Modelling spousal mortality dependence: evidence of heterogeneities and implications

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INTRODUCTION

Motivation

- It has been documented that the two spouses have positive mortality dependence.
- Many pension funds/annuity providers offer reversionary benefit.
- The aim of this paper is to model such a dependence, and discuss its impact on pricing.

Cash flow of a joint annuity: Case A

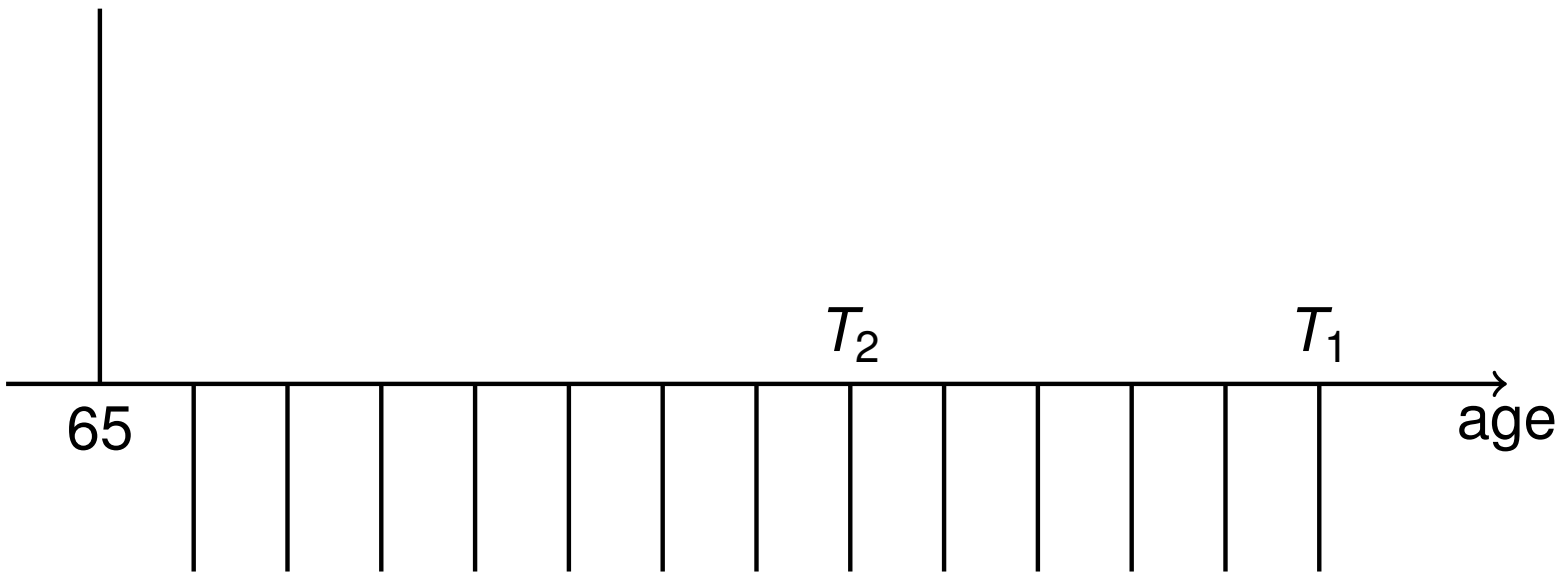


Figure: In Case A, the principal beneficiary dies after the spouse ($T_1 > T_2$). No additional benefit for the spouse.

