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Longevity Risk Mitigation in Pension Design

To Share or to Transfer

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September 29th, 2016.
Longevity 12, Chicago.

The views and opinions expressed herein do not necessarily state or reflect those of Amundi. This research is partly supported by funding from l'Observatoire de l'Epargne Européenne.

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Motivation

Longevity risk endangers the financial security of retirees.

Defining Characteristics

In contrast to mortality risk, i.e., risk of the uncertain time of death given known survival probabilities.

Mis-estimation of future survival probabilities

Investors who accept to bear this risk **command a risk premium** (Bayraktar et al., 2009).

Systematic risk

Longevity Risk

Threat to Retirement Planning

Phasing-out of **DB** schemes

The entity that conventionally bears the risk (i.e., the plan sponsor) no longer does.

Scarcity of longevity-linked **assets**

Maturation of the marketplace for longevity-linked assets is beset by challenges (Tan et al., 2015).

Longevity Risk Mitigation Channels

Benefits are **adjusted** according to **longevity evolution**.

+ A mean of dealing with longevity risk **without involving investors**.

+ Self-sustaining: **Solvency** is always **maintained**.

− **Volatility** of benefits: Subject to **longevity shocks**.

Benefits are **invariant to longevity evolution**, conditional on **provider's solvency**.

+ Longevity risk is **hedged**, i.e., transferred to contract provider.

− **Default** risk: Contract provider has **limited liability**.

− **Cost**lier: Investors only accept to bear the risk, in return for some **financial reward**.

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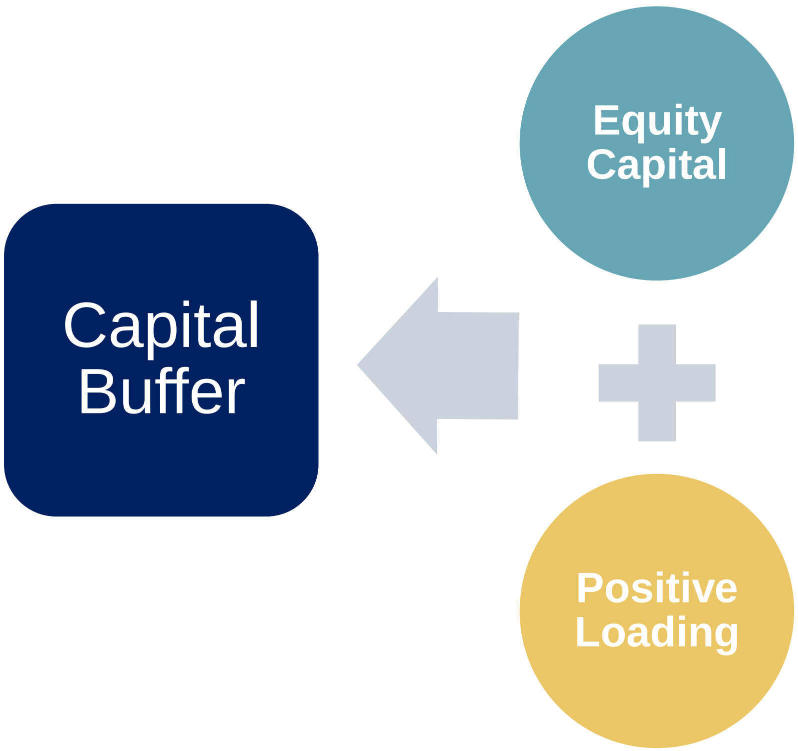
Capital to Enable Annuity Provision

Substantial buffer capital may be necessary to **limit default risk**.

e.g., ≈**18%** of the **contract's value** to **limit the default rate to 1%**.
Contract pays benefit from age 67 to 120, purchased when aged 40 (Maurer et al., 2013).



