Decreasing Bequest Embedded Annuities: A Feasible Pension Product

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Motivation

- Annuity Puzzle shows that most people are not voluntarily selecting annuities as a pension product, even though, in this case, longevity and investment risks are transferred to the insurance companies instead of being beared by retired workers.

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- One of the main reasons for this could be the bequest motive that makes retiring workers prefer a Phased Withdrawal product where the property of savings is maintained, as well as the possibility of eventually leaving some (unknown) amount of money for their heirs.
- Beyond the cost of bearing the longevity and investment risks, the cost of bequest is not "explicit" for the workers at the moment of the retirment, distorting the decision between Phased Withdrawal and Annuity as the optimal pension product.

Money-Back Guaranteed Annuities

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- For example, in a Cash-Refund Annuity, if the annuitant dies after $T \leq T_{\max}$ years, then the heir will receive a bequest of $(P - T * C)$, where P is the premium paid for the annuity and C is the annual life annuity payment. The price of such a bequest embedded annuity could be understood as the sum of the prices for a life annuity for the retiring worker and a life insurance policy for their heirs.

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- This way, the determination of the final price requires solving a recursivity problem since the price of the annuity depends on the expected bequest, which also depends on the price through P and C ($C = P/a^*$, where a^* is the price for $1 of lifetime income). In addition, $a^* = T_{\max} = P/C$, and then the guaranteed period is endogenous.

Money-Back Guaranteed Annuities

- To determine a^*, following Milevsky and Salisbury (2022), we have to solve the recursivity equation:

$$a^*(x,r) = \int_0^{\infty} ({}_s p_x) e^{-rs} ds + \int_0^{a^*(x,r)} (a^*(x,r) - s) e^{-rs} \ ({}_s p_x \mu_{x+s}) ds$$

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- This way, the price for the Cash-Refund Annuity is equal to the sum of the price for the Life Income Annuity plus the price for the "endogenous" life insurance component.

Money-Back Guaranteed Annuities: Loaded Price

- It would be inappropriate to multiply $a^*(x, r)$ by an insurance loading of $(1+\pi)$ to convert them from unloaded to loaded prices, as in the case of the standard Life Income Annuity price ($\widehat{a} = (1+\pi)a$).

$$\frac{\widehat{a}^*(x,r)}{(1+\pi)} = a(x,r) + \int_0^{\widehat{a}^*(x,r)} (\widehat{a}^*(x,r) - s)e^{-rs} \; ({}_s p_x \mu_{x+s})ds$$

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- As pointed out by Milevsky and Salisbury (2022), for sufficiently low levels of r and advanced ages (x), the loaded price for the Cash-Refund Annuity is eventually not viable.

Decreasing Bequest Embedded Annuities for Pensions

- A retirement product with the benefits of traditional annuities (transfering investment and longevity risks to insurance companies), allowing for bequest, should be simple enough to be understood for retiring workers. To do this, the main issue is to make the bequest independent of the price of the annuity.

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- This requires to define in advance the guaranteed period $T_{\max}$, and then the bequest amount as $B = (P - T * P / T_{\max})$. First, we need to price the bequest in terms of how much of the total premium we should pay for it. After that, we use the remaining premium to buy a life annuity for the retiring worker.

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- Under this pricing method, the retiring worker knows from the very beginning the guaranteed period, the amount of the bequest depending on the year of death, as well as the amount of the annuity lifetime income. In addition, the cost of the bequest is totally explicit for the retiring worker in terms of how much lifetime income is giving up.