Case B

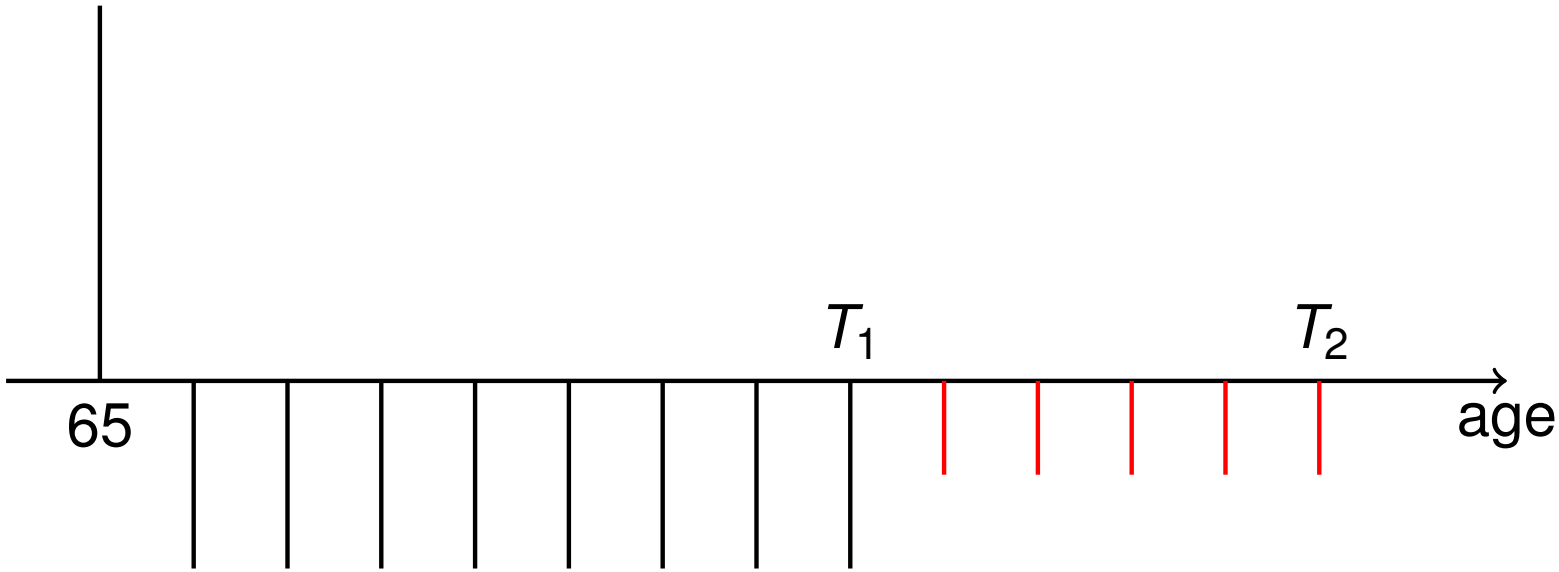


Figure: In Case B, the principal beneficiary dies before the spouse ($T_1 < T_2$). Spouse continuous to be receive a reduced annuity.

Different types of dependencies

The mortality jump can be due to two reasons:

- The broken-heart syndrome, that is a causal effect.
- Spurious risk correlation due to similar risk factors, i.e. observed and unobserved heterogeneities.

Current literature usually neglects the heterogeneities.

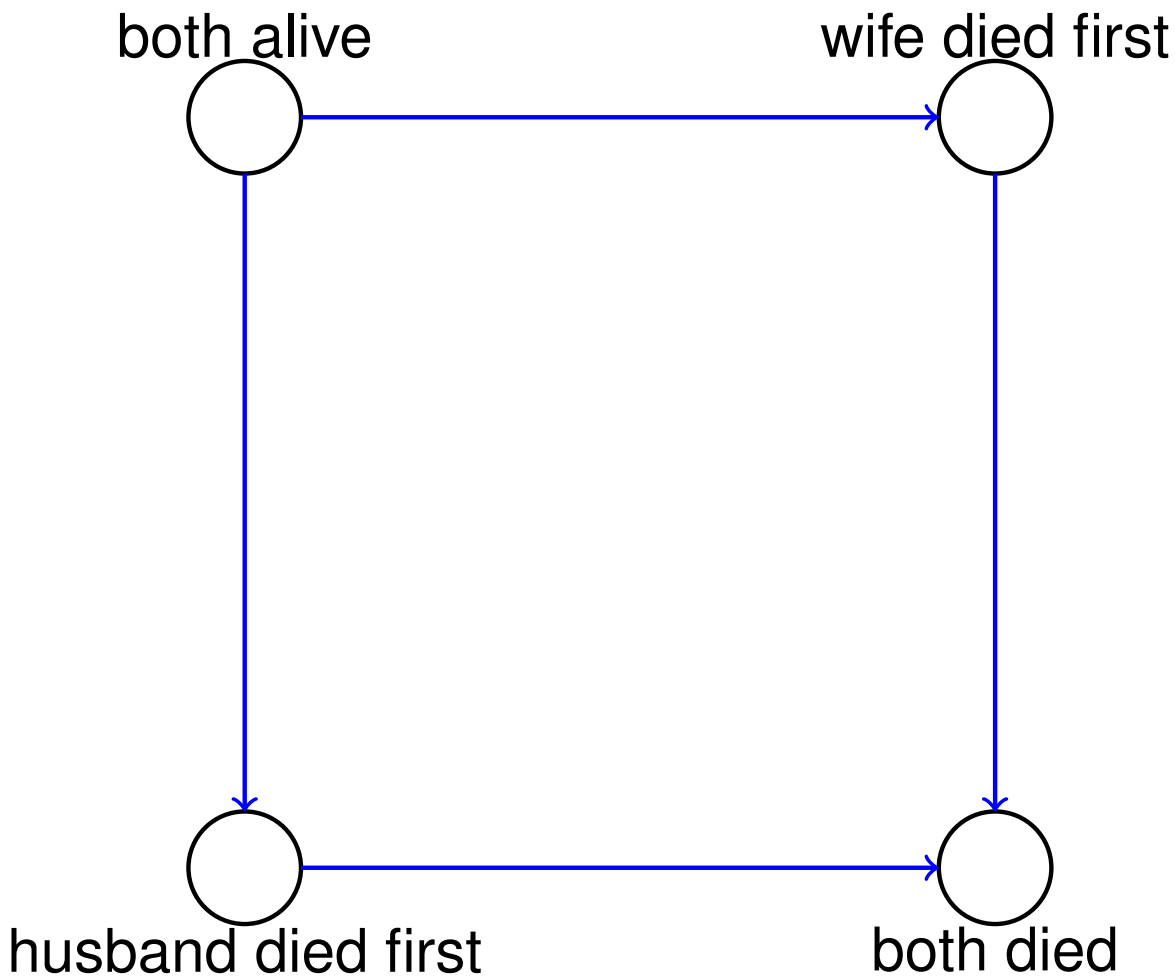
It is important to disentangle these different types of dependencies [see Gourieroux and Lu (2015 IME)].

