Parameter and Model Uncertainties in Pricing Deep-deferred Annuities

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Longevity 11, Lyon, 2015

Outline

Introduction

Deep-deferred annuity

- also known as a longevity annuity and an advanced life delayed annuity;
- not long in the market, but available for purchase inside of 401(k) and IRA plans in the US now;
- regular payments start until the insured survive to high age, say 80;
- provides protection against the risk of outliving your money in late life;

Risk factors

- High age mortality
- Mortality improvement at high ages
- Gender
- Married status

Research Motivations

- Deep-deferred annuity is growing in popularity since Milevsky (2005) introduced it as a longevity annuity; Milevsky (2014) presented its market development; Gong and Webb (2010) showed it is an annuity people might actually buy.
- It's longevity risk concentrated, with cash flow involved in the tail of mortality distribution
- Pricing and hedging of longevity risk involved in deep-deferred annuities are challenging
- There is a research gap in this area

Aim of This Research

Compare parameter and model uncertainties in pricing immediate annuities and deep-deferred annuities to demonstrate risk factors which may not have a very significant impact on immediate annuities but affect the prices and riskiness of deep deferred annuities.

- Critical risk factors in pricing deep-deferred annuities
  - High age mortality
  - Mortality improvement at high ages
- Extent of impact of those factors

The Models

Mortality tail distribution

Three different shapes of tail distribution:

1. force of mortality is increasing and concave upward without any bound
   Gompertz Law : $\mu_x = Be^{ax}$.

2. force of mortality is increasing to a high age, say 105, and capped to have a flat tail
   Cubic model with capped flat tail :
   $\mu_x = \max\left(A + Bx^3 + Cx^2 + Dx,\ ln2\right)$.

3. force of mortality is increasing and approaches a asymptotic maximum value at extreme high ages
   Perk's logistic: $\mu_x = \frac{A+Be^{ax}}{1+Ce^{ax}}$.

Mortality improvement rate model

- Mortality improvement rate model
  We model mortality improvement rates Mitchell et al. (2013):

$$\ln \frac{m_{x,t}}{m_{x,t-1}} = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t} \tag{1}$$

- The projected mortality improvement rates will be applied to the chosen mortality curves to estimate age specific mortality rates in the future.

Parameter estimation for the mortality improvement rate model

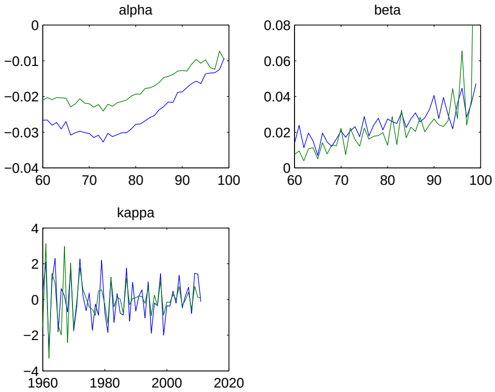

Figure: Fitted α_x, β_x, and κ_t from the Mitchell's mortality improvement rate model for males (green) and females (blue)

Parameter extrapolation for the high age mortality

- Extrapolate α_x
  - try a Gaussian function for α_x, which allows α_x gradually approaches 0.
  - try a polynomial function for α_x, and once α_x reaches 0 at an age we will assume the extrapolated value will be 0 after that age.
  - two extrapolation methods do not have significant difference on pricing annuities, and we chose a Gaussian method.
- Extrapolate β_x
  - $\beta_x' s$ appear to increase linearly, and therefore we fit a linear function for β_x and extrapolate it linearly.

Two population improvement rate models

- Without non-divergence constraint
  Use a single-population mortality improvement model for male and female respective, and then model the κ_t from the two populations by a vector autoregressive model.

- With non-divergence constraint
  Assume that male and female mortality rates will not diverge over the long run, and use the following model:

$$\ln \frac{m_{x,t}^{(i)}}{m_{x,t-1}^{(i)}} = \alpha_x + \beta_x \kappa_t^{(i)}, \text{with}$$

$$\kappa_t^{(1)} - \kappa_t^{(2)} \text{ mean reverting.} \quad (2)$$

where $i = 1,\ 2.$ The two populations share the same α_x and β_x.

Model Risks

Compare model risks

Model risk is demonstrated by the extend of change in immediate annuity incomes and deep-deferred annuity incomes from different models.

