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Price Jump Indicators: Stock Market Empirics during the Crisis

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Abstract

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Keywords: price jump indicators; non-parametric testing; clustering analysis; financial econometrics; Basel Accords.
JEL classification: C14, C58, F37, G15, G17

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Abstract

We analyze the behavior and performance of multiple price jump indicators across markets and over time. By using high-frequency stock market data we identify clusters of price jump indicators that share similar properties in terms of their performance in that they minimize Type I and Type II errors. We show that clusters of price jump indicators formed over the observations do not exhibit equal size. Clusters are stable across stock market indices and accuracy across price jump indicators are both stable over time. There was no significant change in the composition of clusters associated with market activity and the detected numbers of price jumps are stable over time. The recent financial crisis does not seem to affect the overall jumpiness of mature or emerging stock markets. Our results support the stress testing approach of the Basel III Accords in that the jump component of the volatility process does not need to be treated separately for the purpose of stress testing.

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1. Motivation and relevant literature

Time series generated from financial markets contain discontinuities in the price evolution—price jumps—as originally suggested by Press (1967) and more recently by Cont (2001). Price jumps should be incorporated into financial models as in Merton (1976) or Andersen et al. (2002). Specifically, Chernov et al. (2003) show that jump diffusion models are superior to pure diffusion processes, and Daal et al. (2007) and Duan et al. (2006) show the imperative to account for jumps using GARCH models. The identification of price jumps is a critical issue with respect to model specification. A large number of methods and indicators to identify price jumps has emerged, but the literature does not offer a clear consensus yet on how to identify price jumps accurately (see Urga and Dumitru, 2012 for a discussion). The issue of price jump identification has gained additional currency as the recent financial and economic crises heavily impacted developed as well as emerging financial markets (see Acharya and Richardson, 2009 for a discussion) and stress testing has been implemented to a greater extent than before. In this paper we analyze the behavior of multiple price jump indicators applied on high-frequency stock market data. We identify clusters of indicators in terms of their performance and analyze their behavior across markets and over time. We show a great degree of stability in terms of indicators' performance. We also provide evidence that the recent financial crisis did not affect the overall jumpiness of mature or emerging stock markets. Our results support the approach for proper stress testing suggested by the Basel III Accords.

The financial literature is quite specific on the importance of detecting price jumps. The key reason precise detection of price jumps is needed is that their presence has serious consequences for financial risk management and pricing. Pricing with jumps using continuous-time diffusion equations was studied by Broadie and Jain (2008), where the authors consider the pricing of volatility and variance swaps. They conclude that the pricing of swaps significantly differs when jumps are taken into account, thus one cannot appropriately price the risk while ignoring jumps. Arshanapalli et al. (2013) support the need to include the jump component into the risk measures to estimate the proper risk-return relationship. Carr and Wu (2010) use a jump diffusion model to simultaneously price stock options and credit default swaps and find a significant presence of the interplay between credit and market risks. A similar confirmation of the change in the pricing mechanism was also shown by Duffie et al. (2000), Liu et al. (2003), and Johannes (2004). Jarrow and Rosenfeld (1984), Nietert (2001), and Pan (2002) study pricing in the presence of jumps and all of them confirm the presence of the jump risk premium. Following the above reasoning, Nyberg and Wilhelmsson (2009) discuss the importance of including event risk as recommended by the Basel II accord, which suggests employing a VAR model with a continuous component and price jumps representing event risks. However, Andersen et al. (2007) conclude that most of the standard approaches in the financial literature on pricing assets assume a continuous price path. Finally, the stress testing procedures according to the recent Basel Accords suggests focusing on covariance matrices for book assets, while the implicit role of extreme, non-Gaussian events is untouched. Since this assumption is clearly violated in most cases, the results tend to be heavily biased.

The key role price jumps play in financial engineering triggered interest in the financial econometrics literature, especially in how to identify price jumps. Several different approaches evolved over recent years. Generally, we can identify in the literature four groups of price jump indicators. Below we offer only a brief account of each, providing a detailed exposition in Section 2 and in the Appendix. The first group is represented by the work of Mancini (2009), Ait-Sahalia et al. (2009), and Ait-Sahalia and Jacod (2009a, b). These studies use the scaling properties of the different components contributing to the data generating process to derive and analyze the jump statistics. The indicators exhibit optimal asymptotic behavior; however, the finite sample behavior can be more difficult to tackle. The second group, containing indicators based on bipower variation, was initiated by a series of papers: Barndorff-Nielsen and Shephard (2004, 2006), Barndorff-Nielsen et al. (2004), and Barndorff-Nielsen et al. (2008). The method is based on two distinct measures of overall volatility, where the first one takes into account the entire price-time movement while the second one ignores the contribution of the model-dependent price-jump component. This method was further improved by Lee and Mykland (2008). The third group is represented by a test developed by Jiang and Oomen (2008) that relies on the difference between the swap variance and the realized variance. The fourth group of price jump indicators is due to Jiang et al. (2009) and Joulin et al. (2008), who define the scaling properties of the tails of the normalized series distributions. Then, the scaling index enables them to define the jumpiness of the market purely based on how much of the weight lies in the tails and how this weight is distributed.

Given the relatively large number of indicators developed to identify price jumps, the literature is not in accord in terms of which indicators are the most suitable. Jumps are identified with various techniques that yield different results. This issue was brought up by several recent studies that evaluate jump indicators via the Monte Carlo approach. Hanousek et al. (2012) studied 14 different specifications (which are also used in this study) and they classified the indicators by their ability to predict price jumps (with no penalty for false identification) and by their ability not to incorrectly identify jumps (with no attention paid to missed identification). Basically, their approach compares border cases for the optimality of price jump indicators with respect to Type I and Type II errors. In the price jump literature, these two criteria are also referred to as optimality with respect to the power and size of the test. Their comparison technique was based on a pairwise one-sample method, which is in general more robust than two-sample methods. Further, Theodosiou and Zikes

(2011) employ a Monte Carlo study to compare a battery of eight price jump indicators with a focus on ultra-high-frequency data with market micro-structure noise to identify whether a given period is a jump period or not. However, the ultra-high frequency of traded assets may differ substantially from lower frequencies as Christensen, et al. (2011) claim that at the tick level there are almost no jumps. Further, Dumitru and Urga (2012) performed a comprehensive Monte Carlo comparison among nine procedures under alternative sampling frequencies, levels of volatility, persistence in volatility, degree of contamination with microstructure noise, and jump size and intensity. They show that the Lee and Mykland (2008) and Andersen et al. (2007) intraday procedures exhibit the best performance, provided the price process is not very volatile, and propose an improvement to these procedures. Vortelinos and Thomakos (2013) further test seven realized volatility estimators for international equity indices. In addition, Bajgrowicz and Scaillet (2011) suggest in their Monte Carlo study a procedure to avoid spurious jumps by proper thresholding.

The contribution of our paper is threefold. First, we apply a large set of price jump indicators, whose prediction accuracy and performance was compared in the extensive simulation study by Hanousek et al. (2012), to nine stock market indices from both mature and emerging markets. Second, we assign price indicators to groups using a cluster analysis of the predicted number of jumps for each market and time period. Then we analyze the changes in the behavior of those price indicators across countries and different phases of financial market development, including the recent crisis. Namely, we compare the stability of the identified clusters across markets and business cycles. Third, we discuss the proper implementation of stress testing in a regulatory framework, which forms a natural complement to liquidity and capital requirements, as they are outlined in recent Basel III Accords. Wehinger (2012) has an extensive discussion of the Basel III proposals, and Allen et al. (2012) as well as Yan et al. (2012) discuss the capital requirements in detail. In particular, we test whether the clusters are stable over time with respect to the analysis of different markets and different time periods. Specifically, we focus on the recent financial crisis, and show that the price jump component of the volatility process does not need to be treated separately for the purpose of stress testing.

