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Predicting Pension Amount

Continuous benefit model with lifestyle-amount interactions

Presentation at Longevity 19 Conference in Amsterdam

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Introducing the authors

Jibreel Ismail – Actuarial Consultant, Ad Res

- 2020 – NMG Consulting
- 2021 – Ad Res Advanced Reinsurance Services GmbH

Kai Kaufhold – Managing Director, Ad Res

- 1996 – Zurich Re (Cologne),
- 2000 – Manulife Reinsurance, Toronto, from 2003: Cologne
- 2011 – Ad Res Advanced Reinsurance Services GmbH

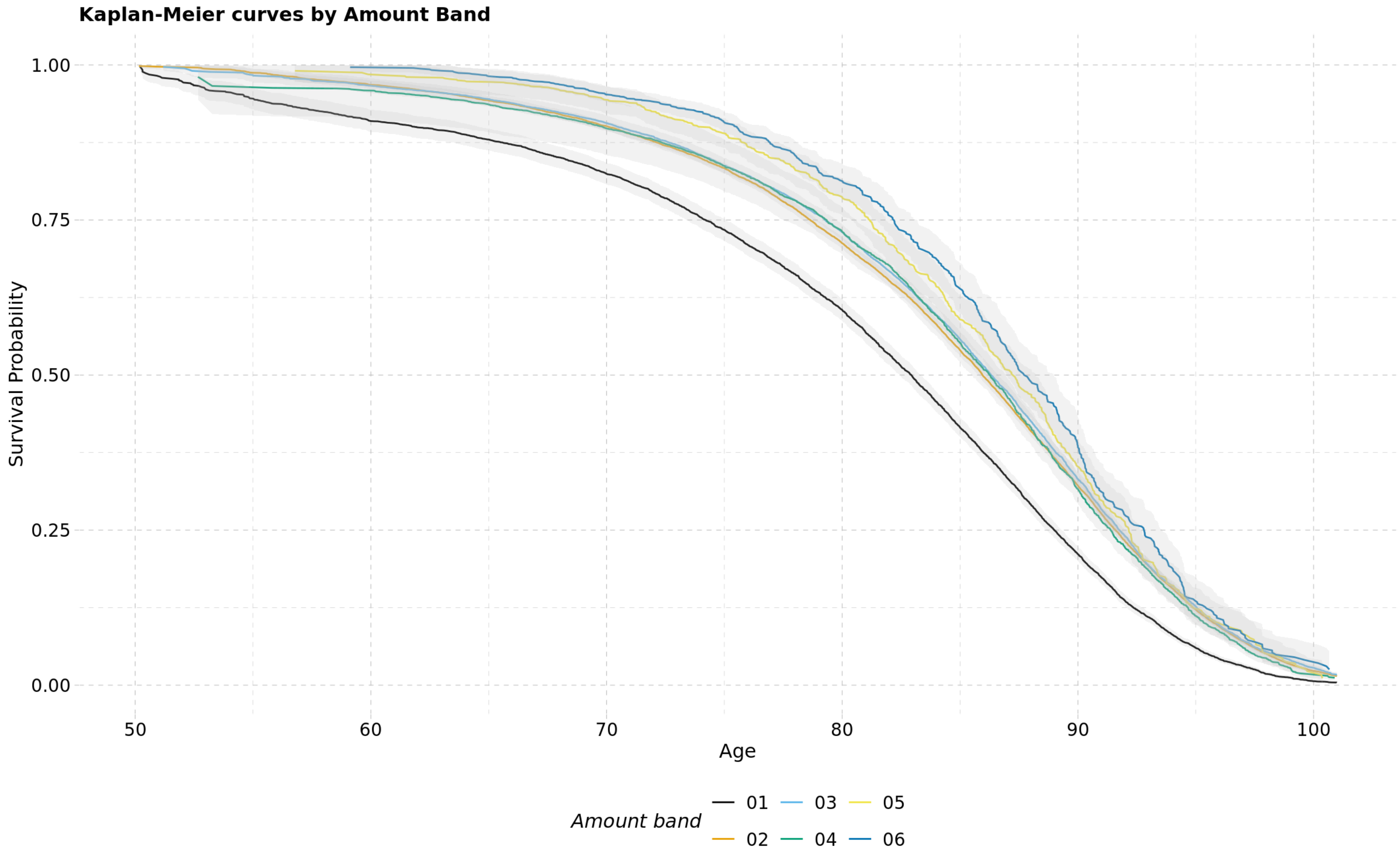
Stefan Ramonat – Principal, Mercer Canada

- 2010 – Morneau Shepell
- 2015 – Mercer Canada

Agenda

- Motivation – Why predict pension amounts?
- Model – Hermite-spline hazards
- Data – Pension scheme data
- Results – Differentiating pension amount models
- Discussion – What's missing?

