Wrocław University of Economics

Internal transfer of longevity risk in households and its impact on household ability to participate in pension-related capital market

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Life-length risk

- Interpretation
  - Risk in negative sense (possibility of negative financial consequences for a household)
  - Individual
    - Longevity (unless bequest is considered)
  - Household
    - Longevity
    - Premature death

Life-length risk

- Interpretation
  - Early death
    - If very early indeed – an insurance event (not in the scope of retirement planning risk)
    - If shortly before retirement – positive effect on household finance
    - **If after retirement, but earlier than expected life length – usu. negative effect**

This is what we are interested in

Rationale

- Classical approach in personal finance:

$$EDU = \sum_{t=0}^{\infty} \left[\frac{1}{(1+\rho)^t} \left({}_t p_{\mathbf{x}(s)} \right) u(C_t) \right]$$

… hardly applicable for a two person household

- Why?

Rationale

- … since:
  - life-length risk (in its financial sense) of a household is two-fold: longevity and premature death

    (unlike risk of an individual which is only related to longevity)
  - consumption depends, amongst others, on how many household members are alive and who of them lives longer
  - retirement investment is assigned to the whole household and it is to the decision of household members whether to buy life annuity for the first or the second person or divide it in any other proportions
  - possible patterns of consumption term structure may end out with a surplus or not, after the last household member dies, and this difference between them must be taken into account

General concept

- 2 adult persons (main members of the household)
- 2 financial goals considered:
  - Retirement
  - Bequest
- 3 sub-classes of consumption:
  - Common (fixed and not assigned to any particular person),
  - Consumption of person 1
  - Consumption of person 1
- 2 investments: of person 1 and person 2 (contributions fixed in real terms)
- 2 available assets: risky and risk-free (returns fixed in real terms)
- Only risk related to length of life taken into consideration (no market, credit, legal risk, etc. considered)

The model

- Decision variables in optimization process
  - Consumption-investment proportion
  - Retirement investment division between investment maturing on the date R_1 and the date R_2
  - Decision which retirement capital is to be assigned to buy life annuity for which person