The paper is organized as follows. In Section 2 we introduce four groups of price jump indicators, summarize their mutual performance, and provide a formal definition of clusters including suitable algorithms. In Section 3 we describe the data set we use. The empirical application on high frequency stock market indices is contained in Section 4, which is followed by conclusions.

2. Methodology: Performance of the price jump indicators and cluster analysis

The studies of price jump identification reviewed in Section 1 are focused primarily on prediction accuracy and the comparison of different sets of price indicators in different (usually simulated) data environments. Simply speaking, researchers analyzed which indicator performs better in the given circumstances, e.g. in a given data set. The studies employing real financial data have tested indicators on data originating in mature and emerging markets, covering both calm and turbulent periods of time. Our goal is to analyze to what extent the price indicators are stable in their predictions. For example, we analyze whether those indicators that have a tendency to predict more price jumps do it in the same fashion across different periods (calm versus turbulent) and type of market (developed or emerging). Hence, we aim to study whether the jump indicators form groups that would show similar and stable behavior across different types of period and across markets.

We proceed in the following manner to achieve our goals. First, we introduce a set of common price jump indicators previously developed in the literature and discuss their known relative performance, which is provided in Hanousek et al. (2012). Second, we formally introduce a cluster methodology to study the relative mutual performance of price jump indicators using real data. The clusters are defined as groups of price jump indicators exhibiting similar detection accuracy, which is defined rigorously using a suitably defined measure. Finally, we describe how to assess the performance of the price jump indicators among themselves.

2.1 Price jump indicators

In our analysis we employ 14 price jump indicators that are divided into four groups. In this section, we list those indicators and briefly refer to the seminal works that develop them. For all indicators, time is measured in equidistant time steps, which are 5-minute periods. We provide a more detailed description and formal notation of the price jump indicators according to the groups in the technical Appendix, Sections A1–A4.

Group 1 contains indicators based on the findings of Mancini (2009) and Ait-Sahalia and Jacod (2009 a, b):

1. Centiles: The price jump is identified as those returns below the 0.5th centile or above the 99.5th centile. Centiles are calculated for the entire sample.

2. Block-centiles: The price jump is identified as those returns below the 0.5th centile or above the 99.5th centile. Each trading day is divided into 15-minute blocks and centiles are calculated for each block separately for the entire sample.

Group 2 is formed by the set of indicators based on bipower variation introduced in Barndorff-Nielsen and Shephard (2004, 2006):

3. $Z_{RJ,TP}$-statistics with a 99% confidence interval (CI) and length of moving window n = 60.

4. $Z_{RJ,TP}$-statistics with a 99% CI and n = 120.
5. Improved $Z_{RJ,TP}$-statistics with a 99% CI and n = 60.
6. Improved $Z_{RJ,TP}$-statistics with a 99% CI and n = 120.
7. ξ-statistics with a 99% CI and n = 60.
8. ξ-statistics with a 99% CI and n = 120.

Group 3 contains measures that identify the presence of price jumps within a given time window using swap variance as in Jiang and Oomen (2008):

9. JO_{Ratio}-statistics with a 99% CI and n = 60.
10. JO_{Ratio}-statistics with a 99% CI and n = 120.
11. Improved JO_{Ratio}-statistics with a 99% CI and n = 60.
12. Improved JO_{Ratio}-statistics with a 99% CI and n = 120.

Group 4 is related to the field of statistical finance (Bouchaud, 2002) or Econophysics. The indicators rely on the scaling properties of price movements and we employ the price-jump index defined by Joulin et al. (2008):

13. Price-jump index as defined in A4.1: The price jump is identified as those returns with $p_{ji} > 4$ and n = 120.
14. Price-jump index as defined in A4.1: The price jump is identified as those returns with $p_{ji} > 4$ and n = 420.

2.2 Clustering methodology

Comparing the relative performance of the price jump indicators with real data is a difficult task since one does not know the specific properties of the real data-generating process, unlike in the case of a standard Monte Carlo experiment. Nevertheless, a suitable non-parametric method to assess the relative performance of jump indicators on real data is the clustering methodology. Clusters, or groups, of price jump indicators that behave similarly in terms of jump detection can be considered empirically as being close to each other and any analysis using members of the same cluster should provide similar or even identical results. Namely, if two price jump indicators are members of the same cluster, and thus have similar detection accuracy, the numbers and timing of the indicated price jumps should be similar in both cases.

The crucial characteristic in the recipe to form clusters is the similarity measure, according to which clusters of similar indicators are formed. Hence, similarity has to be defined as a mathematically rigorous and well-behaving measure with respect to the data set. Kaufman and Rousseeuw (1990) point out that the analysis of clusters is rather an art and in complicated cases the results may differ significantly based on the methods used. In the following account we describe a definition of the algorithm for the construction of clusters using the most robust methods provided in the literature.

Let us denote $\boldsymbol{x}_{\boldsymbol{i}}^{(\boldsymbol{t})}$, an observation of variable *i* at time *t*, where we are interested in the formation of clusters across variables based on the similarity over time *t*. The similarity between two variables can be defined in many ways; see Kaufman and Rousseeuw (1990) for more references. In our case, we use standard Euclidean distance, which is defined for two *n*-dimensional vectors x_i and x_j as

$$d_{ij} = \sqrt{\sum\nolimits_{l=1}^{n} \left(x_i^{(l)} - x_j^{(l)}\right)^2} \, .$$

In fact, the distance measures the dis-similarity between variables, where the larger the distance, the less similar the variables are. In practice, any L-norm would be a suitable candidate for the distance measure.[1]

Then, we may formulate the algorithm for the formation of clusters. We use the hierarchical agglomerative linkage method, which starts from the case where observations for each variable form an individual cluster. In each step, the new cluster is formed by joining exactly one pair of clusters from the previous step, which has the highest similarity—the least mutual distance. For that purpose, we utilize the Lance and Williams (1967) recurrence formula to optimally calculate the distance for every step. The key formula of the method is

$$d_{k,ij} = \alpha_i d_{ki} + \alpha_j d_{kj} + \beta d_{ij} + \gamma |d_{ki} - d_{ki}| \, ,$$

where $d_{k,ij}$ is a distance between cluster k and the newly created cluster formed by joining the clusters i and *j*; d_{ij} is the distance between the clusters i and *j*; and α_i, α_j, γ, and δ are scheme-specific constants. There are many different schemes to link clusters together in the iterative process, each stressing different aspects, i.e., linking based on the median, average or even outliers are possible candidates. We follow the recommendation of Kaufman and Rousseeuw (1990), who state that average-link-based methods work well in many situations and provide reasonably robust results. In the case of the average-link-based method, the scheme-specific constants read

$$\alpha_i = \frac{n_i}{n_i + n_j} \, , \alpha_j = \frac{n_j}{n_i + n_j} \, , \gamma = 0 \, , \delta = 0 \, ,$$

where n_i is the number of observations in cluster i and n_j is the number of observations in cluster j.

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[1] Lance and Williams (1967), Kaufman and Rousseeuw (1990), and Everitt et al. (2011) show that the deviation from the standard Euclidean distance may result in a serious inconsistency in the data, making it difficult to interpret results.

The Lance and Williams (1967) recurrence formula builds a pyramid of clusters starting at the individual level and ending up with a one single global cluster, with a distance between individual steps. The distance may be used to define a certain rule for an optimal number of clusters in the data. The usual approach is to employ the Duda-Hart index (Duda, et al. 2001) or the Calinski-Harabasz pseudo-F index (Calinski and Harabasz, 1974) and decide based on the algorithm. However, it is a well-known problem that the different stopping rules may not give the same result; see the discussion in Kaufman and Rousseeuw (1990), Everitt et al. (2011), Gordon (1999), or Milligan and Cooper (1985). An alternative, suggested in the studies just mentioned, is to use threshold methods. One possible way is to define a cluster as those groups of data whose mutual similarity is not lower than a certain threshold. The other way, preferred in this work, is to set a fixed number of clusters and find the members of such clusters. The latter method is suitable for studying the stability of clusters over time or the variation of other parameter(s); see Kaufman and Rousseeuw (1990) for more details.

