CENTRE FOR ECONOMETRIC ANALYSIS
CEA@Cass

http://www.cass.city.ac.uk/cea/index.html

Cass Business School
Faculty of Finance
106 Bunhill Row
London EC1Y 8TZ

Dynamic Conditional Correlation Models with Asymmetric Multivariate Laplace Innovations

Juan-Pablo Cajigas and Giovanni Urga

CEA@Cass Working Paper Series

WP–CEA–08-2007

Dynamic Conditional Correlation Models with Asymmetric Multivariate Laplace Innovations

Juan-Pablo Cajigas
(Cass Business School, London, UK, and Barclays Capital, London, UK)

Giovanni Urga*
(*Cass Business School, London, UK, and Bergamo University, Italy)*

28 April 2007

*Corresponding author: G.Urga@city.ac.uk. Centre for Econometric Analysis, Faculty of Finance, Cass Business School, 106, Bunhill Row, London, EC1Y 8TZ (U.K.) Tel. 00.44.(0)20.70408698; Fax. 00.44.(0)20.70408881; http://www.cass.city.ac.uk/faculty/g.urga http://www.cass.city.ac.uk/cea/index.html

Dynamic Conditional Correlation Models with Asymmetric Multivariate Laplace Innovations

Abstract. In this paper we propose to estimate multivariate GARCH processes and a class of dynamic conditional correlation (DCC) models assuming that the n-dimensional returns series follow an Asymmetric Multivariate Laplace (AML) distribution. This multivariate distribution is able to capture leptokurtosis and asymmetry which characterize returns from financial assets. It preserves, under general conditions, desirable properties such as finiteness of moments and stability under geometric summation. We illustrate the methodology by fitting a sample of 21 FTSE All-World stock indices and 13 bond return indices. We provide clear evidence that in our data set the asymmetric generalised dynamic conditional correlation (AGDCC)-MGARCH model with AML distribution of innovations overwhelmingly outperforms the class of DCC-MGARCH models that assume normality of innovations.

J.E.L. Classification Number: C32, G0, G1, G2.

Keywords: Dynamic Conditional Correlations, Maximum Likelihood Estimation Method, Value-at-Risk.

1 Introduction

The main aim of this paper is to develop a multivariate time-varying framework for modelling and forecasting cross-market correlations where innovations are assumed to follow an Asymmetric Multivariate Laplace (AML) distribution. A good understanding of the dynamic properties of cross-market correlation (or dependence across markets) is vital for assessing the level of integration between international markets both for investment purposes and for increasing the capacity to produce reliable forecasts. Modelling the dynamics of volatilities of returns from financial assets has been one of the working horses in the development of financial econometrics over the last years (Bollerslev, 2001; Engle, 2001). Nonetheless, most of the advances, especially if we consider the use of the proposed framework for practical purposes, have been seen almost exclusively in univariate cases. The growth in techniques modelling the dynamics of covariances and correlations has lagged considerably behind the growth in modelling time-varying volatility, as evidenced by the shortage of the literature on time-varying correlations compared to that of modelling time-varying volatility. One of the main reasons for this uneven expansion is the "curse of dimensionality", due to the extremely cumbersome problems faced in estimating unrestricted multivariate GARCH (MGARCH) models in highly dimensioned settings[1].

Among the alternative MGARCH specifications recently proposed in the literature, one in particular has proved to be particular suitable to provide a parsimonious, flexible and feasible model that significantly reduces the "curse of dimensionality". This is the dynamic conditional correlation (DCC) model proposed in Engle (2002), Tse and Tsui (2002) and Engle and Sheppard (2001)[2]. In this model, the dynamic variance-covariance matrix of conditional returns is specified as a function of univariate variances and linear correlations. When the model is estimated by maximum likelihood this framework allows to "break" the log-likelihood function into two parts, one for the parameters determining univariate volatilities and another for the parameters determining the correlations (the so-called DCC two-step estimation technique). By using this technique large systems can be consistently estimated with limited computational costs without imposing too many restrictions like in the case of factor models.

A vital assumption of the DCC model is that standardized residuals are normally distributed. Normality allows (Q)MLE to provide feasible and con-

[1] Bauwens et al (2006) provide a comprehensive survey on MGARCH models.

[2] See inter alia Palandri (2005), Hafner, van Dijk, and Franses (2005), Silvennoinen and Terasvirta (2005), Engle and Colacito (2006), Pelletier (2006).

sistent though inefficient DCC coefficients of conditional correlations (Bollerslev and Wooldridge, 1992). Nevertheless, financial time series do not favour this assumption. Where time-varying volatilities are estimated by assuming a normal-GARCH process for the innovations, it is easy to show that even for correctly specified models, statistically significant levels of leptokurtosis and excess kurtosis can still be found.

Current routes of investigation to overcome the limits in the assumption of normality are mainly two. First, there is a strand of literature which aims at using non parametric and semiparametric methods of investigations: Engle and González-Rivera (1991), Drost and Klaassen (1997), González-Rivera (1997), González-Rivera and Drost (1999) for the univariate case. For the few working papers studying the multivariate case (see for instance Hafner and Rombouts, 2004; Long and Ullah, 2005; Hafner, van Dijk, and Franses, 2005) estimation and inference are quite difficult and feasible only for a small number of assets. A second strand of contributions use thick (non normal) distributions to achieve efficiency with implication for the first stage. See for instance Bollerslev (1987), Baillie and Bollerslev (1989), Nelson (1991), Fiorentini, Sentana, Calzolari (2003), Bauwens and Laurent (2004) and Mencia and Sentana (2005).

As already pointed out, returns from financial assets show well defined patterns of leptokurtosis and skewness which cannot be captured by the normality assumption. There are several multivariate distributions in the literature that present high levels of kurtosis as well as asymmetries and that could be used in a MGARCH framework. However, the majority of these distributions are either too complicated to be estimated for GARCH purposes or present undesirable properties (like an infinite variance) that limit their use for financial applications. One multivariate distribution that parsimoniously captures the main features of financial returns and keeps flexibility is the AML distribution, as recently proposed by Kotz, Kozubowski, and Podgorski (2003). In the univariate context, the Laplace or double-exponential distribution has been widely used in financial modelling. Some applications include Madan and Seneta (1990), Madan, Carr and Chang. (1988), Linden (2001), Heyde and Kou (2004), Komunjer (2005) among others. The asymmetric multivariate version used in this paper is defined as a subclass of geometric stable distributions, a characteristic that in the case of the AML distribution can be used to model linear combinations of random variables with univariate symmetric Laplace distributions. This feature is extremely important as it allows to use this distribution in the computation of the parametric-VaR of portfolios of financial assets, characteristic that was thought exclusive of the Pareto-stable distribution and of its most widely used limiting case such as the normal distribution[3].

Our work is in the spirit of Mencia and Sentana (2005) who use a generalized hyperbolic distribution in a model where the variance matrix dynamics follow a conditionally heteroskedastic single factor model and the conditional variance of the factor obeys a univariate GQARCH (1,1) process; and Bauwens and Laurent (2004) who use a type of multivariate skewed Student-t distribution to fit a DCC (1,1) model to two sets of three assets data. As far as we are concerned this is the first work where the AML distribution is used to model the returns of financial assets in a MGARCH setting.

The paper develops as follows. Section 2 briefly presents the seminal DCC framework and its extensions to allow for asymmetries in asset-specific correlations, as proposed by Cappiello, Engle and Sheppard (2004, CES henceforth). The case of DCC models with normally distributed residuals is presented in Section 2.1, while in Section 2.2 we present a framework where the DCC is enriched by the AML distribution. In Section 3, we discuss the implications of estimating the DCC model by maximum likelihood under the AML assumption for innovations. In Section 4 we report the results from an empirical application using a sample of 21 FTSE All-World stock indices and 13 bond return indices. Section 5 concludes.

2 Dynamic Conditional Correlation Models

Consider the n-dimensional returns process $r_t \in \mathbb{R}^{T\times n}$, $t = 1, .., T$ generated as

$$r_t = \mathbf{H}_t^{1/2}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t \tag{1}$$

$$\mathbf{H}_t = Var\left(r_t \mid \Omega_{t-1}\right) \tag{2}$$

where Ω_{t-1} is the information set at time $t-1$, and $\boldsymbol{\varepsilon}_t$ is an i.i.d. process. In the DCC setting $\mathbf{H}_t$ is modelled directly as a function of dynamic univariate variances and dynamic linear correlations,

$$\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t \tag{3}$$

where $\mathbf{D}_t \in \mathbb{R}^{T\times n\times n}$ is a diagonal matrix with elements $\sqrt{h_{it}}, i = 1, ..., n$, $t = 1, .., T$, and $\mathbf{R}_t$ is defined as

$$\mathbf{R}_t = \left(\mathbf{Q}_t^*\right)^{-1}\mathbf{Q}_t\left(\mathbf{Q}_t^*\right)^{-1} \tag{4}$$

[3]The property that linear combination of multivariate AML distributiions are AML is going to be useful "only" for the one-day ahead VaR computation if one uses a GARCH model for the variance. For more than one-day ahead one would need to compute the VaR through simulations. Basle II ask 10-day ahead VaR computations.

We wish to thank Denis Pelletier for bringing this point to our attention.

where $\mathbf{Q}_t^*$ is a diagonal matrix of the form

$$\mathbf{Q}_t^* = (Diag\ Q_t)^{1/2} \tag{5}$$

and

$$\mathbf{Q}_t = \left(1 - \sum_{l=1}^{L} \alpha_l - \sum_{s=1}^{S} \beta_s\right) \overline{\mathbf{Q}} + \sum_{l=1}^{L} \alpha_l \boldsymbol{\varepsilon}_{t-l} \boldsymbol{\varepsilon}_{t-l}^{'} + \sum_{s=1}^{S} \beta_s \mathbf{Q}_{t-s} \tag{6}$$

$\overline{\mathbf{Q}} \in \mathbb{R}^{n \times n}$ is the unconditional variance-covariance matrix of $\boldsymbol{\varepsilon}_t$, i.e. $\overline{\mathbf{Q}} = E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t')$, and α_l and β_s are scalar parameters satisfying $\sum_{l=1}^{L} \alpha_l + \sum_{s=1}^{S} \beta_s < 1$. The specification in (4) secures that $\mathbf{R}_t$ will be a valid correlation matrix while (3) and (6), in addition to the condition of stationarity, secure $\mathbf{H}_t$ to be a positive definite matrix.

The dynamics in (3) is particularly appealing because it allows for a two step estimation that makes feasible the estimation of highly dimensioned processes, estimation that for many non-factor models is usually not possible because of the "curse of dimensionality".

2.1 DCC Models with Normally Distributed Standardised Residuals

To illustrate the two-step estimation technique let us assume first normality for the vector of standardised residuals, i.e. $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{R}_t)$. Denoting $\boldsymbol{\theta}$ as the vector of parameters in the conditional variance-covariance matrix $\mathbf{H}_t$, the log-likelihood $L_T(\boldsymbol{\theta})$ for the T observations of this estimator,

$$L_T(\boldsymbol{\theta}) = \sum_{t=1}^{T} \log f(r_t \mid \boldsymbol{\theta}, \Omega_{t-1}) \tag{7}$$

is given by,

$$L_T(\boldsymbol{\theta}) = -\frac{1}{2} \sum_{t=1}^{T} \left\{ n \log(2\pi) + \log |\mathbf{H}_t| + \mathbf{r}_t^{'} \mathbf{H}_t^{-1} \mathbf{r}_t \right\}. \tag{8}$$

Following (3) we have,

$$L_T(\boldsymbol{\theta}) = -\frac{1}{2} \sum_{t=1}^{T} \left\{ n \log(2\pi) + \log |\mathbf{D}_t \mathbf{R}_t \mathbf{D}_t| + \mathbf{r}_t^{'} (\mathbf{D}_t \mathbf{R}_t \mathbf{D}_t)^{-1} \mathbf{r}_t \right\} \tag{9}$$

$$L_T(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T}\left\{n\log(2\pi) + \log\left|\mathbf{D}_t^2\right| + \log|\mathbf{R}_t| + \boldsymbol{\varepsilon}_t'\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t\right\} \tag{10}$$

with standardised residuals $\boldsymbol{\varepsilon}_t = \mathbf{D}_t^{-1}\mathbf{r}_t$. Engle (2002) proposes to estimate the first stage by assuming $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0},\mathbf{I})$ where $\mathbf{I} \in \mathbb{R}^{n\times n}$ is an identity matrix. By partitioning the vector of parameters in two subsets $\boldsymbol{\theta} = (\boldsymbol{\zeta},\boldsymbol{\varphi})$, where $\boldsymbol{\zeta}$ contains the parameters of the n univariate volatilities and $\boldsymbol{\varphi}$ contains the parameters of the correlations, the log-likelihood function can be expressed as

$$L_T(\boldsymbol{\theta}) = L_T(\boldsymbol{\zeta}) + L_T(\boldsymbol{\varphi} \mid \boldsymbol{\zeta}) \tag{11}$$

The estimation of the first stage consists in the maximization of the function

$$L_T(\boldsymbol{\zeta}) = -\frac{1}{2}\sum_{t=1}^{T} n\log(2\pi) + \log\left|\mathbf{D}_t^2\right| + \mathbf{r}_t'\mathbf{D}_t^{-2}\mathbf{r}_t. \tag{12}$$

Once the vector $\boldsymbol{\zeta}$ is estimated, the vector of standardise residuals $\boldsymbol{\varepsilon}_t = \mathbf{D}_t^{-1}\mathbf{r}_t$ is employed in the second stage, which corresponds to the maximization of the function

$$L(\boldsymbol{\varphi} \mid \boldsymbol{\zeta}) = -\frac{1}{2}\sum_{t=1}^{T} \log|\mathbf{R}_t| + \boldsymbol{\varepsilon}_t'\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t, \tag{13}$$

under the assumption $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0},\mathbf{R}_t)$.

The novelty of this technique is that estimation is speeded up by employing $n+1$ log-likelihood functions[4] instead of one single but nonetheless extremely flat log-likelihood function.