Survival curves by amount band



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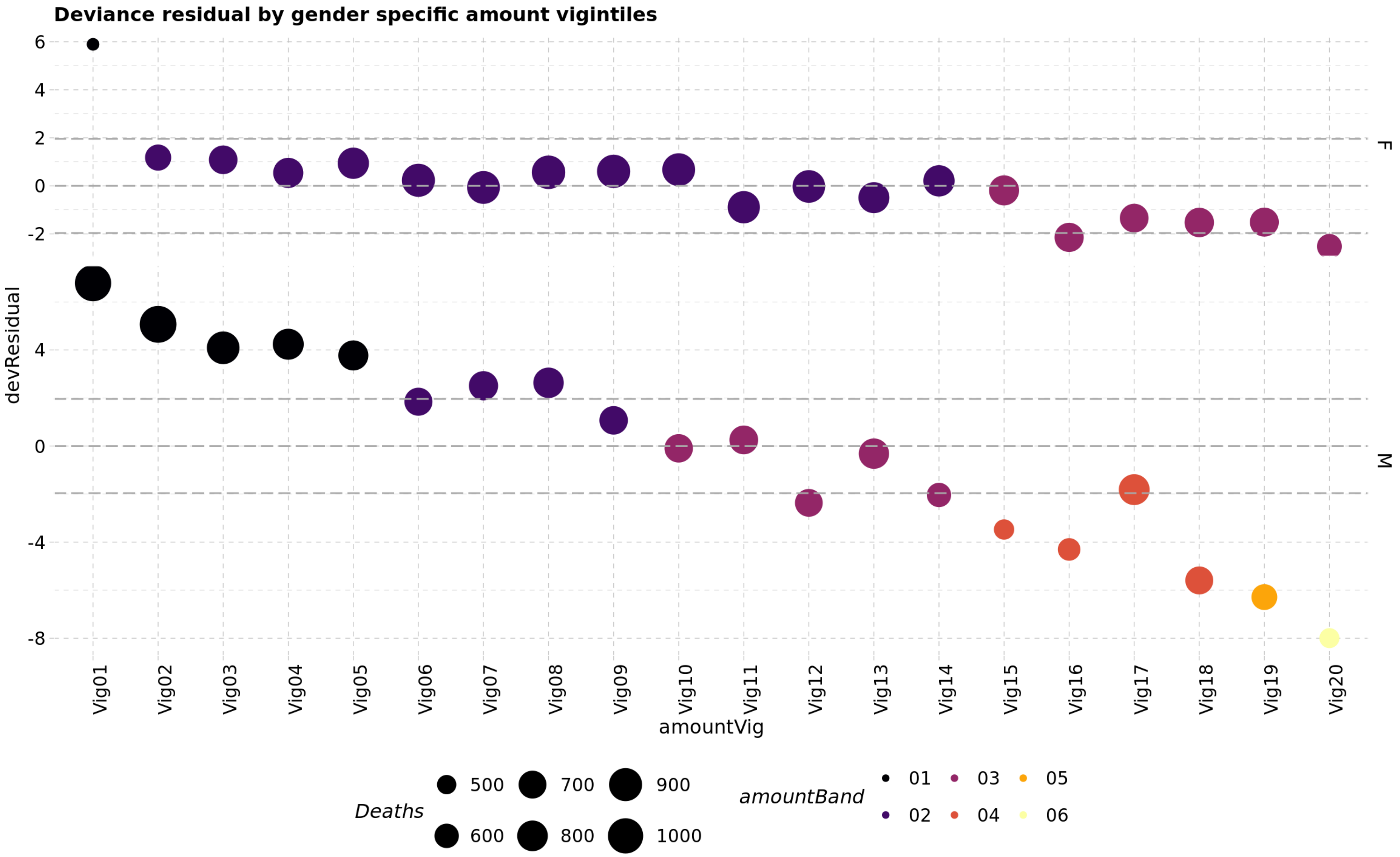
Amounts differentials very important



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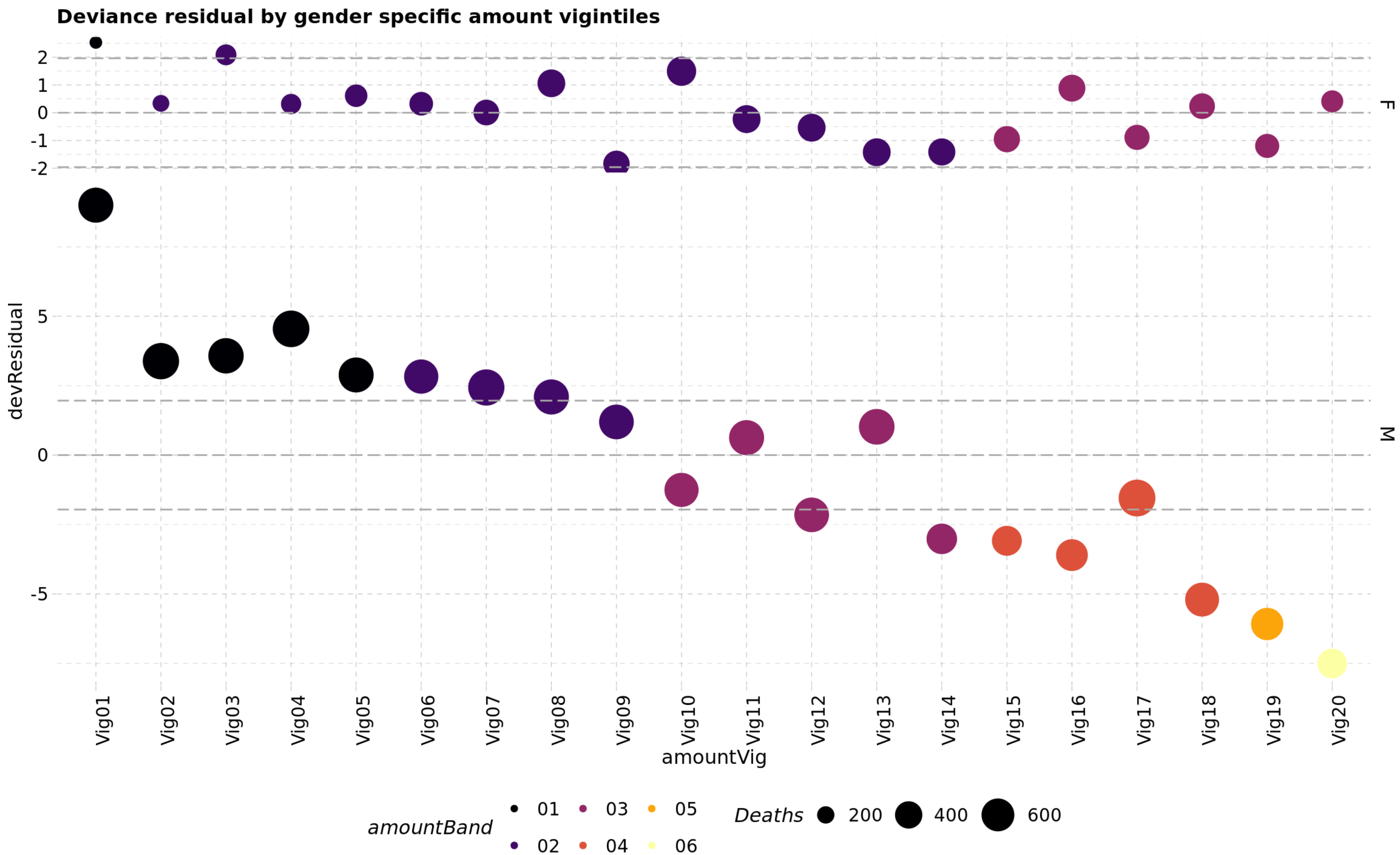
Amounts differentials for pensioners only



