A Practical Approach of Natural Hedging for Insurance Companies

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ABSTRACT

This research investigates a natural hedging strategy and attempts to find an optimal allocation of insurance products that can deal with longevity risks for insurance companies. In this paper, using copula to capture the dependence structure of life insurance and annuity policies, natural hedging strategy is constructed by using different age of policyholder for both life insurance and annuity policies. Consequently, insurance company can hedge the whole portfolios of life insurance and annuity by simply controlling the suitable proportions of premiums at each age of policyholder for both life insurance and annuity products. This paper provides a simple and efficient approach for insurance companies to obtain a feasible natural hedging strategy in practice.

Keywords: Longevity risk, Natural hedging strategy, Experienced mortality rates, Copula

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Introduction

Longevity risk is the risk that the uncertainty in the future development of human life expectancy differs from the anticipated one. As life expectancy of humans worldwide is expected to continue increasing, longevity risk becomes an important financial issue of insurance companies (see, e.g. Olivieri and Pitacco 2003, Cossette et al 2007, and Hari et al 2008; Yang et al., 2010). Currently, lots of actuaries still have mispricing problem with both life and annuity products for the reason of without considering enough mortality improvement. Thus, life insurers may gain profits even as annuity insurers suffer losses due to longevity risk.

Typically, there are two methods to consider the mortality improvement used by actuaries in practice. First, actuaries assume that mortality rates decrease with a constant percentage each year for all ages (e.g. 1%). An overwhelming majority of insurance companies adopt this method in US. Second, they multiply a static life or annuity table by a constant percentage (e.g. 90%). The mispricing problem is more serious in the second method since it is a concept of static mortality and does not consider annual mortality improvement. This mispricing problem commonly exists in the countries with official static life or annuity tables issued by governments or actuarial societies which all insurance companies use to price life or annuity products (e.g., Taiwan, Korean, Japan). The official life or annuity tables are often out of date since the experience data used to build the table might be many years ago. The official life and annuity tables are used for valuation purpose, most of insurance companies employ them to calculate premiums due to the lack of real experience data. An intuitive solution to solve this problem is to construct stochastic mortality models to help companies hedge against longevity risk, for both life insurance and annuity products. A wide range of mortality models have been proposed and discussed in the past two decades (Lee and Carter, 1992; Brouhns et al., 2002; Renshaw and Haberman, 2003; Koissi et al., 2006; Melnikov and Romaniuk, 2006; Cairns et al., 2006; Cairns et al., 2009; Yang et al., 2010). However, this solution is often difficult to apply in practice because of the challenges of market competition. That is, even insurance companies have mortality models to account for actual future improvements in mortality, selling their annuity (life insurance) products using the derived mortality rates would be too expensive (cheap) in a competitive market because most insurance companies currently still use the concept of static mortality table to price their products.

Another possible solution is capital market solution with three kinds of approach: mortality securitization (see, for example, Cox et al. 2006), survivor bonds (e.g., Blake and Burrows, 2001; Denuit et al. 2007), and survivor swaps (e.g., Cairns et al., 2006; Dowd et al. 2006). These studies show that insurance companies could transfer their exposures to the capital markets which have more funding and participators. Although mortality derivatives are convenient, they also encounter obstacles in practice. In particular, these special purpose vehicles demand more attention to customers and counterparties, such that insurance companies incur massive transaction costs.

Natural hedging is an alternative choice to solve the mispricing problem of longevity risk. That is, insurance companies might optimize the allocation of their annuities and life insurance to hedge against longevity risk. Different from mortality securitization, natural hedging strategy does not need a liquid market to hedge longevity risk and hence avoid a large amount of transaction cost. This approach is internal to the insurance company, which makes it more convenient and practical to implement. However, natural hedging strategy remains a relatively new topic in the actuarial field, and not many papers have studied this issue (Wang et al., 2003, Cox and Lin, 2007, Cui, 2009, Wills and Sherris, 2009, Wang et al., 2010, Wang et al., 2013).