Decreasing Bequest Embedded Annuities for Pensions

- The price for the Decreasing Bequest Annuity is given by:

$$a(x,r)_{DBA} = a(x,r) + \int_{0}^{T_{\max}} (T_{\max} - s)e^{-rs} \left({}_{s}p_{x}\mu_{x+s}\right) ds$$

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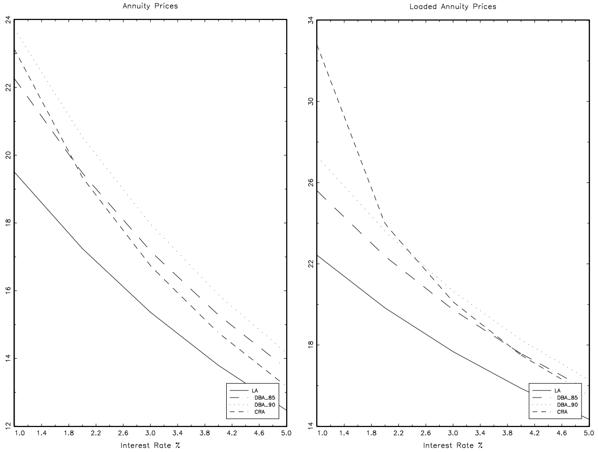
- This way, the price is equal to the sum of the price for the Life Income Annuity plus the price for the "exogenous" life insurance component.

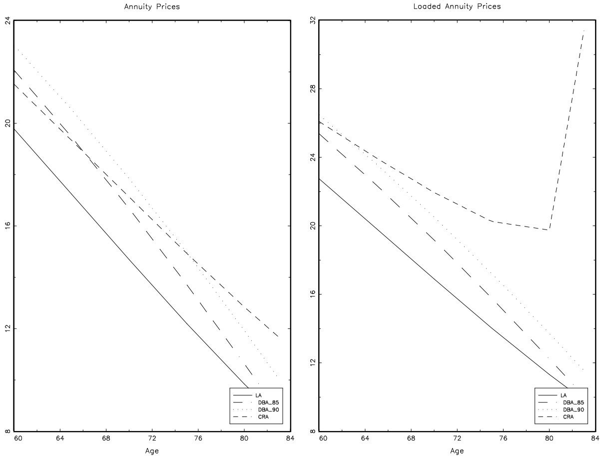
Annuity Prices and Loaded Annuity Prices vs Interest Rate

- Annuity Prices for $x = 65$, Gompertz mortality ($m = 90, b = 10$), and $\pi = 15\%$.



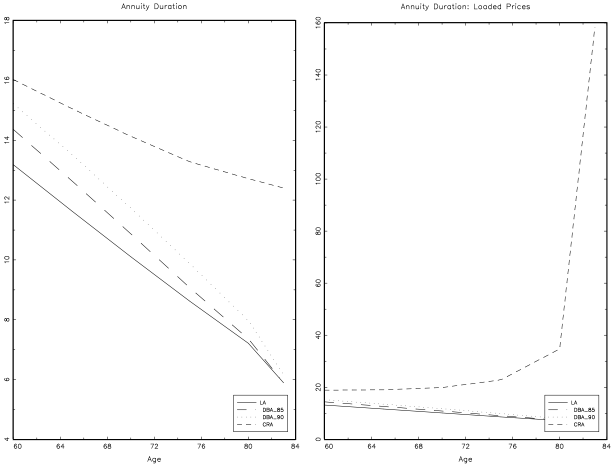
Annuity Prices and Loaded Annuity Prices vs Age

- Annuity Prices with $r = 2\%$, Gompertz mortality ($m = 90$, $b = 10$), and $\pi = 15\%$.



