Valuation of longevity-linked life annuities

Jorge Bravo[1]   Najat El Mekkaoui[2]

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[1] Nova University of Lisbon - Infor. Management School (jbravo@novaims.unl.pt)
[2] University Paris-Dauphine (najat.el-mekkaoui@dauphine.fr)

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Introduction & Motivation

- Uncertainty about the future development of mortality gives rise to longevity risk (idiosyncratic and systematic)
- Conventional life annuities are the classical tool in the management of individual longevity risk
- In a conventional life annuity the annuity provider takes all risk
- Longevity risk management solutions include loss control/financing techniques (reinsurance, longevity swaps, Securitization,...)
- Traditional reinsurance is expensive and has limited capacity
- Regulatory capital charges are expected to increase for annuity providers under Solvency II framework
- What about sharing longevity risk between annuity provider and annuitants?
- What is the size of annuity premium discounts that should be offered to policyholders for them to accept to (partially or totally) sacrifice certainty in their annuity benefits?

Longevity linked life annuities

- A number of alternative structures which allow participants to pool mortality and longevity risk have been proposed in the literature
- Group Self-Annuitization Schemes (Piggott et al., 2005; Valdez et al., 2006; Qiao and Sherris, 2013)
  - Pool participants are insured against individual longevity risk but agree to share systematic mortality and longevity risks
- Annuity Overlay Fund (Donnelly et al., 2014)
  - investment risk is separated from mortality risk, each individual retains absolute control over their own investments, the fund does not protect against longevity risk
- Mortality indexed annuity (Richter and Weber, 2009)
- Longevity-indexed life annuities (Denuit et al., 2011, Bravo et al., 2009)
  - benefits are adjusted over time according to a longevity index

Conventional life annuity

- Consider an individual buying a conventional immediate life annuity at age x_0 in calendar year t_0
- The random life annuity single premium, computed using a cohort approach, is

$$a_{x_0}^{[\mathcal{F}_0]}(t_0) = \mathbb{E}\left(\sum_{k=1}^{T_{x_0}(t_0)} Z(0,k) \,|\mathcal{F}_0 \right) = \sum_{k=1}^{\omega - x_0} Z(0,k) \cdot {}_k\, p_{x_0}^{[\mathcal{F}_0]}(t_0)$$

where

$${}_k p_{x_0}^{[\mathcal{F}_0]}(t_0) = \prod_{j=0}^{k-1} \left[1 - q_{x_0+j}(t_0 + j)\right]$$

${}_k p_{x_0}^{[\mathcal{F}_0]}(t_0)$: probability of a person aged x_0 in year t_0 being alive at age $x_0 + k$, based on a forecast of some reference population
$Z(t, T)$: zero coupon bond price

- Consider now a longevity-linked life annuity (LLLA) structured as in Denuit et al. (2011)
- Annual annuity payments depend on a longevity index

$$\mathcal{I}_{t_0+k} = \frac{{}_k p_{x_0}^{[\mathcal{F}_0]}(t_0)}{{}_k p_{x_0}^{[\mathcal{F}_k]}(t_0)} = \prod_{j=0}^{k-1} \frac{p_{x_0+j}^{[\mathcal{F}_0]}(t_0+j)}{p_{x_0+j}^{[\mathcal{F}_k]}(t_0+j)}, \quad k = 1, ..., \omega - x_0 \qquad (1)$$

- If the contract specifies an annual benefit payment equal to one unit of currency, the annuitant receives a stream of payments $\mathcal{I}_{t_0+1}$, $\mathcal{I}_{t_0+2}, ..., \mathcal{I}_{t_0+\omega-x_0}$ as long as he/she survives
- More generally, define the following adjustment mechanism:

$$b_{t_0+k} = b_{t_0} f_{t_0+k}\left(\mathcal{I}_{t_0+k}\right), \qquad k = 1, ..., \omega - x_0, \qquad (2)$$

where b_{t_0} denotes the benefit determined at annuity inception and where f_{t_0+k} is a suitable function

- The random life annuity single premium under a LLLA is

$$a_{x_0}^{LLLA}\left(t_0|\mathcal{I}_{t_0+k}\right)=\sum_{k=1}^{\omega-x_0} Z\left(0,k\right)\cdot{}_k\, p_{x_0}^{[\mathcal{F}_k]}\left(t_0\right)\cdot b_{t_0} f_{t_0+k}\left(\mathcal{I}_{t_0+k}\right) \quad (3)$$