Finally, choosing the rule based on a fixed number of clusters allows us to explicitly study the stability of clusters. The literature usually considers stability in the context of the clustering methodology as being a property of the algorithm rather than the data (see, for example, Lange et al., 2004). In these cases, the data are a priori stable and the clustering algorithm is chosen in such a way that it produces stable clusters for randomly drawn subsamples. In our case, we alter this methodology and assume that the structure of the underlying data may slowly change over time. This implies a possible change in the composition of clusters. In particular, let $x_i^{(t)}$ be the observation for the i-th variable at time t, where t runs over a discrete set of values $t \in G \equiv \{1, \dots, T\}$. Stability over time, as studied in this paper, means that we partition G into N connected and non-overlapping partitions G_n, which satisfy $G = \bigcup_{n=1}^{N} G_n$, and analyze the composition of clusters over the different partitions G_n as well as for the clusters based on the entire sample G. In this approach, we assume that the algorithm itself is stable over the partition and all the changes across partitions are due to a variation in the data-generating process. We use the above-described method to form clusters of the price jump indicators employed in this study in terms of their performance. The estimated numbers of price jumps per unit of time (5 minutes) for a set of price jump indicators are the variables upon which we construct the clusters.

2.3 Performance of price jump indicators

The variety of price jump indicators poses a question: which one is able to provide the most accurate identification of a price-jump? To answer the question, we follow Hanousek et al. (2012), who used the same set of price jump indicators and performed an extensive Monte Carlo comparison study with the aim to identify the optimal price jump indicators with respect to Type I and Type II errors. Let us briefly summarize their results, which is the input for our empirical analysis and clustering methodology. In particular, they used the sequential McNemar test, which is a robust one-sample approach optimal to numerical simulations, and identified the following indicators as dominating the rest in terms of their prediction accuracy.

Type I optimality

The first comparison criterion refers to Type I error.[2] Optimization with respect to Type I error is to maximize the number of correctly identified price jumps, with no penalty for false identification. In the framework of price jumps, this criterion is usually referred to as optimization with respect to the power of the test. In a simulation study, Hanousek et al. (2012) showed that the best indicator was indicator No. 1 based on centiles due to Mancini (2009) and Ait-Sahalia and Jacod (2009 a, b). This indicator dominated in nearly all simulation cases.

Type II optimality

The second comparison criterion refers to Type II error.[3] Optimization with respect to Type II error means to minimize the false identification of non-occurring price jumps. In the literature, this criterion is usually presented as the optimality with respect to the size of the test. In a simulation study by Hanousek et al. (2012), the best performing indicator was indicator No. 8 due to Lee and Mykland (2008) with the ξ-statistics with 99% CI and $n = 120$. In addition, the analysis shows that even the indicator version with time window $n=60$ (indicator No. 7) performs well since these two statistics are in many cases statistically indistinguishable.

The above-mentioned optimal price jump indicators therefore could serve as benchmarks for empirical analysis. In particular, the price jump indicators that for a given sub-sample detect the same (correct) number of price jumps as these two benchmarks behave similarly as the indicators optimal with respect to Type I and Type II errors.

Let us comment on the performance of the price jump indicators that were not optimal, based on the Monte Carlo assessment in Hanousek et al. (2012). The results show that indicator No. 8, which was optimal with respect to Type II error, was second-best with respect to Type I error. Further, in Type II optimality, the best-performing indicator was No. 8, where the second-best was actually No 7. However, both these indicators are from the same family and differ only by parametric

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[2] In diagnostic analysis it is usually called the false positive probability.

[3] In diagnostic analysis it is usually called the false negative probability.

specification. The third-best optimal indicator was No. 1. The results thus suggest that a pair of indicators, No. 1 and No. 8, dominates both optimality criteria.

The performance of other indicators in terms of determining dominance conclusion was a minor consideration. There were five price jump indicators in total that each had the best result at least once with respect to Type I error—indicators Nos. 1, 2, 7, 8, and 14. Then, there were three price jump indicators in total that each had the best result at least once with respect to Type II error—indicators Nos. 1, 7, and 8. This underlines the conclusion that it is empirically relevant to focus on the behavior of the two optimal price jump indicators and consider them as the benchmarks to which others will be compared.

3. Data

We employ data from nine stock market indices at a 5-minute frequency spanning the period from January 2007 to December 2010. We use six developed stock market indices in the following countries: Japan (NKY index), Germany (DAX index), France (CAC index), the United Kingdom (the FTSE100 index UKX), and the USA (the S&P 500 index SPX and the Dow Jones Industrial Average index INDU). We also use three emerging stock market indices: the Czech Republic (PX index), Poland (WIG index), and Hungary (BUX index). Data originate from Datastream and Bloomberg. Hence, our data set is formed by a wide range of different stock market indices including both mature and emerging markets, markets with different market micro-structure and levels of regulations, and even stock exchanges with and without a regular lunch break.

The data in our study are processed as follows: for each stock market index, we take the last price available for every fifth minute. We consider prices from regular trading hours only.[4] Then, we construct for every time step a time series of log-returns given as $r_t = \log\left(P_t / P_{t-1}\right)$.[5] Finally, appending the log-returns constructed independently for every trading day into one series, we construct the continuous time series of log-returns by linking the trading days with log-returns together. By such construction, we actually omit the overnight returns.

4. Results and economic application

[4] We mention this since many stock exchanges have a pre-opening trading period, trading after closing hours, or a final auction. We do not consider such activity in our analysis.

[5] Note that log-returns are not defined as they are not available whenever there is a break in the trading period. This discards all discontinuities due to non-trading activities. Namely, there are big price changes overnight, which emerged because of the trading activity in different markets. For example, prices at the European stock markets change overnight due to trading in North America and Asia. In this study, we are interested in the price jumps coming from trading activity only.

In the following section, we employ a battery of 14 price jump indicators for a set of nine stock market indices sampled at a 5-minute frequency[6] over a period ranging from 2007 to 2010. We then employ the clustering methodology and form clusters based on the similarity between the monthly figures estimated for each stock market index and each price jump indicator individually. We use standard Euclidean distance and the average-link-based method in recurrence formula of Lance and Williams (1967). The stopping rule is based on a fixed number of five clusters, which allows us to test the stability of the components across clusters.

4.1 Price jump indicators

We employ the 14 price jump indicators described earlier to estimate the number of price jumps per calendar month in the sample. For that we need to make two adjustments. First, for centile-based price jump indicators—price jump indicators Nos. 1 and 2—we need a certain period to estimate the centiles. We take the calendar quarter as the period over which we estimate threshold centiles. Therefore, each quarter shares the same level of centiles that we use to construct the thresholds for price jumps. Second, the remaining price jump indicators require a certain history to get appropriate price jump statistics. This poses the question how to consider the initial moments of every month. For that purpose, when we estimate the price jump statistics at the beginning of every month, we take a certain part of the previous month to be able to identify a price jump even at the very first moments of the month. This also requires starting our sample even before January 2007 and thus we have to employ data from December 2006, as was mentioned above.

Price jump indicators allow us to obtain 14 different estimates of the number of price jumps per month. The simulation analysis suggests that these numbers will differ across the price jump indicators. We therefore analyze how the estimated numbers based on the real time series differ from each other using the clustering analysis.

4.2 Clustering of the price jump indicators: Stock market indices

Let us denote $x_{i,j}^{(t)}$ as an observed number of identified price jumps by price jump indicator i at stock market index j for month t. Such a classification of the data allows us to study the formation of clusters across three dimensions: time, stock market indices and price jump indicators. We further use formula of Lance and Williams (1967) with standard Euclidean distance and an average-link-based hierarchical agglomerative method to form five clusters.