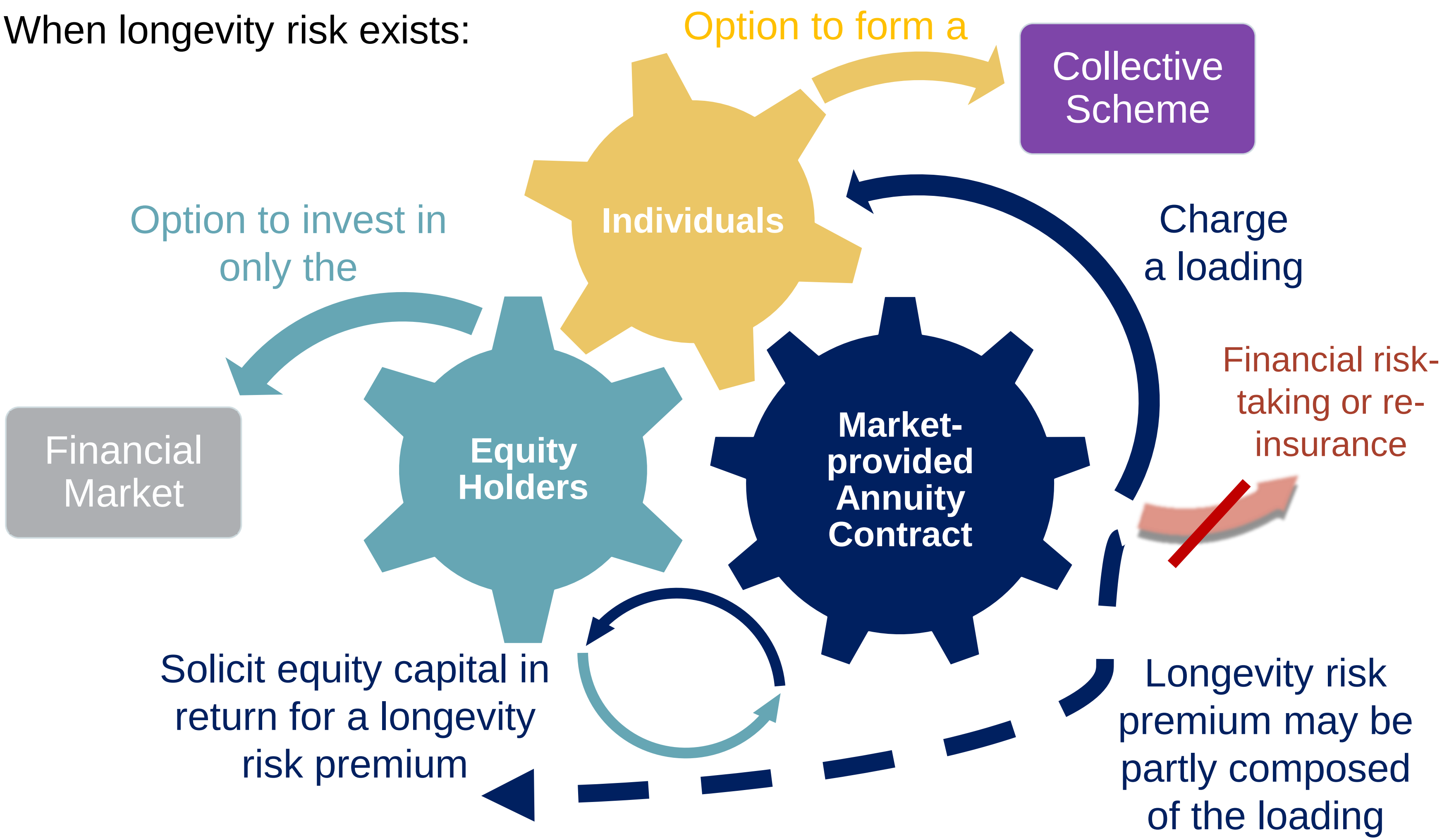
The need to generate a **longevity risk premium** to compensate equity holders.

Decision Factors

The need to preserve policy holders who may **prefer the collective scheme** if the loading is high.

Objective: Enhance the Modeling of the Market Solution

When longevity risk exists:



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Related Literature

1. Group-Self-Annuitization (Pigott et al., 2005) vs. other schemes (e.g., conventional annuities):
   - **Preclude** longevity risk
     – Stamos, 2008; Donnelly et al., 2013.
   - **Disregard** the annuity provider's **business model**
     – Stamos, 2008; Donnelly et al., 2013; Milevsky and Salisbury, 2015**.**
   - **Impose** the insurer's **default risk**
     – Hanewald et al., 2013.
   - Overarching conclusion: **Preference** for the **collective scheme** is **increasing** in the **loading**
     – Hanewald et al., 2013; Boyle et al., 2015.