Must control for

- Gender
- Surviving spouses
- Ill-Health retirals

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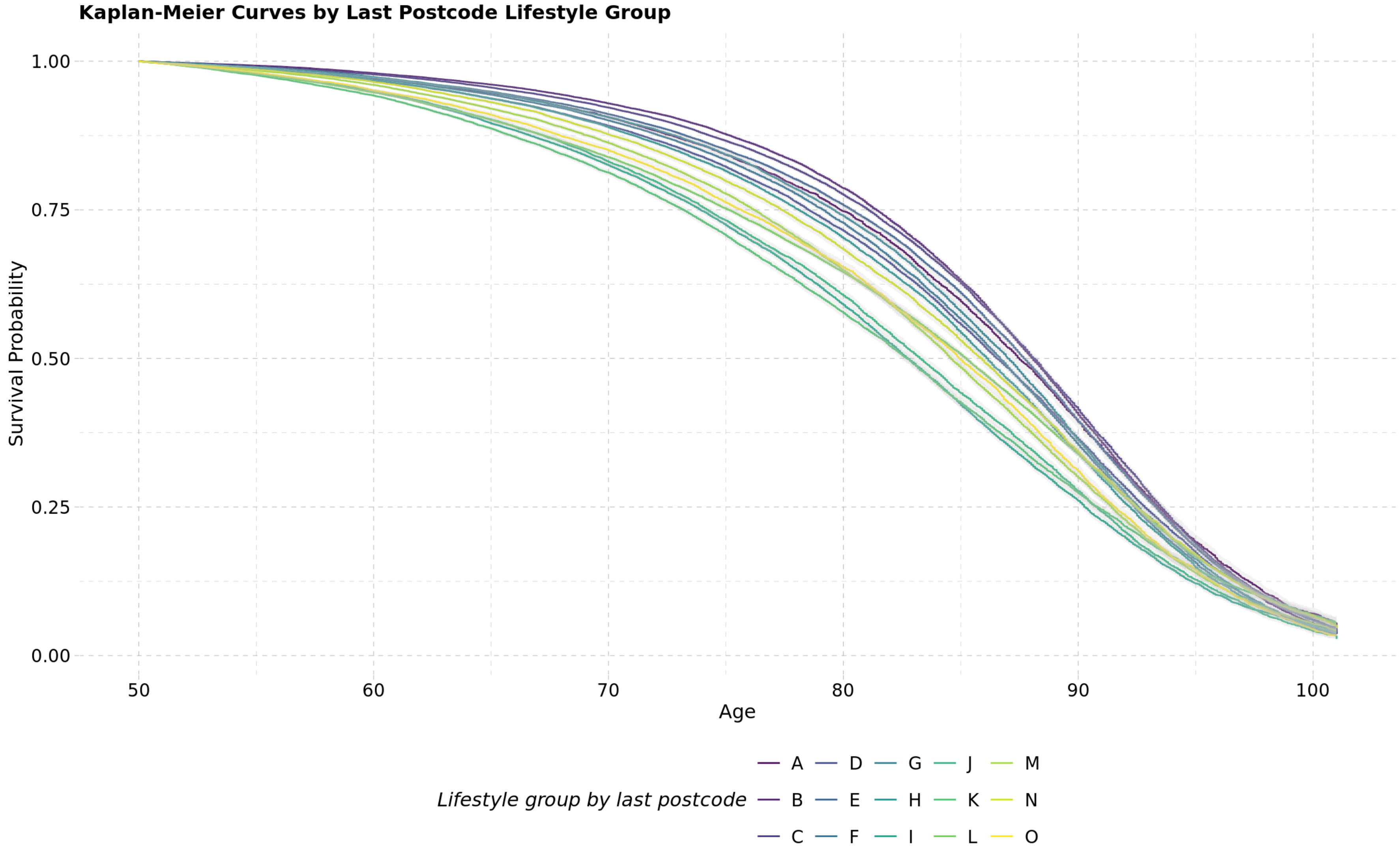
Postcodes are predictive, too.



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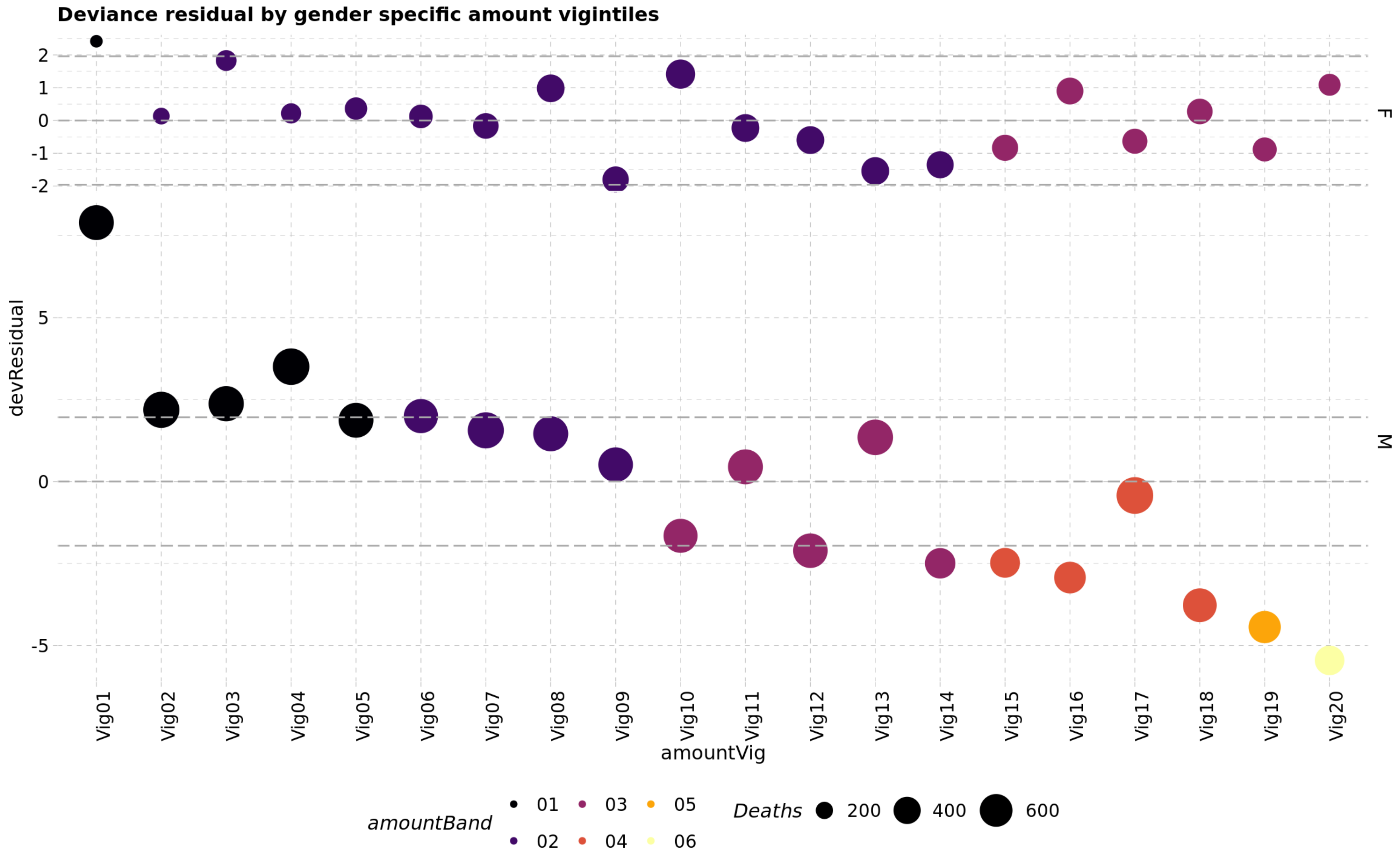
After controlling for Postcode Lifestyle