To show consistency of the two-step DCC estimator Engle (2002) employs the results in Newey and McFadden (1994) for the two-step Generalised Method of Moments (GMM). The result follows from that Maximum Likelihood estimation can be considered a special case of the GMM when the moment conditions are set equal to the scores of the log-likelihoods

$$\nabla_{\boldsymbol{\zeta}} L_T(\boldsymbol{\zeta}) = \mathbf{0} \tag{14}$$

$$\nabla_{\boldsymbol{\varphi}|\boldsymbol{\zeta}} L_T(\boldsymbol{\varphi} \mid \boldsymbol{\zeta}) = \mathbf{0}. \tag{15}$$

The specification in (6) can be enriched by allowing for asymmetries in conditional correlations and covariances as well as for asset-specific correlations,

[4] The first stage implies the estimation of n univariate volatility processes and the second stage the estimation of one single correlation process.

as proposed in Cappiello, Engle and Sheppard (2004). We will refer to this general model as the Asymmetric Generalised Dynamic Conditional Correlation (AGDCC) (L,S,U) model, where L corresponds to the number of autoregressive lags, S corresponds to the number of persistence lags, and U corresponds to the number of asymmetric shock lags[5]. The specification of the matrix $\mathbf{Q}_t$ for the AGDCC (1,1,1) case is given by

$$\begin{aligned} \mathbf{Q}_t &= \left(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}\right) + \mathbf{A}'\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}\mathbf{A} + \\ &\quad + \mathbf{B}'\mathbf{Q}_{t-1}\mathbf{B} + \mathbf{G}'\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}'_{t-1}\mathbf{G} \end{aligned} \tag{16}$$

where $\mathbf{A}, \mathbf{B}$, and $\mathbf{G}$ are diagonal parameter matrices ($\mathbf{A}, \mathbf{B}, \mathbf{G} \in \mathbb{R}^{\mathbf{n}\times\mathbf{n}}$) with elements a_{ii}, b_{ii} and g_{ii} respectively, $\boldsymbol{\eta}_\tau = I\left[\boldsymbol{\varepsilon}_\tau < 0\right] \circ \boldsymbol{\varepsilon}_\tau$, " $\circ$ " denotes the Hadamard product and $\overline{\mathbf{N}} = E(\boldsymbol{\eta}_t\boldsymbol{\eta}'_t)$. $\mathbf{Q}_t$ will be positive-definite if

$$\left(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}\right)$$

is positive definite[6].

The AGDCC (L,S,U) model (16) nests several specifications:

1. DCC (L,S) model : $\mathbf{G} = [0], \mathbf{A} = \sqrt{a}, \mathbf{B} = \sqrt{b}$
2. ADCC (L,S,U) model: $\mathbf{G} = \sqrt{g}, \mathbf{A} = \sqrt{a}, \mathbf{B} = \sqrt{b}$
3. GDCC (L,S) model: $\mathbf{G} = [0]$.

2.2 DCC Models with Asymmetric Multivariate Laplace Distributed Standardised Residuals

The hypothesis of normality of the vector of standardised residuals makes (Q)MLE feasible, providing consistent though inefficient estimates of the dynamic conditional correlation coefficients (Bollerslev and Wooldridge, 1992). However, normality is not a satisfactory property for financial time series. This has important implications not only for the econometric properties of parameter estimates, but also for the use of the models in applications such as portfolio allocation, VaR and Expected Shortfall analyses.

[5] This is an extension of the Asymmetric Dynamic Conditional Correlation (ADCC) (L,S,U) model presented in Cappiello *et al* (2004) . An alternative generalisation that allow for asset-specific correlations and group-specific correlations is in Hafner and Franses (2003, "Generalised DCC" model).

[6] Because this cannot always be guaranteed, Hafner and Franses (2003) propose to replace this expression by $\left(1 - \overline{a}^2 - \overline{b}^2\right)\overline{\mathbf{Q}}$. Though $\mathbf{Q}_t$ is then positive-definite, this substitution sacrifices the correlation-targeting approach implicit in (16).

In this paper, we propose to estimate conditional correlations assuming that the innovations follow the asymmetric Laplace distribution proposed by Kozubowski and Podgorski (2001) as a subclass of geometric stable distributions. In particular, we adopt the results in Kotz, Kozubowski and Podgorski (2003), who generalised the laws of an Asymmetric Laplace to the multivariate case[7].

The density function of the AML distribution allowing for time dependency in $\mathbf{H}_t$ and $\mathbf{r}_t$ is given by,

$$f(\mathbf{r}) = \frac{2\exp\left(\mathbf{r}_t'\mathbf{H}^{-1}\mathbf{m}\right)}{(2\pi)^{n/2}\left|\mathbf{H}\right|^{1/2}}\left(\frac{\mathbf{r}_t'\mathbf{H}^{-1}\mathbf{r}_t}{2+\mathbf{m}'\mathbf{H}^{-1}\mathbf{m}}\right)^{v/2} K_v\left(\sqrt{(2+\mathbf{m}'\mathbf{H}^{-1}\mathbf{m})(\mathbf{r}_t'\mathbf{H}^{-1}\mathbf{r}_t)}\right) \tag{19}$$

where $v = (2-n)/2$ and $K_v(u)$ is the modified Bessel function of the third kind defined by $K_v(u) = \frac{(u/2)^v\Gamma(1/2)}{\Gamma(v+1/2)}\int_1^\infty e^{-ut}(t^2-1)^{v-1/2}dt$, $u>0, v\geq -1/2$. The vector $\mathbf{m}$ is the location parameter and the matrix $\mathbf{H}$ is the scale parameter of this distribution. These quantities are related: $\mathbf{m} \equiv \mathbf{Hb}$ where $\mathbf{b} \in \mathbb{R}^n$.

A very important characteristic of the AML distribution is that it is unimodal with the mode equal to zero. Because of this the $\mathbf{m}$ parameter does not only determines the mean of the distribution, but also its level of asymmetry. When $\mathbf{m} = \mathbf{0}$ the distribution is symmetric collapsing as it

[7] In the geometric stable model, the return $r_{f(p)}$ is considered to be the sum of smaller returns $r^{(i)}$ over the period of time $f(p)$ which is a stopping time random variable with geometric probability function $P(f(p) = j) = p(1-p)^{j-1}$, $j = 1, 2, ...$ The geometric stable distribution can be approximated to a normalised geometric stable model sum when the p parameter of the stopping time function $f(p)$ approaches zero. More formally, the random array $\mathbf{X}$ has a geometric stable distribution in $\mathbb{R}^n$ if and only if,

$$a(p)\sum_{i=1}^{f(p)}\left(\boldsymbol{\kappa}(p) + \mathbf{r}^{(i)}\right) \xrightarrow{d} \mathbf{X}, \qquad \text{as } p \to 0 \tag{17}$$

where $\left\{\{\mathbf{r}^{(d)} = \left(r_1^{(d)}, ..., r_n^{(d)}\right), d \geq 1\right\}$ is a sequence of i.i.d. random vectors in $\Re^n$ independent of $f(p)$, $a(p) > 0$, $\boldsymbol{\kappa}(p) \in \mathbb{R}^n$, and $\xrightarrow{d}$ denotes convergence in distribution. Kozubowski and Podgorski (2001) show that when each vector in $\mathbf{r}$ has mean m_i, $i = 1, ...n$, a variance σ_{ij}, $i = 1, ...n$, $j = 1, ...n$, and for $a(p) = \sqrt{p}$ and $\boldsymbol{\kappa}(p) = \mathbf{m}\left(\sqrt{p}-1\right)$, the random variable $\mathbf{X}$ defined by the convergence in distribution property in (17) has an AML distribution with the characteristic function,

$$\Psi(\mathbf{t}) = \frac{1}{1+\frac{1}{2}\mathbf{t}'\mathbf{H}\mathbf{t} - i\mathbf{t}'\mathbf{m}} \tag{18}$$

where $\mathbf{t} \in \mathbb{R}^n$, and $\mathbf{H} \in \mathbb{R}^{n\times n}$ is a positive-definite matrix.

can clearly be seen in equation (18) to the elliptical case (see discussion in Johnson and Kotz, 1972).

Figures 1-6 present alternative bivariate AML densities with alternative **m** vectors and correlation levels.

[**Insert Figures 1-6 here**]

As shown in Kotz, Kozubowski and Podgorski (2003), AML distributions can also be obtained as a limiting case of the Generalised Hyperbolic (GH) distribution, introduced by Barndorff-Nielsen (1977). These are location-scale mixtures of normal distributions, i.e. if $\mathbf{X}$ has a GH distribution in $\mathbb{R}^n$ then,

$$\mathbf{X} \overset{D}{=} \boldsymbol{\mu} + \mathbf{m}\boldsymbol{\xi} + \boldsymbol{\xi}^{1/2}\mathbf{Z} \tag{20}$$

where "$\overset{D}{=}$" denotes equality in distribution, $\mathbf{Z} \sim N_n(\mathbf{0}, \mathbf{H})$, $\boldsymbol{\mu} \in \mathbb{R}^n$, and $\boldsymbol{\xi}$ is a generalised inverse Gaussian (GIG) variable with parameters ν, γ, and δ, i.e. $\boldsymbol{\xi} \sim GIG(\nu, \gamma, \delta)$. AML distributions appear when $\boldsymbol{\mu} = \mathbf{0}$ and when $\boldsymbol{\xi}$ is not $GIG(\nu, \gamma, \delta)$ but standard exponential, i.e. $\boldsymbol{\xi} \sim EXP(1)$. Note that the limiting case $GIG(1, 0, 2)$ is equivalent to $EXP(1)$.[8]

The representation of the AML distribution as a location-scale mixture of normal distributions is given by,

$$\mathbf{X} \overset{D}{=} \mathbf{m}\boldsymbol{\xi} + \boldsymbol{\xi}^{1/2}\mathbf{Z} \tag{21}$$

where in this case $\boldsymbol{\xi} \sim EXP(1)$. From this it can easily be seen that $E(\mathbf{X}) = \mathbf{m}$ and $Var(\mathbf{X}) = \mathbf{H} + \mathbf{m}\mathbf{m}'$. This is of particular importance for the estimation of the MGARCH model. Contrary to the Gaussian case, the variance of a random variable with AML distribution does not coincide with the scale parameter of the distribution. Note that $Var(\mathbf{X}) = \mathbf{H}$ only when the distribution is elliptical, i.e. when $\mathbf{m} = \mathbf{0}$).

[8] Mencia and Sentana (2005) analyse the GH distribution in multivariate conditionally heteroskedastic dynamic regression models. The dynamics of the conditional covariance matrix H_t are given by a single factor model with a GQARCH(1,1) specification for the common factor, and a time-invariant diagonal matrix for the idiosyncratic terms. Given that $\mu = 0$ because the mean of the returns has been removed prior to estimation, the only difference with the AML distribution resides in the employed mixing distribution. The generalised inverse Gaussian distribution allows for flexible tail modelling but at the cost of limiting the inclusion of rich dynamics for the conditional variance matrix because of the "course of dimensionality". For the case of a highly parameterised specification like the AGDCC model the estimation using the GH distribution is extremely difficult.

In contrast with the majority of GH distributions, the AML distribution in the special case $\mathbf{m} = \mathbf{0}$ is stable, just as the normal. This condition implies an important property necessary for the modelling of financial portfolios known as the additivity property, which is basically the concept that a linear combination of independent random variables with stability index α is also stable with the same parameter α (See Khindanova, Rachev and Schwartz, 2001).

Pareto stable distributions are stable under random summation. Formally, the random variable $\mathbf{X}$ is said to be Pareto stable if for any $a_i > 0$, $i = 1, ..., d$, there exist a constant $c = d^{1/\alpha}$ and $\mathbf{u}_d \in \mathbb{R}^n$ for any $d \geq 2$ such that,

$$a_1\mathbf{X}^{(1)}+...+a_d\mathbf{X}^{(d)} \stackrel{D}{=} c\mathbf{X} + \mathbf{u}_d \tag{22}$$

where $\mathbf{X}^{(1)}, ..., \mathbf{X}^{(d)}$ are independent copies of $\mathbf{X}$. In an alike way Laplace laws are stable, but under geometric summation instead of random summation. To be able to preserve stability we have to constraint the normalising constants $a(p)$ and $\boldsymbol{\kappa}(p)$ in (17) to,

$$a(p) = \sqrt[\alpha]{p}, \quad \boldsymbol{\kappa}(p) = \mathbf{0} \tag{23}$$

The first condition implies that for the case of the AML distribution $\alpha = 2$. This is the same α value of the normal distribution which is the only Pareto-stable distribution with a finite second moment. The second condition $\boldsymbol{\kappa}(p) = \mathbf{0}$ implies $\mathbf{m} = \mathbf{0}$, restricting the use of the distribution for portfolio-VaR applications to the symmetric case.

3 Estimating DCC Models with AML Distributed Standardised Residuals

We turn now to the estimation of DCC models employing AML distributions. The likelihood function $L_T^{AML}(\boldsymbol{\theta})$ assuming a AML distribution for the conditional returns is proportional to

$$\begin{aligned} L_T^{AML}(\boldsymbol{\theta}) \quad = \quad & \sum_{t=1}^{T} \left\{ \mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{m} - \frac{1}{2}\ln|\mathbf{H}_t| + \right. \\ & \frac{v}{2}\left(\ln(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t) - \ln(2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m})\right) \\ & \left. + \ln\left[K_v\left(\sqrt{(2+\mathbf{m}'\mathbf{H}_t^{-1}\mathbf{m})(\mathbf{r}_t'\mathbf{H}_t^{-1}\mathbf{r}_t)}\right)\right] \right\}. \end{aligned} \tag{24}$$

From $\mathbf{H}_t \equiv \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$, equation (24) can be written as

$$
\begin{aligned}
L_T^{AML}(\boldsymbol{\theta}) \quad = \quad & \sum_{t=1}^{T}\Bigg\{\mathbf{r}_t'(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)^{-1}\mathbf{m} - \frac{1}{2}\ln|(\mathbf{D}_t\mathbf{R}_t\mathbf{D}_t)| \qquad (25)\\
& +\frac{v}{2}\left(\ln(\mathbf{r}_t'\mathbf{D}_t^{-1}\mathbf{R}_t^{-1}\mathbf{D}_t^{-1}\mathbf{r}_t) - \ln(2+\mathbf{m}'(\mathbf{D_t R_t D_t})^{-1}\mathbf{m})\right)\\
& +\ln\left[K_v\left(\sqrt{(2+\mathbf{m}'(\mathbf{D_t R_t D_t})^{-1}\mathbf{m})(\mathbf{r}_t'\mathbf{D_t^{-1}R_t^{-1}D_t^{-1}}\mathbf{r}_t)}\right)\right]\Bigg\}
\end{aligned}
$$

The $\mathbf{m}$ parameter cannot be estimated in the first step because it is a function of the conditional covariances which are estimated only in the second stage. Thus, assume $\mathbf{R}_t = I$ and $\mathbf{m} = \mathbf{0}$ and let us denote with $\boldsymbol{\zeta}$ the set of parameters in the matrix $\mathbf{D}_t$. The first stage likelihood function is

$$
\begin{aligned}
L_T^{AML}(\boldsymbol{\zeta}) \quad = \quad & \sum_{t=1}^{T}\Bigg\{-\frac{1}{2}\ln\left|\mathbf{D}_t^2\right| + \frac{v}{2}\left(\ln(\mathbf{r}_t'\mathbf{D}_t^{-2}\mathbf{r}_t) - \ln(2)\right) \qquad (26)\\
& +\ln\left[K_v\left(\sqrt{2(\mathbf{r}_t'\mathbf{D_t^{-2}}\mathbf{r}_t)}\right)\right]\Bigg\}
\end{aligned}
$$

Contrary to the normal case, $L_T^{AML}(\boldsymbol{\zeta})$ cannot be expressed as the sum of n-log-likelihood functions, i.e. the parameters in $\boldsymbol{\zeta}$ have to be estimated maximizing one single log-likelihood function. This, however, does allow to continue to use the two-step estimation technique[9] although it does extend the computing time for estimation.