The model

- Value function

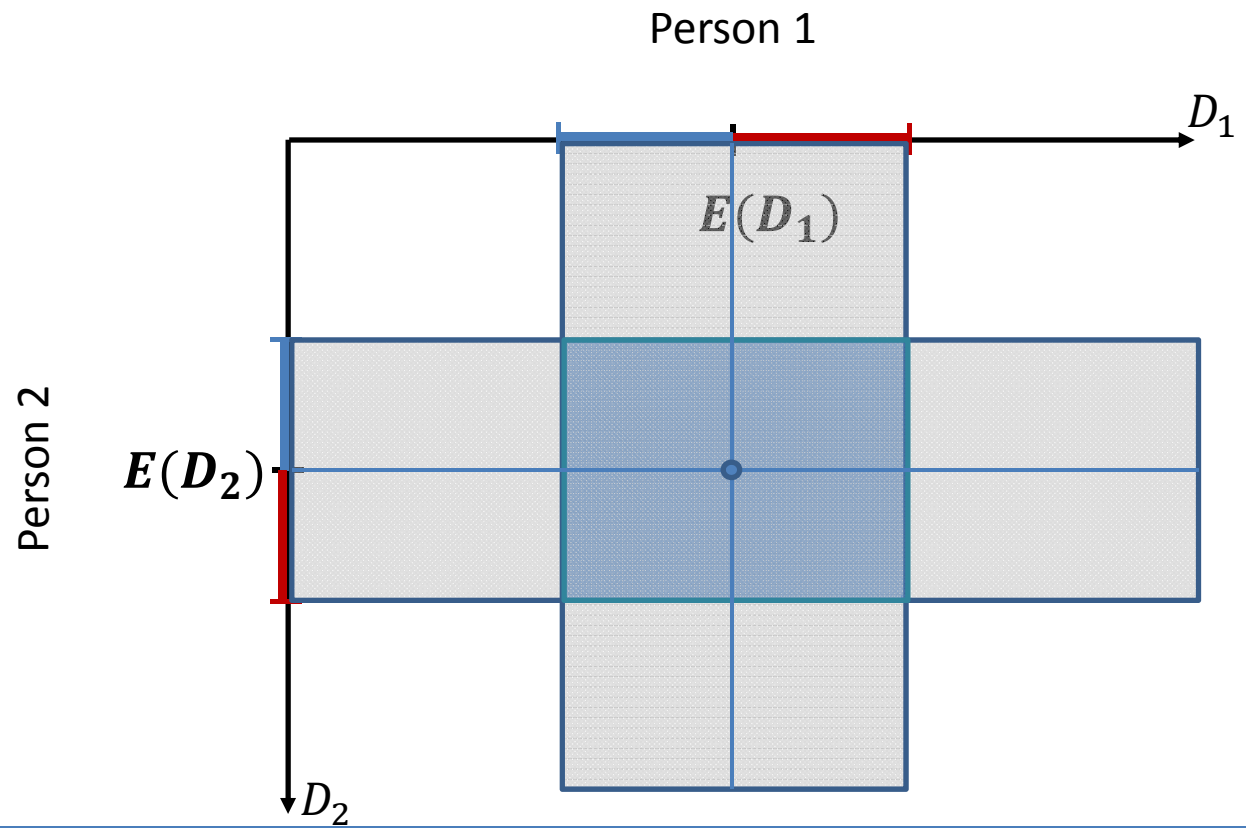
$$V(c_0, v; A) =$$

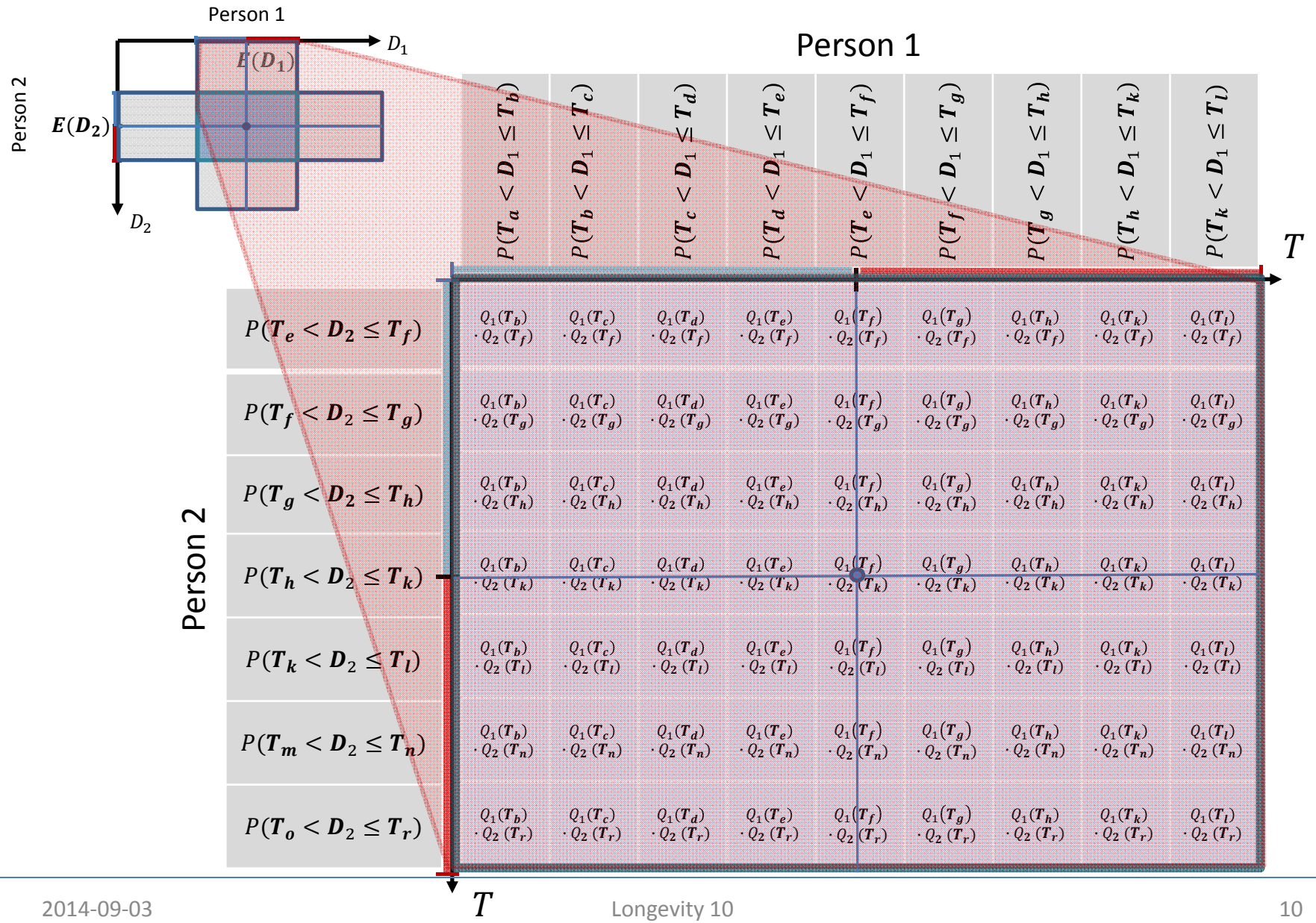
$$= \sum_{D_2^*=E(D2)-\gamma^*}^{E(D2)+\delta^*} \sum_{D_1^*=E(D1)-\gamma^*}^{E(D1)+\delta^*} p\left[\alpha\left(\sum_{t=0}^{max\{D_1^*,D_2^*\}} \frac{1}{(1+r_c)^t} u\big(C(t; D_1^*, D_2^*)\big)\big(\gamma(t)+\delta(t)\big)\right)+\right.$$

$$\left.\beta \frac{1}{(1+r_B)^{max\{D_1^*;D_1^*\}}} u\big(B(max\{D_1^*, D_2^*\}; D_1^*, D_2^*)\big)\right]$$

$$\longrightarrow max$$

Simplification




Person 1
D_1
$E(D_1)$
Person 2
$E(D_2)$
D_2
Person 1
$P(T_a < D_1 \leq T_b)$
$P(T_b < D_1 \leq T_c)$
$P(T_c < D_1 \leq T_d)$
$P(T_d < D_1 \leq T_e)$
$P(T_e < D_1 \leq T_f)$
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$P(T_g < D_1 \leq T_h)$
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Person 2
$P(T_e < D_2 \leq T_f)$
$P(T_f < D_2 \leq T_g)$
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The model

Household vs. disjoint variant

	Household	Disjoint variant
Fixed costs (consumption)	Joint	Divided into two equal parts
Variable costs (consumption)	Individually assigned to a person but then covered jointly	Individually assigned to a person
Incomes	Individually assigned but spent jointly	Individually assigned
Investment	The difference between joint income and joint consumption of the household	The difference between individual income and individual consumption
Private pension plan	Individually assigned but the accumulated capital may be used for purchasing life annuity for any person	Individually assigned and the accumulated capital is used to purchase life annuity for this particular person
Private retirement	Individually assigned but spent jointly	Individually assigned

The model

- The mechanism of risk transfer

  Risk transfer is implied by capital transfer between household members

  – In the disjoint variant, if the income differences are high, one person may underinvest her/his private retirement or even not be able to invest at all, whereas the second person overinvests

  – Even if incomes are equal and sufficient the differences in expected lifetime are count:

    - In a household there is a possibility of buying life annuity for the person who is expected to live longer with the whole capital accumulated by the two persons
    - In the disjoint variant each person buys annuity for her or himself with the capital accumulated by him or herself

Numerical example

- Assumptions
  - Household
    - Person 1 - Male:
      - 34 years old
      - Annual income: 31.000
      - Life expectancy: 74 years old (from statistical tables)
    - Person 2 - Female:
      - 32 years old
      - Annual income: 19.000
      - Life expectancy: 82 years old (from statistical tables)
  - Consumption
    - Common consumption: 20.000
    - Individual consumption of Person 2 is higher then consumption of Person 1 (10%)