The existing literatures on risk modeling of natural hedging commonly have two flaws. The first and critical flaw is that natural hedging is not remarkable in practice in the past. Previous literatures can only deal with a simple case: hedging of a group of certain age life insurance requires a group of another certain age annuity. That is, if an insurance company tries to employ a natural hedging strategy, it needs a one-by-one pair of each age to construct the corresponding hedging strategy. This simple strategy can help insurers to understand the ideas of natural hedging, but it is very difficult to apply for the complicated portfolio consisting of life and annuity policies with different genders, ages, and face amounts in practice. The second flaw is to neglect the dependence structure of longevity risk, we need a mortality model to project the future mortality rates for life insurance and annuity simultaneously. As pointed out by Wilson (2001), due to global convergence in mortality levels, it is improper to prepare mortality forecasts for individual national populations in isolation from one another, and even more so for the regions within a country.

The main contribution of this paper is to obtain an optimal natural hedging strategy under which insurance companies can hedge the entire portfolios of life insurance and annuity by simply controlling the suitable proportions of premiums in each age for both life and annuity products. In addition, we use experienced mortality rates from life insurance companies rather than population mortality rates. This data set includes more than 50,000,000 policies, collected from the incidence data of all Taiwanese life insurance companies. Using experienced mortality rates makes our proposed model more realistic for the application of insurance companies in practice. Finally, following Wang et al. (2015), we extend the Lee-Carter model (Lee and Carter, 1992) to a multi-population framework to capture the mortality dependence between life insurance mortality rates and the annuity mortality rates. Consqeuently, this paper aims to solve all these two shortcomings of previous literatures and provides a practical natural hedging strategy for insurance companies.

The reminder of this article is organized as follows: Section 2 provides a Lee-Carter model under a multi-population framework. Section 3 contains the optimal natural hedging strategy under which insurance companies can hedge the entire portfolios of life insurance and annuity. Section 4 is numerical analysis. In Section 5, we summarize our findings and offer some conclusions and suggestions for further research.

Parameter Calibration of Mortality Data

In this section, we specify the model for the mortality rate and interest rate. We then introduce the copula method to capture the mortality dependence between life insurance mortality rates and the annuity mortality rates.

2.1. Mortality model

We use the LC model (Lee and Carter, 1992) to project the mortality process. The Census Bureau population forecast similarly has used it as a benchmark for long-run forecasts of U.S. life expectancy. The two most recent Social Security Technical Advisory Panels suggest that trustees should adopt this method or tactics consistent with it (Lee and Miller, 2001). For our study, the central death rate for age *x* at time *t*, $m_{x,t}$, follows the

$$\ln(m_{x,t}) = \alpha_x + \beta_x k_t + e_{x,t} , \tag{1}$$

where $m_{x,t}$ represent the central death rate for a person aged *x* at time *t*, α_x is the average age-specific mortality factor, β_x is the age-specific improving factor, and k_t is the time-varying mortality index. The time effect index k_t can be estimated by using an ARIMA (p,1,q) process.

Many articles investigate the fit of Lee-Carter parameters; the two most popular methods are singular value decomposition (SVD) and approximation; Lee and Carter (1992) use SVD to find parameters. However, Wilmoth (1993) modifies approximation method, weighted least squares, to avoid the zero-cell problem. Therefore, we use the approximation method to estimate the parameters of the Lee-Carter model, because we recognize some missing values in our data.

We simulate 100,000 paths of simulations of mortality rates for each age of policyholders to catch the possible evolution of future mortality rates according to the calibrated parameters of the Lee-Carter model.

2.2. Mortlaity Dependence between life insurance and the annuity

In this paper, the mortality data, collected from the Taiwan Insurance Institute (TII), include more than 50,000,000 policies issued by life insurance companies in Taiwan. This mortality data set contains policy information of all insurance companies in Taiwan for more than thirty years. The original data are categorized by age, gender, and sorts. We use these original data to construct four Lee-Carter mortality tables: female annuity (fa), male annuity (ma), female life insurance (fl), and male life insurance (ml). The maximal age of a policyholder in the original data is about 85 years. We calibrate the parameters of our model using the approximation method, depicting in Figure 1.