Defining $\boldsymbol{\varepsilon}_t = \mathbf{r}_t'\mathbf{D}_t^{-1}$ and $\boldsymbol{\varepsilon}_t^* = \mathbf{m}'\mathbf{D}_t^{-1}$, the second-stage log-likelihood is given by

$$
\begin{aligned}
L_T^{AML}(\varphi|\,\boldsymbol{\zeta}) \quad = \quad & \sum_{t=1}^{T}\Bigg\{\boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}(\boldsymbol{\varepsilon}_t^*)' - \frac{1}{2}\ln|\mathbf{R}_t| + \qquad (27)\\
& \frac{v}{2}\left(\ln(\boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t') - \ln(2+\boldsymbol{\varepsilon}_t^*\mathbf{R}_t^{-1}(\boldsymbol{\varepsilon}_t^*)')\right)\\
& +\ln K_v\left(\sqrt{(2+\boldsymbol{\varepsilon}_t^*\mathbf{R}_t^{-1}(\boldsymbol{\varepsilon}_t^*)')(\boldsymbol{\varepsilon}_t\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t')}\right)\Bigg\}
\end{aligned}
$$

Expression (26) and (27) can be computed by calculating the one dimensional integral of the third component of the functions. Both functions were estimated via maximum likelihood estimation method. We choose this

[9] The parameter estimates in the vector $\widehat{\boldsymbol{\zeta}}$ will still be consistent if the specification for the distribution of the returns is correct.

method because it yields consistent and asymptotically efficient parameter estimates when the assumed distribution is correctly specified. The use of a flexible distribution like the AML is in this regard very important. This distribution captures excess kurtosis and asymmetries which are usual features in the return of financial assets. Other works like Fiorentini et al (2003) have relaxed normality using the multivariate Student-t distribution, and although this distribution allows for excess kurtosis it can not deal with skewness, risking a possible inconsistency of parameter estimates.

In a companion paper (Cajigas, Kao and Urga, 2007), we provide condition for consistency of the MLE for MGARCH models when an AML distribution is assumed for standardised residuals. Consistency is proved valid for a variety of specific cases of DCC models.

Finally, the computation of (26) and (27) is feasible but requires calculation of some functionals and is thus computationally quite costly. An alternative is the numerical solution suggested in Kotz, Kozubowski and Podgorski (2003).

4 Empirical Application

The application reported in this section is intended to provide evidence about the appropriateness of the use of the AML-DCC type specification compared to the normal-DCC model. We focus on specification tests for the distribution of standardised residuals and on the features of parameter estimates. We present also a small Value-at-Risk exercise to compare the behavior of both models. A comprehensive set of risk management applications is reported in Cajigas (2006).

We consider shares indices of 21 countries listed in the FTSE All-World Indices and bond indices of 13 countries constructed by datastream. We refer the interested reader to Cappiello *et al* (2004) for a detailed description of the data[10]. The frequency is weekly and spans over the period 08/01/1987-07/02/2002 (785 observations). The 21 countries of the share indices are: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Singapore, Spain, Sweden, Switzerland, United Kingdom, and the United States. The 12 countries of the bond indices are Austria, Belgium, Canada, Denmark, France, Germany, Ireland, Japan, Netherlands, Sweden, Switzerland, and the United Kingdom.

[10] We wish to thank Kevin Sheppard for providing us with the dataset.

Weekly returns for bonds and shares were calculated through log differences using Friday to Friday closing prices and filtered by removing the mean

$$r_{jt} = \log\left(\frac{P_{jt}}{P_{jt-1}}\right) - \frac{1}{T}\sum_{i=1}^{T}\log\left(\frac{P_{ji}}{P_{ji-1}}\right), \quad j = 1, ...n \tag{28}$$

where P_{jt} is the price of assets j at time t.

We estimate the four models described above: AGDDC (1,1,1), GDCC(1,1), ADCC (1,1,1), and the DCC (1,1)[11]. Tables 1a-1b report parameter estimates of the joint GARCH (1,1) processes for the univariate volatilities, and the skewness and kurtosis of the returns standardised by their estimated standard deviation.

To evaluate the parametric assumptions for the univariate case we compare the Kolmogorov-Smirnov distances between residuals standardised by volatilities estimated in the first stage, with the two implicit univariate normal and Laplace distributions. Tables 2a-2b report these statistics.

[Insert Tables 1a,b-2a,b here]

In this case there is no apparent advantage in the use of the Laplace distribution. The results for the more relevant multivariate case nonetheless are more encouraging. To evaluate the multivariate distributions we implemented the visual diagnostic proposed in Kawakatsu (2006). The idea is based on the fact that if $r_t \sim N(0, H_t)$, then $r_t H_t^{-1} r_t'$ has a $\chi^2(n)$ distribution. Although we do not know the distribution of $r_t H_t^{-1} r_t'$ when $r_t \sim AML(m, H_t)$ we generated an empirical distribution in order to perform the comparison.

Figure 7 presents the qq-plots of the sample quantiles of $r_t H_t^{-1} r_t'$ for the four normal models against the quantiles of $\chi^2(n)$, and the qq-plots of the sample quantiles of $r_t H_t^{-1} r_t'$ for the four AML models against the quantiles of the empirical distribution. From a visual inspection it is clear that the assumption of a AML distribution is more appropriate than the one of normality.

[Insert Figure 7 here]

To reinforce our findings about the inconvenience of the assumption of multivariate normality we also performed the omnibus test of Doornik and

[11] For sake of simplicity, in what follows we do not report anymore the number of lags structure.

Hansen (1994). Multivariate normality was overwhelmingly rejected for the raw and standardised data after fitting the normal DCC, ADCC, GDCC, and AGDCC models. All p-values are ≈ 0, and thus we do not report them in the paper.

Before estimating the models for the conditional correlation we evaluated the constancy of correlation performing the LM test of Tse (2000). We overwhelmingly reject the null of constant correlation with a p-value = 0.000, and in this case too we do not report the results.

Tables 3a,b reports the correlation parameter estimates for the DCC and ADCC models, and the vector of asymmetry coefficients $\mathbf{b}$.

[Insert Tables 3a,b here]

We found that the parameter estimates of the DCC (1,1) model are very similar to those reported in CES. This is not the case of the ADCC model. For this model CES report a much higher level for the persistence parameter (0.94816 for the normal case against 0.5217 for the AML case).

The correlation parameter estimates for the GDCC and AGDCC specifications are reported in Table 4a.b and Table 5a,b. Overall, we found high levels of persistence but not as pronounced as in CES. For the case of normal innovations the range of the β parameter in the GDCC model goes from 0.9186 (Canada shares) to 0.9759 (Austria bonds), while for the case of AML innovations is much more open; it goes from 0.1764 (Germany shares) to 0.9748 (New Zealand shares). The parameter estimates of the AGDCC model also show a higher degree of heterogeneity across indices when the AML distribution is assumed. All asymmetric parameters in the AGDCC model were highly significant.

Table 6 reports the Log-likelihood values for each one of the four models.

[Insert Tables 4-6 here]

In contrast to the case described in CES where the innovations are assumed normal, the inclusion of asymmetric terms or diagonal components does not increase the log-likelihood.

We follow Engle and Sheppard (2001) and employ the minimum variance portfolio criterion as a specification test of the models. We compare the variance of the portfolios formed by all the securities in the array $\mathbf{X}_t$ estimated with the eight models (four models assuming normality and four models assuming the AML distribution). The weight vector at time t for each one of the portfolios is given by,

$$ {}^{m_i}\mathbf{w}_t = \frac{{}^{m_i}\mathbf{H}_t^{-1}\boldsymbol{\iota}}{\boldsymbol{\iota}'{}^{m_i}\mathbf{H}_t^{-1}\boldsymbol{\iota}} \tag{29} $$

where $i = 1, ..., 8$ and $\boldsymbol{\iota}$ is an (n x 1) vector of ones. The variance of each portfolio will be given by $\mathbf{V}_t = \mathbf{w}_t'\mathbf{H}_t\mathbf{w}_t$. If ${}^{m_i}\mathbf{H}_t$ is accurately specified, then model m_i should give the minimum variance portfolio. Figures 8a and 8b show the eight series of ${}^{m_i}\mathbf{V}_t$ and Table 7 presents the average portfolio volatilities.

[Insert Table 7 here]
[Insert Figures 8a-8b here]

We also implement a Value-at-Risk (VaR) exercise as specifcation test for the models. Consider the portfolio return

$$ \mathbf{r}_p = \sum_{i=1}^{n} w_i r_i = \mathbf{w}'\mathbf{r} \tag{30} $$

where $w_1 + ... + w_n = 1$. The VaR at the α level is the solution to,

$$ \alpha = \int_{-\infty}^{VaR} f(\mathbf{r}_p) d\mathbf{r}_p \tag{31} $$

where $f(\mathbf{r}_p)$ is the density function of $\mathbf{r}_p$. In the special case where $f(\mathbf{r}_p)$ is the density of the AML distribution the conditional VaR implicit in (31) is

$$ VaR_t = \left(\mathbf{w}_t'\mathbf{H}_t\mathbf{w}_t\right)^{1/2} L_\alpha \tag{32} $$

where L_α is the α-th quantile of the univariate standard Laplace distribution.

We consider three constant vectors of weights $\mathbf{w}$:

$$ \mathbf{w^1} = \begin{bmatrix} w_1^1 = 0.035714286 \\ . \\ . \\ . \\ w_{21}^1 = 0.035714286 \\ w_{22}^1 = 0.019230769 \\ . \\ . \\ . \\ w_{34}^1 = 0.019230769 \end{bmatrix} \tag{33} $$

$$\mathbf{w^2} = \begin{bmatrix} w_1^2 = 0.029411765 \\ . \\ . \\ . \\ w_{34}^2 = 0.029411765 \end{bmatrix} \tag{34}$$

$$\mathbf{w^3} = \begin{bmatrix} w_1^3 = 0.011904762 \\ . \\ . \\ . \\ w_{21}^3 = 0.011904762 \\ w_{22}^3 = 0.057692308 \\ . \\ . \\ . \\ w_{34}^3 = 0.057692308 \end{bmatrix} \tag{35}$$

$\mathbf{w}^1$ corresponds to the case where the 21 FTSE All-World indices constitute the 75% of the portfolio and the 13 Bond indices constitute the remaining 25%. $\mathbf{w}^2$ corresponds to the case where the 34 indices have the same weight in the portfolio, and $\mathbf{w}^3$ corresponds to the case where the 21 FTSE All-World indices constitute the 25% of the portfolio and the 13 Bond indices constitute the remaining 75%. Given that we are considering four MGARCH models for the dynamics of $\mathbf{H}_t$ and two different innovation densities (the normal and the AML), we have in total 24 different variance portfolios to analyse.

We computed the conditional VaR for the 24 cases at the 1% level using the entire sample of 785 observations.

The Markov tests proposed in this work are designed to detect clustering in the violations of the VaR measures, where a violation is defined as the event where the ex-post portfolio loss exceeds the ex-ante VaR. Clearly, given the parametric model-based nature of the VaR methodology employed in this exercise, a correct dynamic specification of the portfolio volatility and a correct distribution for conditional returns are necessary to secure a right specification of the VaR technique.

We consider the unconditional coverage (*uc*), independence (*ind*), and conditional coverage (*cc*) test of Christoffersen and Pelletier (2004). Consider the hit sequence of VaR violations defined as

$$I_t = \begin{cases} 1, \text{if } r_t < -VaR(\alpha) \\ 0, \text{ else} \end{cases} \tag{36}$$

against the alternative that the sequence is iid Bernoulli with parameter π, where π is the ratio of the number of violations over the number of observations. If the VaR method is correct the empirical failure rate π must be equal to α.

The *ind* test tests explicitly the assumption of independence of the hit sequence,

$$H_{0,ind:}\pi_{01} = \pi_{11} \tag{37}$$

where π_{ij} is the probability of an i on day $t-1$ being followed by a j on day t. Neither the *uc* test nor the *ind* test are complete by their own, the first one tests that on a average the coverage implicit by the VaR model is correct, while the second tests the clustering effect on the failures without testing the correct number of failures. The *cc* test combines both tests:

$$H_{0,cc:}\pi_{01} = \pi_{11} = \alpha. \tag{38}$$

Under the null, the likelihood ratio test of unconditional coverage (LR_{uc}) and the likelihood ratio test of independence (LR_{ind}) are χ^2 with one degree of freedom. Under the null the likelihood ratio test of conditional coverage (LR_{cc}) is χ^2 with two degrees of freedom.

Tables 8, 9, and 10 present the failure rates and p-values of the *uc*, *ind*, and *cc* tests for the four MGARCH models and for the three portfolios $\mathbf{w}^1, \mathbf{w}^2$, and $\mathbf{w}^3$.

[Insert Tables 8-10 here]

The performance of the scalar models (DCC and ADCC) across portfolios is very similar. The estimated VaR models in general capture quite well the clustering of violations. The models with AML innovations are superior to the models with normal innovations for the cases of the w^1 and w^2 portfolios. For the w^1 portfolio the results for the independence test are quite mixed. For the w^1 portfolio we found a very poor performance of the models regarding the unconditional and conditional coverage.

Plots with the distribution of conditional correlations for four pairs of correlations as well as descriptive statistics for the estimated series are presented in Figures 9-12.

[Insert Figures 9-12 here]

First, we observe that the distribution of the correlation across models is very unstable. Across the four pairs the ADCC model with AML innovations highlights for its extreme level of kurtosis (225.11 for the UK shares-US shares pair, 24.79 for the Japan bonds-UK bonds pair, 30.49 for the UK shares-Mexico shares pair, and 20.92 for the UK bonds-Switzerland bonds pair). This leptokurtosis is a result of very small volatilities (0.84%, 0.72%, 0.82%, and 0.52% respectively) and one single positive jump registered on Black Monday in 1987.

Plots with the conditional correlation series for four pairs of correlations are presented in Figures 13-16.

[**Insert Figures 13-16 here**]

In general, the kurtosis registered for the dynamic correlation estimated assuming an AML distribution is higher than that estimated assuming normality. In Table 11 we present a comparison of the levels of kurtosis between correlations assuming the two types of distributions for the asset pairs considered in Figures 9-16.