Numerical example

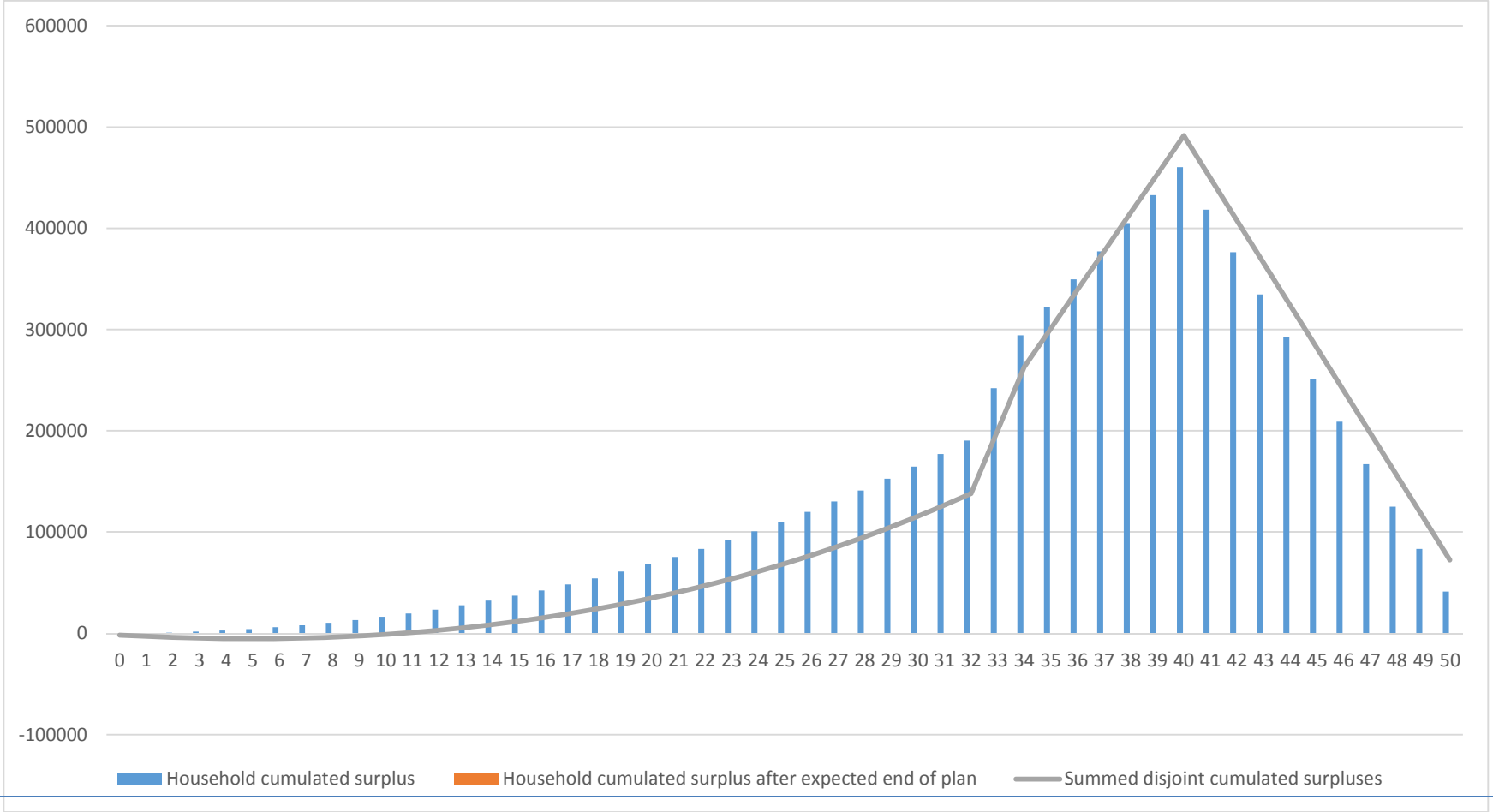
- Parameters
  - Replacement rate
    - Male: 40%
    - Female: 35%
  - Retirement age
    - both persons 67 years old
  - Expected rate of return on risky investment
    - 2,00% (in real terms)
  - Risk-free rate
    - 0,00% (in real terms – inflation indexed)

Numerical example

- Parameters
  - Income growth rate
    - Both persons 2,00% (in real terms)
  - Consumption growth rate
    - 1,60% (in real terms) for common consumption
    - 2,00% (in real terms) for individual consumption
  - Discount rates
    - Consumption discount rate: 10,00%
    - Bequest discount rate: 2,00%
  - Consumption and bequest parameters
    - $\alpha = 0,75$
    - $\beta = 0,25$

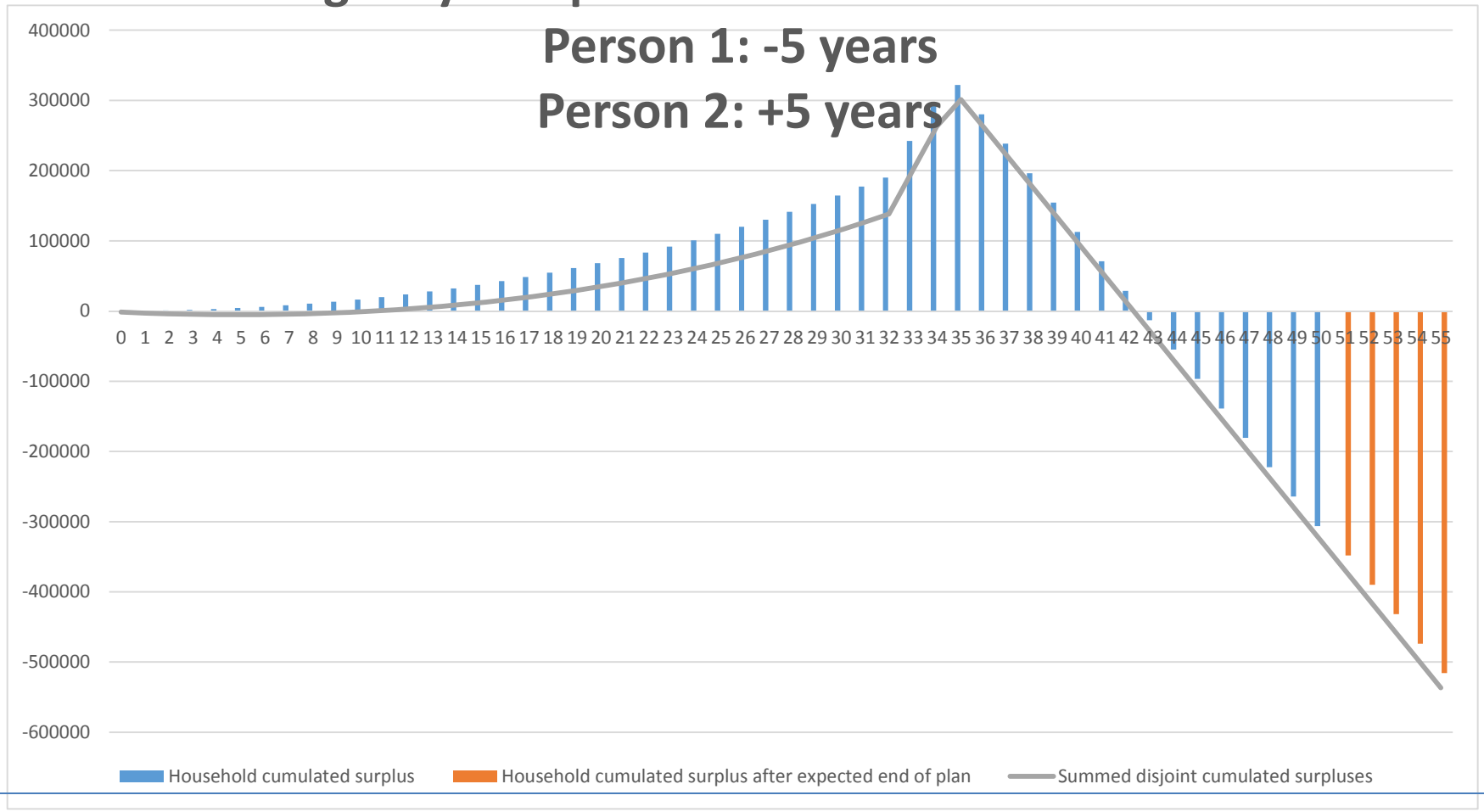
Optimized financial plan without risk aversion (household vs. disjoint variant)