2. Longevity-indexed vs. non-indexed contracts
   - Omission of insurer's equity holders:
     Assume that the buffer capital is **entirely composed** of **loading charged to individuals**.
     – Richter and Weber, 2011; Maurer et al., 2013.

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Summary of Findings (1/2)

If the annuity provider sells zero-loading contracts:

Individuals

Exhibit **marginal** preference for the **collective scheme.**

Collective Scheme

Annuity Contract

Certainly Equivalent Loading of **-35 to -5.2 bps**

Equity Holders

Find that **longevity risk exposure** is an **inferior** investment opportunity.

No Longevity Risk Exposure

Longevity Risk Exposure

50% higher Sharpe ratio

Lower expected excess return

Higher standard deviation

The figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

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Summary of Findings (2/2)

Consequence:

- The market-provided annuity contract **would not co-exist** with the collective scheme.

Outcome is **robust** to:

- Individuals' risk aversion levels (e.g., γ = 2, 5, and 8);
- Deferral period (e.g., 40 years, 20 years, and immediate); and
- Stock exposure (e.g., 0%, 20%, 40%, 60%, glide path).

Individuals exhibit **preference for the annuity contract** if:

- They are **highly risk-averse** (e.g., γ = 10, 15, and 20):
  - Certainty Equivalent Loading (CEL): 0.3 to 61.6 bps
- The **uncertainty** surrounding life expectancies is heightened:
  I. Doubled variance to the errors of the longevity time trend: CEL = 3.2%, zero-default-risk.
  II. Higher uncertainty of survival probability at older ages: CEL = 46.1 bps, zero-default-risk.

Unless otherwise stated, the figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

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Model Description

Financial Market

- Constant interest rate, r
- Stochastic stock market index: $dS_t = S_t(r + \lambda_S \sigma_S)dt + S_t \sigma_S dZ_{S,t}$

Longevity Model

- Lee and Carter (1992): $ln\left(m_{x,t}\right) = a_x + b_x k_t + \varepsilon_{x,t}$
- Time trend, k_t follows an ARIMA(0,1,0) process.
- Omission of mortality (i.e., micro-longevity) risk.

Individual Preference

- Choose a contract at age 25 in year t_0.
- Receive retirement benefits, Ξ_t, between ages 66 to 95, conditional on survival at time t.
- CRRA Utility: $\int_{t_R}^{T} e^{-\beta t} \frac{\Xi_t^{1-\gamma}}{1-\gamma} \times_{t-t_0} p_{25} dt$

t_R is the year when the individual is aged 66.
T is the year when the individual is aged 95.

Financial Contracts for Retirement (1/2): DVA

The DVA and the GSA treat **financial market risk identically** (i.e., fully borne by the individuals), but **differ** on the **longevity risk distribution.**

Deferred Variable Annuity (DVA)

- Parametrized by the **Assumed Interest Rate** (AIR).
- **Indexed** to a reference portfolio.
- Similar to the variable annuity studied in the literature, e.g., Koijen et al., 2009; Maurer et al., 2013.

- **Entitlements** are determined using **longevity forecasts** on the date of contract sale, **benefits received are equivalent to entitlements** while provider is solvent.
- **Default** occurs if the DVA provider's $Value\ of\ assets < Value\ of\ liabilities$
  - In default, individuals **recover the residual wealth** of the provider, which they use to buy a portfolio of **equally-weighted bonds** of maturities starting from the retirement year (or present year if retirement has begun) to the year of maximum age.
- The annuity provider can **fully hedge financial market risk** by adopting the reference portfolio's investment policy.