Outline

- We estimate a model which distinguishes them.
- We find that heterogeneities explain a significant proportion of the dependency. These heterogeneities can be either observed or unobserved.
- We find important over/under estimations of the standard model depending on the characteristics of the couple.

THE MODEL

The status of the couple



Before the first death: spurious risk correlation

The mortality intensities are:

$$\theta_1(t|U,V,T_1>t,T_2>t,X,c_1,c_2)=\lambda_1(t+c_1)\exp(\beta_1'X)U,$$
$$\theta_2(t|U,V,T_1>t,T_2>t,X,c_1,c_2)=\lambda_2(t+c_2)\exp(\beta_2'X)V,$$

where:

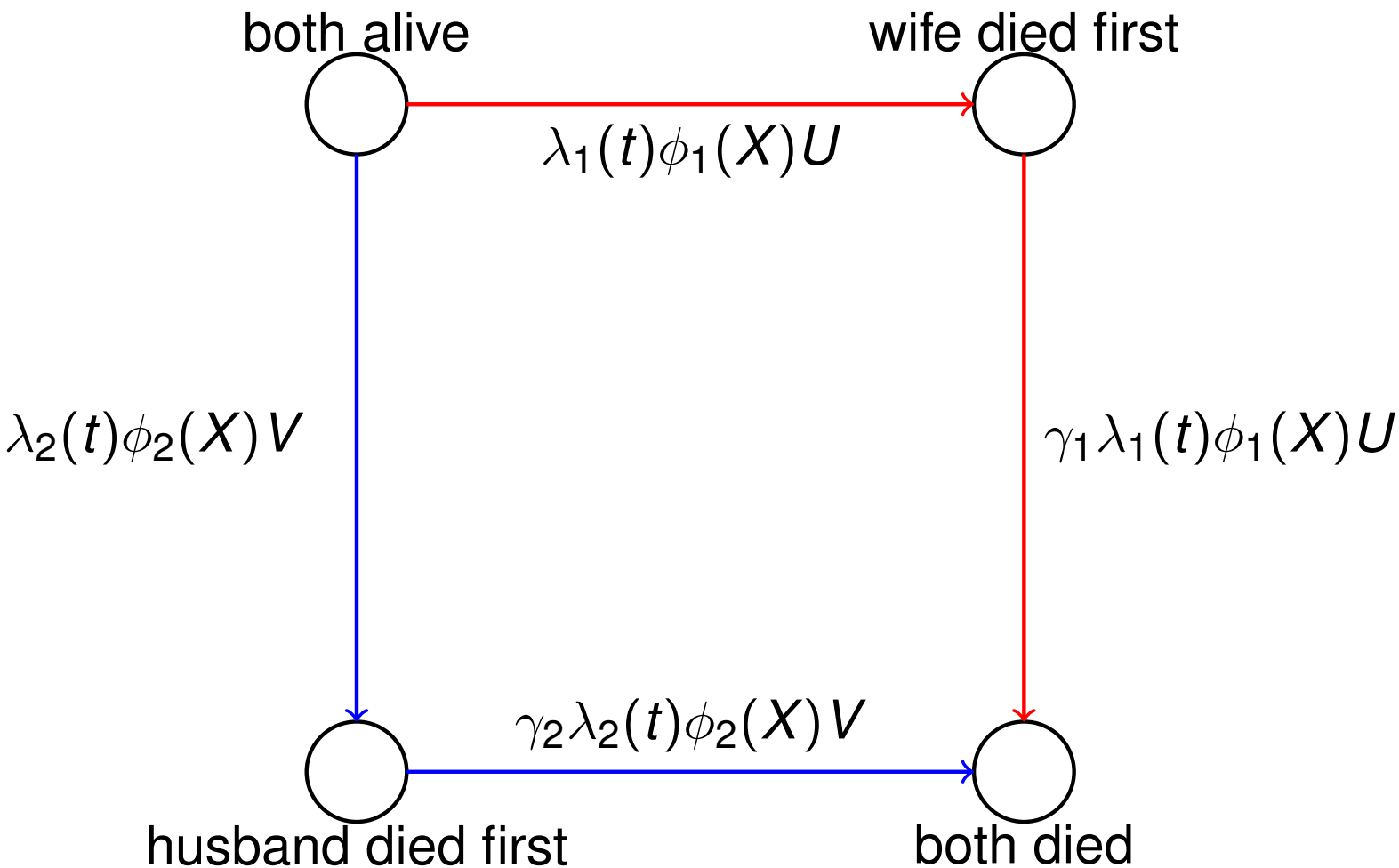
- T_1, T_2 are duration, c_1, c_2 are ages at inception;
- X is the covariate(s), including ages at inception and dates of birth;
- Thus there can be duration effects, age effects, and even cohort effects;
- U, V are the unobserved individual heterogeneities; they introduce *spurious dependence* between T_1, T_2;
- λ_1, λ_2 are baseline intensities;