under expected survival times



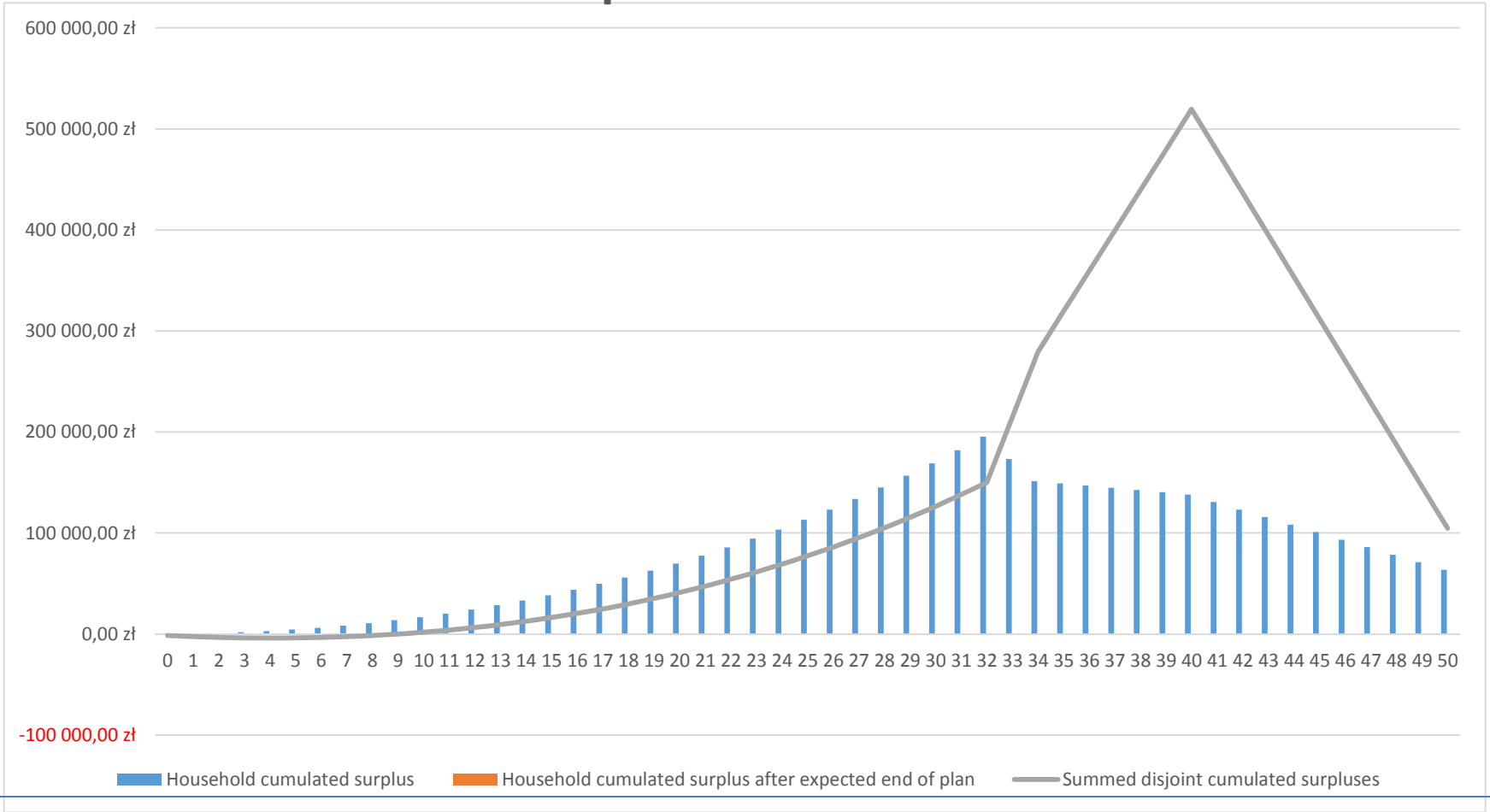
Optimized financial plan without risk aversion (household vs. disjoint variant)