- Assume $b_{t_0}=1$ and $f_{t_0+k}\left(\mathcal{I}_{t_0+k}\right)=\mathcal{I}_{t_0+k}$, $\forall k\in[1,\omega-x_0]$
- From (1) we can write

$${}_k p_{x_0}^{[\mathcal{F}_0]}\left(t_0\right)={}_k\, p_{x_0}^{[\mathcal{F}_k]}\left(t_0\right)\cdot\mathcal{I}_{t_0+k}$$

so that $a_{x_0}^{LLLA}\left(t_0|\mathcal{I}_{t_0+k}\right)$ becomes

$$\sum_{k=1}^{\omega-x_0} Z\left(0,k\right)\cdot{}_k\, p_{x_0}^{[\mathcal{F}_k]}\left(t_0\right)\cdot\mathcal{I}_{t_0+k}=\sum_{k=1}^{\omega-x_0} Z\left(0,k\right)\cdot{}_k\, p_{x_0}^{[\mathcal{F}_0]}\left(t_0\right)$$

- Without loss of generality, assume that:
  - $b_{t_0} = 1$
  - $f_{t_0+k}\left(\mathcal{I}_{t_0+k}\right) = \mathcal{I}_{t_0+k}$
- Benefit payments at time $t_0 + k$ $\left(k = 1, ..., \omega - x_0\right)$ can then expressed as:

CASE A: $\mathcal{I}_{t_0+k} = 1$

- mortality rates evolve has expected

$$b_{t_0+k} = \mathcal{I}_{t_0+k} = 1 \tag{4}$$

Corollary

The longevity-linked life annuity resembles a traditional life annuity with level benefit

CASE B: $\mathcal{I}_{t_0+k} < 1$

- Observed longevity improvements are higher than predicted
- The annuity benefit payment is given by the current value of the longevity index capped by its inception value, i.e.,

$$b_{t_0+k} = \min\left(1, \mathcal{I}_{t_0+k}\right) \tag{5}$$

- This benefit can be expressed in terms of the final (maturity) payoff of an European-style longevity put option on the value of the longevity index with strike the minimum (initial) amount guaranteed, i.e.,

$$b_{t_0+k} = 1 - \underbrace{\max\left(1 - \mathcal{I}_{t_0+k}; 0\right)}_{\text{longevity floorlet}} \tag{6}$$

CASE C: $\mathcal{I}_{t_0+k} > 1$

- Observed longevity improvements are actually worse than forecasted
- Benefit payments at time $t_0 + k$ are floored by the annuity benefit at inception

$$b_{t_0+k} = \max\left(1, \mathcal{I}_{t_0+k}\right) \tag{7}$$

- The benefit can be expressed in terms of the final payoff the final payoff of a long position in a conventional level life annuity and a long position in an European-style longevity caplet with strike equal to the annuity benefit at inception and underlying $\mathcal{I}_{t_0+k}$

$$b_{t_0+k} = 1 + \underbrace{\max\left(\mathcal{I}_{t_0+k} - 1; 0\right)}_{\text{longevity caplet}} \tag{8}$$

- Objective: delimitate the systematic longevity risk beared by the policyholder in order to eliminate the possibility that annuity benefits may become extremely low in a scenario in which observed longevity improvements are significantly higher than predicted
- The longevity index is replaced by its capped version

$$\mathcal{I}_{t_0+k}\left(\mathcal{I}_{t_0+k}^{\min}, \mathcal{I}_{t_0+k}^{\max}\right) = \max\left\{\min\left(\mathcal{I}_{t_0+k}; \mathcal{I}_{t_0+k}^{\max}\right); \mathcal{I}_{t_0+k}^{\min}\right\}$$

with $0 < \mathcal{I}_{t_0+k}^{\min} < 1 < \mathcal{I}_{t_0+k}^{\max}$.