- use annual annuity incomes from $10,000 lump sum premium payment.
- be gender-specific
- measure ratio of difference

$$\text{Difference ratio} = \frac{\text{highest annuity income} - \text{lowest annuity income}}{\text{average annuity income from all models}}$$

- interest rate $=4\%$.
- use Japanese mortality data from years 1960 to 2012

Different mortality tail distribution

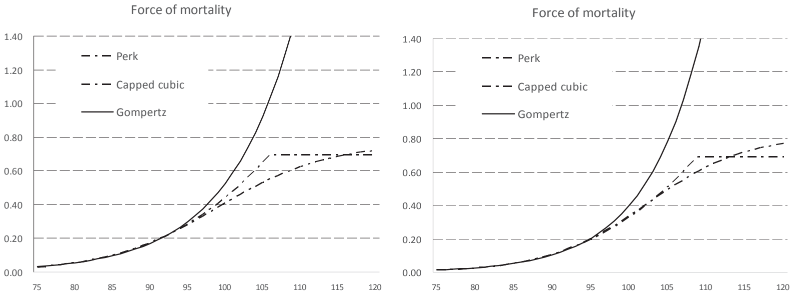

Figure: Three different models for mortality tail distribution, for ages 75-99, males (Left) and females (Right)

Effect of different mortality tail distribution

Males

	Gomp. ($)	Cubic ($)	Perk ($)	Ratio of difference (%)
$\ddot{a}_{65}$	759	759	759	0.04
${}_{20}\ddot{a}_{65}$	8063	8118	8111	0.68
${}_{25}\ddot{a}_{65}$	24642	24868	24774	0.91
${}_{30}\ddot{a}_{65}$	123205	118027	115355	6.60

Females

	Gomp. ($)	Cubic ($)	Perk ($)	Ratio of difference (%)
$\ddot{a}_{65}$	649	649	649	0.07
${}_{20}\ddot{a}_{65}$	4317	4319	4318	0.05
${}_{25}\ddot{a}_{65}$	10201	10249	10211	0.46
${}_{30}\ddot{a}_{65}$	35542	34480	34549	3.05

Table: Annual annuity incomes from different mortality tail distributions

Effect of Mortality improvement model

Males

	Gomp. ($)	Cubic ($)	Perk ($)	Ratio of difference(%)
$\ddot{a}_{65}$	720	720	719	0.09
$_{20}\ddot{a}_{65}$	5910	5918	5903	0.26
$_{25}\ddot{a}_{65}$	15049	15007	14903	0.97
$_{30}\ddot{a}_{65}$	55920	53048	51632	8.01

Females

	Gomp. ($)	Cubic ($)	Perk ($)	Ratio of difference(%)
$\ddot{a}_{65}$	610	609	609	0.19
$_{20}\ddot{a}_{65}$	3219	3195	3195	0.76
$_{25}\ddot{a}_{65}$	6439	6354	6345	1.47
$_{30}\ddot{a}_{65}$	16990	16113	16154	5.34

Table: Annual annuity incomes for different mortality curves and single population mortality improvement model

Effect of two population mortality improvement model

Males

	Gomp. ($)	Cubic ($)	Perk ($)	Ratio of difference(%)
$\ddot{a}_{65}$	718	718	717	0.13
$_{20}\ddot{a}_{65}$	5799	5794	5773	0.44
$_{25}\ddot{a}_{65}$	14408	14293	14162	1.72
$_{30}\ddot{a}_{65}$	49486	46425	44972	9.61

Females

	Gomp. ($)	Cubic ($)	Perk ($)	Ratio of difference(%)
$\ddot{a}_{65}$	611	610	610	0.21
$_{20}\ddot{a}_{65}$	3239	3213	3212	0.84
$_{25}\ddot{a}_{65}$	6487	6392	6382	1.63
$_{30}\ddot{a}_{65}$	17018	16084	16114	5.69

Table: Annual annuity incomes from different mortality curves and two population mortality improvement model without non-divergence constraint

Effect of two population mortality improvement model with non-divergence constraint

Males

	Gomp. ($)	Cubic ($)	Perk ($)	Ratio of difference(%)
$\ddot{a}_{65}$	706	706	705	0.14
${}_{20}\ddot{a}_{65}$	5302	5294	5275	0.50
${}_{25}\ddot{a}_{65}$	12617	12514	12404	1.70
${}_{30}\ddot{a}_{65}$	41950	39583	38430	8.80

Females

	Gomp. ($)	Cubic ($)	Perk ($)	Ratio of difference(%)
$\ddot{a}_{65}$	614	613	613	0.18
${}_{20}\ddot{a}_{65}$	3304	3281	3280	0.72
${}_{25}\ddot{a}_{65}$	6681	6599	6589	1.39
${}_{30}\ddot{a}_{65}$	17856	16947	16986	5.27

Table: Annual annuity incomes from different mortality curves and two population mortality improvement model with non-divergence constraint

Findings in model risk

- Different mortality tail distribution will not affect immediate annuity at retirement age but significantly affect deep-deferred annuities, especially for those with payments deferred to age 90 and above.
- Assumption of future mortality improvement has fairly significant difference in immediate annuity incomes while such difference is tremendous in deep-deferred annuity incomes, especially for males.
- Choosing a two-population mortality improvement model or not is trivial for immediate annuity but significant for deep-deferred annuities.
- Generally speaking, considering non-divergence constraint in a two-population model is more important than a two-population model per se. Once again, the effect is much more significant on deep-deferred annuities.