Longevity and premature-death risk realized

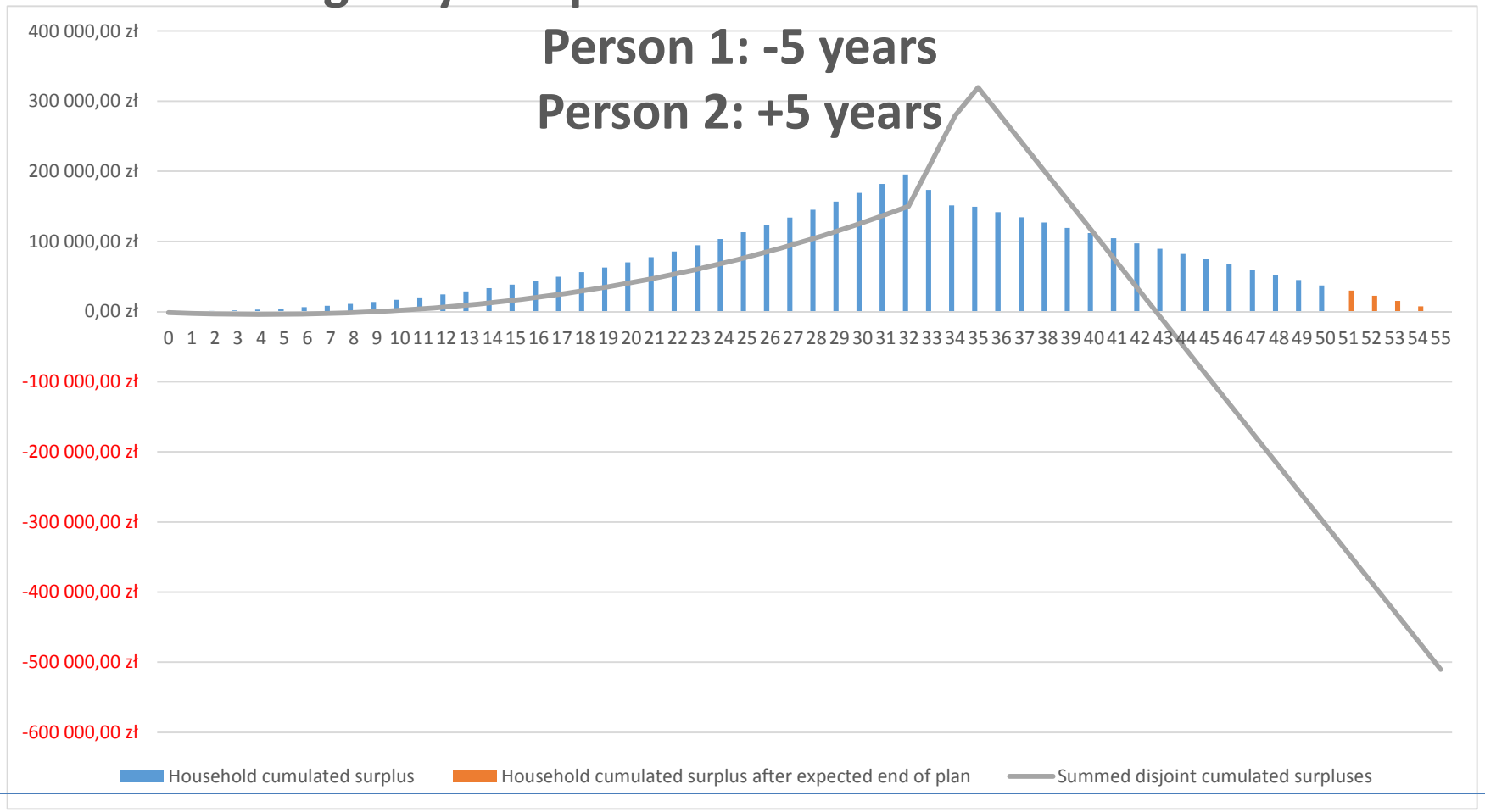


Optimized financial plan with risk aversion (γ=5, δ=5) (household vs. disjoint variant)

under expected survival times



Optimized financial plan with risk aversion (γ=5, δ=5) (household vs. disjoint variant)