- The upper and lower barriers for the longevity index may change over time or be constant during the whole contract
- In this latter case, $\mathcal{I}_{t_0+k}^{\max} = \mathcal{I}^{\max}$ and $\mathcal{I}_{t_0+k}^{\min} = \mathcal{I}^{\min}$, $k = 1, \ldots, \omega - x_0$

Alternative decompositions for the annuity payments in terms of European-style longevity options:

1. A **protective longevity collar**, i.e., a combination between the underlying, a long position in a longevity floorlet with strike $\mathcal{I}^{\min}_{t_0+k}$ and a short position in a longevity caplet with strike $\mathcal{I}^{\max}_{t_0+k}$

$$b_{t_0+k} = \mathcal{I}_{t_0+k} + \underbrace{\max\left(\mathcal{I}^{\min}_{t_0+k} - \mathcal{I}_{t_0+k}; 0\right)}_{\text{longevity floorlet}} - \underbrace{\max\left(\mathcal{I}_{t_0+k} - \mathcal{I}^{\max}_{t_0+k}; 0\right)}_{\text{longevity caplet}}$$

2. A **long position in a longevity caplet spread**, i.e., a combination between the minimum guarantee, a long position in a longevity caplet with strike $\mathcal{I}^{\min}_{t_0+k}$ and a short position in a longevity caplet with strike $\mathcal{I}^{\max}_{t_0+k}$

$$b_{t_0+k} = \mathcal{I}^{\min}_{t_0+k} + \max\left(\mathcal{I}_{t_0+k} - \mathcal{I}^{\min}_{t_0+k}; 0\right) - \max\left(\mathcal{I}_{t_0+k} - \mathcal{I}^{\max}_{t_0+k}; 0\right)$$

Capped Longevity-linked life annuities

Alternative decompositions for the annuity payments in terms of European-style longevity options:

3. A **short position in a longevity floorlet spread**, i.e., a combination between the maximum benefit, a long position in a longevity floorlet with strike $\mathcal{I}_{t_0+k}^{\min}$ and a short position in a longevity floolet with strike $\mathcal{I}_{t_0+k}^{\max}$

$$b_{t_0+k} = \mathcal{I}_{t_0+k}^{\max} + \max\left(\mathcal{I}_{t_0+k}^{\min} - \mathcal{I}_{t_0+k}; 0\right) - \max\left(\mathcal{I}_{t_0+k}^{\max} - \mathcal{I}_{t_0+k}; 0\right)$$

Remark: Contrary to traditional financial options that give the buyer the right, but not the obligation, to buy or sell the underlying asset at a specific strike price on or before a certain date, the longevity options embeeded in a LLLA will be automatically exercised if expiring in-the-money.

Structuring products using longevity options

- By combining a long position in a longevity caplet with a short position in a longevity floorlet we are able to build a **synthetic long forward position** in the longevity index, i.e., for $b_{t_0} = 1$

$$\max\left(\mathcal{I}_{t_0+k} - 1; 0\right) - \max\left(1 - \mathcal{I}_{t_0+k}; 0\right) = \left(\mathcal{I}_{t_0+k} - 1\right)$$

- In a symmetrically designed longevity-linked life annuity at any time $t_0 + k$ $(k = 1, ..., \omega - x_0)$ the benefit payment can be expressed as a combination of a long position in a conventional life annuity with a long position in a synthetic forward on the longevity index, i.e.,

$$\begin{aligned} b_{t_0+k} &= 1 + \max\left(\mathcal{I}_{t_0+k} - 1; 0\right) - \max\left(1 - \mathcal{I}_{t_0+k}; 0\right) \\ &= 1 + \left(\mathcal{I}_{t_0+k} - 1\right) = \mathcal{I}_{t_0+k} \end{aligned}$$

Pricing longevity-linked life annuities

- The single premium of a symmetrically designed LLLA can be calculated by adding the price difference between an European-style longevity cap $\mathcal{L}^{CAP}$ and the corresponding longevity floor $\mathcal{L}^{FLOOR}$ (both with maturity $\omega - x_0$, underlying asset $\mathcal{I}_{t_0+k}$ and constant strike equal to 1) from a conventional life annuity premium

$$a_{x_0}^{LLLA}\left(t_0|\mathcal{I}_{t_0+k}\right) = a_{x_0}^{[\mathcal{F}_0]}\left(t_0|\mathcal{I}_{t_0+k}\right) + \mathcal{L}^{CAP} - \mathcal{L}^{FLOOR} \tag{9}$$