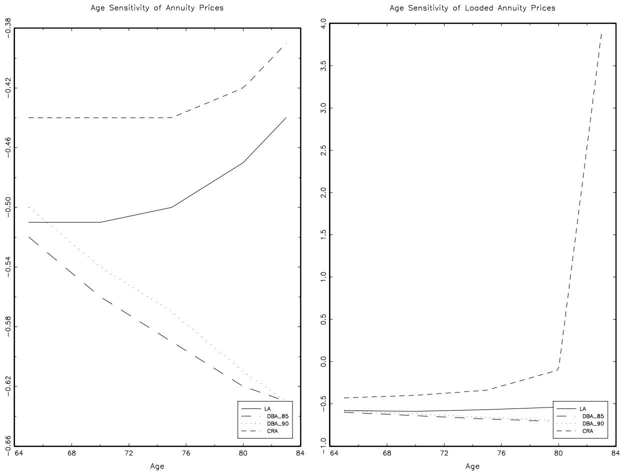
Duration of Annuity Prices

- Annuity Duration is approximated as: $D = -\frac{\left[\frac{a(x,r+\Delta r)}{a(x,r)} - 1\right]}{\Delta r}$



Age Sensitivity of Annuity Prices

- Age Sensitivity is defined as: $\frac{\partial a(x,r)}{\partial x} = \frac{a(x+\Delta x,r)-a(x,r)}{\Delta x}$



Pension Cost: Single Decreasing Bequest Annuities (Male)

- The following table shows how more expensive a Decreasing Bequest Annuity is compared to the corresponding Life Annuity for a male with Gompertz Mortality ($m = 90, b = 10$), for different: retirement age (x), guaranteed period ($T_{\max}$), and interest rates.

$Age(x)$	$T_{\max}$	$r = 2\%$	$r = 3\%$	$r = 4\%$
60	25	11.57%	10.30%	9.22%
65	20	12.87%	11.76%	10.79%
70	15	13.48%	12.62%	11.85%
60	30	16.45%	14.18%	12.34%
65	25	19.05%	16.89%	15.09%
70	20	21.41%	19.49%	17.83%

Pension Cost: Joint Decreasing Bequest Annuities (Female Beneficiary, Receiving 60% of Male Annuity)

- The following table shows the case for joint annuities, similar to the table above, considering Female Gompertz Mortality ($m = 92$, $b = 10$), and age (y).

$Age(x)$	$Age(y)$	$T_{\max}$	$r = 2\%$	$r = 3\%$	$r = 4\%$
60	57	25	1.29%	1.10%	0.95%
65	62	20	1.38%	1.22%	1.08%
70	67	15	1.31%	1.20%	1.10%
60	57	30	2.72%	2.23%	1.85%
65	62	25	3.10%	2.65%	2.27%
70	67	20	3.33%	2.95%	2.62%

Solvency Capital Requirement (SCR)

- Following Chen and Rach (2022), we define the Solvency Capital Requirement for each period as the difference between the Best Estimated Liability under a longevity shock and under the current mortality table: $SCR_X(t) = BEL_X(t|shock) - BEL_X(t)$

$$BEL_X(t|shock) = {}_t p_x \int_t^{\infty} e^{-r(s-t)} \; {}_{s-t}p_{x+t}^{1-\varepsilon_L} C(s) ds$$

$$+ {}_t p_x \int_t^{\infty} e^{-r(s-t)} \; {}_{s-t}p_{x+t}^{1-\varepsilon_L} \mu_{x+s} (1-\varepsilon_L) B(s) ds$$

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Risk Margin (RM)

- And the Risk Margin is given by,

$$RM_X = CoC \sum_{t=0}^{\infty} e^{-r(t+1)} SCR_X(t)$$

r	LA	DBA_{85}	$(DBA_{85}/LA - 1)$	DBA_{90}	$(DBA_{90}/LA - 1)$
2%	5.6060	4.2213	-24.70%	3.5171	-37.26%
3%	4.4626	3.3538	-24.85%	2.8252	-36.69%
4%	3.5840	2.6838	-25.12%	2.2814	-36.35%

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- The following table shows the RM_X for a Life Annuity, and two Decreasing Bequest Annuities with a guaranteed period $T_{\max} = 85$ and 90, Male Gompertz Mortality ($m = 92, b = 10$), $P = 100$, $x = 65$, $CoC = 6\%$, and $\varepsilon_L = 0.20$.