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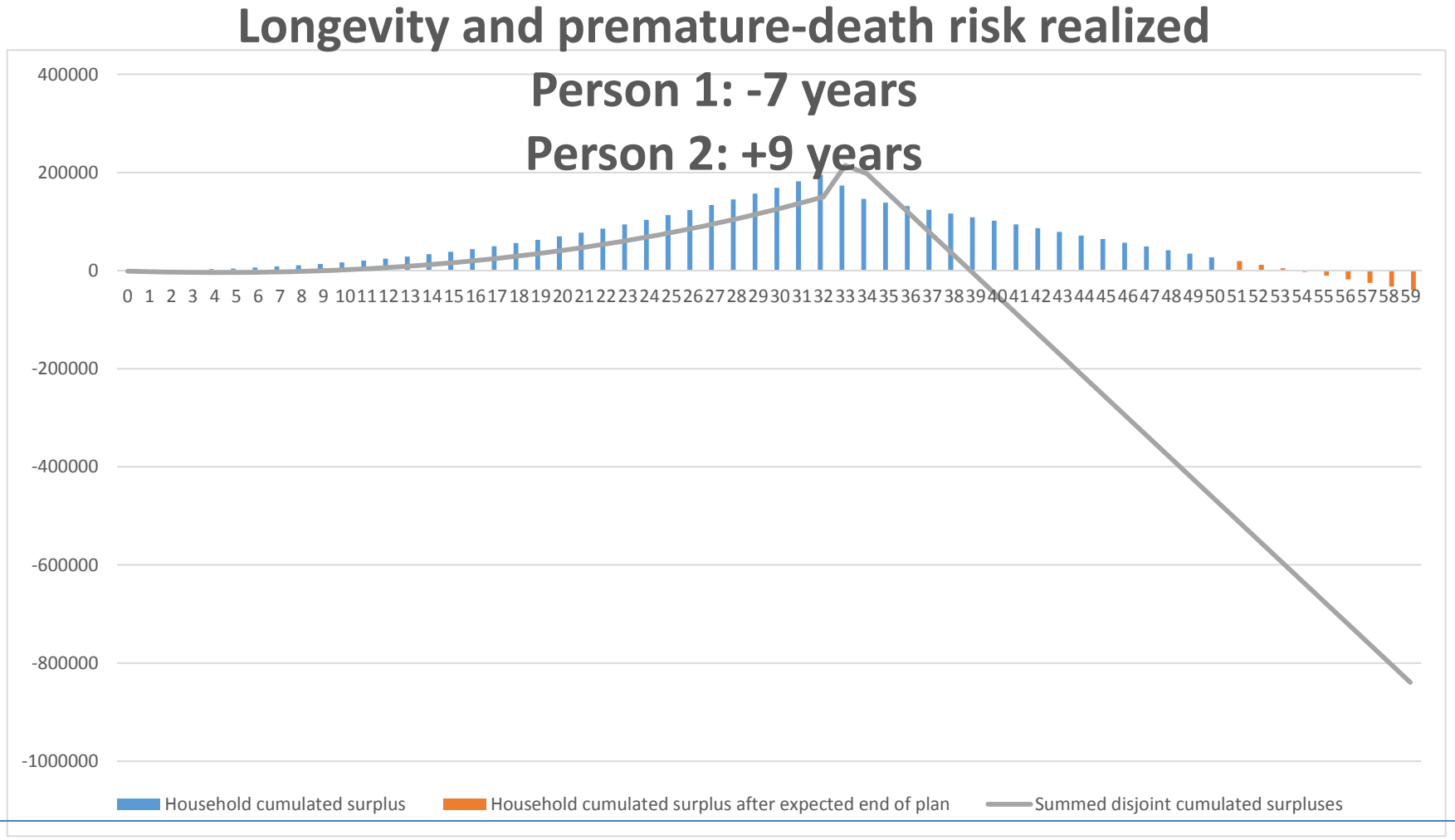
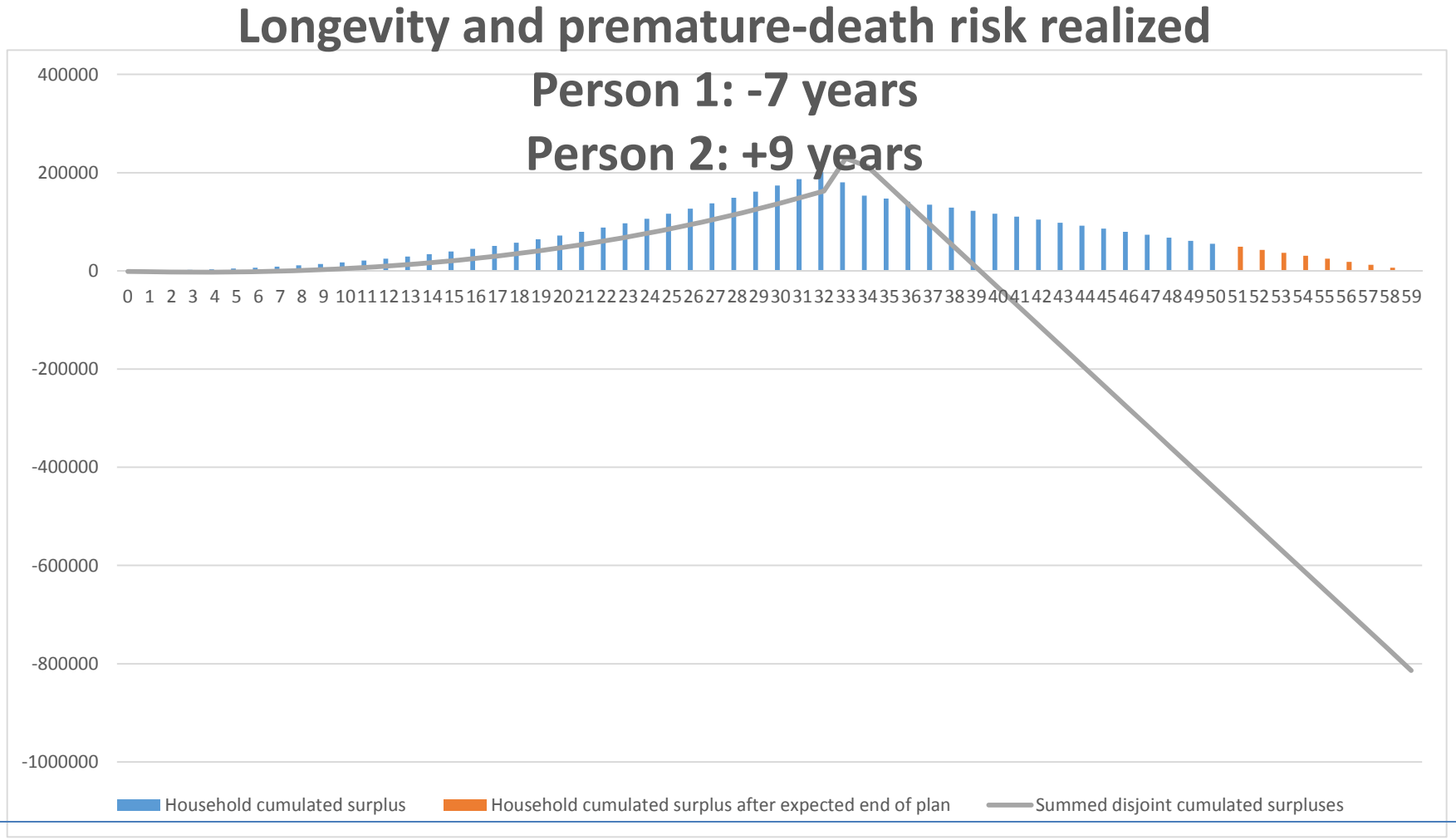
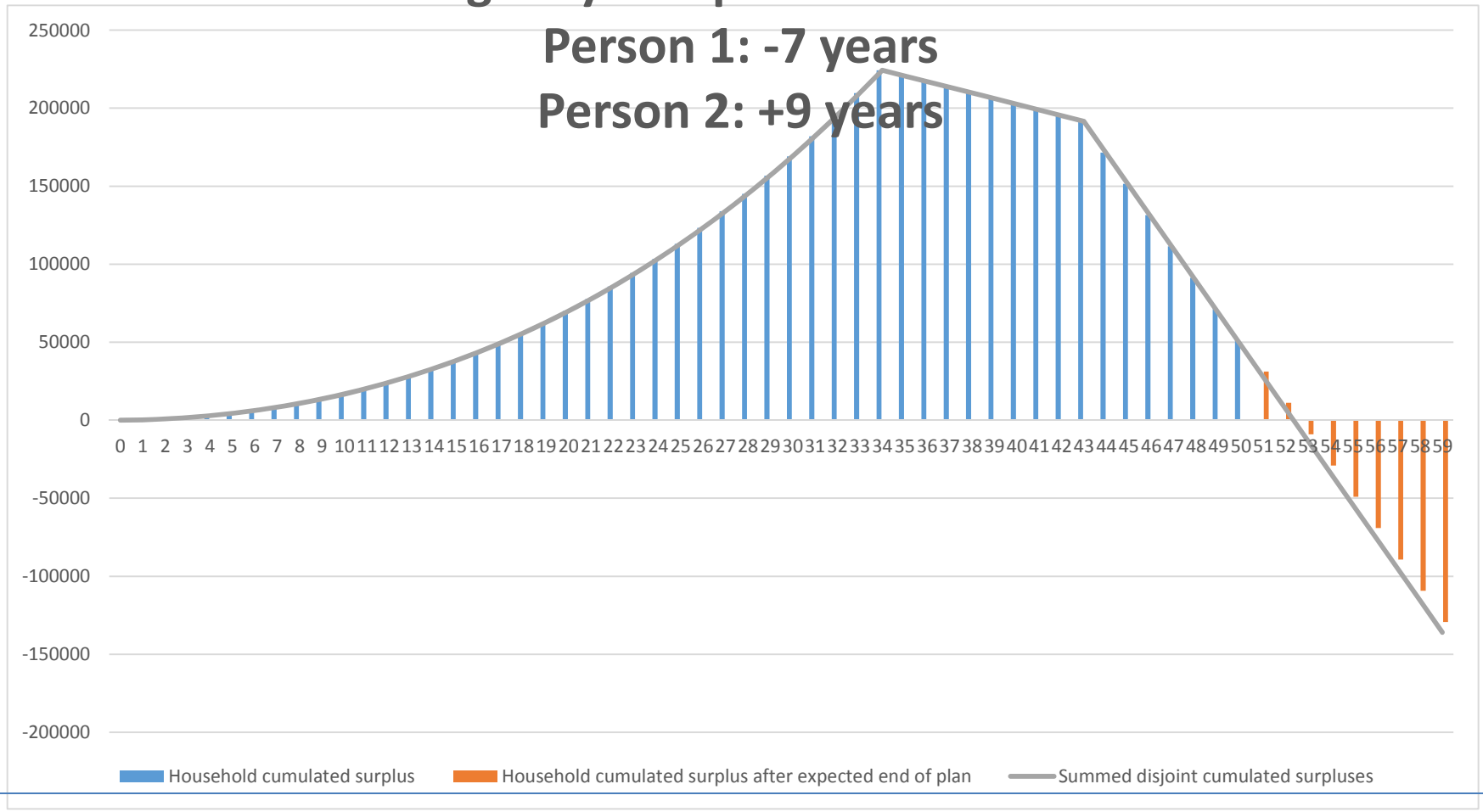