where

$$\mathcal{L}^{CAP} = \sum_{k=1}^{\omega-x_0} Z\left(0,k\right)_k\, p_{x_0}^{[\mathcal{F}_k]}(t_0) \max\left(\mathcal{I}_{t_0+k} - 1; 0\right) \tag{10}$$

$$\mathcal{L}^{FLOOR} = \sum_{k=1}^{\omega-x_0} Z\left(0,k\right)_k\, p_{x_0}^{[\mathcal{F}_k]}(t_0) \max\left(1 - \mathcal{I}_{t_0+k}; 0\right) \tag{11}$$

Pricing longevity-linked life annuities

- Similar to an interest rate cap, the embedded longevity cap (floor) entails a series of European call (put) options or longevity caplets (floorlets), each of which expiring at each period the longevity linked life annuity is in existence
- Finally, the premium for a Capped longevity-linked life annuity (CLLLA), $a_{x_0}^{CLLLA} \equiv a_{x_0}^{CLLLA}\left(t_0 | \mathcal{I}_{t_0+k}, \mathcal{I}_{t_0+k}^{\max}, \mathcal{I}_{t_0+k}^{\min}\right)$ can be written as

$$a_{x_0}^{CLLLA} = \sum_{k=1}^{\omega - x_0} Z\left(0, k\right)_k \, p_{x_0}^{[\mathcal{F}_k]}(t_0) \max\left\{\min\left(\mathcal{I}_{t_0+k}; \mathcal{I}_{t_0+k}^{\max}\right); \mathcal{I}_{t_0+k}^{\min}\right\}$$

- The premium $a_{x_0}^{CLLLA}\left(t_0 | \mathcal{I}_{t_0+k}, \mathcal{I}_{t_0+k}^{\max}, \mathcal{I}_{t_0+k}^{\min}\right)$ represents the systematic risk remaining with the annuity provider

The valuation framework

- The valuation of longevity-linked derivatives with non-linear payoff structure has received little attention in the literature:
  - Blake et al. (2006a) suggest that the payoff of an option should be linked to a survivor index or futures price
  - Lin and Cox (2007) study the pricing of a longevity call option linked to a population longevity index for older ages by combining a GBM and a compound Poisson process allowing for jumps
  - Cui (2008) uses the Equivalent Utility Pricing Principle to valuate longevity options
  - Dawson et al. (2010) derive a closed-formula for European swaptions
  - Milidonis et al. (2011) price longevity options using a Markov regime-switching model
  - Wang and Yang (2013) price survivor floors using an extension of the Lee-Carter model
  - Boyer and Stentoft (2013) evaluate both European and American-style short-term survivor moving strike options using risk-neutral simulation methods

The valuation framework

- Because of market incompleteness, the valuation of Longevity-linked securities is difficult
- Various methods have been proposed to approximate the prices of longevity-linked securities in an incomplete market
  - arbitrage-free pricing framework (e.g., Cairns et al. 2006)
  - Instantaneous Sharpe ratio (e..g, Milevsky et al. 2005)
  - Equivalent Utility Pricing Principle (e.g., Cui 2008)
  - CAPM- and CCAPM-based approach (Friedberg and Webb 2005)
  - Distortion operators, e.g., **Wang transform** (e.g., Lin and Cox (2005), Dowd et al. (2006), Blake et al. (2006b), Denuit, Devolder, and Goderniaux (2007)
  - Risk-neutral simulation approach (Boyer and Stentoft, 2013)

The valuation framework

- We use the risk-neutral simulation approach to price the longevity options embedded in LLLA
- Risk-adjusted distributions of future mortality rates are simulated using the Wang transform and assuming the dynamics of mortality rates is modelled using the log bilinear Lee-Carter model under a Poisson setting
- The Wang-distortion methodology is simple to implement in practice, has nice properties and preserves Gaussianity
- Wang (1996) defines the following risk adjusted distribution:

$$\tilde{F}(x) = \Phi\left[\Phi^{-1}\left(F(x) - \lambda\right)\right]$$

  where $F(x)$ is the CDF of variable X, Φ is the normal CDF, and λ is the market price of risk
- If applied on a Gaussian distributed variable X, i.e. $X \sim N\left(\mu, \sigma^2\right)$, the Wang operator yields a transformed variable $\tilde{X}$ that also follows a Normal distribution with $N\left(\mu - \lambda\sigma, \sigma^2\right)$