[Insert Figure 1]

Let $m_x(t)=m_{x+t,t}$; $m_x^{A,1}(t)$ ($m_x^{A,2}(t)$) denotes the mortality rate of the male(female) annuity aged $x+t$ at time t; $m_x^{L,1}(t)$ ($m_x^{L,2}(t)$) denotes the mortality rate of the male(female) life insuranceaged $x+t$ at time t. Consequently, we have

$$\ln m_{x,t}^{s,g}=\alpha_x^{s,g}+\beta_x^{s,g}k_t^{s,g}+e_{x,t}^{s,g}\ ,\quad s=L\ or\ A,\ g=1\ or\ 2\ , \qquad (2)$$

where

$$k_t^{s,g}-k_{t-1}^{s,g}=\omega+\sum_{h=1}^{P}\varphi_h^{s,g}\left(k_{t-h}^{s,g}-k_{t-h-1}^{s,g}\right)+\sum_{h=0}^{Q}\psi_h^{s,g}\varepsilon_{t-h}^{s,g}\ ,\quad s=L\ or\ A,\ g=1\ or\ 2\ , \qquad (3)$$

Li and Lee (2005) point out that the populations of the world are becoming more closely linked by communication, transportation, trade, technology, and disease. Observing a portfolio of annuities for couples, Frees et al. (1996) demonstrate that the times of death are highly correlated. Recently, Luciano et al. (2008) have used copula methods to capture the dependency between the survival times of members of a couple. Accordingly, to capture the dependence structure of mortality rates of female annuity, male annuity, female life insurance, and male life insurance, we assume that the dependence structure of four mortality rates risk factors $e_{x,t}^{A,1}$, $e_{x,t}^{A,2}$, $e_{x,t}^{L,1}$ and $e_{x,t}^{L,2}$ is as follows:

$$P\left(\varepsilon_{x,t}^{A,1}\le x_1,\varepsilon_{x,t}^{A,2}\le x_2,\varepsilon_{x,t}^{L,1}\le x_3,\varepsilon_{x,t}^{L,2}\le x_4\middle|\Im_{t-1}\right)$$

$$=C\left(F^{A,1}\left(x_1\middle|\Im_{t-1}\right),F^{A,2}\left(x_2\middle|\Im_{t-1}\right),F^{L,1}\left(x_3\middle|\Im_{t-1}\right),F^{L,2}\left(x_4\middle|\Im_{t-1}\right)\middle|\Im_{t-1}\right), \qquad (4)$$

where $F^{s,g}$ is the marginal conditional cdf of $\varepsilon^{s,g}$ and C is a conditional copula function. Following Wang et al. (2015), we use the symmetric multivariate copulas—Gaussian copula and Student's t copula—to characterize the symmetric mortality dependence and the asymmetric copulas—skewed t copula—to characterize the asymmetric mortality dependence. We will empirically test whether the dependence structure of four mortality rates risk factors display symmetry or asymmetry in the dependence structure.

2.2. Interest rate model

Several models depict the local process for the short-term interest rate. To avoid the problem of a negative nominal interest rate, we assume that the time-*t* spot rate $r(t)$ for a filtered probability space $(\Omega, F, P, (F_t)_{t=0}^{T})$ follows the Cox, Ingersoll and Ross (CIR; 1985) model,

$$dr(t) = \kappa_r(\theta_r - r(t))dt + \sigma_r\sqrt{r(t)}dW_r(t), \tag{5}$$

where F_t, $t \in [0,T]$, is the smallest sigma field, such that $r(t)$ and the housing price $H(t)$ is known and measurable; P is the physical (real-world) probability measure; $(F_t)_{t=0}^{T}$ is the right-continuous natural filtration, such that $F_t \subset F_u$, $t \leq u$; θ_r is the long-run short interest rate; κ_r is the speed of reversion; σ_r is the instantaneous volatility; and $W_r(t)$ is a standard Brownian motion.