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[6] We opt for 5-minute data since at the tick level there are almost no jumps to be identified (Christensen et al., 2011) and at lower frequencies the data might be too aggregated.

In our analysis we form five clusters that are characterized as follows, with the numbers in parentheses denoting the numbers and types of price jump indicators in a particular cluster:

Cluster 1 (8): Type II-optimal indicators

Majority of tested indicators (Nos. 7, 8, 9, 10, 11, 12, 13, and 14)

Cluster 2 (2): Type I-optimal indicators – centiles based

Price jump indicators based on the extreme returns threshold using centiles (Nos. 1 and 2)

Cluster 3 (2): Higher-moments indicators

Price jump indicators based on the $Z_{RJ,TP}$ statistics with both time windows (Nos. 3 and 4)

Cluster 4 (1): Higher-moments with short memory indicator

Price jump indicator with improved $Z_{RJ,TP}$ statistics using short-run memory (No. 5)

Cluster 5 (1): Higher-moments with long memory indicator

Price jump indicator with improved $Z_{RJ,TP}$ statistics using long-run memory (No. 6)

For the clustering analysis on the stock market indices we employ a data set that contains for each stock market index and price jump indicator a time series of the estimated number of price jumps per month. We have further sorted clusters according to the mean of the number of price jumps they provide. For example, cluster No. 1 gives on average less price jumps per month than clusters No. 2–5.

As a robustness check we have also analyzed different numbers of clusters. Specifically, we estimated the results when 4 and 6 clusters of indicators were formed. The qualitative results remained the same, though. In particular, Cluster 1 did not change its composition for either of the two alternative specifications, while in case of four clusters, Cluster 4 and Cluster 5 merged together. In the case when 6 clusters were formed, Cluster 3 was divided into two smaller clusters. Details are available upon request.

4.3 Global clustering

As noted above, we use the clustering algorithm to form clusters of price jump indicators for each stock market index and the entire time series. Table 1 provides an overview of the clustering analysis by stock market indices. We perform the analysis by stock market indices separately as well as against clustering for all stock market indices together. Therefore, we can distinguish the composition of clusters on the level of individual indices and its difference from global clustering.

The following observations can be made based on the cluster division above. First, the analysis reveals the existence of a large Cluster 1, which contains more than half of the price jump indicators used in the study. In terms of the number of detected jumps, the indicators in Cluster 1 seem to behave in the same way as the optimal indicator (No. 8; see section 4.2) with respect to the criteria based on Type II error. However, this similarity in the number of detected jumps does not mean that indicators detect the same jumps. This finding suggests that there is a stable subset of price jump indicators that seem to be optimal. They provide an estimated number of price jumps close to the number indicated by the optimal price jump indicator (No. 8).

We further focus our attention on Cluster 1 and investigate how much the identified price jumps overlap with each other using different price jump indicators from this particular cluster. Table 2 contains the statistics of the overlapped price jump indicators for Cluster 1.[7] We present the number of price jumps correctly identified by the optimal indicator (No. 8) and then the number of price jumps identified by other price jump indicators followed by the number of price jumps jointly identified together with the optimal price jump indicator. The results clearly suggest that the overlap between the price jump indicators in the same cluster is not very large; in very few cases it exceeds 25%. Hence, the price jump indicators in the same cluster tend to provide false accuracy.

In addition, the results clearly suggest that the PX index (the Prague Stock Exchange) deviates from the sample. This suggests a different price generating process, which is in agreement with recent findings of Hanousek and Novotny (2012), who show the presence of the "PX Puzzle", a reverse scaling behavior of this stock market index with respect to other regional stock market indices from both emerging and mature markets.

4.4 Stability of clusters over time

In the next step, we answer the question whether the clusters are composed from the same price jump indicators over time. For that purpose, we perform the clustering analysis by year. Table 3 contains an estimation of clusters by year. In this approach, we assume that the clustering algorithm itself is stable on a year time scale and any difference in the composition of the estimated clusters is due to the temporal change of the data-generating process. This assumption seems reasonable since qualitative results obtained with different numbers of fixed clusters are identical.

In particular, we focus our attention on the two years that were the most economically distinct: 2007 and 2009. The year 2007 can be, in terms of our data sample, considered a period of financial stability. On the other hand, 2009, marked by the fall of Lehman Brothers, is in general

[7] The numbers for other clusters are available upon request.

considered as a year when financial turmoil emerged in full strength. In general, our results suggest the stability of clustering over years.

Further, we assess the transition dynamics of the price indicators during the financial crisis. In Table 4 we present the number of cases when price jump indicators remain in the same cluster (diagonal terms) and the number of cases when price jump indicators changed clusters during the financial crisis (off-diagonal terms). For the analysis we employ a Stuart-Maxwell test statistic as a symmetry test to build the contingency Table 4. The Stuart-Maxwell test is in this case asymptotically equal to χ_3^2 and takes a value of 4.41. This corresponds to a *p*-value of 0.22, which means that we cannot reject the null hypothesis that the table is symmetric. We therefore see that the size of the clusters remained the same and, despite mild migration, the clusters remain stable despite the financial crisis.

4.5 Stability of clusters with respect to stock market indices

In Section 4.4 we documented the stability of clustering over different stock market indices (with the exception of the PX index). To reinforce this finding of stability we present a contingency Table 5 that captures the number of cases when clustering on an individual level deviates from global clustering. Analogously to the previous test, the reported Stuart-Maxwell test statistic for the symmetry of this contingency table is asymptotically equal to χ_3^2 and takes a value of 4.09. The corresponding *p*-value is 0.39, i.e., we cannot reject the null hypothesis that the table is symmetric. The table thus suggests that global clustering is in agreement with individual clustering and cannot be claimed as an average of the individual level.

4.6 Price jumps in time

Finally, we construct a series of the estimated number of price jumps per month for each cluster and each stock market index and analyze the evolution of the series over time. If the number of price jumps changes over time, then the series would tend to deviate from the mean and exhibit non-stationarity. In Table 6 we present augmented Dickey-Fuller test statistics on the series of monthly estimated numbers of price jumps for each cluster and each stock market index. The null hypothesis states that a time series with numbers of price jumps per month contains a unit root and the alternative states that a time series is stationary. The McKinnon *p*-value is in all cases $p<0.001$, which means that at a very high significance level, we can reject the null hypothesis and conclude that each time series is stationary.

The results suggest that the number of price jumps per month for each cluster and each stock market index is stable over time. This means that we do not see a link between the financial crisis and the frequency of price jumps. This result is robust since it was confirmed for all price jump indicators.

Still, there is the question whether the difference in price jump detection among clusters remains stable over time. For that purpose, we repeat the augmented Dickey-Fuller test for the series of the difference between the estimated numbers of price jumps using different clusters. Table 7 contains augmented Dickey-Fuller test statistics on differences in monthly estimated numbers of price jumps between clusters for each stock market index. The first column on the left denotes the clusters for which we consider a difference. The null hypothesis states that a time series with differences in the number of price jumps per month contains a unit root and the alternative states that a time series is stationary. Since the p-value for all the entries is $p < 0.001$, we can reject the null hypothesis and consider each time series to be stationary. This means that the difference between the estimated number of price jumps using different clusters is stable over time. For the sake of clarity, Figure 1 contains the estimated number of price jumps for each stock market index and each cluster per month and thus the figure graphically complements the analysis.

Our results clearly support the approach chosen by the Basel Accords, i.e., do not propose any specific treatment to cover the risk associated with the price jumps. The overall scaling of the volatility during the distress period adequately covers the increased diffusive variation as well as price jump components. The correlation among price jumps, on the other hand, belongs to scenario stress testing, which helps to mimic the extreme correlation among extreme price movements. We can therefore conclude that the Basel III accords that are coming into force, together with properly chosen scenario stress testing, do not go against empirically observed stylized facts.