Financial Contracts for Retirement (2/2): GSA

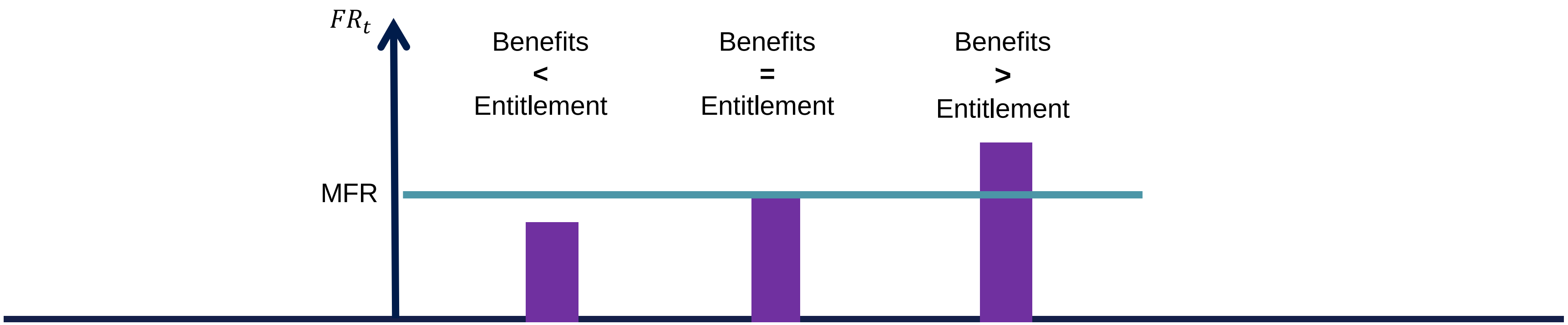
The DVA and the GSA treat **financial market risk identically** (i.e., fully borne by the individuals), but **differ** on the **longevity risk distribution.**

Group Self-Annuitization (GSA)

- **Entitlement** calculation is **identical** to that of a DVA with zero loading.
  - Parametrized by the Assumed Interest Rate (AIR).
  - Indexed to a Reference Portfolio.

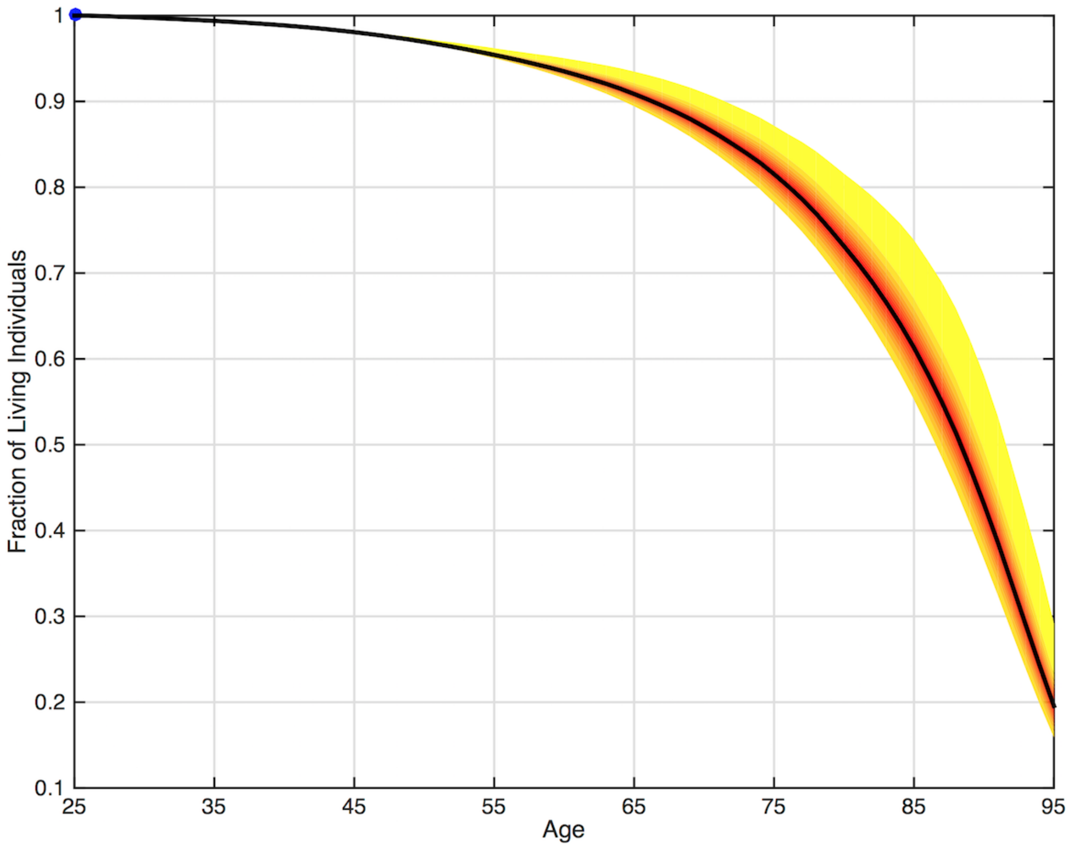
- **Entitlements are adjusted** each year by this ratio to determine the benefits paid-out.