Optimized financial plan with risk aversion (γ=7, δ=9) (household vs. disjoint variant)



Optimized financial plan with risk aversion (γ=5, δ=5) (household - 2 identical persons vs. disjoint variant)

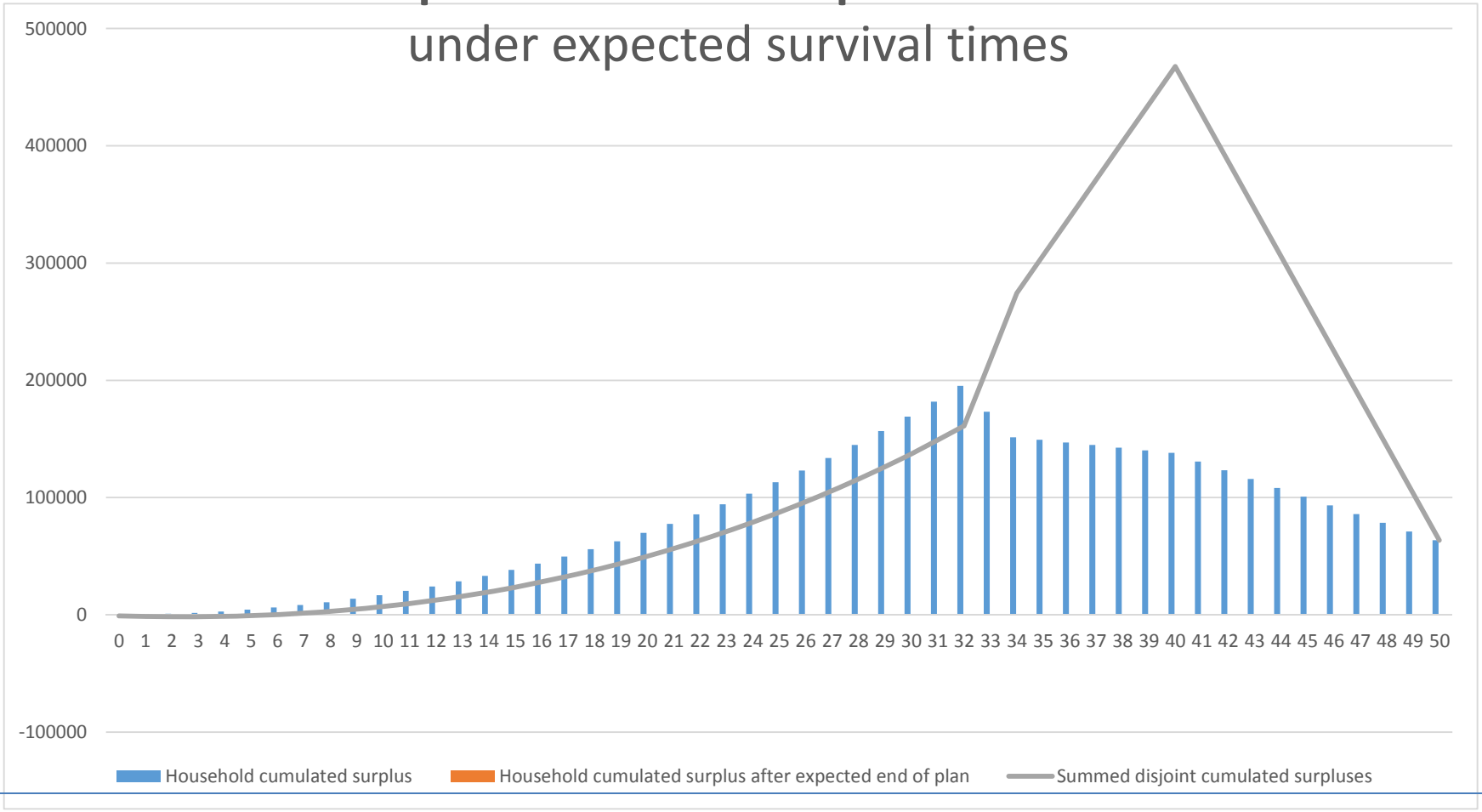
More severe longevity and premature-death risk realized



Optimized financial plan with risk aversion (γ=5, δ=5) (household vs. disjoint variant)