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Motivation

- Mortality differentials by pension amount are unavoidable
- Differentials by pension amount are messy
- Do amount differentials vary by geo-demographic lifestyle group?
- Gaps in predictive power of pension amount:
  - Pension size band is flawed predictor
  - Salary & tenure
  - Multiple DB pension pots
  - Split between DC and DB (depends on age and retirement date)
- LexisNexis reference dataset without pension amount but postcode

Model Specification

Hermite-spline Hazard Rate Model with Interactions

Modelling Approach

1. Hermite-spline approach to modelling mortality hazard by age
2. Transform benefit amounts
3. Express mortality differential by amount as continuous function
   - Discount for benefit amounts $s > 0$

See Richards (2022), as well as previous presentation!

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Hermite-splines for modelling mortality

$$\log(\mu_{x,y}) = \alpha\, h_{00}(t) + \omega\, h_{01}(t) + m_0\, h_{10}(t) + \eta\, h_q(t) + \delta\, y$$

$$t = (x - x_0)/(x_1 - x_0)$$

$$h_{00}(t) = (1 + 2t)(1 - t)^2$$

$$h_{01}(t) = t^2(3 - 2t)$$

$$h_{10}(t) = t(1 - t)^2$$

$$h_q(t) = 16t^2(1 - t)^2$$

The coefficients each have an intuitive meaning:

α – describes the log mortality at beginning of age range x_0

ω – describes the log mortality at the end of the age range x_1

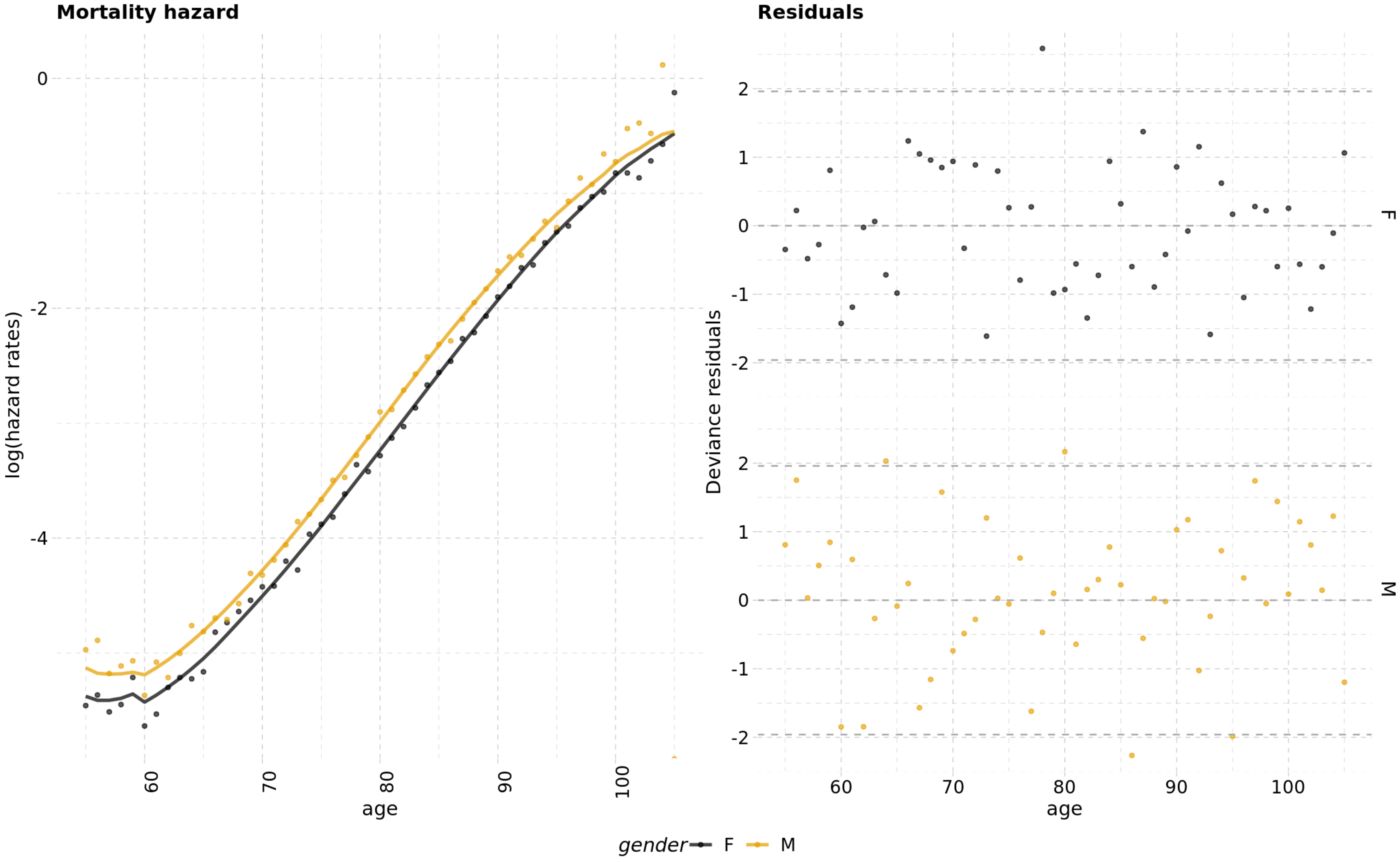
m_0 – describes the slope at the beginning of the age range x_0