This is called mixed proportional hazard competing risks model.

After the first death: the broken-heart syndrome

- After the death of male (resp. female), the mortality of the spouse is multiplied by a constant γ_1 (resp. γ_2).
- These parameters capture the real duration dependence, the ”contagion".
- Usually $\gamma_1 \neq \gamma_2$.

(Conditional) transition intensities



The parameters of interest are:

- Baseline intensities λ_1, λ_2, which we specify as exponential functions.
- The distribution of (U, V), which we specify in a semi-parametric way.
- Regression coefficients β_1, β_2.
- Jump parameters γ_1, γ_2.

The data

- A French joint annuity portfolio kindly provided by Scor.
- Population size: 27000 couples.
- Observation period: 2005-2013.
- Available information: ages at inception, amount of the annuity, dates of birth, potential dates of death (with truncations and censoring).
- The model is estimated by maximum likelihood. See the paper for details.

EMPIRICAL RESULTS

What we find

- Asymmetric jumps: $\gamma_1 = 1.53 > \gamma_2 = 1.28$.
- The unobserved heterogeneities of the two spouses are highly positively correlated.
- They account for a significant portion of the dependency that cannot be explained by the broken-heart syndrome.
- Moreover the parameters γ_1, γ_2 are over-estimated if unobserved heterogeneities are omitted.

Comparison of three models

Model	unobserved heterogeneity	mortality jump
1	Yes	Yes
2	No	Yes
3	No	No

Table: Summary of the three models

We compare their forecasts of the following quantities:

- Residual life expectancies.
- The price of Contract 1 (resp. 2), for a male (female) principal beneficiary, with a $1/3$ discount for the spouse if she/he survives longer.

Case 1: Male is aged 60, female 65				
Model	male residual life expectancy	female residual life expectancy	Price Contract No. 1	Price Contract No. 2
1	23.06	24.47	25.56	26.26
2	22.86	24.28	25.35	26.05
3	23.53	23.02	25.93	25.68

Case 2: Male is aged 70, female 65				
Model	male residual life expectancy	female residual life expectancy	Price Contract No. 1	Price Contract No. 2
1	15.37	23.93	20.23	24.51
2	15.24	23.64	20.02	24.23
3	15.22	23.02	20.04	23.94

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Discussions

- The standard model that omits dependencies lead to huge error.
- This error can be either positive, or negative, depending on the age of both spouses.
- Omitting unobserved heterogeneities leads to under-estimation of the life expectancies.

CONCLUSION

- We estimated a model which captures the joint dependency of the two lifetimes.
- We found evidence of heterogeneities on the mortality rates, besides the broken-heart syndrome. We assessed their impacts.
- Finally, the findings are also useful for the pricing of single life insurance.

THANKS FOR YOUR ATTENTION.
QUESTIONS/SUGGESTIONS?