[**Insert Table 11 here**]

5 Conclusions

In this paper we proposed a multivariate GARCH asymmetric generalised dynamic conditional correlation model where the vector of standardised residuals is assumed to follow an asymmetric multivariate Laplace distribution. This multivariate distribution is able to capture leptokurtosis and asymmetry which characterise returns from financial assets. This is the only distribution (besides the normal) with desirable properties such as additivity and finiteness of moments. In addition, contrary to the majority of (geometric) stable distributions, it has a density function with a closed-form that makes the maximum likelihood estimation method easy to implement. Very importantly, we show that the two-step approach of the DCC model is preserved when innovations are modelled via AML distributions, while this is not the case for a multivariate Student-t.

The empirical validity of the model we propose is tested by fitting the sample of 21 FTSE All-World stock indices and 12 bond return indices of Cappiello, Engle and Sheppard (2004). We provide clear evidence that this distribution overwhelmingly outperforms the case in which we assume normality of innovations. The empirical validity of this form is also tested in the context of a Value-at-Risk (VaR) model. By performing a conditional-VaR analysis, we obtained mixed results. Though all models capture quite well the clustering of violations of the VaR levels, they performed quite poorly when they were tested for the level of failure rates. But when we evaluate the independence of hit sequences, once again the models with asymmetric multivariate Laplace innovations outperformed models where normality of the innovations is assumed.

The empirical application presented in this paper was carried out in order to compare and evaluate the behaviour of parameter estimates computed with several specifications. A more comprehensive set of empirical implementations to risk management to examine the benefits of our proposed framework is beyond the scope of this paper. Of course, it is the object of companion analyses.

Acknowledgment

We wish to thank participants in the international conference "*Common Features in London*" (Cass Business School, London, 16-17 December 2004), in particular Robert Engle, Neil Shephard, Kevin Sheppard; the "*1st Italian Congress of Econometrics and Empirical Economics*" (Universita' Ca' Foscari, Venice, 24-25 January 2005); the "*Simulation Based and Finite Sample Inference in Finance II*" Conference (Québec City, 29-30 April 2005), in particular our discussant Tongshu Ma; the ESRC Econometric Study Group Conference (Bristol, 14-16 July, 2005); the CEA@Cass/ESRC seminar on "*The use of copulas and DCC models to measure dependence in finance*" (Cass Business School, 17 March 2006), in particular Valentina Corradi, Andrew Patton, and Fabio Trojani; the Finance Seminar (Tilburg, 20 March 2006) in particular Ralph S.J. Koijen; the Bag Lunch Seminar (Finance Department, Kellogg Management School), in particular Torben Andersen, Ravi Jagannathan, Ernst Schaumburg; the CEA@Cass/ESRC seminar on "*Using thick distributions, DCC and copula models*" (Cass Business School, London,19 May 2006), in particular Enrique Sentana and Javier Mencia; the 4th OxMetrics Conference (Cass Business School, London, 15-16 September 2006), in particular Andrew Harvey, David Hendry and Sebastien Laurent, for useful discussions and comments which helped to progress with this paper. Special thanks to Denis Pelletier, Fabio Trojani and Radu Tunaru for detailed comments on a previous version of the manuscript, and to Chihwa Kao for long discussions on the identification and consistency issue. Kevin Sheppard and Lorenzo Cappiello provided the dataset and useful insights to estimate the various models. However, the usual disclaimer applies. We wish to ackowledge financial support from Centre for Econometric Analysis at Cass and INQUIRE UK 2004. This article represents the views of the authors and not of INQUIRE UK.

References

Baillie, R. T. and Bollerslev, T. (1989), "The Message in Daily Exchange Rates: A Conditional Variance Tale", *Journal of Business & Economic Statistics*, 7, 297-305.

Barndorff-Nielsen, O. (1977), “Exponentially decreasing distributions for the logarithm of particle size. *Proc. Roy. Soc. Lond. A* 353, 401-419.

Bauwens, L., and Laurent, S. (2004), “A new class of multivariate skew densities, with application to GARCH models”, mimeo.

Bauwens, L., S. Laurent and J. Rombouts (2006), "Multivariate GARCH models: a survey", *Journal of Applied Econometrics*, 21, 79-109.

Bollerslev, T. (1987), "A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return", *Review of Economics and Statistics*, 69, 542-547

Bollerslev, T. (2001), “Financial econometrics: past developments and future challenges”, *Journal of Econometrics*, 100, 41-51.

Bollerslev, T. and J. M. Wooldridge (1992), "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances”, *Econometric Reviews,* 11, 143-172.

Cajigas, J. (2006), *A Multivariate GARCH Model for the Non-Normal Behaviour of Financial Assets*, PhD Dissertation, Cass Business School.

Cajigas, J., C. Kao and Urga, G. (2007), Asymptotic properties of the MLE in AML-DCC models", mimeo, Centre for Econometric Analysis, Cass Business School.

Cappiello, L., Engle, R. and K. Sheppard (2004), “Asymmetric dynamics in the correlations of global equity and bond returns”, ECB Working Paper No. 204.

Christoffersen, P. and Pelletier, D. (2004), “Backtesting Value-at-Risk: a duration-based approach”, *Journal of Financial Econometrics*, 2, 84-108.

Doornik, J. A. and Hansen, H. (1994), “An omnibus test for univariate and multivariate normality”, mimeo, Nuffield College, Oxford.

Drost, C.D. and Klaassen, C.A.J. (1997), “Efficient estimation in semiparametric GARCH models”, *Journal of Econometrics*, 81, 193-221.

Dufour, J-M. (2006), “Monte Carlo tests with nuisance parameters: a general approach to finite-sample Inference and nonstandard asymptotics”, *Journal of Econometrics*, 133, 443-477.

Engle, R. (2001), “Financial econometrics – A new discipline with new methods”, *Journal of Econometrics*, 100, 53-56.

Engle, R. (2002), “Dynamic conditional correlation - A simple class of multivariate GARCH models”, *Journal of Business & Economic Statistics,* 20, 339- 350.

Engle, R., and Colacito, R. (2006), "Testing and valuing dynamic correlations for asset allocation," *Journal of Business & Economic Statistics*, 2, 238-253.

Engle, R. and Gonzalez-Rivera, G. (1991), “Semiparametric ARCH models”, *Journal of Business & Economic Statistics*, 9, 345-359.

Engle, R. and Sheppard, K. (2001), “Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH”, NBER Working Paper No. W8554

Fiorentini, G., E. Sentana and Calzolari, G. (2003), "Maximum likelihood estimation and inference in multivariate conditionally heteroskedastic dynamic regression models with Student-t innovations", *Journal of Business & Economic Statistics,* 21, 532-546.

Gonzalez-Rivera, G. (1997), “A note on adaptation in ARCH models”, *Econometric Reviews*, 16, 55-68.

Gonzalez-Rivera, G. and Drost, C.D. (1999), “Efficiency comparisons of maximum- likelihood-based estimators in GARCH models”, *Journal of Econometrics*, 93, 93-111.

Hafner, C. M. and Franses, H. P. (2003), “A generalized DCC model for many asset returns”, Econometric Institute Report EI 2003-18, Erasmus University Rotterdam.

Hafner, C. M., van Dijk, D., and Franses, H. P. (2005), "Semi-parametric modelling of correlation dynamics", Econometric Institute Report EI 2005-26, Erasmus University Rotterdam.

Hafner, C. M. and Rombouts, J. V. K. (2004), “Semiparametric multivariate volatility models”, mimeo, Econometric Institute, Erasmus University Rotterdam,.

Heyde, C. C. and Kou, S. G. (2004), “On the controversy over tailweight of distributions”, *Operations Research Letters*, 32, 399-408.

Johnson, N. L. and Kotz, S. (1972), *Distributions in Statistics: Continuous Multivariate Distributions,* Wiley, New York.

Kawakatsu, H. (2006), "Matrix exponential GARCH", *Journal of Econometrics,* 134, 95-128 .

Khindanova, I., Rachev, S., and E. Schwartz. (2001), “Stable modeling of Value at Risk”, *Mathematical and Computer Modeling,* 34, 1223-1259.

Komunjer, I. (2005), "Quasi-maximum likelihood estimation for conditional quantiles", *Journal of Econometrics*, 128, 137-164.

Kotz, S., Kozubowski, T. J., and K. Podgorski (2003), *The Laplace Distribution and Generalizations. A Revisit with Applications to Communications, Economics, Engineering, and Finance*, Birkhauser, Berlin.

Kozubowski, T. J., and Podgorski, K. (2001), “Asymmetric Laplace laws and modeling financial data”, *Mathematical and Computer Modelling,* 34, 1003-1021.

Linden, M. (2001), “A model for stock return distribution”, *International Journal of Finance and Economics*, 6, 159-169.

Long, X. and Ullah, A. (2005), "Nonparametric and semiparametric multivariate GARCH model", mimeo, Department of Economics, University of California, Riverside.

Madan, D. B. and Seneta, E. (1990), “The variance gamma model for share market returns”, *Journal of Business,* 63, 511-524.

Madan, D. B., Carr, P. P., and Chang, E. C. (1998), “The variance gamma process and option pricing”, *European Finance Rev., 2,* 79-105.

Mencia, F. J., and Sentana, E. (2005), “Estimation and testing of dynamic models with generalised hyperbolic innovations”, CEMFI working paper No. 0411

Nelson, D.V. (1991), “Conditional heteroskedasticity in asset returns: A new approach”, *Econometrica*, 59, 347-370.

Newey, W., and McFadden, D. (1994), “Large sample estimation and hypothesis testing,” in *Handbook of Econometrics* (Vol. 4), eds. R. Engle and D. McFadden, New York: Elsevier Science, pp. 2113–2245.

Palandri, A.(2005), "Sequential conditional correlations: inference and evaluation," mimeo, Department of Economics, Duke University.

Pelletier, D. (2006), "Regime switching for dynamic correlations", *Journal of Econometrics*, 131, 445-473.

Silvennoinen, A. and T. Terasvirta (2005), "Multivariate autoregressive conditional heteroskedasticity with smooth transitions in conditional correlations", SSE/EFI Working Paper Series in Economics and Finance, N. 577.

Tse, I. K. (2000), “A tests for constant correlations in a multivariate GARCH model”, *Journal of Econometrics*, 98, 107-127.

Tse, Y. K. and Tsui, A. K. C. (2002), "A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations," *Journal of Business & Economic Statistics*, 20, 351-362.

Tables

Table 1a: Parameter estimates for **stock indices** for the univariate GARCH models and skewness and kurtosis of the returns standardised by their estimated standard deviation using the AML distribution.

	ϖ	α	β	Stand. Skew.	Stand. Kurt.
Australia	0.000019	0.0296	0.9466	-1.25	11.06
Austria	0.000051	0.1092	0.8452	-0.36	4.21
Belgium	0.000013	0.0497	0.9257	-0.47	4.51
Canada	0.000031	0.0800	0.8650	-1.09	10.77
Denmark	0.000034	0.0771	0.8802	0.05	4.01
France	0.000056	0.0733	0.8487	-0.25	3.58
Germany	0.000019	0.0527	0.9194	-0.56	4.90
H.K.	0.000220	0.1342	0.7470	-1.21	9.30
Ireland	0.000022	0.0395	0.9389	-1.05	10.71
Italy	0.000013	0.0362	0.9590	-0.20	4.37
Japan	0.000151	0.1540	0.7623	0.05	3.90
Mexico	0.000174	0.1015	0.8550	-0.49	6.56
Netherlands	0.000021	0.4681	0.9064	-1.11	9.84
New Zealand	0.000207	0.0380	0.7758	-0.58	6.05
Norway	0.000049	0.0671	0.8923	-1.026	10.82
Singapore	0.000142	0.1282	0.7934	-1.25	11.58
Spain	0.000102	0.0876	0.8068	-0.50	5.77
Sweden	0.000053	0.0787	0.8833	-0.55	5.43
Switzerland	0.000051	0.0340	0.8846	-1.18	13.77
U.K.	0.000015	0.0539	0.9181	-0.99	7.72
U.S.A.	0.000039	0.0308	0.8951	-1.35	14.67

Table 1b: Parameter estimates for **bonds** for the univariate GARCH models and skewness and kurtosis of the returns standardised by their estimated standard deviation using the AML distribution.

	ϖ	α	β	Stand. Skew.	Stand. Kurt.
Austria	0.000072	0.0818	0.529	0.13	3.62
Belgium	0.000021	0.0705	0.8013	0.12	3.73
Canada	0.000045	0.1466	0.5880	0.00	3.70
Denmark	0.000009	0.0466	0.8975	0.11	3.69
France	0.000042	0.0780	0.6891	0.09	3.27
Germany	0.000062	0.0847	0.5846	0.22	3.86
Ireland	0.000016	0.0653	0.8543	-0.29	4.15
Japan	0.000057	0.0854	0.7169	0.59	5.76
Netherlands	0.000022	0.0495	0.8208	0.26	4.07
Sweden	0.000001	0.0316	0.9638	0.01	3.42
Switzerland	0.000136	0.0638	0.3963	0.24	3.87
U.K.	0.000003	0.0318	0.9507	-0.25	5.113
U.S.A.	0.000020	0.0710	0.4411	0.64	8.342

Table 2a. Kolmogorov-Smirnov statistics for stock indices.

	Normal	Laplace
Australia	0.0529	0.0622
Austria	0.0445	0.0501
Belgium	0.0559	0.0665
Canada	0.0721	0.0577
Denmark	0.0524	0.0485
France	0.0510	0.0717
Germany	0.0557	0.0850
H.K.	0.0744	0.0808
Ireland	0.066	0.0592
Italy	0.029	0.0497
Japan	0.0386	0.0434
Mexico	0.0832	0.0432
Netherlands	0.0758	0.0762
New Zealand	0.0443	0.0425
Norway	0.0661	0.0599
Singapore	0.0690	0.0501
Spain	0.0520	0.0454
Sweden	0.0614	0.0681
Switzerland	0.0786	0.0556
U.K.	0.0965	0.0685
U.S.A.	0.0650	0.0589

Table 2b. Kolmogorov-Smirnov statistics for bonds.

	Normal	Laplace
Austria	0.0557	0.0769
Belgium	0.0545	0.0707
Canada	0.0751	0.0483
Denmark	0.0619	0.0600
France	0.0483	0.0658
Germany	0.0490	0.0745
Ireland	0.0709	0.0667
Japan	0.0575	0.0724
Netherlands	0.0489	0.0694
Sweden	0.0602	0.0509
Switzerland	0.0420	0.0616
U.K.	0.0729	0.0527

Table 3a: Parameter estimates for the DCC(1,1) model and ADCC(1,1,1) models using the AML distribution.