Log bilinear Lee-Carter model under a Poisson setting

- Brouhns et al. (2002), Renshaw and Haberman (2003)
- The model is defined by

$$D_{x,t} \sim \mathcal{P}oisson\left(\mu_x\left(t\right)E_{x,t}\right) \tag{12}$$

with

$$\mu_x\left(t\right) = \exp\left(\alpha_x + \beta_x\kappa_t\right), \tag{13}$$

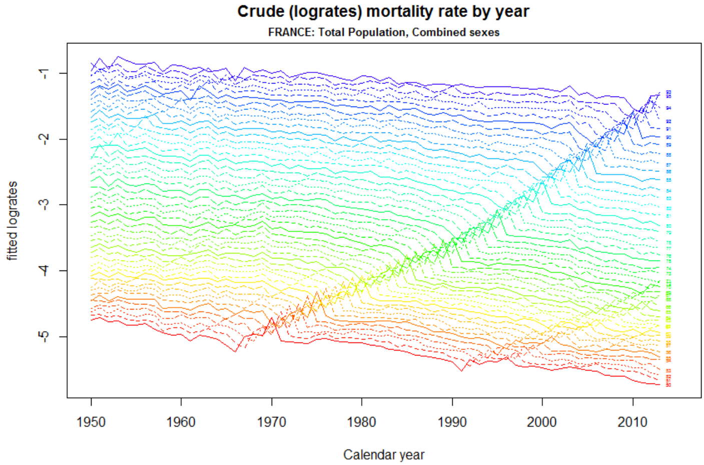
s.t. identification constraints

$$\sum_{t=t_{\min}}^{t_{\max}} \kappa_t = 0 \quad \text{and} \quad \sum_{x=x_{\min}}^{x_{\max}} \beta_x = 1.$$

- Parameters estimated using an iterative algorithm (Goodman, 1979)
- Denuit and Goderniaux (2005) closure method was applied, with $p_{125} = 0$

Estimation results for the Poisson Lee–Carter model

- Mortality Data: French total population 1950-2013, ages 50-95
- Source: Human Mortality Database (2015)



Estimation results for the Poisson Lee–Carter model

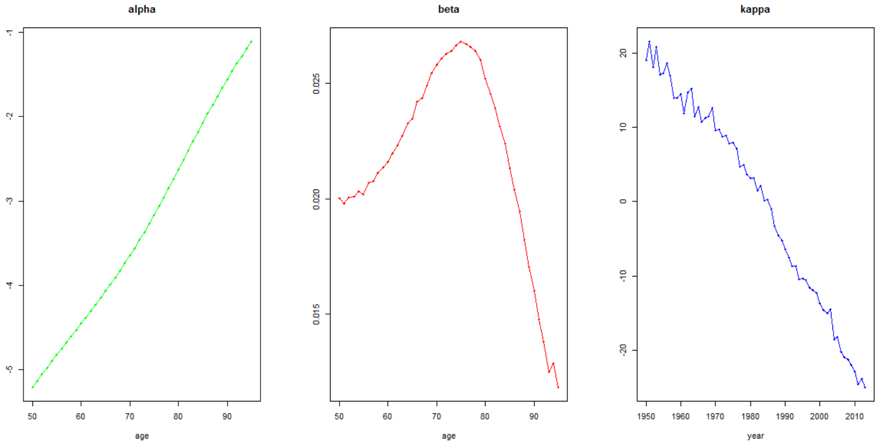


Figure: Poisson Lee-Carter parameter estimates

Forecasting/Simulating with the Poisson Lee–Carter model

- We model κ_t using a random walk with drift time series model

$$\kappa_t = \theta + \kappa_{t-1} + \epsilon_t, \quad \epsilon_t \sim N\left(0, \sigma^2\right)$$

θ	$s.e.\ (\theta)$	σ	AIC	BIC	R^2
-0.6995	0.1818	2.083	229.01	233.29	0.989

Table: Parameter estimates and goodness-of-fit measures for ARIMA(0,1,0) model using data for France from 1950 to 2013.