$$\frac{Funding\ Ratio\ in\ year\ t\ (FR_t)}{Minimum\ Funding\ Requirement\ (MFR)}$$



Longevity Risk Visualized

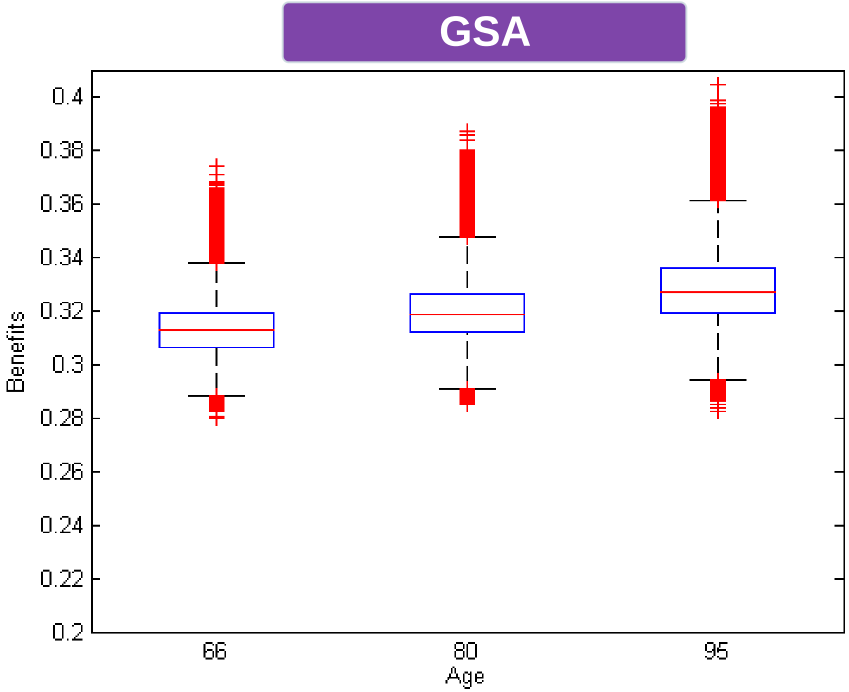
Lee and Carter (1992) model calibrated on U.S. female mortality data from 1980 to 2013, from the Human Mortality Database.

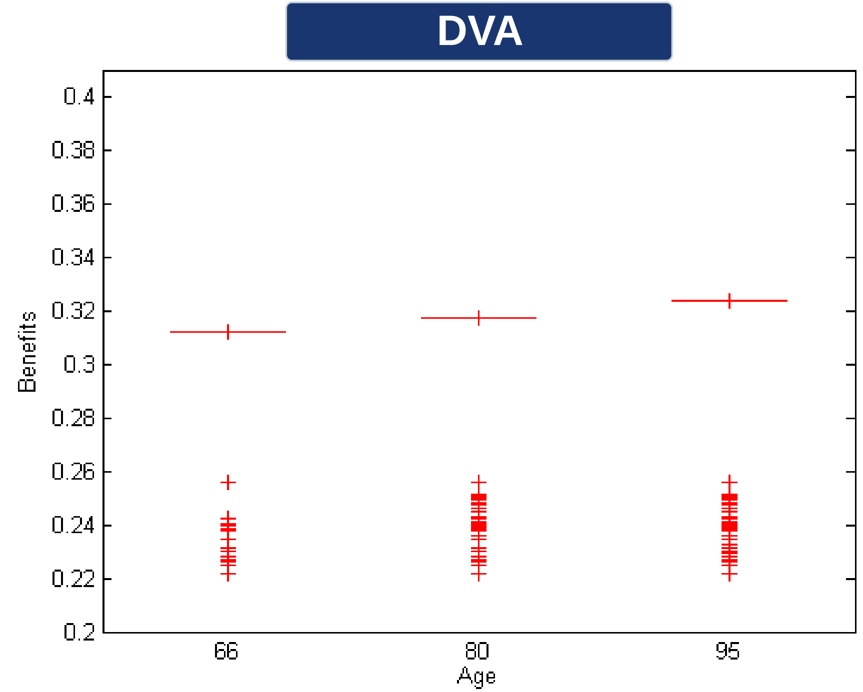


The fan plot is based on 10,000 replications.