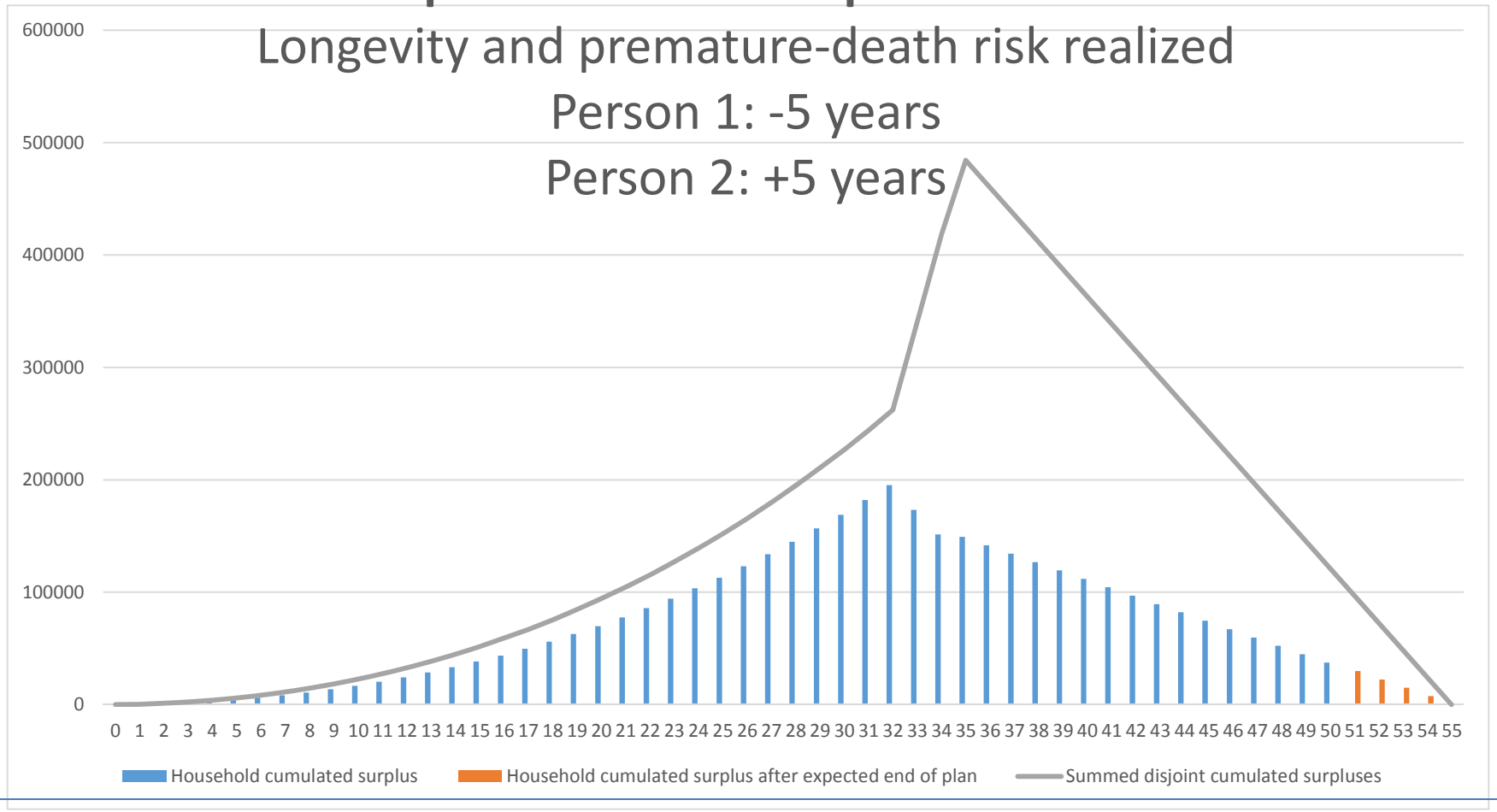
Enforced equal cumulated surplus levels at the end

under expected survival times



Optimized financial plan with risk aversion (γ=5, δ=5) (household vs. disjoint variant)

Enforced equal cumulated surplus levels at the end



Optimized financial plan with risk aversion (γ=5, δ=5) (household vs. disjoint variant)

Longevity and premature-death risk realized

	C1	C2	Iv1	Iv2	Iv1+Iv2	Sp1 (0)	Sp2 (0)	Household Sp [0,0]	Household Sp [-5,5]
2-person household	9 429	10 372	0	10 199	**10 199**	0	0	63 596	103
disjoint variant	9 429	10 372	11 571	0	11 571	0	-1 372	104 684	-510 319
disjoint variant (adjusted final effect at ED)	10 478	9 834	10 522	0	10 522	0	-834	63 597	-----
disjoint variant (adjusted final effect at D1=ED1-5; D2=ED2+5	6 278	14 714	2 722	15 286	18 008	0	0	-----	104

In 2-person household variant there is the lowest joint investment required.

When the persons differ in incomes a significant shortfall may be incurred by this person if it is treated separately (ceteris paribus) whereas this shortfall filled up in the household variant.

Optimized financial plan with risk aversion (γ=5, δ=5) (household vs. disjoint variant)

Longevity and premature-death risk realized

	C1	C2	Iv1	Iv2	Iv1+Iv2	Sp1 (0)	Sp2 (0)	Household Sp [0,0]	Household Sp [-5,5]
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If we enforce the same final financial effect (bequest) on the disjoint variant (*adjusted disjoint variant*) the required initial level of investment is much higher than for the household. This even leads to a shortfall on the side of one person for the very beginning of the plan.

The more adverse scenario adjusted disjoint variant is suited the higher required investment level at start.

Optimized financial plan with risk aversion (γ=5, δ=5) (household vs. disjoint variant)

Longevity and premature-death risk realized

	C1	C2	Iv1	Iv2	Iv1+Iv2	Sp1 (0)	Sp2 (0)	Household Sp [0,0]	Household Sp [-5,5]
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For the same level of investment a household may obtain better retirement conditions than 2 persons treated separately.

For the same level of retirement income a household needs to invest less in the accumulation phase of a life cycle.

There is possible such situation that a household is able to afford investing into private pension instruments whereas in the disjoin variant at least one person cannot afford any investment at all.

Conclusions

- Input of our model:
  - optimization of retirement planning dedicated to two person households
  - intuitive and easily applicable risk aversion factors
  - narrowing the optimization area
  - different discount rates for consumption and bequest
  - portfolio effect obtained thanks to internal risk transfer within the household

Conclusions

- Household members do not need to fully cover the retirement gap (overlapping coverage of common household costs).
- Household financial plan is much less sensitive to deviations from expected values of stochastic factors than the disjoint variant (other things being equal)
- Joint treatment of household finance allows to reduce joint retirement investment contribution for a given level of consumption or increase joint consumption for a given level of joint investment contribution
  - This allows, among others, to invest into private pensions instruments by households whose members would not be able to do it if treated separately

Conclusions

- Transfer of funds between household members smoothes away differences in human capital (both resulting from the potential to generate income from job and from the differences in expected life times)
- There is a kind of portfolio effect because the difference in sensitivity of joint and disjoint variant to changes of risk factors is the higher to the higher extend the household members differ from each other
  - If the household member are identical (age, sex, income etc.) then sensitivities to premature-death and longevity risk are also identical for both household and disjoint variant.
- There is a potential for development of pension-plan instruments dedicated to households
  - Possibility of choice of the moment when accumulated capital is exchange into life annuity (retirement date of the firs or the second person)
  - Possibility of choice to which person the life annuity is assigned
  - Or may be even a life annuity for a household, such that payouts stop when the last household member dies, whichever of the two.