5. Conclusions

We employed 14 price jump indicators, whose detection accuracy was previously studied in Hanousek et al. (2012) in an extensive Monte Carlo setup, for nine stock market indices sampled at a 5-minute frequency. The indices cover both mature and emerging stock markets, which usually escape the attention of the mainstream empirical price jump literature. We then utilize Lance and Williams' recurrence formula and form clusters of price jump indicators based on the similarity in the observed monthly figures of estimated price jumps. The similarity is based on standard Euclidean distance and clusters are formed using the average-link-based method.

The clusters of price jump indicators formed over the observations do not exhibit equal size. There is one large cluster consisting of half of the price jump indicators. Further, we employ the prior knowledge from the simulation study and identify two important clusters, whose members behave like Type I- and Type II-error-optimal indicators. We then analyze the mutual overlap inside the cluster, which reveals that the actual overlap in identified price jumps is very low. This suggests weak accuracy across price jump indicators. This has serious consequences for meta-analysis. In particular, observing the same figures of estimated price jumps among different price jump indicators and/or different time series may be spurious without any real consequence.

Further, the results support the hypothesis that the formed clusters are stable across stock market indices and over time. In particular, we support the hypothesis that there was no significant change in the composition of clusters due to the recent financial crisis, as can be anticipated from the insensitivity of the early warning indicators to higher moments of capital-market performance. We further test the stationarity of the observed monthly figures of price jumps per cluster. The results based on the augmented Dickey-Fuller test supports the stationarity of the estimated monthly figures. In addition, we have complemented the analysis by testing for stationarity across the relative differences, which provides the same conclusion.

Our findings have interesting implications with regard to the effect of financial crisis on stock markets. The fact that the detected number of price jumps does not change over time suggests that the recent financial crisis did not affect the overall jumpiness of mature or emerging stock markets. The findings also suggest that despite popular belief, the rate of price jump arrivals at stock markets did not change during the crisis and therefore they do not need to be treated separately for the purpose of stress testing. The Basel III Accords emphasize modeling the covariance matrix properly, while they include rare events in the domain of scenario testing. Our results support this perception of rare events and do not suggest controlling for them separately as is done with covariance matrices.

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Appendix A

A1 Group 1: Truncation threshold

The first set contains indicators based on the findings of Mancini (2009) and Ait-Sahalia and Jacod (2009 a, b). The price process is assumed to be decomposed into a Gaussian component, corresponding to normal (white) noise, and a non-homogenous Poisson component, corresponding to price jumps. Therefore, when a significant price jump appears, the price increment is dominated by the non-homogenous Poisson element. On the other hand, when price movements are governed solely by Gaussian noise, the average and/or maximum magnitudes of such increments can be estimated. These properties can be used to set a threshold value that will effectively distinguish the two components, i.e. predict jumps.

In reality, however, we do not know the proper threshold values. Mancini (2009) and Ait-Sahalia and Jacod (2009 a, b) suggest some data-driven methods to decide the threshold values. These methods have well-behaving asymptotic properties. However, their empirical applications can be questioned due to an argument raised by Bollerslev, et al. (2009), who claim that the ratio between the volatility and number of price jumps varies over time and is hard to capture in finite samples. An additional approach is to use the overall centiles of returns and decide on the extreme returns based on the crossing of the threshold. The intraday variation may be then mimicked by splitting the trading days into blocks and calculating the centiles independently for each block. This approach is close to what was suggested by Boudt, et al. (2011).

Formally, let us assume that the underlying price increment process is given as $\Delta S=\sigma\Delta X+\Delta J$, where the price increment is $\Delta S = S_t - S_{t-1}$. We further assume that we observe the realization of prices in equidistant time steps Δt. In this definition, X corresponds to the Brownian motion and J to a β-stable process. The increments of the two components can be expressed as $\Delta X = (\Delta t)^{\frac{1}{2}} X_1$ and $\Delta J = (\Delta t)^{1/\beta} J_1$ with equalities in distribution.

The different magnitudes in the two components can be used to discriminate between the noise components and the price jumps coming solely from the J-process.[8] The big price jumps cause $\Delta S = \Delta J$ (in distribution) with X having a negligible effect, while in the presence of no big price jumps, which is most of the time, $\Delta S = \sigma(\Delta t)^{1/2} X_1$. Therefore, we can, for a given Δt, choose a

[8] The J-process contributes to a large amount of small price jumps; however, we want to focus on big price jumps only. The goal is not to completely determine the properties of the J-process but rather to determine how to discriminate extreme price movements.

threshold value equal to $\alpha(\Delta t)^{\gamma}$, with $\alpha > 0$ and $\gamma \in (0, \frac{1}{2})$, such that if $\Delta S > \alpha(\Delta t)^{\gamma}$ then *ΔS* is at a given moment dominated by *J* with a certain probability.

This argument can be reverted: Assuming the knowledge of the rate of the arrival of big jumps, we can imply a corresponding threshold using centiles. Centiles, therefore, serve as a prior to form a threshold for discriminating price jumps from the noise. The overall price volatility changes during a given day, therefore we divide the trading day into blocks and then form thresholds over the same blocks from different trading days. Based on the above the following indicators are constructed:

A1.1 Global centiles

We employ the 99.5th/0.5th centiles as the upper/lower thresholds and define price jumps as those returns that are higher/lower than a given upper/lower centile.

A1.2 Centiles over block-windows

We divide the trading day into 15-minute-long blocks. For each block, we define price jumps as in A1.1.

A2 Group 2: Bipower variation

The second set of indicators is based on bipower variation, introduced in Barndorff-Nielsen and Shephard (2004, 2006). Specifically, it employs the difference between the two measures of variation: realized variation and bipower variation. In a standard way we assume that the price generating process can be decomposed into two components—regular white noise and price jumps. The identification of price jumps uses the fact that the realized variation measures the variation of both components, while bipower variation measures the variation coming from the white noise only. This measure can be applied in two different ways (*ex-post* and *ex-ante*).

First, Huang and Tauchen (2005) introduced a statistics to determine the presence of price jumps within a given time window. This statistics, known as the max-adjusted statistics, can be thus used to identify the exact moment when a price jump occurs by employing different window bandwidths. Nevertheless, if two price jumps are separated by an interval shorter than the given window, the second price jump cannot be identified. In this study we use approach of Andersen et al. (2010) and modify it to adjust for our rolling window framework: in iterated algorithm the identified price jumps are replaced by averages within the moving window and the process continues until we find no jumps. The second approach, constructed by Lee and Mykland (2008), directly employs bipower variation in a sequential setup. The goal of this approach is to identify price jumps at the moments when they occurred. Their statistics compares the current price movement with the bipower variation calculated over a moving window with a given number of preceding observations, excluding the current one.

Barndorff-Nielsen and Shephard (2004, 2006) define two different measures for variation: realized variation defined as $RV_t = \sum_{i=2}^{n} r_i^2$ and bipower variation defined as $BV_t = \mu_1^{-2}\left(\frac{n-1}{n-2}\right)\sum_{i=3}^{n}|r_i||r_{i-1}|$, with $\mu_\alpha = E\left(|Z|^\alpha\right)$ for Z ~ N (0,1), or generally $\mu_\alpha = 2^{\alpha/2}\tilde{A}\left(\frac{\alpha+1}{2}\right)/\sqrt{\pi}$. Their combinations serve as building blocks for various price jump statistics introduced below.