	DCC(1,1)	ADCC(1,1,1)
α	0.00987	0.0045
β	0.9571	0.5217
γ		0.0863
Shares	β_i	b
Australia	0.2617	0.0000
Austria	-0.3961	0.0000
Belgium	-0.0526	0.0000
Canada	-0.5051	0.0000
Denmark	0.5120	0.0000
France	0.4909	0.0000
Germany	0.0861	0.0000
H.K.	-1.1099	0.0000
Ireland	0.2934	0.0000
Italy	0.4822	0.0000
Japan	0.0931	0.0000
Mexico	0.1047	0.0000
Netherlands	-1.1287	0.0000
New Zealand	0.2909	0.0000
Norway	0.0544	0.0000
Singapore	-0.1068	0.0000
Spain	-0.3626	0.0001
Sweden	0.0493	0.0000
Switzerland	-0.0780	0.0000
U.K.	-0.5037	0.0000
U.S.A.	0.6387	0.0000

Notes to Table 2.3a: DCC(1,1) model: $Q_t = \left(\overline{\mathbf{Q}} - \boldsymbol{\alpha}\overline{\mathbf{Q}} - \boldsymbol{\beta}\overline{\mathbf{Q}}\right) + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1}$; ADCC(1,1,1) model: $Q_t = \left(\overline{\mathbf{Q}} - \boldsymbol{\alpha}\overline{\mathbf{Q}} - \boldsymbol{\beta}\overline{\mathbf{Q}} - \gamma\overline{\mathbf{N}}\right) + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1} + \gamma\eta_{t-1}\eta'_{t-1}$

Table 3b: Parameter estimates for the DCC(1,1) model and ADCC(1,1,1) models using the AML distribution.

	DCC(1,1)	ADCC(1,1,1)
α	0.00987	0.0045
β	0.9571	0.5217
γ		0.0863
Bonds	β_i	b
Austria	-2.3187	0.0000
Belgium	0.9697	0.0000
Canada	0.2407	0.0000
Denmark	0.4694	0.0000
France	-2.641	0.0000
Germany	5.4709	0.0000
Ireland	-3.5647	0.0000
Japan	1.0291	1.1962
Netherlands	0.7855	0.0000
Sweden	0.5523	0.0000
Switzerland	-0.4615	0.0000
U.K.	-0.8344	0.0000
U.S.A.	2.7438	1.8035

Notes to Table 3b: DCC(1,1) model: $Q_t = \left(\overline{\mathbf{Q}} - \boldsymbol{\alpha}\overline{\mathbf{Q}} - \boldsymbol{\beta}\overline{\mathbf{Q}}\right) + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1}$; ADCC(1,1,1) model: $Q_t = \left(\overline{\mathbf{Q}} - \boldsymbol{\alpha}\overline{\mathbf{Q}} - \boldsymbol{\beta}\overline{\mathbf{Q}} - \gamma\overline{\mathbf{N}}\right) + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1} + \gamma\eta_{t-1}\eta'_{t-1}$

Table 4a. Parameter estimates for the GDCC (1,1) model using the AML distribution.

	GDCC(1,1)		
Shares	α_i	β_i	b
Australia	0.0130	0.9103	0.0119
Austria	0.0205	0.9793	0.0000
Belgium	0.0002*	0.5437	0.0000
Canada	0.3563	0.6414	0.1949
Denmark	0.0766	0.7250	0.0000
France	0.0760	0.8274	0.0000
Germany	0.0147*	0.1764	0.0000
H.K.	0.0004*	0.4351	0.0072
Ireland	0.0090	0.7565	0.0000
Italy	0.3110	0.5661	0.0000
Japan	0.1250	0.3615	0.0000
Mexico	0.0000*	0.8883	0.0000
Netherlands	0.0000*	0.7923	0.0000
New Zealand	0.0252	0.9748	0.0113
Norway	0.0002	0.6731	0.0128
Singapore	0.0007*	0.8896	0.1428
Spain	0.0150	0.5150	0.0000
Sweden	0.0222	0.7109	0.0000
Switzerland	0.0595	0.2155	0.0000
U.K.	0.0456	0.9498	0.0000
U.S.A.	0.0399	0.9202	0.0000

Notes to Table 4a: GDCC model: $Q_t = \left(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}\right) + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'Q_{t-1}B$; * indicates 5% significance level

Table 4b. Parameter estimates for the GDCC (1,1) model using the AML distribution.

	GDCC(1,1)		
Bonds	α_i	β_i	b
Austria	0.0735	0.7585	0.0000
Belgium	0.0318	0.8927	0.0000
Canada	0.0208	0.9786	0.4967
Denmark	0.0550	0.8268	0.0000
France	0.0627	0.8444	0.0000
Germany	0.1022	0.7773	0.0000
Ireland	0.1536	0.8069	0.0000
Japan	0.0888	0.8189	0.0756
Netherlands	0.0950	0.7977	0.0000
Sweden	0.0003*	0.8813	0.0000
Switzerland	0.0318	0.9670	0.0000
U.K.	0.0228	0.9203	0.0000
U.S.A.	0.0082	0.9896	3.1433

Notes to Table 4b: GDCC model: $Q_t = \left(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}\right) + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'Q_{t-1}B$; * indicates 5% level of significance.

Table 5a. Parameter estimates for the AGDCC (1,1,1) model using the AML distribution.

Shares	α_i	g_i	β_i	b
Australia	0.0006*	0.0568	0.8440	0.0000
Austria	0.0027*	0.0742	0.9231	0.0000
Belgium	0.0002*	0.1990	0.6655	0.0000
Canada	0.0015*	0.2283	0.7701	0.0000
Denmark	0.0050	0.0946	0.9003	0.0000
France	0.0050	0.0864	0.8346	0.0000
Germany	0.0004*	0.1369	0.6508	0.0000
H.K.	0.1014	0.0053*	0.6299	0.0000
Ireland	0.0002*	0.1135	0.6294	0.0000
Italy	0.0025*	0.0309	0.9666	0.0000
Japan	0.0260	0.0000	0.9740	0.5942
Mexico	0.0376*	0.0080	0.7502	0.0000
Netherlands	0.0145	0.0524	0.7463	0.0000
New Zealand	0.0002*	0.0093	0.9905	0.0000
Norway	0.0002	0.0073	0.7881	0.0000
Singapore	0.0518	0.0201	0.9280	0.0000
Spain	0.0237	0.0105*	0.5845	0.0000
Sweden	0.0386	0.0540	0.8854	0.0000
Switzerland	0.0171	0.1301	0.8528	0.0000
U.K.	0.0822	0.0228	0.6062	0.0000
U.S.A.	0.0070*	0.0379	0.8427	0.0000

Note to Table 5a: AGDCC (1,1,1) model: $Q_t = \left(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}\right) + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'Q_{t-1}B + G'\eta_{t-1}\eta_{t-1}'G$; * indicates 5% level of significance.

Table 5b. Parameter estimates for the AGDCC (1,1,1) model using the AML distribution.

Bonds	α_i	g_i	β_i	b
Austria	0.1238	0.1482	0.6814	0.0000
Belgium	0.0854	0.1572	0.7059	0.0000
Canada	0.0238	0.0000*	0.9762	0.0000
Denmark	0.0641	0.1767	0.7592	0.0000
France	0.2588	0.0000*	0.6494	0.0000
Germany	0.1305	0.1449	0.6906	0.0000
Ireland	0.0796	0.1684	0.7518	0.0000
Japan	0.0004*	0.0000*	0.5485	0.0000
Netherlands	0.1268	0.1473	0.7247	0.0000
Sweden	0.0003*	0.1929	0.6866	0.0000
Switzerland	0.0036*	0.2820	0.7142	0.0000
U.K.	0.0007*	0.2625	0.7002	0.0000
U.S.A.	0.0009*	0.0345	0.8109	2.1780

Note to Table 5b: AGDCC (1,1,1) model: $Q_t = \left(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}\right) + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'Q_{t-1}B + G'\eta_{t-1}\eta'_{t-1}G$; * indicates 5% level of significance.

Table 6 . Log-likelihood values for the four estimated models

Model	Log-likelihood
DCC(1,1)	-24027
ADCC(1,1,1)	-25381
GDCC(1,1)	-25269
AGDCC(1,1,1)	-26808

Table 7. Average variance of the portfolios with alternative models

Model	Average Variance
AML-DCC	2.64E-005
AML-ADCC	2.28E-005
AML-GDCC	2.61E-005
AML-AGDCC	2.28E-005

Note to Table 7: Average variance of the portfolios formed by all securities in the sample data estimated with the four models assuming the AML distribution.

Table 8. VaR Analysis: Portfolio w1

PORTOLIO W1	Failure rate	uc	ind	cc
Normal-DCC	0.015287	0.16723	0.010686	0.014813
Laplace-DCC	0.0076433	0.48872	0.032332	0.079653
Normal-ADCC	0.014013	0.28667	0.0071606	0.015242
Laplace-ADCC	0.0076433	0.48872	0.032332	0.079653
Normal-GDCC	0.024204	0.00071497	0.079946	0.00070455
Laplace-GDCC	0.011465	0.68686	0.002787	0.010544
Normal-AGDCC	0.015287	0.16723	0.010686	0.014813
Laplace-AGDCC	0.0076433	0.48872	0.032332	0.079653

Note to Table 8: Portfolio w1: 75% formed by the 21 FTSE All-World indices and 25% by the 13 Bond indices. VaR(1%) failure rates and p-values for the hit sequence Markov tests: unconditional coverage (uc) test, independence (ind) test, and conditional coverage (cc)

Table 9. VaR Analysis: Portfolio w2

PORTOLIO W2	Failure rate	uc	ind	cc
Normal-DCC	0.012739	0.45939	0.0045904	0.013695
Laplace-DCC	0.0076433	0.48872	0.032332	0.079653
Normal-ADCC	0.012739	0.45939	0.0045904	0.013695
Laplace-ADCC	0.0076433	0.48872	0.032332	0.079653
Normal-GDCC	0.025478	0.00026545	0.098754	0.00033125
Laplace-GDCC	0.012739	0.45939	0.0045904	0.013695
Normal-AGDCC	0.014013	0.28667	0.57605	0.48484
Laplace-AGDCC	0.0076433	0.48872	0.032332	0.079653

Note to Table 9: Portfolio w2: The 34 assets have the same weight. VaR (1%) failure rates and p-values for the hit sequence Markov tests: unconditional coverage (uc) test, independence (ind) test, and conditional coverage (cc)

Table 10. VaR Analysis: Portfolio w3

PORTOLIO W3	Failure rate	uc	ind	cc
Normal-DCC	0.012739	0.45939	0.0045904	0.013695
Laplace-DCC	0.0038217	0.046627	0.87941	0.13657
Normal-ADCC	0.012739	0.45939	0.61145	0.66849
Laplace-ADCC	0.0050955	0.12729	0.83959	0.30632
Normal-GDCC	0.017834	0.046926	0.47581	0.10772
Laplace-GDCC	0.0089172	0.75609	0.72265	0.89474
Normal-AGDCC	0.011465	0.68686	0.64774	0.83058
Laplace-AGDCC	0.0050955	0.12729	0.83959	0.30632

Note to Table 10: Portfolio w3: 25% formed by the 21 FTSE All-World indices and 75% by the 13 Bond indices. VaR (1%) failure rates and p-values for the hit sequence Markov tests: unconditional coverage (uc) test, independence (ind) test, and conditional coverage (cc)

Table 11. Kurtosis of the conditional correlations.

Model	Pair 1	Pair 2	Pair 3	Pair 4
Normal-DCC	3.93	2.15	3.14	2.97
AML-DCC	5.55	2.34	3.58	3.00
Normal-ADCC	11.69	5.66	8.50	6.04
AML-ADCC	225.11	24.79	30.49	20.92
Normal-GDCC	3.42	2.06	2.82	2.93
AML-GDCC	8.79	3.45	7.55	6.20
Normal-AGDCC	5.03	5.29	7.09	4.60
AML-AGDCC	4.97	8.34	6.05	4.12

Note to Table 11: Kurtosis of the conditional correlation series created with the eight models for four pairs of assets. Pair 1 corresponds to U.K. shares-U.S.A. shares, Pair 2 to U.K. bonds-Japan bonds, Pair 3 to U.K. sahres-Mexico shares, and Pair 4 to U.K. bonds-Switzerland bonds

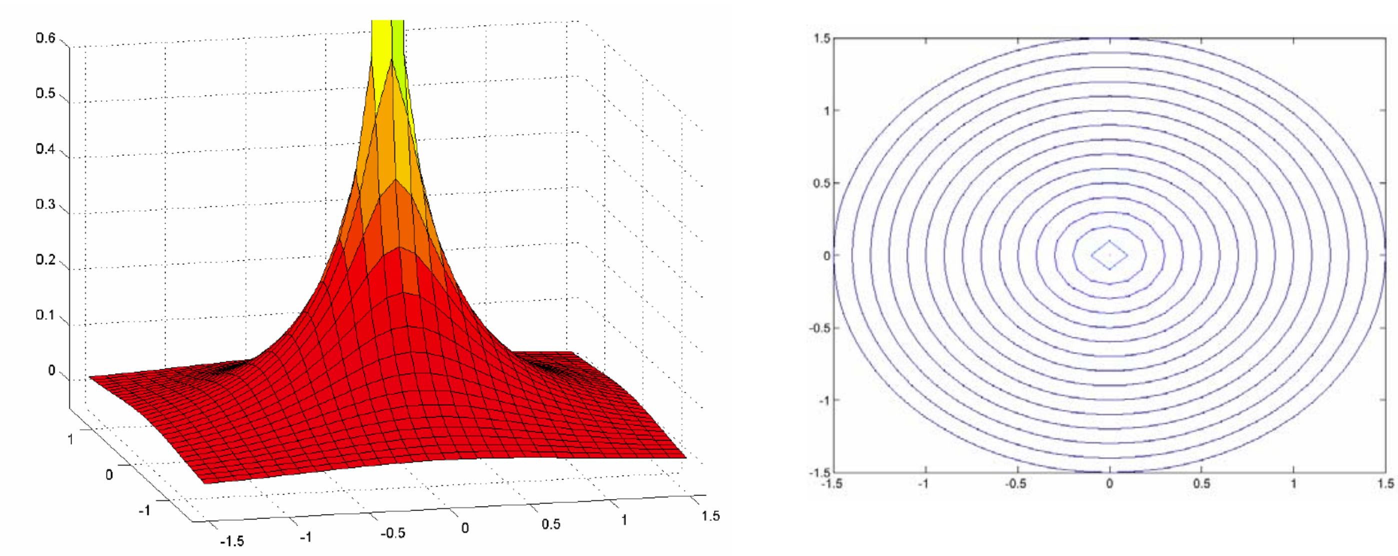


Figure 1. Bivariate Asymmetric Laplace density and contours with $m_1 = m_2 = 0$, $\sigma_1 = \sigma_2 = 1$, and $\rho = 0$