η – influences the log mortality in the middle of the age range

δ – describes an age-independent trend by calendar time y

See Richards (2019) and Ramonat (2024)

Captures age-shape of mortality



Transforming amounts

- Gaussian transform $\tau_1(\lambda_0, s) = 2\Phi\left(se^{\lambda_0}\right) - 1$
- Logistic transform $\tau_2(\lambda_0, s) = {}^{se^{\lambda_0}}\!/_{1+se^{\lambda_0}}$
- Exponential transform $\tau_3(\lambda_0, s) = 1 - \exp\left(-se^{\lambda_0}\right)$
- Inverse exponential transform $\tau_4(\lambda_0, s) = 2\left(1 + \exp\left(-se^{\lambda_0}\right)\right)^{-1} - 1$
- Inverse tangent transform $\tau_5(\lambda_0, s) = \frac{2}{\pi}\tan^{-1} se^{\lambda_0}$

See Richards (2022)

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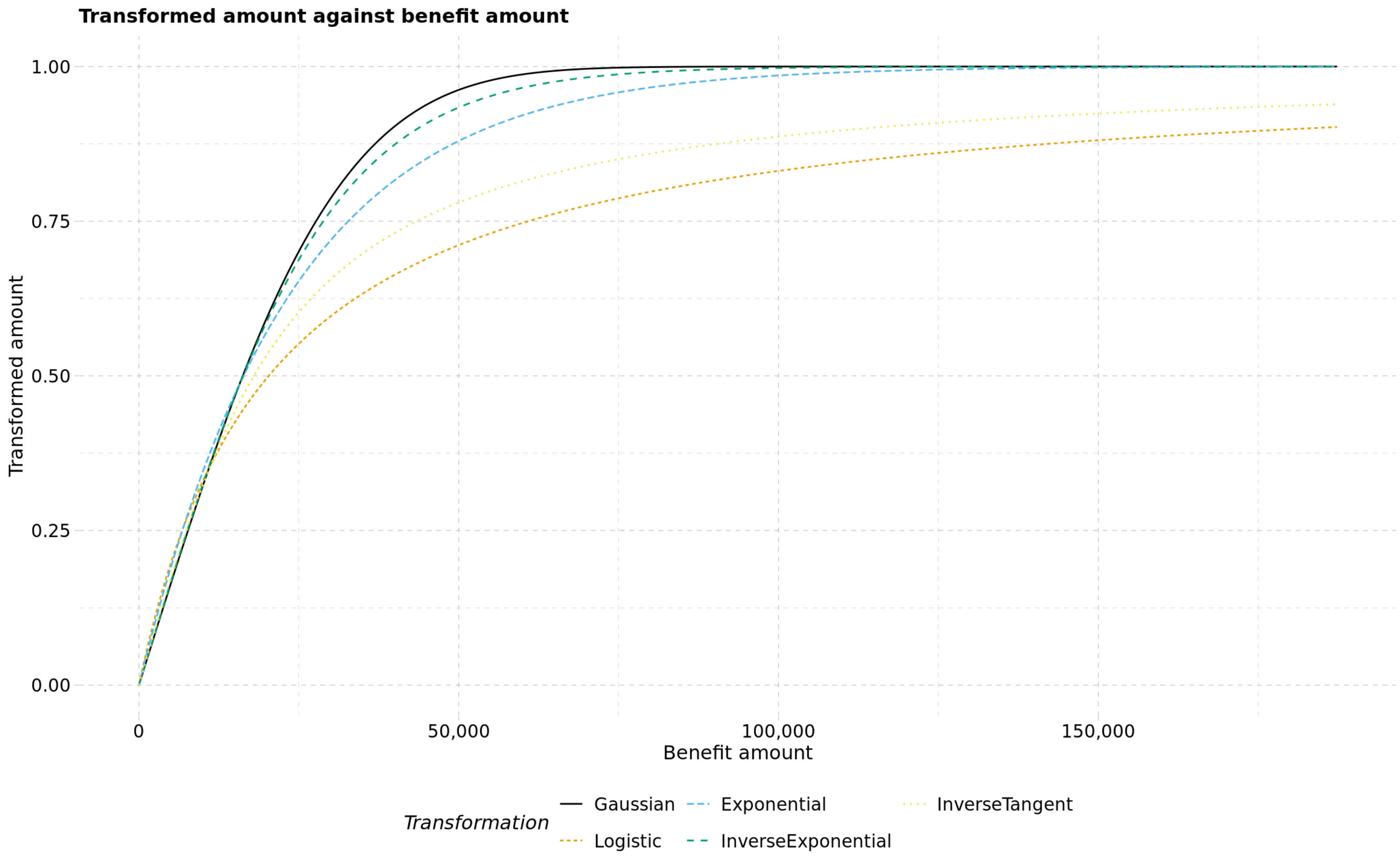
Amount transforms



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Modelling amounts continuously