- To simulate risk-adjusted distributions of future mortality rates, we use draws from a $\epsilon \sim N\left(-\lambda\sigma, \sigma^2\right)$ distribution, with $\lambda \in [0.0, 0.3]$
- A simulation consists of $N = 10000$ trajectories for κ_t generated recursively, one for each year necessary to price the longevity derivatives

Pricing results: Longevity floor

Panel A: Conventional life annuity pure premium $(r = 0\%)$

Age	50	55	60	65	70	75	80	85	90
$a_{x_0}^{[\mathcal{F}_0]}(t_0)$	36.80	31.80	26.98	22.34	17.89	13.70	9.93	6.79	4.44

Panel B: Longevity Floor price

Age	50	55	60	65	70	75	80	85	90
$\lambda = 0.0$	0.51	0.46	0.40	0.34	0.27	0.21	0.14	0.09	0.05
$\lambda = 0.1$	1.17	1.01	0.84	0.68	0.52	0.37	0.24	0.14	0.08
$\lambda = 0.2$	2.08	1.76	1.44	1.13	0.84	0.58	0.37	0.21	0.11
$\lambda = 0.3$	3.08	2.60	2.13	1.66	1.23	0.84	0.51	0.28	0.14

Panel C: Longevity Floor price as a % of $a_{x_0}^{[\mathcal{F}_0]}(t_0)$ (in b.p)

Age	50	55	60	65	70	75	80	85	90
$\lambda = 0.0$	137	143	148	151	153	151	144	132	115
$\lambda = 0.1$	317	316	312	303	289	269	242	207	171
$\lambda = 0.2$	565	553	534	508	472	425	368	304	241
$\lambda = 0.3$	837	819	789	745	685	609	518	417	323

Pricing results: Capped Longevity floor

Panel A: Capped Longevity Floor price ($r = 0\%$)

Bounds/Age	50	55	60	65	70	75	80	85	90
[0.0, 1]	3.08	2.60	2.13	1.66	1.23	0.84	0.51	0.28	0.14
[0.9, 1]	1.53	1.33	1.12	0.92	0.71	0.52	0.34	0.21	0.11
[0.8, 1]	2.21	1.91	1.60	1.29	0.98	0.70	0.45	0.26	0.14
[0.7, 1]	2.60	2.24	1.86	1.48	1.11	0.78	0.49	0.27	0.14
[0.6, 1]	2.84	2.42	2.00	1.58	1.18	0.81	0.51	0.28	0.14
[0.5, 1]	2.97	2.52	2.07	1.63	1.21	0.83	0.51	0.28	0.14
[0.4, 1]	3.03	2.57	2.11	1.65	1.22	0.83	0.51	0.28	0.14
[0.3, 1]	3.07	2.59	2.12	1.66	1.22	0.83	0.51	0.28	0.14
[0.2, 1]	3.08	2.60	2.13	1.66	1.23	0.84	0.51	0.28	0.14
[0.1, 1]	3.08	2.60	2.13	1.66	1.23	0.84	0.51	0.28	0.14

Table: Life annuity pure premium and longevity floor price for cohorts of individuals with selected ages between 50 and 90 years old in 2013. Results are based on a flat yield curve at zero percent and a longevity risk premium equal to 0.3.

Pricing results: Capped Longevity floor

Panel B: Capped Longevity Floor price as a % of $a_{x_0}^{[\mathcal{F}_0]}(t_0)$ (in b.p)

Bounds/Age	50	55	60	65	70	75	80	85	90
[0.0, 1]	837	819	789	745	685	609	518	417	323
[0.9, 1]	417	418	416	410	398	378	347	304	256
[0.8, 1]	601	601	593	577	550	510	452	379	305
[0.7, 1]	708	703	689	663	623	567	493	405	318
[0.6, 1]	770	761	741	708	659	593	509	413	322
[0.5, 1]	806	793	768	730	675	604	515	416	323
[0.4, 1]	825	809	781	740	682	608	517	417	323
[0.3, 1]	833	816	787	743	685	609	518	417	323
[0.2, 1]	836	818	788	744	685	609	518	417	323
[0.1, 1]	837	819	789	745	685	609	518	417	323

Table: Life annuity pure premium and longevity floor price for cohorts of individuals with selected ages between 50 and 90 years old in 2013. Results are based on a flat yield curve at zero percent and a longevity risk premium equal to 0.3.