A2.1 Max-adjusted statistics

Although the combination of the variations can be used to identify periods with higher propensity for jumps, one needs higher moments to form explicit statistics tests. There are at least two possible ways to estimate the higher moments: Anderson, et al. (2007) introduced tripower quarticity, $TP_j = n\mu_{4/3}^{-3}\left(\frac{n-1}{n-3}\right)\sum_{i=j-n+4}^{j}|r_i|^{4/3}|r_{i-1}|^{4/3}|r_{i-2}|^{4/3}$, to assess the conditional standard deviation, while Barndorff-Nielsen and Shephard (2004, 2006) employed quadpower quarticity, $QP_j = n\mu_1^{-4}\left(\frac{n-1}{n-4}\right)\sum_{i=j-n+5}^{j}|r_i||r_{i-1}||r_{i-2}||r_{i-3}|$.

According to Huang and Tauchen (2005), the best statistics is $Z_{RJ,TP}$, defined as $Z_{RJ,TP} = \frac{RJ}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2+\pi-5\right)\left(\frac{1}{n}\right)\max\left(1,\frac{TP}{BV^2}\right)}}$ with $RJ_j = (RV_j - BV_j)/RV_j$ and asymptotically $Z_{RJ,TP} \sim N(0,1)$. The null hypothesis states that there is no jump in a given period. If the statistics exceeds the critical value $\Phi^{-1}(\alpha)$, then we reject the null hypothesis of no price jump at confidence level α. The definition works with backward-looking realized and bipower variations: $RV_j = \sum_{i=j-n+2}^{j} r_i^2$ and $BV_j = \mu_1^{-2}\left(\frac{n-1}{n-2}\right)\sum_{i=j-n+3}^{j}|r_i||r_{i-1}|$. Observing a significant jump at time step j means that somewhere in a window of length n ending at time step j is at least one significant price jump. Thus, the change between two consecutive periods with no price jump and periods with a price jump can serve as an indicator for the moments when a jump is identified for the first time. Further, we employ two different time windows, n = 60 and n = 120, to have more robust estimates. Finally, price jumps are those prices for which $Z_{t-1} \leq \Phi^{-1}(\alpha)$ and $Z_t > \Phi^{-1}(\alpha)$.

A2.2 Max-adjusted statistics: Improved identification method

The improved identification method is based on the procedure developed by Andersen et al. (2010), who implemented price jump detection in two steps: In the first step, they identify trading days that tend to have price jumps. In the second step, they find the biggest intraday return, mark it as a price jump and then repeat the first step without taking this return into account. In our implementation, the improved algorithm works as follows: returns identified as price jumps are replaced by the average value calculated over the same length as was used for identification. The replaced value is not a price jump.

A2.3 Lee-Mykland

The bipower-based statistics of Lee and Mykland (2008) is given as $L(i) = \frac{r_i}{\hat{\sigma}(i)}$, with

$\hat{\sigma}^2(i) = \frac{1}{n-2}\sum_{j=i-n+2}^{i-1}|r_i||r_{i-1}|$. Then, the statistics gives us $\frac{\max_i |L(i)| - C_n}{S_n} \to \xi$, where ξ has a cumulative distribution function $P(\zeta \leq x) = \exp(-e^{-x})$, and the two constants are given as

$C_n = \frac{(2\log n)^{1/2}}{c} - \frac{\log\pi + \log(\log n)}{2c(2\log n)^{1/2}}$ and $S_n = \frac{1}{c(2\log n)^{1/2}}$, where in both cases $c = \frac{\sqrt{2}}{\sqrt{\pi}}$. Whenever the ξ-statistics exceeds the critical value ξ_{CV}, we reject the null hypothesis of no price jump at time t_i. Lee and Mykland recommend $n_{15\text{-}min}$ = 156 and $n_{5\text{-}min}$ = 270. In our analysis, we use n = 60 and n = 120.

A3 Group 3: Jiang-Oomen statistics

The third group of indicators identifies the presence of price jumps within a given time window using swap variance (Jiang and Oomen, 2008). It is claimed to be less sensitive to intraday volatility patterns than classical bipower variation. The setup of this price jump indicator is similar to the previous case: We employ a moving window, where we test for the presence of price jumps in every time window using a swap variance-based statistics. Analogous to the previous case, we employ an Andersen et al. (2010) iterative algorithm, where we replace the identified price jumps with moving averages and thus allow for the identification of consecutive price jumps.

The Jiang and Oomen (2008) statistics is based on swap variance, defined as

$SwV = 2\sum_{i=2}^{n}(R_i - r_i)$, where $R_i = \frac{P_i - P_{i-1}}{P_i}$, $P_i = \exp(p_i)$ and $r_i = p_i - p_{i-1}$.

A3.1 Jiang-Oomen statistics-based price jump indicator

The Jiang-Oomen statistics is defined as $JO_{Ratio} = \frac{nBV}{\sqrt{\Omega_{SwV}}}\left(1 - \frac{RV}{SwV}\right)$, where the realized variation RV and bipower variation BV are defined as above. The statistics is asymptotically equal to $z \sim N(0,1)$ and tests the null hypothesis that a given window does not contain any price jump. The indicator for a price jump is defined as those price movements for which $JO_{t-1} \leq \Phi^{-1}(\alpha)$ and $JO_t > \Phi^{-1}(\alpha)$. The same comments as for the max-adjusted statistics apply. We use two price-jump indicators with n = 60 and n = 120.

A3.2 Jiang-Oomen statistics: Improved identification method

We use the same improvement technique as in section A2.1.

A4 Group 4: Statistical finance

The last group of identification techniques originates in the field of statistical finance, as referred to by Bouchaud (2002), which is also known as Econophysics. This group of indicators relies on the scaling properties of price movements. We employ the price-jump index defined by Joulin et al. (2008). The price index is defined as the absolute returns normalized with respect to the L_1 variance, i.e., the variance defined as an average of absolute returns over a given moving window. The price jump index has, as the literature confirms (Joulin et al., 2008), certain scaling properties of the tail part of its distribution. Thus, we define price jumps as those returns for which the price jump index exceeds a certain empirically determined threshold. This group of indicators employs the scaling techniques of returns (see Stanley and Mantegna, 2000, and references therein).

A4.1 Price jump index

The price-jump index is defined as $pij_i = \frac{|r_i|}{\frac{1}{n}\sum_{j=i-n+1}^{i}|r_i|}$, or the absolute returns normalized by the variance in the L1 measure. The empirical observations suggest (Joulin et al., 2008) that the scaling properties behave as $P(pij>s) \sim s^{-\alpha}$. Therefore, we define a price jump as a price return where the price jump index exceeds a given threshold $\hat{s}$. In our analysis, we choose $\hat{s}$ = 4 and $n = 120$ and $n = 420$.

Table 1: Clustering of price jump indicators.

No.	Developed markets			European markets			Emerging CEE markets			GC
	INDU	SPX	NKY	CAC	DAX	UKX	BUX	PX	WIG	
1	2	2	1	2	2	2	2	1	1	**2**
2	2	2	1	2	2	2	2	1	1	2
3	3	3	3	3	3	3	3	3	3	3
4	3	3	2	3	3	3	3	2	2	3
5	4	4	4	4	5	4	5	5	4	4
6	5	5	5	5	4	5	4	4	5	5
7	1	1	1	1	1	1	1	3	1	1
8	1	1	1	1	1	1	1	2	1	**1**
9	1	1	1	1	1	1	1	2	1	1
10	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	2	1	1
12	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	2	1	1
14	1	1	1	1	1	1	1	2	1	1

Note: The table presents the clusters of the monthly figures of estimated price jumps for each stock market independently. Clusters are sorted according to the mean of the number of price jumps per month they provide. Bold entries are price jump indicators optimal with respect to Type I error (No. 1) and Type II error (No. 8). GC stands for the global cluster for all the stock market indices. In particular, for each stock market index, we have calculated average-link-based clusters with standard Euclidean distance for 14 monthly time series of estimated price jumps corresponding to each price jump indicator. Finally, we have calculated clusters for all the stock market indices together. Abbreviation glossary: INDU—the Dow Jones Industrial Average Index, USA; SPX—the S&P 500 Index, USA; NKY—the Nikkei Index, Japan; CAC—the CAC 40 Index, France; DAX—the DAX 30 Index, Germany; UKX—the FTSE 100 Index, United Kingdom; BUX—the BUX Index, Hungary; PX—the PX Index, the Czech Republic; WIG—the WIG Index, Poland.