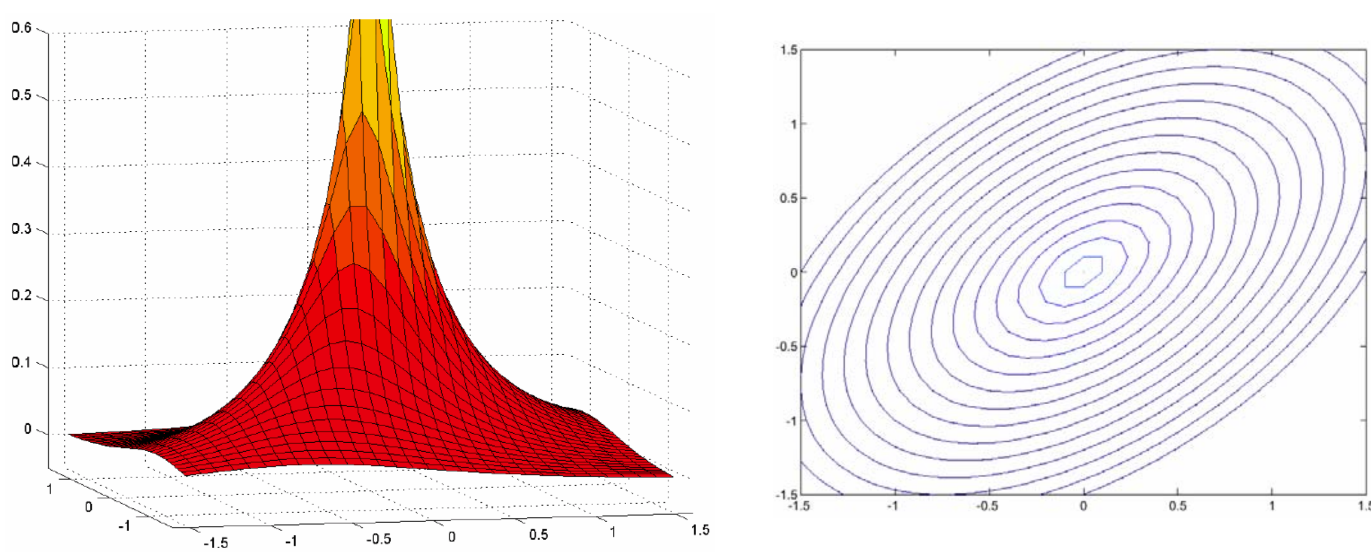


Figure 2. Bivariate Asymmetric Laplace density and contours with $m_1 = m_2 = 0$, $\sigma_1 = \sigma_2 = 1$, and $\rho = 0.5$

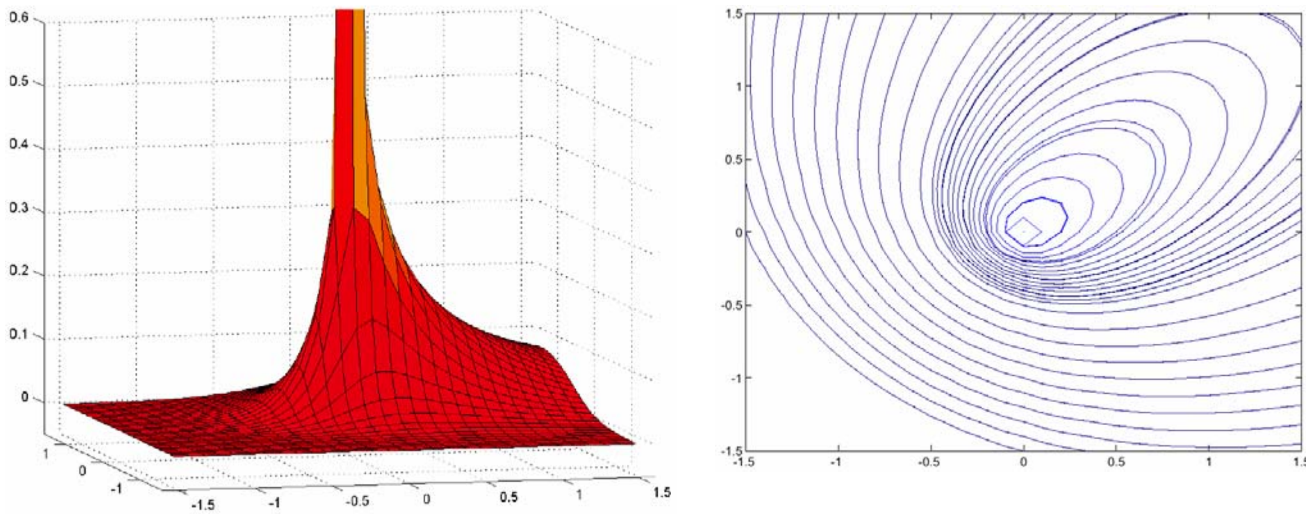


Figure 3. Bivariate Asymmetric Laplace density and contours with $m_1 = m_2 = 2$, $\sigma_1 = \sigma_2 = 1$, and $\rho = 0$

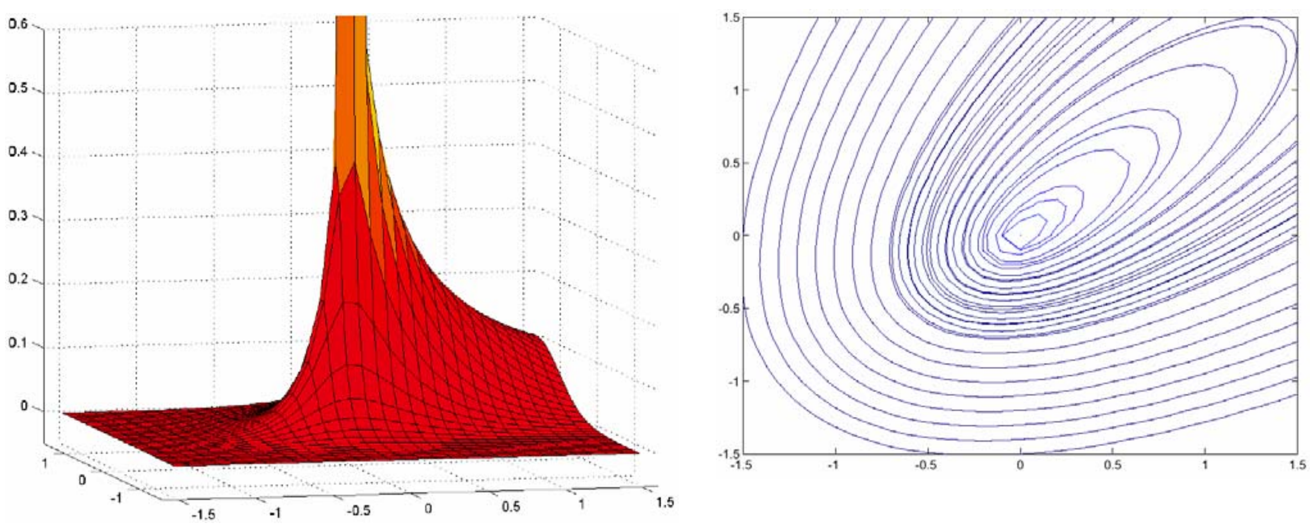


Figure 4. Bivariate Asymmetric Laplace density and contours with $m_1 = m_2 = 2$, $\sigma_1 = \sigma_2 = 1$, and $\rho = 0.5$

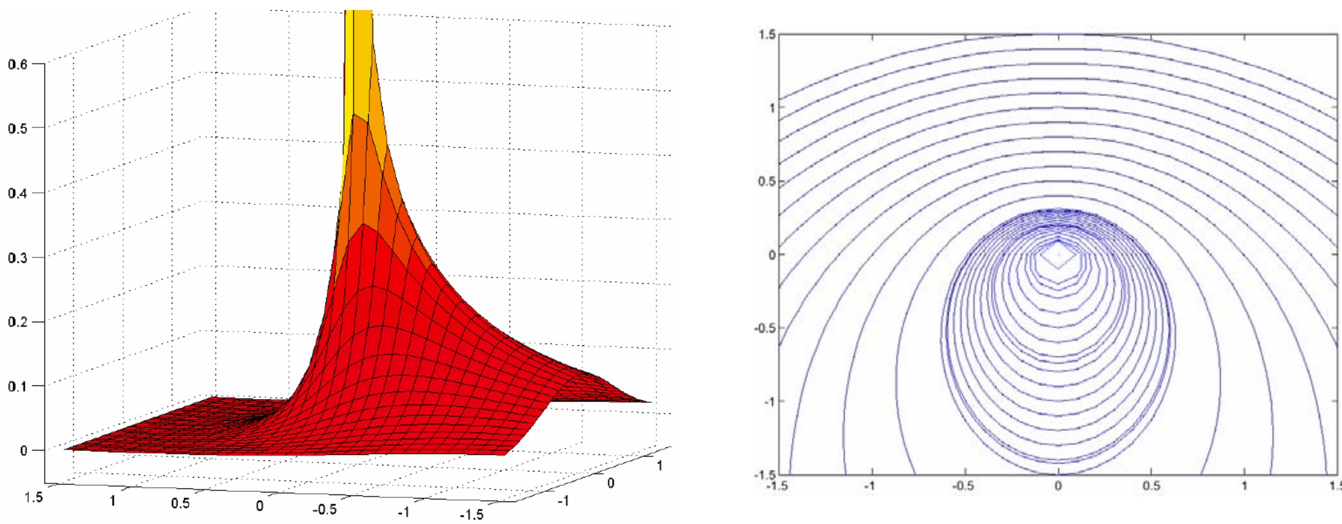


Figure 5. Bivariate Asymmetric Laplace density and contours with m_1 = -2, m_2 = 0, σ_1 = σ_2 = 1, and ρ = 0

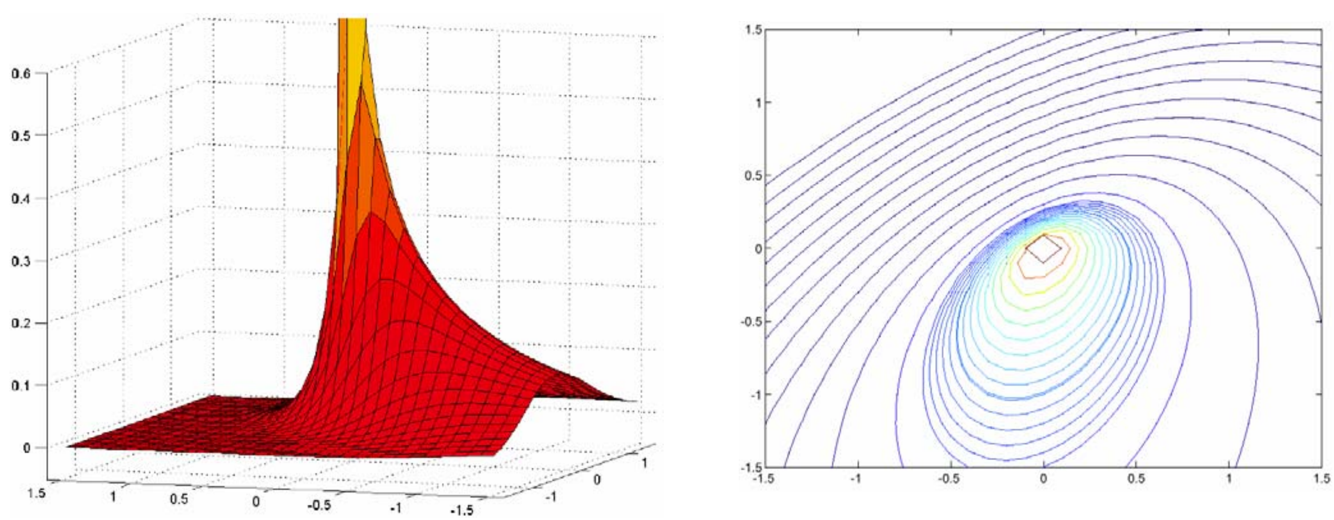


Figure 6. Bivariate Asymmetric Laplace density and contours with m_1 = -2, m_2 = 0, σ_1 = σ_2 = 1, and ρ = 0.5

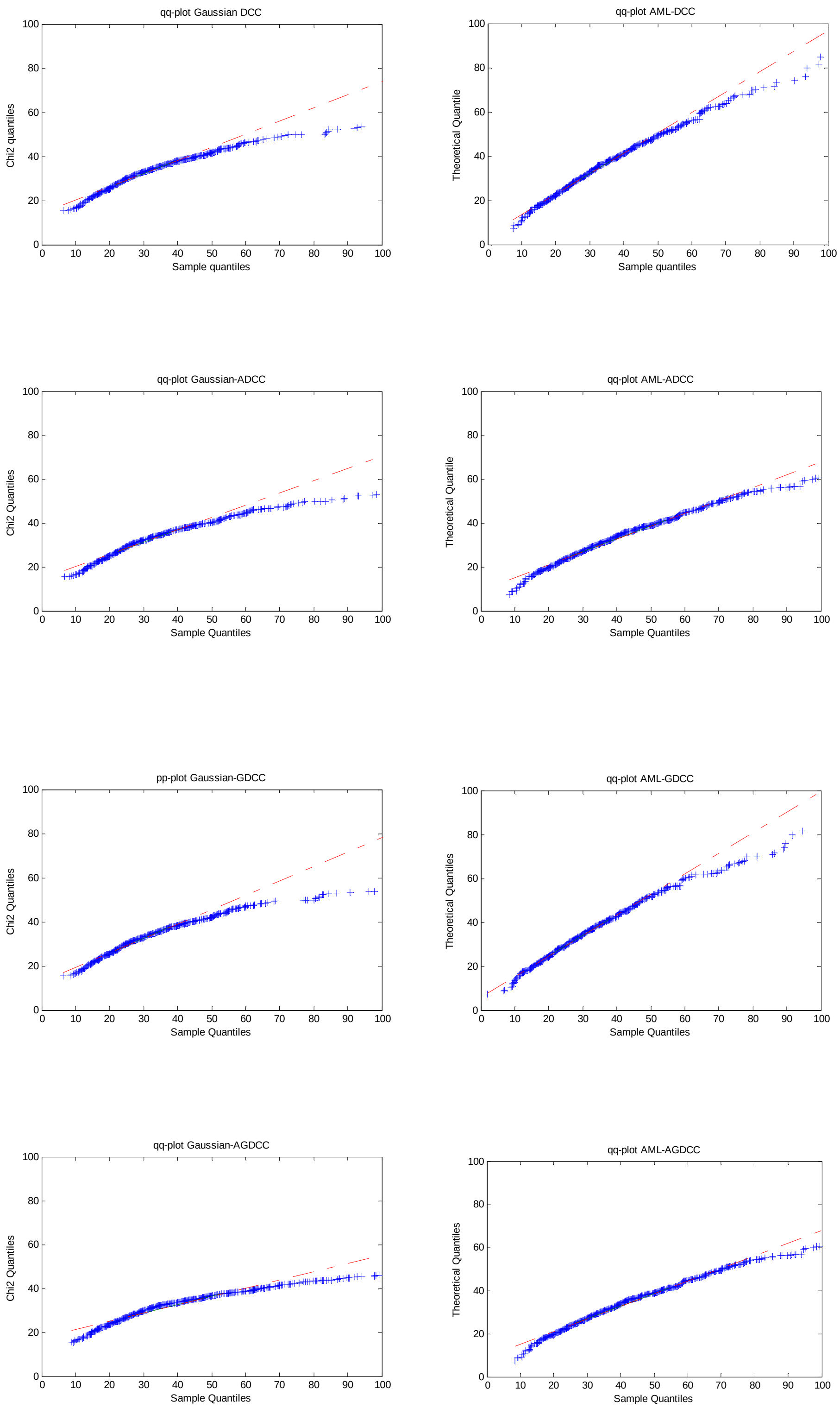
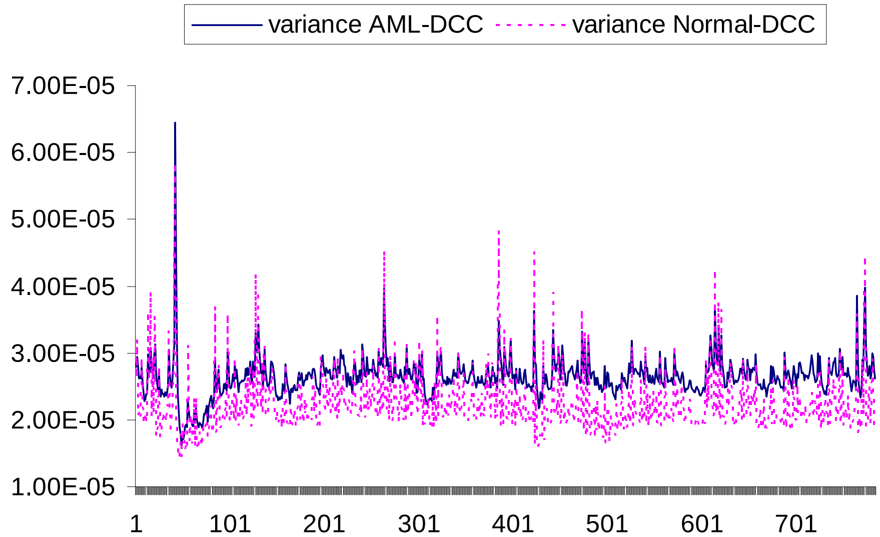