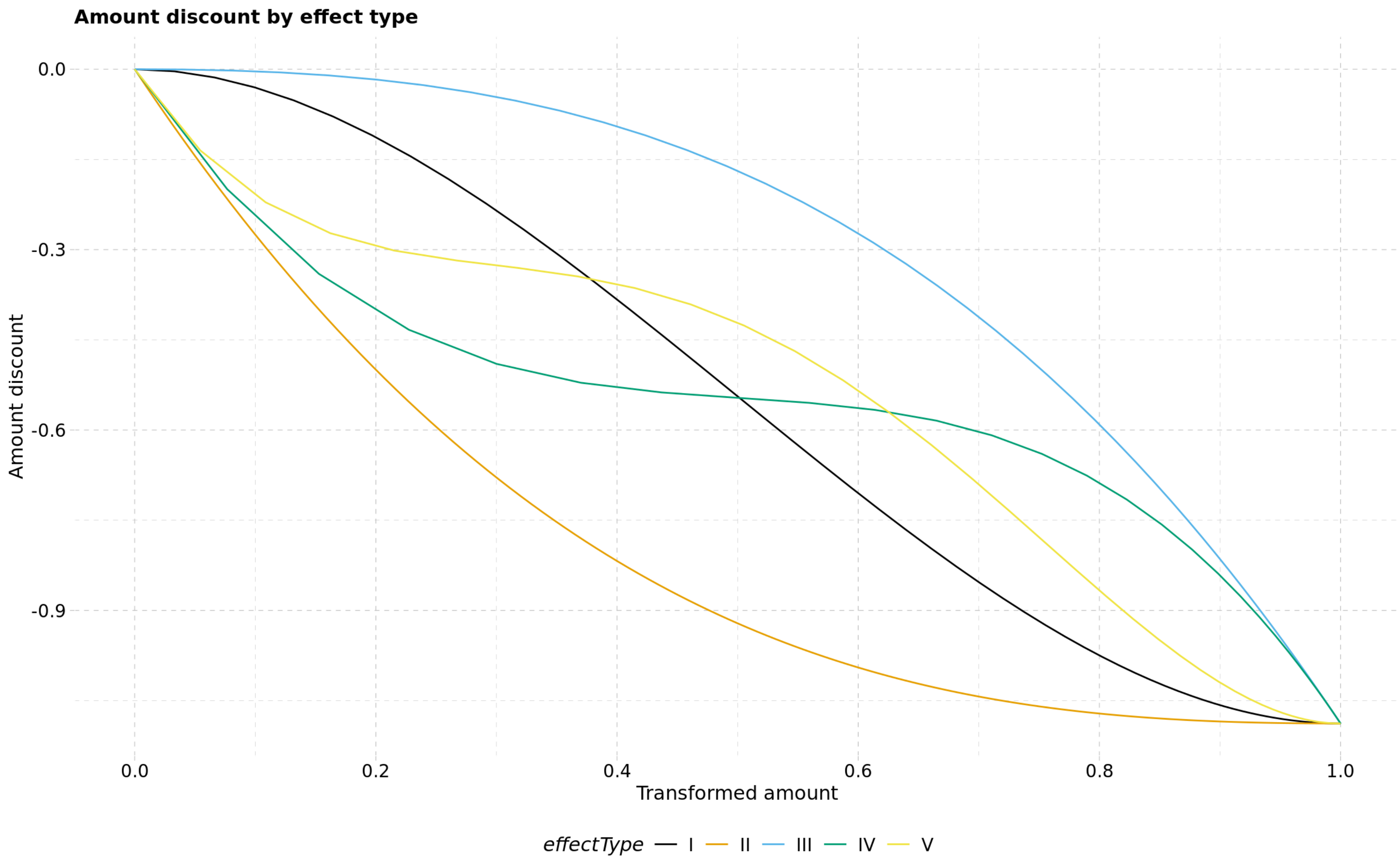
$$\log(\mu^*_{x,y}) = \log(\mu_{x,y}) + \nu\, h_{01}(u) + n_0\, h_{10}(u) + n_1\, h_{01}(u)$$

Coefficients

- $u = \tau_i(\lambda_0, s)$ – transformed benefit amount s
- λ_0 – shape factor for benefit amount transformation τ_i
- ν – ultimate mortality discount for “infinite” amount
- n_0 – slope at amount $=\ 0$
- n_1 – slope at ultimate amount

See Richards (2022)

Amount discount functions



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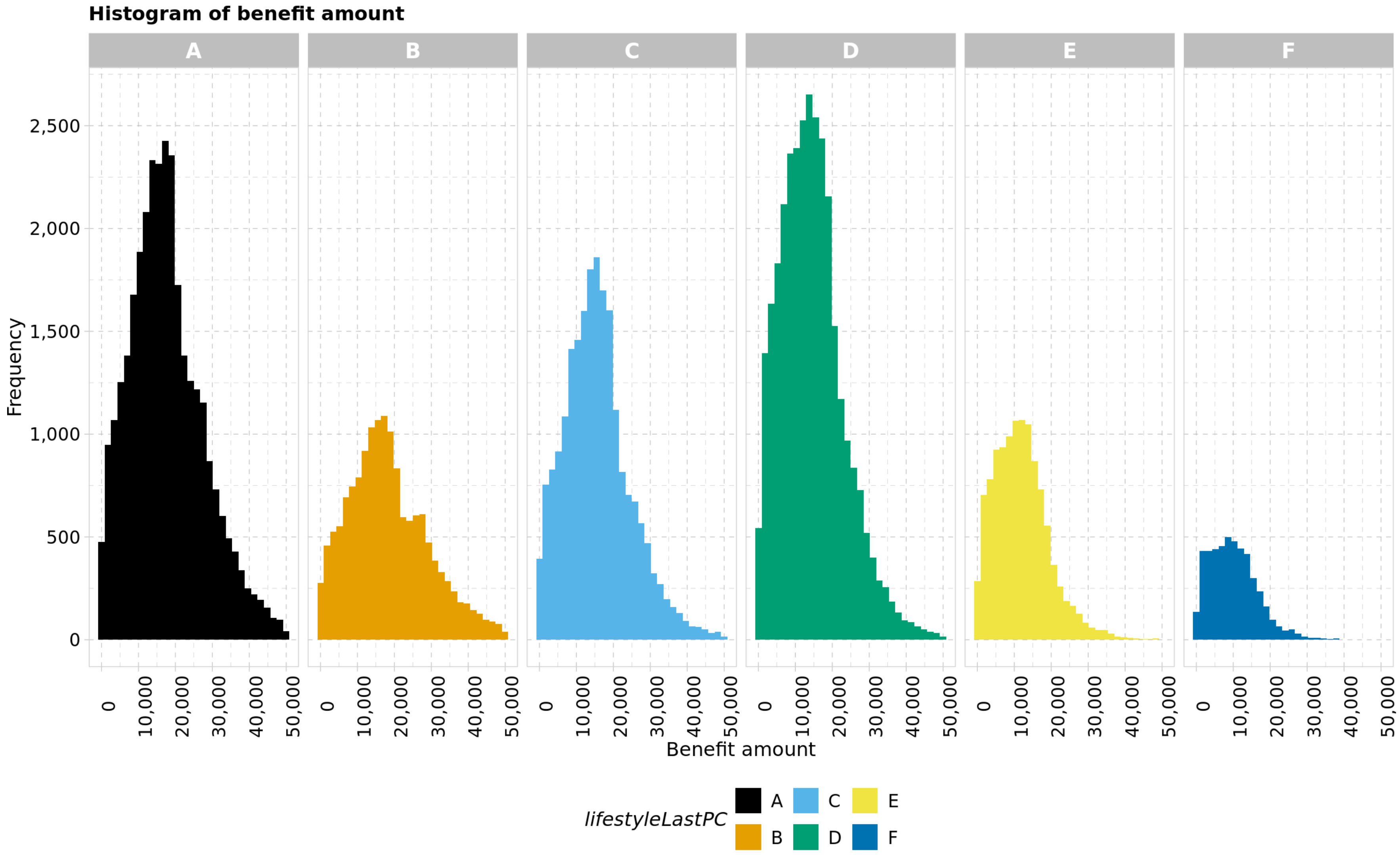
Let’s talk about the data!