Table 2: Overlap of price jumps in No. 1 (the biggest cluster)

No.	Developed markets			European markets			Emerging CEE markets		
	INDU	SPX	NKY	CAC	DAX	UKX	BUX	PX	WIG
8	401	419	298	469	549	380	516	999	674
7	598/364	645/379	381/257	640/401	767/500	577/328	933/481	1,613/922	1,038/615
9	660/91	638/78	395/63	539/93	614/94	520/62	690/79	848/199	713/142
10	347/61	335/56	254/42	319/64	321/51	257/44	291/54	363/106	370/83
11	896/105	850/89	550/71	734/98	827/112	699/72	918/87	1,147/228	1,000/170
12	476/72	455/65	373/50	435/70	462/63	354/51	378/59	515/129	554/101
13	524/180	523/169	355/131	582/193	646/239	533/183	733/233	1,210/462	850/321
14	578/173	562/159	405/127	583/175	693/233	559/174	865/230	1,213/453	865/320

Note: The table illustrates the overlap inside Cluster 1, which is the biggest cluster, containing eight price jump indicators in total including the one optimal with respect to Type II error. In the first row, which corresponds to the optimal indicator No. 8, there are figures of estimated price jumps for each stock market index over a period of four years. In each of the subsequent seven rows, there are pairs of numbers: The first figure denotes the total number of price jumps identified for the stock market index by the particular price jump indicator. The second figure denotes how many price jumps were jointly identified by both the optimal price jump indicator and the one corresponding to the row. For example, for the INDU index, the optimal indicator No. 8 captured 401 price jumps over a period of four years, indicator No. 7 captured 598 price jumps, and 364 were identified jointly. Abbreviation glossary: INDU—the Dow Jones Industrial Average Index, USA; SPX—the S&P 500 Index, USA; NKY—the Nikkei Index, Japan; CAC—the CAC 40 Index, France; DAX—the DAX 30 Index, Germany; UKX—the FTSE 100 Index, United Kingdom; BUX—the BUX Index, Hungary; PX—the PX Index, the Czech Republic; WIG—the WIG Index, Poland.

Table 3: Clusters of price jump indicators using averages by year.

No.	Developed markets			European markets			Emerging CEE markets			GC
	INDU	SPX	NKY	CAC	DAX	UKX	BUX	PX	WIG	
1	2 2 2 2	2 2 1 2	2 2 1 1	2 2 1 2	2 2 1 1	2 2 2 3	1 2 2 1	1 2 1 1	2 1 1 1	2
2	2 2 2 2	2 2 1 2	2 2 1 1	2 2 1 2	2 2 1 1	2 2 2 3	1 2 2 1	1 2 1 1	2 1 1 1	2
3	3 3 3 3	3 3 3 3	3 3 3 2	3 3 3 3	3 3 3 3	3 3 3 4	3 3 3 3	3 3 3 3	3 3 3 3	3
4	3 3 3 3	3 3 3 3	3 3 2 3	3 3 2 3	1 3 2 2	3 3 3 4	2 3 3 3	2 2 1 2	3 2 2 2	3
5	4 4 4 4	4 5 4 4	4 5 4 4	4 4 4 4	5 5 5 4	5 4 4 5	4 5 5 5	5 5 5 5	4 4 5 5	4
6	5 5 5 5	5 4 5 5	5 4 5 5	5 5 5 5	4 4 4 5	4 5 5 5	5 4 4 4	4 4 4 4	5 5 4 4	5
7	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 2 2	1 1 1 2	1 1 2 2	3 2 2 2	2 1 1 1	1
8	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 2	1 1 1 2	1 1 1 2	2 2 1 2	1 1 1 1	1
9	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 3 1 1	1 1 2 1	1 2 1 2	2 1 1 1	1
10	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1
11	1 1 2 1	1 1 2 1	1 1 1 1	1 1 1 1	1 3 1 1	1 3 1 1	1 1 2 3	2 2 1 2	2 1 1 1	1
12	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1
13	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 2	1 1 1 2	1 1 2 2	2 2 1 2	2 1 1 1	1
14	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 2	1 1 1 2	1 1 2 2	2 2 1 2	2 1 1 1	1

Note: The table extends the results of Table 1, presenting the clusters of the monthly figures of estimated price jumps for each stock market independently by year. For each year, clusters are sorted according to the mean of the number of price jumps per month they provide. GC stands for the global cluster common for all the stock market indices. In particular, for each stock market index and each year, we have calculated average-link-based clusters with standard Euclidean distance for 14 monthly time series of estimated price jumps corresponding to each price jump indicator. Finally, we have calculated clusters for the all the stock market indices together. For example, the first entry of INDU—2 2 2 2—means that price jump indicator No. 1 applied on the data of the Dow Jones Industrial Average index belongs in 2007 to Cluster 2, in 2008 to Cluster 2, in 2009 to Cluster 2, and in 2010 to Cluster 2. Price jump indicator No. 2, when applied to all the stock market indices, belongs to Cluster 2. Abbreviation glossary: INDU—the Dow Jones Industrial Average Index, USA; SPX—the S&P 500 Index, USA; NKY—the Nikkei Index, Japan; CAC—the CAC 40 Index, France; DAX—the DAX 30 Index, Germany; UKX—the FTSE 100 Index, United Kingdom; BUX—the BUX Index, Hungary; PX—the PX Index, the Czech Republic; WIG—the WIG Index, Poland.

Table 4: Contingency table for clustering over years.

2007/2009	1	2	3	4	5
1	56	11	0	0	0
2	20	4	1	0	0
3	0	4	12	0	0
4	0	0	0	6	3
5	0	0	0	3	6

Note: The table captures the migration of price jump indicators across clusters for all stock market indices between two years: 2007 (a period with stable markets) and 2009 (a period when financial turmoil emerged in full strength). Each entry indicates the number of cases when the price jump indicator for a given stock market index was in 2007 in the cluster denoted by the row and in 2009 in the cluster denoted by the column. For example, 56 price jump indicators were in Cluster 1 for all the stock market indices for both years and 20 price jump indicators migrated from Cluster 2 in 2007 to Cluster 1 in 2009. The remaining entries should be interpreted in a similar way. The symmetric table denotes no significant trend in migration between years.

Table 5: Stability of clustering with respect to individual stock market indices.

Global/Individual	1	2	3	4	5
1	66	5	1	0	0
2	6	12	0	0	0
3	0	3	15	0	0
4	0	0	0	6	3
5	0	0	0	3	6

Note: The table captures the correspondence of the global cluster formed for all the stock markets together and for the clusters calculated using the individual stock market indices. Each entry in the table indicates the number of cases when the cluster calculated using the individual stock market indices corresponds to the cluster calculated based on the entire sample. For example, 66 price jump indicators belong to Cluster 1 using the individual stock market indices and also belong to Cluster 1 using the entire sample. Further, there are 6 cases when price jump indicators belong to Cluster 2 calculated using the individual stock market indices, while using the entire sample, they belong to Cluster 1. The remaining entries should be interpreted in a similar way. The symmetric table denotes no significant difference between global clustering and clustering at the individual stock market index level.

Table 6: Augmented Dickey-Fuller: Estimated number of price jumps per month.