Boxplot of Benefits

- DVA provider's **equity capital is 10%** the best estimated value of the contracts sold (i.e., to coincide with the 90% average leverage ratio of life insurers in the U.S. between 1998 and 2011[1])
- The ensuing **cumulative default rates are low**: < 0.0084%.



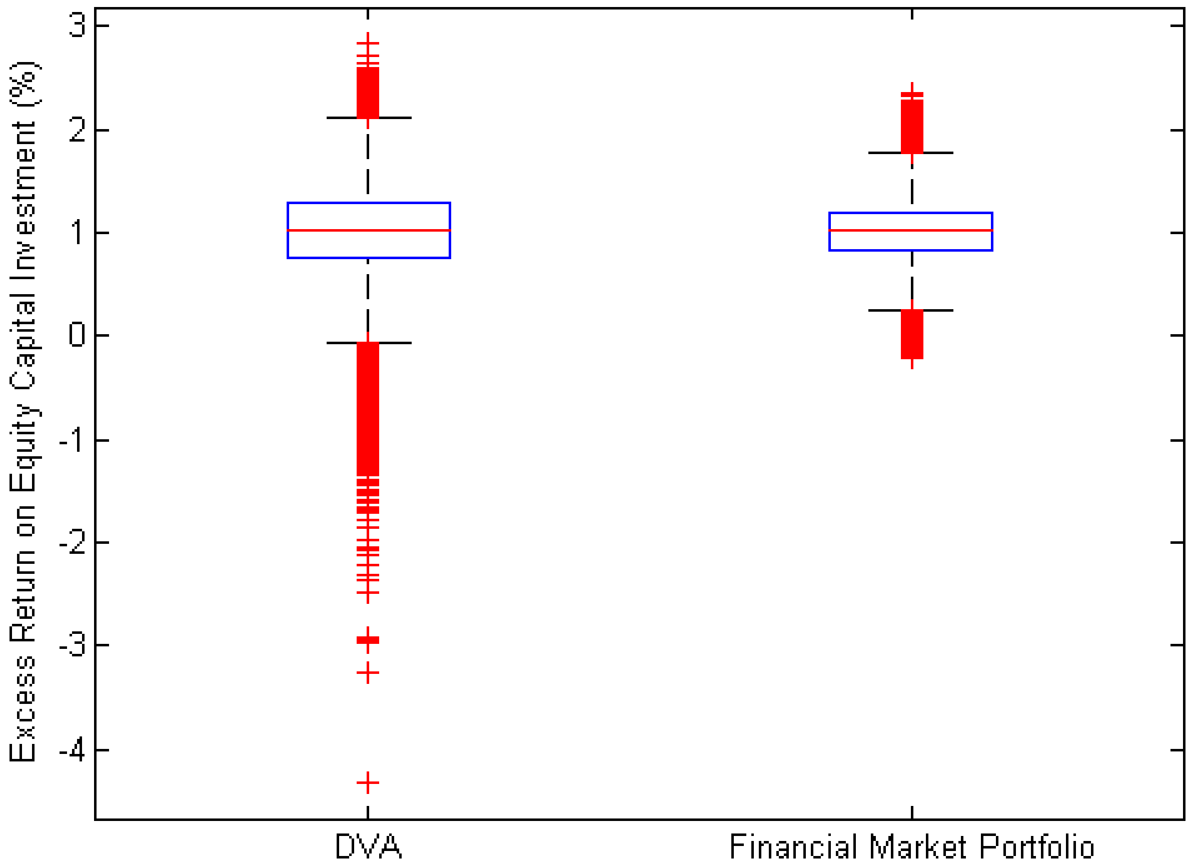


Note: Annuitization capital at age 25 is normalized to 1. Financial market return is constant at 3.62%.

[1]A.M. Best data from Koijen and Yogo (2015)

Figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 100% invested in the money market account.

Boxplot of Equity Holders' Excess Return



Note: Annualized values.