Individual-level mortality experience data from multiple pension schemes

Pension Scheme Data (for calibrating): Years 2015 – 2019, ages 50 – 100

Risk Group	Lives	Exposure	Deaths
All	171,823	729,972	19,935
Females	54,599	224,761	6,719
Males	117,224	505,211	13,216
Lifestyle Group A	45,667	197,941	4,958
Lifestyle Group B	21,835	92,813	2,032
Lifestyle Group C	30,924	131,076	3,084
Lifestyle Group D	47,463	199,660	5,742
Lifestyle Group E	18,189	76,349	2,712
Lifestyle Group F	7,745	32,133	1,407

Amount distributions by Lifestyle Group



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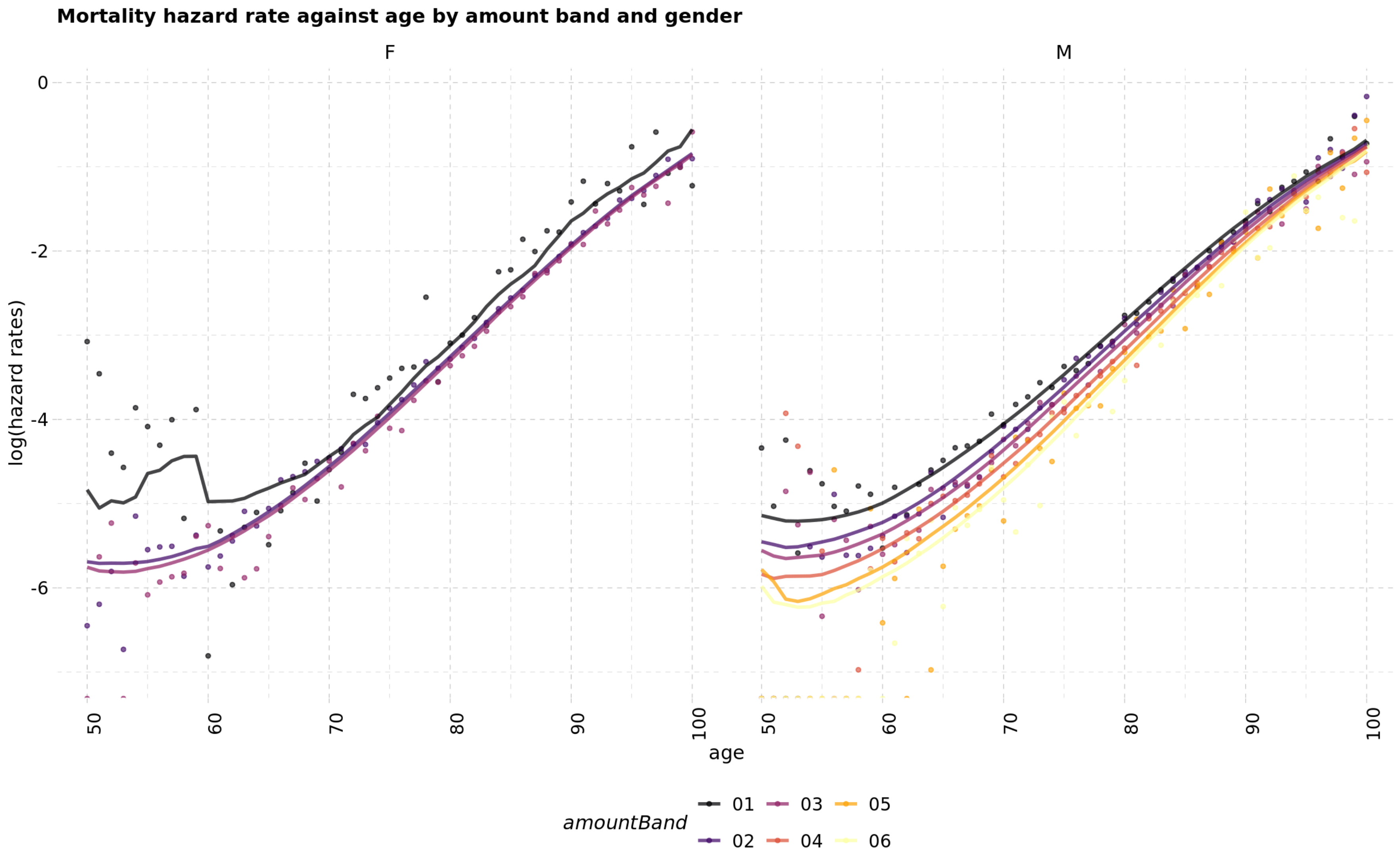
Results

Fitting an amounts model which varies by lifestyle group

Continuous amount model works.