r	LA	DBA_{85}	$(DBA_{85}/LA - 1)$	DBA_{90}	$(DBA_{90}/LA - 1)$
2%	5.6060	4.2213	-24.70%	3.5171	-37.26%
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Solvency Capital Requirement (SCR) for Unbundled Products

- Following Chen and Rach (2022), we define the Best Estimated Liability for the only Life Annuity $BEL_C(t|\varepsilon_L)$ and only Bequest $BEL_B(t|\varepsilon_M)$, under the corresponding Longevity and Mortality shocks,

$$BEL_C(t|\varepsilon_L) = {}_t p_x \int_t^{\infty} e^{-r(s-t)} \; {}_{s-t}p_{x+t}^{1-\varepsilon_L} C(s) ds$$

$$BEL_B(t|\varepsilon_M) = {}_t p_x \int_t^{\infty} e^{-r(s-t)} \; {}_{s-t}p_{x+t}^{1-\varepsilon_M} \mu_{x+s}(1-\varepsilon_M) B(s) ds$$

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- With $SCR_X(t)$ and RM_X defined as before.

Risk Margin (RM): Bundled vs Independent Unbundled Products

- The following tables show the RM_X for Decreasing Bequest Annuities ($T_{\max} = 85$ and 90) and for the two independent Life Annuity and Decreasing Bequest products. Male Gompertz Mortality ($m = 92, b = 10$), $P = 100$, $x = 65$, $CoC = 6\%$, $\varepsilon_L = 0.20$, and $\varepsilon_M = -0.15$.

r	DBA_{85}	LA	DB	$LA + DB$	$(DBA_{85}/(LA + DB) - 1)$
2%	4.2213	4.9667	0.5280	5.4947	-23.18%
3%	3.3538	3.9930	0.4538	4.4468	-24.58%
4%	2.6838	3.2350	0.3921	3.6271	-26.01%

r	DBA_{90}	LA	DB	$LA + DB$	$(DBA_{90}/(LA + DB) - 1)$
2%	3.5171	4.7088	0.8152	5.5240	-36.33%
3%	2.8252	3.8176	0.6822	4.4998	-37.22%
4%	2.2814	3.1142	0.5752	3.6894	-38.16%

Risk Margin (RM): Bundled vs Diversified Unbundled Products

- Allowing for diversification between Longevity and Mortality risks, the Solvency Capital Requirement for the unbundled products is computed as:

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r	DBA_{85}	$LA + DB$	$(DBA_{85}/(LA+DB) - 1)$
2%	4.2213	4.8779	-13.46%
3%	3.3538	3.9179	-14.40%
4%	2.6838	3.1712	-15.37%

r	DBA_{90}	$LA + DB$	$(DBA_{90}/(LA+DB) - 1)$
2%	3.5171	4.6006	-23.55%
3%	2.8252	3.7277	-24.21%
4%	2.2814	3.0391	-24.93%

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- The annuity load (π) charged on commercial annuities consists of four elements: i) adverse selection, ii) risk margins, iii) administrative costs, and iv) profits.
- This way, the lower RM_X for Decreasing Bequest Annuities should translate into a higher MWR for the annuitant compared to the Life Annuity case.

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- A Decreasing Bequest Annuity is equivalent to a bundled product, combining a Life Annuity with Life Insurance for the heirs.

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- On the other hand, the pension cost of the Decreasing Bequest is explicit for the retiring worker, allowing for an informed decision on this matter.
- Joint Bequest Annuities are not expensive given that the bequest is paid only after the second death, and if this event occurs before T_{max}.

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- From the insurer's point of view, this Decreasing Bequest Annuity allows for lower Solvency Capital Requirements and Risk Margins because of the natural hedging between longevity and mortality risk for the bundled product.

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- This reduction in RM_X, if translated into a lower loading factor by the insurer, should imply a better deal for annuitants given the higher MWR associated with the Decreasing Bequest Annuity.