Pricing results: High interest rate scenario

Panel A: Conventional life annuity pure premium $(r = 3\%)$

Age	50	55	60	65	70	75	80	85	90
$a_{x_0}^{[\mathcal{F}_0]}(t_0)$	21.12	19.37	17.45	15.34	13.03	10.56	8.07	5.78	3.92

Panel B: Longevity Floor price

Age	50	55	60	65	70	75	80	85	90
$\lambda = 0.0$	0.17	0.17	0.17	0.16	0.15	0.13	0.10	0.07	0.04
$\lambda = 0.1$	0.37	0.36	0.35	0.32	0.28	0.22	0.16	0.10	0.06
$\lambda = 0.2$	0.64	0.63	0.59	0.53	0.44	0.35	0.24	0.15	0.08
$\lambda = 0.3$	0.94	0.91	0.86	0.76	0.64	0.49	0.34	0.20	0.11

Panel C: Longevity Floor price as a % of $a_{x_0}^{[\mathcal{F}_0]}(t_0)$ (in b.p)

Age	50	55	60	65	70	75	80	85	90
$\lambda = 0.0$	79	88	98	107	115	120	121	115	103
$\lambda = 0.1$	175	188	199	208	212	209	199	178	153
$\lambda = 0.2$	305	323	336	343	341	327	299	259	214
$\lambda = 0.3$	446	472	490	498	490	464	417	353	284

Pricing results: High interest rate scenario

Panel A: Capped Longevity Floor price ($r = 3\%$)

Bounds/Age	50	55	60	65	70	75	80	85	90
[0.0, 1]	0.94	0.91	0.86	0.76	0.64	0.49	0.34	0.20	0.11
[0.9, 1]	0.55	0.54	0.51	0.47	0.41	0.33	0.24	0.16	0.09
[0.8, 1]	0.74	0.73	0.69	0.63	0.54	0.43	0.30	0.19	0.11
[0.7, 1]	0.84	0.82	0.78	0.70	0.60	0.46	0.32	0.20	0.11
[0.6, 1]	0.89	0.87	0.82	0.74	0.62	0.48	0.33	0.20	0.11
[0.5, 1]	0.92	0.90	0.84	0.75	0.63	0.49	0.34	0.20	0.11
[0.4, 1]	0.93	0.91	0.85	0.76	0.64	0.49	0.34	0.20	0.11
[0.3, 1]	0.94	0.91	0.85	0.76	0.64	0.49	0.34	0.20	0.11
[0.2, 1]	0.94	0.91	0.86	0.76	0.64	0.49	0.34	0.20	0.11
[0.1, 1]	0.94	0.91	0.86	0.76	0.64	0.49	0.34	0.20	0.11

Table: Conventional and Capped longevity-linked life annuity pure premium and prices for longevity floor options for cohorts of individuals with selected ages between 50 and 90 years old. Results are based on a flat yield curve at three percent and a longevity risk premium equal to 0.3.

Pricing results: High interest rate scenario

Panel B: Capped Longevity Floor price as a % of $a_{x_0}^{[\mathcal{F}_0]}(t_0)$ (in b.p)

Bounds/Age	50	55	60	65	70	75	80	85	90
[0.0, 1]	446	472	490	498	490	464	417	353	284
[0.9, 1]	261	279	294	306	313	311	297	269	232
[0.8, 1]	350	375	395	409	413	403	374	326	271
[0.7, 1]	397	423	444	457	457	440	402	344	281
[0.6, 1]	422	449	470	480	477	455	412	350	284
[0.5, 1]	436	463	482	491	485	461	416	352	284
[0.4, 1]	443	469	488	495	489	463	417	352	284
[0.3, 1]	445	471	490	497	490	464	417	353	284
[0.2, 1]	446	472	490	498	490	464	417	353	284
[0.1, 1]	446	472	490	498	490	464	417	353	284

Table: Conventional and Capped longevity-linked life annuity pure premium and prices for longevity floor options for cohorts of individuals with selected ages between 50 and 90 years old. Results are based on a flat yield curve at three percent and a longevity risk premium equal to 0.3.