The figure corresponds to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

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Key Statistics

Individuals	Equity Holders

Certainty Equivalent Loading (CEL)

- The **proportional loading** on the DVA contract for which the individual derives the **same expected utility** under the DVA and under the GSA.

θ (%)	γ		
	2	**5**	**8**
0	-35 [-35.1, -35.0]	-20 [-20.0, -19.9]	-5.5 [-5.6, -5.5]
20	-34.9 [-35.0, -34.9]	-20 [-20.1, -20.0]	-5.2 [-5.2, -5.1]

Values are in basis points.

Sharpe Ratio (SR)

- The ratio of the annualized investment return in excess of the annualized return on the money market account, over its annualized standard deviation.

Statistic	γ		
	2	**5**	**8**
$\widehat{R^{(A_{exs})}}$ (%)	1.44 [1.44, 1.44]	1.44 [1.44, 1.45]	1.44 [1.44, 1.45]
$\widehat{\sigma^{(A_{exs})}}$ (%)	5.04 [5.03, 5.06]	4.95 [4.94, 4.96]	4.95 [4.94, 4.96]
$\widehat{SR^{(A)}}$	0.29 [0.29, 0.29]	0.29 [0.29, 0.29]	0.29 [0.29, 0.29]

Reference portfolio: 20% in stock.

$$R^{exs} = 1.43\%$$
$$\sigma^{exs} = 3.17\%$$
$$SR = 0.45$$

The values in parentheses are the 99% bootstrapped confidence intervals.

Figures correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

Sensitivity Analysis (1/3)

Baseline Case: γ = 5; Cumulative Default Rate = 0.038%; CEL = -20 bps.

- Individuals who are **highly risk-averse** prefer the DVA.
  - e.g., γ = 20, CEL = 62 bps.

- If the DVA provider has a **higher leverage ratio**, then individuals prefer the GSA more.
  - e.g., Initial capital is halved to 5%.
  - γ = 5, annual default rate rises to 5%, CEL decreases to -12.9%.

No Material Effect

- Deferral Period (40 years, 20 years, or an immediate annuity)
  - Shorter deferral period allows for **more accurate survival probabilities forecast** but **more imminent longevity shocks** to utility.

- Stock Exposure
  - 0, 20, 40, 60 and glide path (90% at age 25, diminishing to 30% by age 66).

Values correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

Sensitivity Analysis (2/3)

Sensitivity surrounding the longevity model:

Doubled Time Trend Errors' Variance

- Time trend process:
  $k_t = c + k_{t-1} + \delta_t$
- $\delta \sim N(0, 2\widehat{\sigma_\delta^2})$

Drift Parameter Uncertainty

- $k_t = c + k_{t-1} + \delta_t$
- $\hat{c}$ is estimated by maximum likelihood, and is distributed as $\hat{c} \sim N(c, \sigma_c^2)$
- For the l^{th} replication, draw a c_l from the distribution $N\left(\hat{c}, \widehat{\sigma_c^2}\right)$

Alternate Longevity Model

- Cairns, Blake and Dowd (2006)
- $logit(q_{t,x}) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x})$

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Sensitivity Analysis (3/3)

Sensitivity surrounding the longevity model:

Doubled Time Trend Errors' Variance

- **Default rates increase** from 0.0038% to 3.41%: CEL = **-7.7%**;
- If capital is raised sufficiently to **eliminate default risk**: CEL = **3.2%**;
- More volatile excess returns to equity holders.

Drift Parameter Uncertainty

- **No material change** to default rates, CEL, and equity holders' investment performance.

Alternate Longevity Model

- **Higher uncertainty** on the likelihood of **survival at older ages**;
- With default, which rises to 0.48%: CEL = **-50.3** bps;
- Absent default: CEL = **46.1** bps;
- More volatile excess return to equity holders.

Values correspond to contracts for individuals with $\gamma = 5$ and the underlying portfolio is 20% invested in the risky stock index, 80% invested in the money market account.

Conclusion (1/2)

- We investigate **longevity risk mitigation** in retirement planning under two arrangements:
  - **Distributing** the risk among individuals, (GSA) or,
  - **Insuring** the risk with a market-provided annuity contract (DVA).

- We model not only **individual preference** but also the **annuity provider's business** to underscore the involvement of equity holders in enabling the market solution.

- **Individuals prefer** the arrangement (i.e., DVA or GSA) that yields a **higher expected utility**.