Parameter Risks

Test parameter risks

- Use bootstrap method
  - bootstrap estimates of mortality curve parameters
  - bootstrap estimates of the Mitchell's mortality improvement model and the extrapolation of estimated parameters
  - bootstrap estimates of two-population Mitchell's mortality improvement model and the extrapolation of estimated parameters
- Evaluate the variance in estimated annuity prices

Parameter risk in the mortality curves

	Males			
	$\ddot{a}_{65}$	${}_{20}\ddot{a}_{65}$	${}_{25}\ddot{a}_{65}$	${}_{30}\ddot{a}_{65}$
Mean	13.173	1.236	0.404	0.084
S.D.	0.013	0.009	0.005	0.003
Coefficient of Deviation	0.001	0.007	0.012	0.037
	Females			
	$\ddot{a}_{65}$	${}_{20}\ddot{a}_{65}$	${}_{25}\ddot{a}_{65}$	${}_{30}\ddot{a}_{65}$
Mean	15.405	2.315	0.978	0.287
S.D.	0.030	0.020	0.012	0.006
Coefficient of Deviation	0.002	0.009	0.012	0.022

Table: Mean and standard deviation of simulated annuity prices from different mortality curves with parameter uncertainties

Parameter risk after adding mortality improvement model

	$\ddot{a}_{65}$	${}_{20}\ddot{a}_{65}$	${}_{25}\ddot{a}_{65}$	${}_{30}\ddot{a}_{65}$
Males				
	Without parameter uncertainties			
Mean	13.896	1.692	0.667	0.187
S.D.	0.085	0.065	0.047	0.028
	With parameter uncertainties			
Mean	14.045	1.812	0.755	0.235
S.D.	0.337	0.259	0.186	0.102
Females				
	Without parameter uncertainties			
Mean	16.417	3.122	1.568	0.609
S.D.	0.146	0.129	0.108	0.075
	With parameter uncertainties			
Mean	16.221	2.970	1.461	0.557
S.D.	0.354	0.314	0.260	0.176

Table: Parameter uncertainties in mortality curves with mortality improvement rate model

Parameter risk after using two-population mortality model

	$\ddot{a}_{65}$	${}_{20}\ddot{a}_{65}$	${}_{25}\ddot{a}_{65}$	${}_{30}\ddot{a}_{65}$
Males				
	Without parameter uncertainties			
Mean	13.923	1.711	0.681	0.195
S.D.	0.081	0.062	0.045	0.027
	With parameter uncertainties			
Mean	13.891	1.692	0.671	0.193
S.D.	0.224	0.176	0.130	0.078
Females				
	Without parameter uncertainties			
Mean	16.384	3.094	1.545	0.594
S.D.	0.167	0.147	0.124	0.085
	With parameter uncertainties			
Mean	16.378	3.091	1.547	0.603
S.D.	0.366	0.328	0.279	0.195

Table: Parameter uncertainties in mortality curves with two-population mortality improvement rate model, no non-divergence constraint

	$\ddot{a}_{65}$	${}_{20}\ddot{a}_{65}$	${}_{25}\ddot{a}_{65}$	${}_{30}\ddot{a}_{65}$
Males				
	Without parameter uncertainties			
Mean	14.169	1.890	0.799	0.250
S.D.	0.127	0.099	0.073	0.044
	With parameter uncertainties			
Mean	14.201	1.9354	0.847	0.294
S.D.	0.412	0.332	0.258	0.171
Females				
	Without parameter uncertainties			
Mean	16.313	3.041	1.510	0.580
S.D.	0.119	0.106	0.090	0.066
	With parameter uncertainties			
Mean	16.392	3.12	1.586	0.646
S.D.	0.350	0.320	0.280	0.214

Table: Parameter uncertainties in mortality curves with two-population mortality improvement rate model, non-divergence constraint

Findings in parameter risk

- Parameter uncertainties in mortality tail distribution have more significant effect on deep-deferred annuities than on immediate annuities, especially those for males with payments deferred to very high ages.
- After incorporating mortality improvement rate models, parameter risk increase dramatically, when pricing both immediate annuities and deep-deferred annuities.
- Replacing a single population mortality improvement models by a two-population mortality improvement model will not cause significant extra parameter risk.
- Parameter risk involved in adding non-divergence constraint is a not trivial issue.

Conclusions

Concluding remarks

- Mortality improvement assumption is more critical than the shape of mortality curve when pricing immediate annuities for the young old;
- Deep-deferred annuities are not only sensitive to mortality improvement assumption but also the shape of mortality curve .
- Parameter uncertainty in mortality curve and mortality improvement model should be taken into account when pricing deep-deferred annuities.
- Non-divergence constraint assumption is non-trivial when modeling male and female mortality mortality improvement.

Future work

- Investigate the sensitivity to dependence assumption for a husband and wife's future lifetime of joint-life or last-survivor deep deferred annuities.
- Deep-deferred annuities are risky products. It would be significant contribution to work on standards of choosing appropriate mortality models for this type of products.
- A follow-up challenging topic would be the securitization of longevity risk involved in longevity annuities

Thank You !

Q & A