	Developed markets			European markets			Emerging CEE markets		
	INDU	SPX	NKY	CAC	DAX	UKX	BUX	PX	WIG
No. 1	-5.404	-4.906	-6.163	-5.459	-4.459	-3.442	-5.224	-4.999	-7.803
No. 2	-9.691	-9.229	-6.594	-9.015	-8.708	-8.965	-7.212	-7.796	-8.747
No. 3	-6.082	-6.347	-4.500	-6.484	-5.127	-4.348	-4.949	-5.964	-7.772
No. 4	-5.822	-6.091	-4.286	-6.607	-4.886	-5.129	-6.463	-6.323	-7.618
No. 5	-5.831	-6.466	-5.432	-6.610	-5.446	-5.425	-6.189	-6.475	-6.983

Note: The table contains the augmented Dickey-Fuller test statistics for the monthly estimated numbers of prices for each cluster and each stock market index. The time series for each cluster is calculated as a mean over all estimated figures from the price jump indicators belonging to a particular cluster. The test statistics tests the null hypothesis that there is a unit root in the data, with the alternative stating that the time series is stationary. McKinnon p-values are in all cases <0.001 and we may therefore reject the null hypothesis in favor of the alternative. Abbreviation glossary: INDU—the Dow Jones Industrial Average Index, USA; SPX—the S&P 500 Index, USA; NKY—the Nikkei Index, Japan; CAC—the CAC 40 Index, France; DAX—the DAX 30 Index, Germany; UKX—the FTSE 100 Index, United Kingdom; BUX—the BUX Index, Hungary; PX—the PX Index, the Czech Republic; WIG—the WIG Index, Poland.

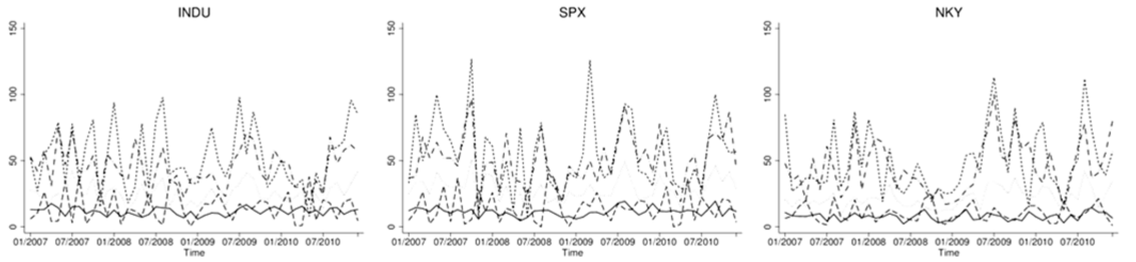
Table 7: Augmented Dickey-Fuller: differences in estimated number of price jumps per month.

	Developed markets			European markets			Emerging CEE markets		
	INDU	SPX	NKY	CAC	DAX	UKX	BUX	PX	WIG
No. 1/2	-9.319	-8.812	-7.340	-8.417	-7.297	-8.017	-7.009	-7.393	-7.909
No. 1/3	-6.312	-6.862	-4.154	-7.019	-6.278	-4.232	-5.303	-6.216	-6.480
No. 1/4	-6.116	-6.423	-4.276	-7.064	-4.928	-5.320	-6.408	-6.444	-6.860
No. 1/5	-5.865	-6.602	-5.378	-6.677	-5.668	-5.374	-6.629	-6.461	-6.803
No. 2/3	-9.618	-8.198	-4.912	-8.934	-7.309	-7.146	-6.238	-6.704	-8.248
No. 2/4	-7.727	-7.022	-4.245	-8.143	-5.491	-6.065	-6.479	-6.499	-7.539
No. 2/5	-7.256	-7.399	-5.191	-7.829	-5.355	-6.804	-6.119	-6.944	-7.095
No 3/4	-6.255	-5.764	-5.264	-6.379	-4.556	-7.028	-7.272	-6.668	-7.983
No. 3/5	-6.137	-6.488	-5.775	-7.002	-5.843	-6.074	-7.040	-6.359	-6.146
No. 4/5	-5.998	-6.910	-6.797	-6.630	-5.790	-6.951	-7.861	-7.038	-6.650

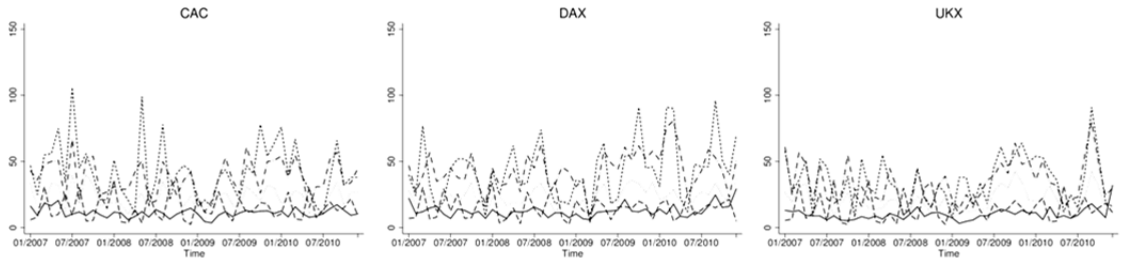
Note: The table contains the augmented Dickey-Fuller test statistics for the monthly differences in the estimated numbers of prices for each cluster and each stock market index. The time series for each cluster is calculated as a mean over all estimated figures from the price jump indicators belonging to a particular cluster. Then, we calculate the difference between clusters, where the notation is such that "No. 1/2" denotes the difference in estimated figures for Cluster 1 minus the figures for Cluster 2. The test statistics tests the null hypothesis that there is a unit root in the data, with the alternative stating that the time series is stationary. McKinnon p-values are in all cases <0.001 and we may therefore reject the null hypothesis in favor of the alternative. Abbreviation glossary: INDU—the Dow Jones Industrial Average Index, USA; SPX—the S&P 500 Index, USA; NKY—the Nikkei Index, Japan; CAC—the CAC 40 Index, France; DAX—the DAX 30 Index, Germany; UKX—the FTSE 100 Index, United Kingdom; BUX—the BUX Index, Hungary; PX—the PX Index, the Czech Republic; WIG—the WIG Index, Poland.

Figure 1: Cluster analysis: Frequency of price jumps per month.

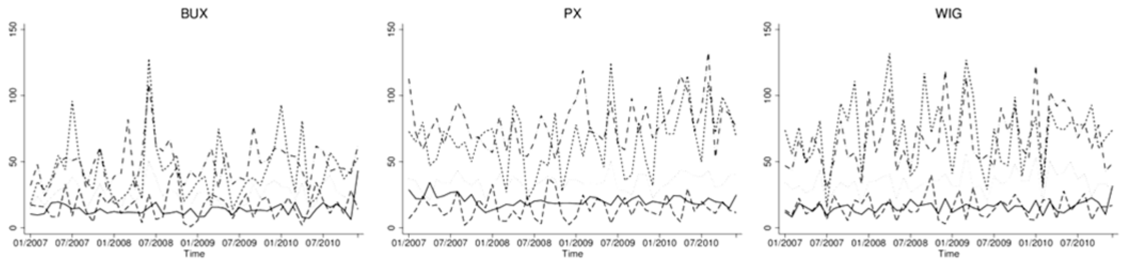
Developed Markets



European Markets



Emerging CEE Markets



Note: The figures depict the number of price jumps per month using different clusters. The notation is as follows: solid line for Cluster 1, dashed line for Cluster 2, dotted line for Cluster 3, dash-dot line for Cluster 4, and short-dash line for Cluster 5. The clusters were sorted based on the mean number of price jumps per month over the entire sample. The first row includes the USA and Japan, the second row includes developed European markets, and the third row captures the emerging CEE markets. Abbreviation glossary: INDU—the Dow Jones Industrial Average Index, USA; SPX—the S&P 500 Index, USA; NKY—the Nikkei Index, Japan; CAC—the CAC 40 Index, France; DAX—the DAX 30 Index, Germany; UKX—the FTSE 100 Index, United Kingdom; BUX—the BUX Index, Hungary; PX—the PX Index, the Czech Republic; WIG—the WIG Index, Poland.