Pricing results: Deferred longevity options

- We now consider the results for the longevity (floor) options embedded in a deferred longevity-linked life annuity
- Similar to a conventional deferred annuity, in a deferred longevity-linked life annuity benefits are linked to the longevity index (1) but the annuitant does not begin to receive payments until some future date $t_0 + d$, where d is the deferment period in years $(d = 5, 10, 15, 20)$
- The price of an European-style deferred longevity floor $\mathcal{L}^{FLOOR}$, underlying asset $\mathcal{I}_{t_0+k}$ and constant strike equal to one unit of currency is calculated as

$$\mathcal{L}^{FLOOR} = \sum_{k=d+1}^{\omega - x_0 - d} Z(0,k)\, {}_k p_{x_0}^{[\mathcal{F}_k]}(t_0) \max\left(1 - \mathcal{I}_{t_0+k}; 0\right)$$

Pricing results: Deferred longevity options

Panel A: Conventional Deferred life annuity pure premium $(r = 0\%; \lambda = 0.3)$

Age	50	55	60	65	70	75	80	85	90
$d = 5$	31.71	26.71	21.90	17.27	12.85	8.74	5.18	2.53	1.03
$d = 10$	26.86	21.93	17.20	12.70	8.49	4.84	2.14	0.70	0.16
$d = 15$	22.15	17.34	12.76	8.51	4.80	2.06	0.62	0.11	0.01
$d = 20$	17.61	12.97	8.66	4.90	2.10	0.62	0.10	0.01	0.00

Panel B: Deferred Longevity Floor price

Age	50	55	60	65	70	75	80	85	90
$d = 5$	3.01	2.52	2.03	1.56	1.11	0.71	0.37	0.14	0.04
$d = 10$	2.93	2.43	1.93	1.44	0.96	0.52	0.19	0.04	0.00
$d = 15$	2.83	2.31	1.78	1.24	0.70	0.27	0.05	0.00	0.00
$d = 20$	2.69	2.13	1.54	0.92	0.37	0.08	0.01	0.00	0.00

Pricing results: Deferred longevity options

Panel C: Deferred Longevity Floor price as a % of Longevity Floor price

Age	50	55	60	65	70	75	80	85	90
$d = 5$	97.78	96.85	95.63	93.87	90.75	84.51	71.79	50.59	27.23
$d = 10$	95.28	93.37	90.72	86.36	78.06	61.97	36.93	13.77	2.91
$d = 15$	91.93	88.67	83.57	74.45	57.48	32.03	10.02	1.46	0.06
$d = 20$	87.39	81.85	72.43	55.55	30.53	9.07	1.13	0.03	0.00

Table: Results are based on a flat yield curve at zero percent and a longevity risk premium equal to 0.3.

Pricing results: Summary

- For a given strike price the longevity floor option prices decrease with the age of the policyholder at inception, i.e., increase with time to maturity
- This relation augments as the risk premium increases
- For all ages (maturities), that the higher the risk premium λ the more valuable are the longevity options
- Option prices increase faster the higher the risk premium λ for ages in the range 50-80 since these are the ages for which the mortality rates are more affected by changes in the time trend parameter κ_t because of higher estimates of β_x
- The embedded longevity option premium ranges between 1.15% and 8.37% of a conventional life annuity premium
- The results for capped LLLA show that allowing for a 30% reduction in annuity payments transfers most of the risk to policyholders
- An increase in interest rates reduces longevity option prices, particularly for a low market price of longevity risk
- Deferred longevity floors must be cheaper than longevity floors.

Final remarks

- Linking the annuity benefit to the mortality experienced in the group of annuitants offers an alternative longevity risk management tool to annuity providers
- Call and put options embedded in a longevity-linked life annuity gives the annuity provider the right to periodically adjust the benefit payments if the observed survivorship rates are different from those expected at the contract initiation
- Longevity-linked life annuity contracts (and GSA and PAF schemes) include embedded longevity options that have to be taken into account in pricing/reserving
- Further research
  - Allow for model & parameter risk in pricing the longevity options
  - Estimate the impact of considering alternative reference life tables
  - Allow for alternative risk sharing arrangements between annuity providers and annuitants
  - Allow for yield curve risk