)} - \frac{\sum_{x=x_0+1}^{x_r} s_{i-1}\big(x, -(x_r-x)\big) \cdot (1+r)^{x_r-x}}{A^{i-1}(x_r)}\right]}{\sum_{x=x_0+1}^{x_r}\big[s_i\big(x, \mathrm{t}-(x_r-x)\big) - s_i\big(x, \mathrm{t}-(x_r-x)\big)\big] \cdot (1+r)^{x_r-x}}$$

Strategy D: quasi NDC

Dual contribution rates but now the allocated rate is fixed for all the classes and the paid contribution are class specific (variable DC by class).
The computation of benefits are not differentiated by class but long-lived people have to pay more (explicit tax).

$$b_i^{(D)}(x_r, t) = \frac{\sum_{h=x_0+1}^{x_r} \pi^* \cdot s_i\big(x, t-(x_r - x)\big) \cdot (1+r)^{x_r - x}}{A(x_r)}$$

with a variable paid contribution rate (tax for long livers):

$$\pi_i = \pi^* \frac{A_i(x_r)}{A(x_r)}$$

We have therefore: $\pi_1 < \pi_2 < \cdots < \pi^* < \cdots < \pi_m$

Properties of Strategy D

Proposition 5:
In a static environment with heterogeneous population and using canonical parameters, the quasi NDC with class specific paid contribution rates rates $\pi_i = \pi^* \frac{A_i(x_r)}{A(x_r)}$ satisfies criteria R1, R2, R3.

We could also consider a progressive version with paid contributions defined by salary slices

Conclusions

Summary

- Longevity heterogeneity breaks standard NDC design even in the simple steady case
- The conflict is structural, not parametric

Three possible strategies (and their progressive version):

- Dual rates
- Ex-post annuity differentiation
- Ex-ante contribution differentiation

Future research

- What happens in a non stationary environment ?
- Fairness between classes defined through actuarial expectations: Other criteria of fairness? Utility tools?
- Differentiated mortality also before retirement age : what to do ?
  - integration of survival dividends (Arnold et al., 2024)
  - differentiated paid contribution for short lived people

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