Figure 7. qq-plots of the distribution of $r_t H_t^{-1} r'_t$ from the multivariate normal MLE (left hand side plots) and from the AML MLE (right hand side plots) for the four type of DCC models.



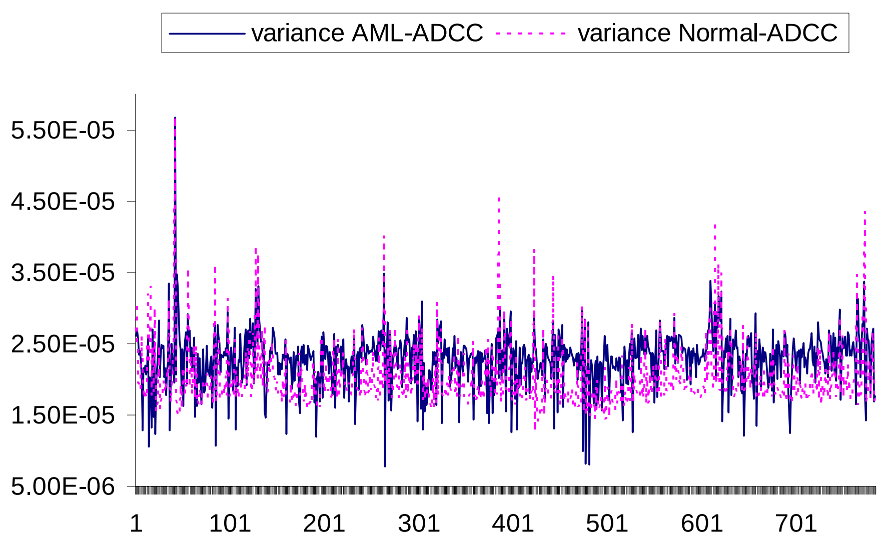
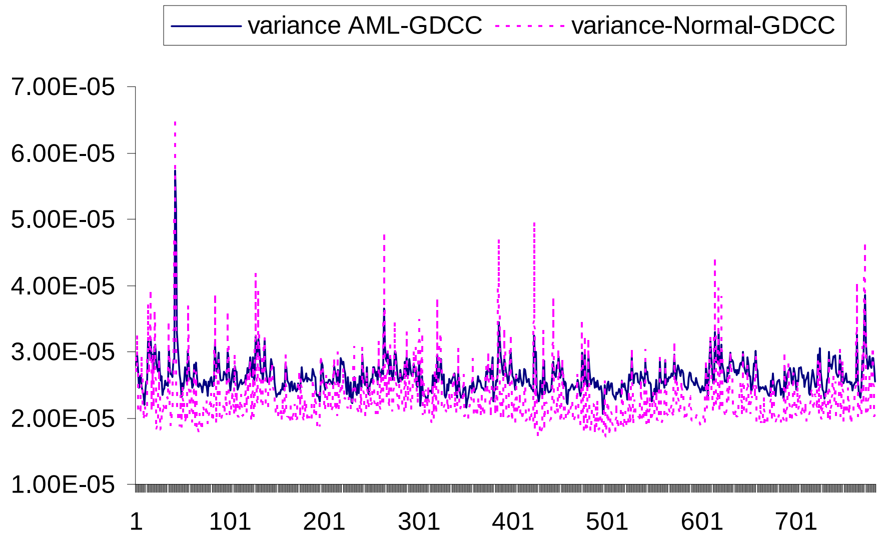


Figure 8a. Series of the variances of the portfolios composed of all assets in the sample data for the AML-DCC, Normal-DCC, AML-ADCC and Normal-ADCC models.



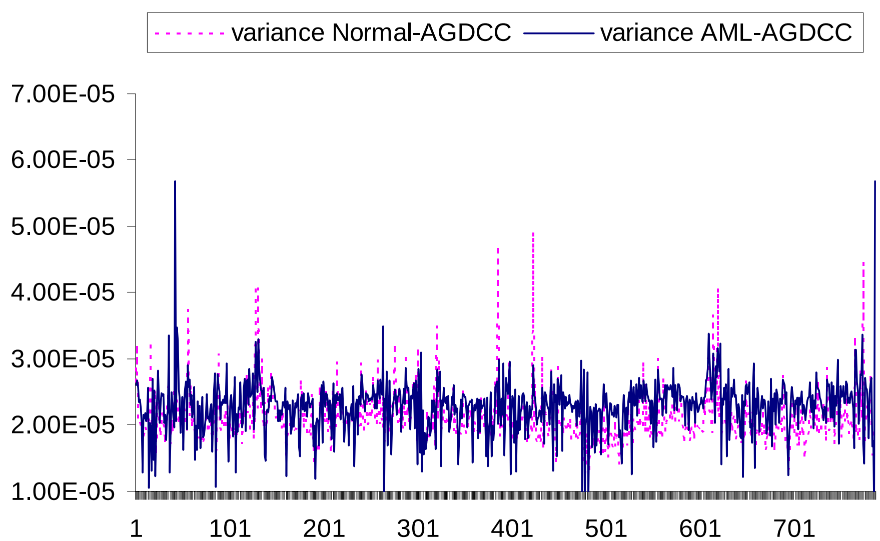
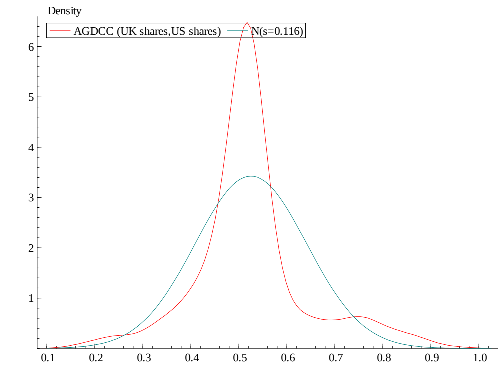
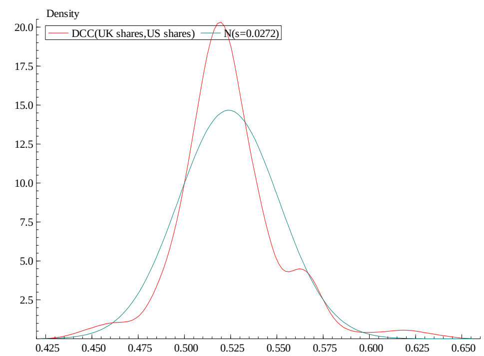


Figure 8b. Series of the variances of the portfolios composed of all assets in the sample data for the AML-GDCC, Normal-GDCC, AML-AGDCC and Normal-AGDCC models

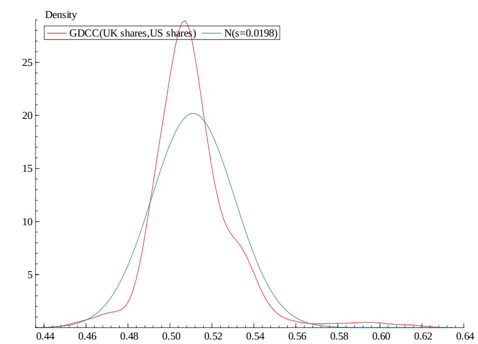
Mean 0.52519
Std.Devn. 0.11643
Skewness 0.59744
Excess Kurtosis 1.9738
Minimum 0.16910
Maximum 0.95840
Asymptotic test: Chi^2(2) = 174.12 [0.0000]**
Normality test: Chi^2(2) = 60.841 [0.0000]**



Mean 0.52390
Std.Devn. 0.027194
Skewness 0.72778
Excess Kurtosis 2.5516
Minimum 0.43860
Maximum 0.64040
Asymptotic test: Chi^2(2) = 282.24 [0.0000]**
Normality test: Chi^2(2) = 78.158 [0.0000]**



Mean 0.51106
Std.Devn. 0.019759
Skewness 1.5059
Excess Kurtosis 5.7961
Minimum 0.45240
Maximum 0.61900
Asymptotic test: Chi^2(2) = 1395.5 [0.0000]**
Normality test: Chi^2(2) = 175.42 [0.0000]**



Mean 0.52361
Std.Devn. 0.0084942
Skewness 10.454
Excess Kurtosis 222.11
Minimum 0.47520
Maximum 0.69710
Asymptotic test: Chi^2(2) =1.6279e+006 [0.0000]**
Normality test: Chi^2(2) = 5200.4 [0.0000]**

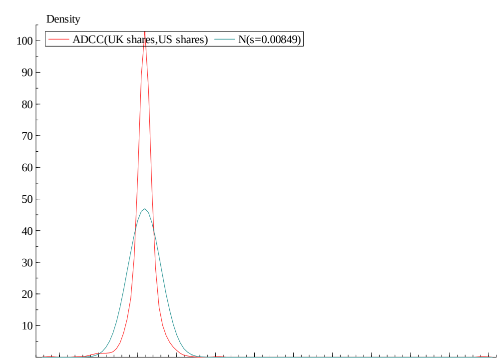
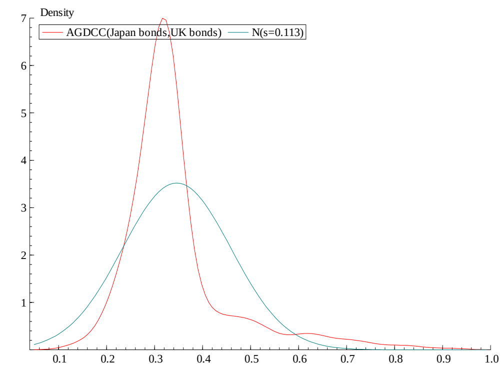
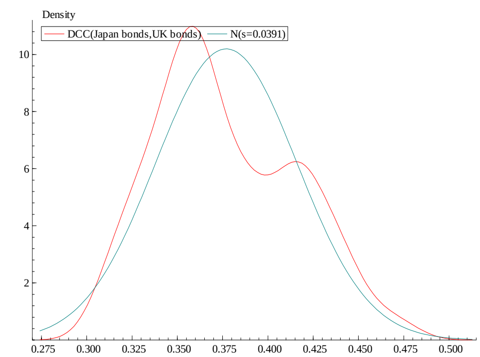


Figure 9. Distribution of the dynamic correlation between the return of UK shares and US shares employing the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

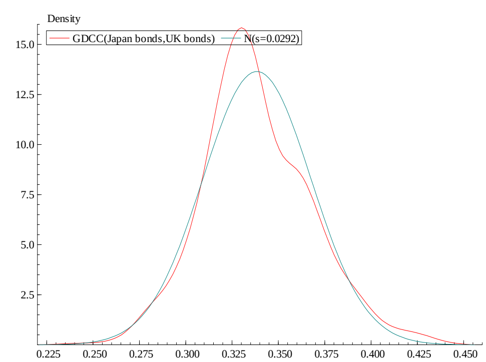
Mean	0.34610
Std.Devn.	0.11333
Skewness	2.0983
Excess Kurtosis	5.3426
Minimum	0.12380
Maximum	0.92800
Asymptotic test: Chi^2(2) =	1509.7 [0.0000]**
Normality test: Chi^2(2) =	995.61 [0.0000]**



Mean	0.37703
Std.Devn.	0.039126
Skewness	0.37672
Excess Kurtosis	-0.65933
Minimum	0.30290
Maximum	0.48150
Asymptotic test: Chi^2(2) =	32.786 [0.0000]**
Normality test: Chi^2(2) =	66.152 [0.0000]**



Mean	0.33844
Std.Devn.	0.029234
Skewness	0.35843
Excess Kurtosis	0.44919
Minimum	0.23960
Maximum	0.43860
Asymptotic test: Chi^2(2) =	23.408 [0.0000]**
Normality test: Chi^2(2) =	16.522 [0.0003]**



Mean	0.38789
Std.Devn.	0.0072434
Skewness	1.7742
Excess Kurtosis	21.791
Minimum	0.34220
Maximum	0.45930
Asymptotic test: Chi^2(2) =	15944. [0.0000]**
Normality test: Chi^2(2) =	895.29 [0.0000]**