- Equity holders' **willingness to provide capital** depends on the **Sharpe ratio** of the investment opportunities that bear the **same financial market risks**, but are either **exposed to, or not exposed to longevity risk**.

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Conclusion (2/2)

- We find that when the DVA is priced at its **best estimate**:
  - Individuals have a **slight preference** for the GSA;
  - Equity holders attain a **lower Sharpe ratio** when **exposed to longevity risk**.

- Market-provided annuity contracts **would not co-exist** with collective schemes.

- Preference for the GSA is **insensitive** to:
  - Contract deferral period;
  - Exposure to stock market risk.

- **Heightened longevity risk** only enhances the appeal of a DVA to the individual if the provider **restrains default risk**.
  - **Sharpe ratio** of equity holders remains **inferior** to the Sharpe ratio of the investment in the financial market only;
  - Aggravated longevity risk leads to higher variability of the equity holder payoff as well.

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Appendices

DVA Contract Details: Entitlement

- The DVA entitlement in period $t, t_R \leq t \leq T$, conditional on the individual's survival, is given by:

$$\Xi^{DVA}(h^*, F, t, x) \quad := \quad \frac{\exp\left(-h^*\left(\theta_t, t\right) \times \left(t - t_R\right)\right)}{A^{DVA}\left(h^*, F, t_0, x\right)} \times \frac{W_t^{Ref}\left(\theta\right)}{W_{t_0}^{Ref}\left(\theta\right)} \qquad (1)$$

$$W_t^{Ref}(\theta) = \text{value of the reference portfolio at time } t$$

$$\theta = \text{investment policy of the reference portfolio, } \{\theta_t\}_{t=t_0}^{T}$$

$$h^* = \text{optimal } AIR\text{, Equation (3)}$$

$$A^{DVA} = \text{per unit contract cost, Equation (2)}$$

$$F = \text{contract loading}$$

$$t_0 = \text{date of annuitization}$$

$$t_R = \text{retirement year}$$

DVA Contract Details: Per Unit Cost

- The per unit contract cost (i.e., annuity factor) is given by:

$$A^{DVA}\left(h^*, F, t_0, x\right) \quad := \quad (1+F) \int_{t=t_R}^{T} {}_{t-t_0}p_x^{(t_0)} \exp\left(-h^*\left(\theta_t, t\right) \times \left(t - t_R\right)\right) \mathrm{d}t \quad (2)$$

$$\begin{aligned} {}_{t-t_0}p_x^{(t_0)} &= \text{conditional probability in year } t_0 \\ &\quad \text{that a living individual of age } x \\ &\quad \text{lives for at least } t - t_0 \text{ more years} \\ h^* &= \text{optimal } AIR\text{, Equation (3)} \\ F &= \text{loading factor} \end{aligned}$$

DVA Contract Details: Optimal AIR

- The optimal Assumed Interest Rate (AIR) maximizes the individual's expected utility in retirement under the DVA contract, when the reference portfolio follows the investment policy $\theta = \{\theta_t\}_{t=t_0}^{T}$

$$h^*(\theta_t, t) := r + \frac{\beta - r}{\gamma} - \frac{1-\gamma}{\gamma}\theta_t \sigma_S \left(\lambda_S - \frac{\gamma \theta_t \sigma_S}{2}\right) \quad (3)$$

r = constant short rate

β = subjective discount factor

γ = risk aversion parameter

θ_t = fraction of the reference portfolio allocated to the stock index at time t

σ_S = diffusion term of the stock index

λ_S = Sharpe ratio of the stock index

Certainty Equivalent Loading

- The Certainty Equivalent Loading (CEL) is the value such that Equation (4) holds.

$$\begin{aligned}
\mathbb{E}\left[U\left(\frac{1}{1+CEL}\times\Xi^{DVA}|_{F=0}\right)\right] &= \mathbb{E}\left[U\left(\Xi^{GSA}\right)\right] \qquad (4)\\
\Xi^{DVA}|_{F=0} &= \text{Retirement benefits of}\\
&\quad \text{a } DVA \text{ with zero loading, } F=0\\
\Xi^{GSA} &= \text{Retirement benefits of a } GSA\\
U(.) &= \text{Utility function}
\end{aligned}$$

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