In a discrete-time setup, we assume that the spot rate between time t and $t+\Delta t$ is fixed at $r(t)$ but may vary from one band to the next. Consequently, the spot rate dynamic is governed by

$$r(t+\Delta t) - r(t) = \kappa_r(\theta_r - r(t))\Delta t + \sigma_r\sqrt{r(t)}\Delta W_r(t). \tag{6}$$

Natural Hedging Strategy

In this paper, we aim to provide a simple and efficient method for insurance companies to obtain a feasible strategy to meet their requirement of natural hedging in practice. The most convenient way of natural hedging for insurance companies is to simply control proportions of premium held in each age of policyholder for both life insurance and annuity policies. Therefore, this approach provides an optimal strategy showing that insurance companies should hold how much proportions of premiums in each age of policyholders for both life and annuity products to meet the purpose of natural hedging. So, with this optimal strategy, insurance company can hedge the whole portfolios of life and annuity by simply controlling the suitable proportions of premiums in each age of policyholders. For the purpose of hedging mispricing risk due to underestimating mortality improvement, we first calculate the profit or deficit of life and annuity for each age of policyholder of a random simulation as follow:

$$\pi_{i,j} = \frac{Px_{i,j}|_{static} - Px_{i,j}|_{dynamic}}{Px_{i,j}|_{static}}, \tag{7}$$

$$\text{for } i = \begin{cases} 1\sim 13, \text{the policyholder of male} - \text{annuity aged } (18+5i) \\ 14\sim 26, \text{the policyholder of male} - \text{insurance aged } (18+5(i-13)) \\ 27\sim 39, \text{the policyholder of female} - \text{annuity aged } \left(18+5(i-26)\right) \\ 40\sim 52, \text{the policyholder of female} - \text{insurance aged } \left(18+5(i-39)\right) \end{cases}$$

where $Px_{i,j}|_{static}$ is the premium calculated by static mortality table of the j^{th} simulation for a certain age of life or annuity policyholder and $Px_{i,j}|_{dynamic}$ is the premium calculated by Lee Carter model of the j^{th} simulation for a certain age of life or annuity policyholder. According to the experience data, we assume the age range of policyholders is between 21 and 85, although people older than 75 may not be allowed to purchase life policies in practice. In order to reduce the numbers of decision variables, we set every five ages of policyholder as a group. For example, if j=1 meaning the annuity policyholder between age 21 and 25. We simulate 100000 paths for each age of policyholder to calculate the profit matrix of life and annuity portfolio as follow.

$$\pi_i = \sum_{j=1}^{100000} \pi_{i,j}, \tag{8}$$

$$\Pi = [\pi_1, \pi_2, \dots, \pi_{52}]^T \ , \tag{9}$$

Following the definition of the above definition of i=1~52, The vector of decision variables, $W = [w_1, w_2, \dots, w_{52}]^T$, are defined as follow. w_i represents the proportion of premiums which should be held for the i^{th}ith group of policyholder in the life and annuity portfolio. For example, w_1 is the proportion of premiums for the male annuity policyholder of age between 21 and 25. w_{40} is the proportion of premiums for the female life policyholder of age between 21 and 25. As a result, the return of life and annuity portfolio is equal to $R_p = W^T \Pi$.

In this paper, we adapt the following natural hedging strategies. The first one is to minimize volatility of mispricing in natural hedging. We set the objective function as follow:

$$\begin{aligned} &Min\ V(R_p) \\ &\text{subject to } 0 \le W \le 1, W^T 1 = 1 \ , \end{aligned} \tag{10}$$

Minimizing volatility of mispricing is important. However, without considering volatility and profit simultaneously, insurance companies may suffer deficit from mispricing due to ignoring the constraint of non-negative profit. Therefore, we further add a constraint to avoid the insurance companies suffer a deficit from mispricing due to mortality improvement as follow.

$$\begin{aligned} &Min\ V(R_p) \\ &subject\ to\ W^T \Pi \ge 0, W^T 1 = 1, 0 \le W \le 1, \end{aligned} \tag{11}$$

This objective function is momentous for insurance companies since the optimal decision can allow them to have a non-negative profit with low risk. Furthermore, it is important to consider the premium structure of insurance companies. For example, the overwhelming majority of premium amounts in the current premium structure in Taiwan is life insurance policy. As a result, we will further consider the insurer's current premium structure as an additional constraint.

Numercial Analysis

To be continued.

Conclusions and Suggestions

To be continued.

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