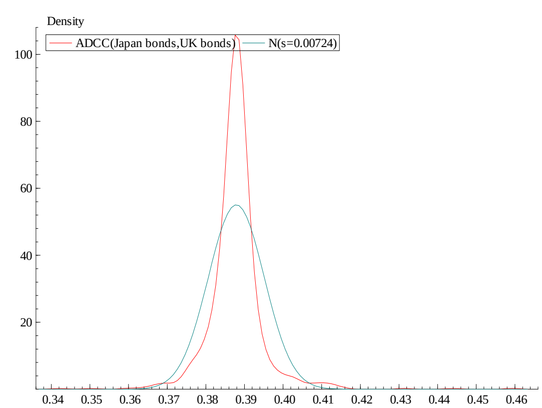
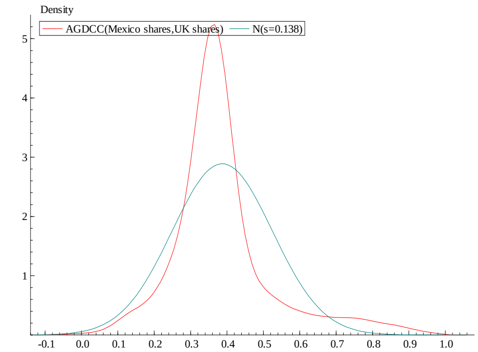
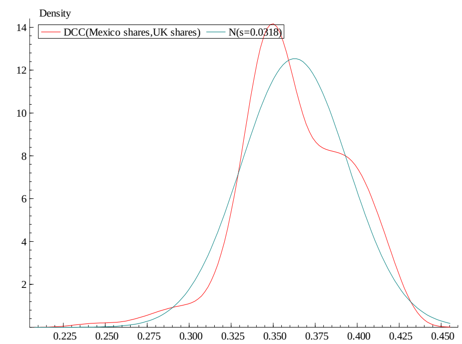


Figure 10. Distribution of the dynamic correlation between the return of Japan bonds and UK bonds employing the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

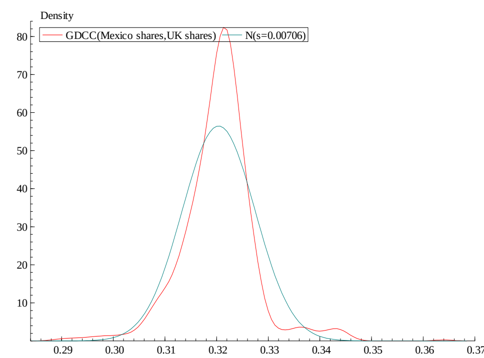
Mean	0.38693
Std.Devn.	0.13806
Skewness	1.3358
Excess Kurtosis	3.0517
Minimum	-0.029900
Maximum	0.94950
Asymptotic test:	Chi^2(2) = 538.06 [0.0000]**
Normality test:	Chi^2(2) = 220.01 [0.0000]**



Mean	0.36282
Std.Devn.	0.031816
Skewness	-0.32559
Excess Kurtosis	0.58357
Minimum	0.23340
Maximum	0.43110
Asymptotic test:	Chi^2(2) = 24.786 [0.0000]**
Normality test:	Chi^2(2) = 16.112 [0.0003]**



Mean	0.32040
Std.Devn.	0.0070628
Skewness	0.23460
Excess Kurtosis	4.5504
Minimum	0.28930
Maximum	0.36390
Asymptotic test:	Chi^2(2) = 684.45 [0.0000]**
Normality test:	Chi^2(2) = 281.82 [0.0000]**



Mean	0.36575
Std.Devn.	0.0082164
Skewness	-0.16322
Excess Kurtosis	27.493
Minimum	0.29340
Maximum	0.44070
Asymptotic test:	Chi^2(2) = 24726. [0.0000]**
Normality test:	Chi^2(2) = 2597.8 [0.0000]**

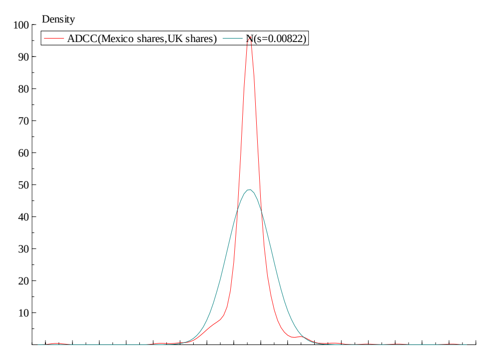
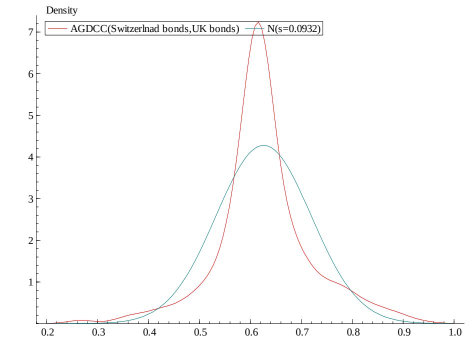
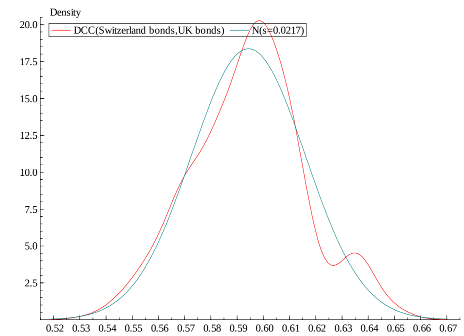


Figure 11. Distribution of the dynamic correlation between the return of UK shares and Mexico shares employing the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

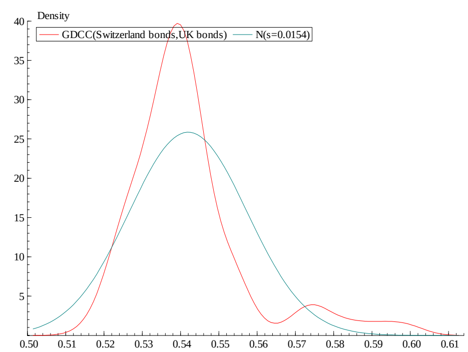
Mean 0.62607
Std.Devn. 0.093218
Skewness 0.050799
Excess Kurtosis 2.1194
Minimum 0.25990
Maximum 0.94750
Asymptotic test: Chi^2(2) = 147.26 [0.0000]**
Normality test: Chi^2(2) = 94.468 [0.0000]**



Mean 0.59432
Std.Devn. 0.021719
Skewness 0.058313
Excess Kurtosis 0.0034808
Minimum 0.53060
Maximum 0.65380
Asymptotic test: Chi^2(2) = 0.44528 [0.8004]
Normality test: Chi^2(2) = 0.47967 [0.7868]



Mean 0.54196
Std.Devn. 0.015424
Skewness 1.6521
Excess Kurtosis 3.2051
Minimum 0.51350
Maximum 0.60200
Asymptotic test: Chi^2(2) = 693.11 [0.0000]**
Normality test: Chi^2(2) = 579.90 [0.0000]**



Mean 0.62043
Std.Devn. 0.0052324
Skewness 0.30366
Excess Kurtosis 17.926
Minimum 0.58740
Maximum 0.67080
Asymptotic test: Chi^2(2) = 10523. [0.0000]**
Normality test: Chi^2(2) = 1609.0 [0.0000]**

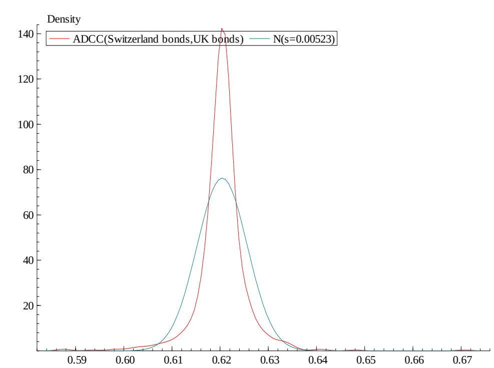


Figure 12. Distribution of the dynamic correlation between the return of UK bonds and Switzerland shares employing the AGDCC, DCC, GDCC, and ADCC models with AML distributions.

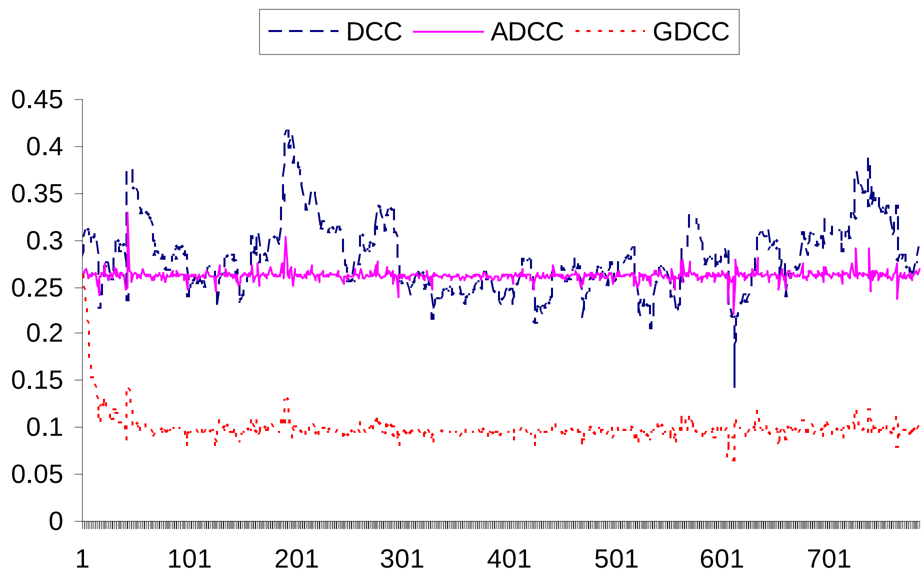


Figure 13. Plot of the correlation series between the returns of "Japan shares" and "U.K. shares" estimated with the DCC (1,1), ADCC(1,1,1) and GDCC(1,1) models and with the AML distribution.

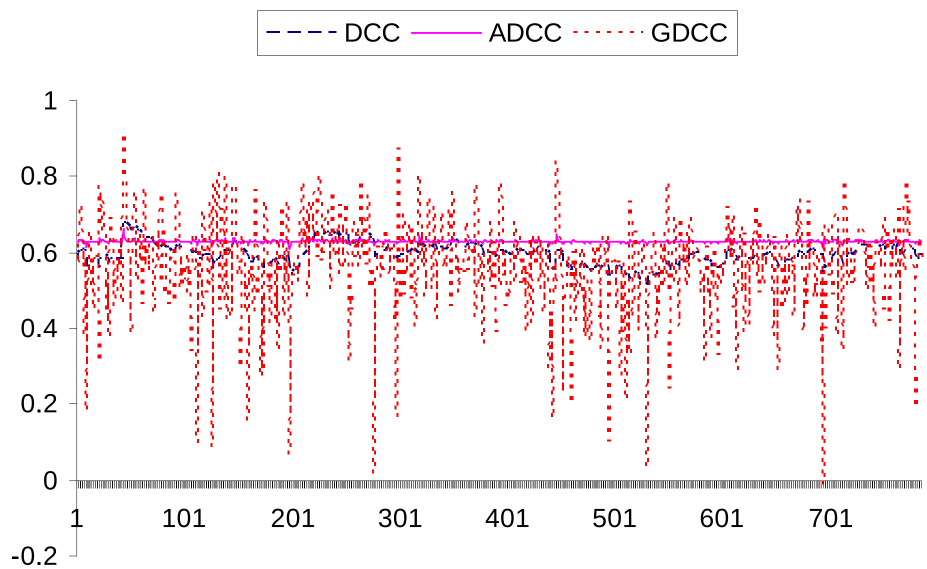


Figure 14. Plot of the correlation series between the returns of "Switzerland bonds" and "U.K. bonds" estimated with the DCC (1,1), ADCC(1,1,1) and GDCC(1,1) models and with the AML distribution.

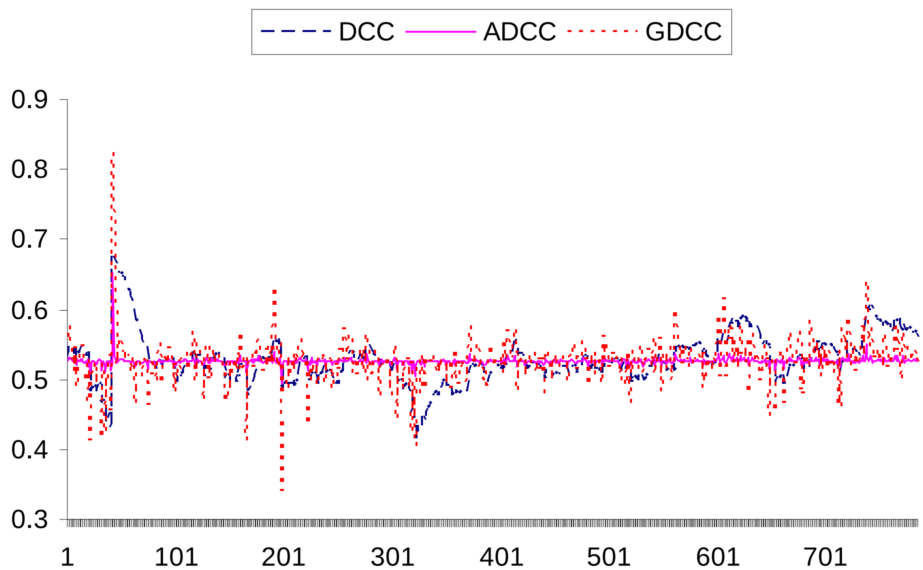


Figure 15. Plot of the correlation series between the returns of “U.S. shares” and “U.K. shares” estimated with the DCC (1,1), ADCC(1,1,1) and GDCC(1,1) models and with the AML distribution.

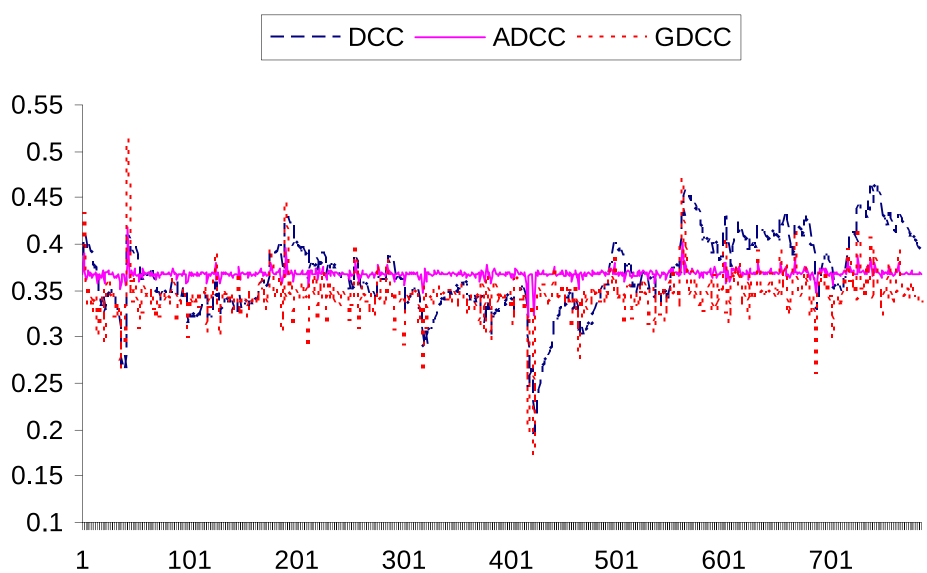


Figure 16. Plot of the correlation series between the returns of “Mexico shares” and “U.K. shares” estimated with the DCC (1,1), ADCC(1,1,1) and GDCC(1,1) models and with the AML distribution.