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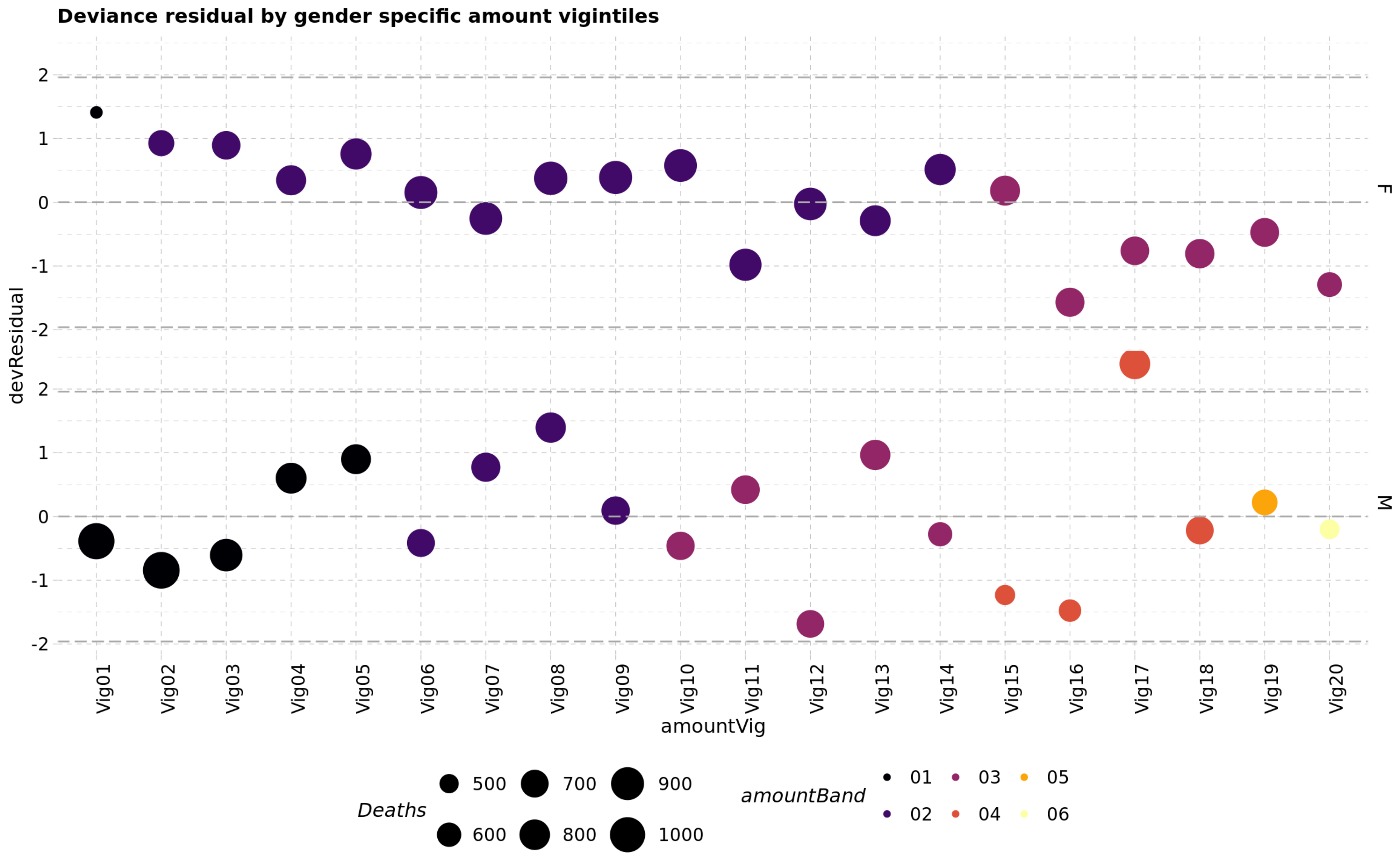
Continuous amount model works.



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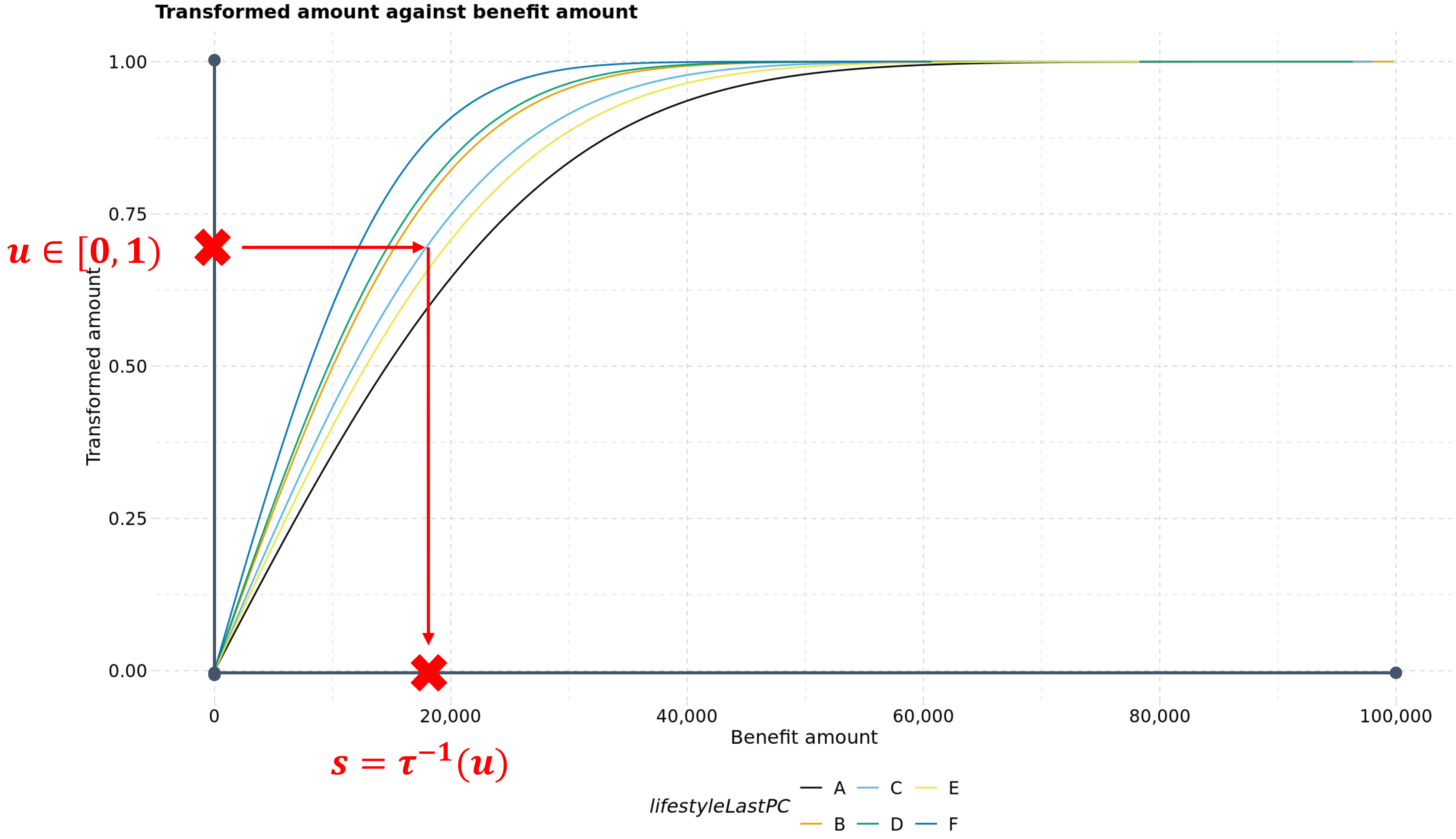
Amount transforms by Lifestyle Group (males only)

Lifestyle Group	Lambda-Estimate	Standard Error
A	-10.01	± 0.28
B	-9.56	± 0.27
C	-9.72	± 0.28
D	-9.53	± 0.21
E	-9.74	± 0.32
F	-9.30	± 0.31

Transforms by Lifestyle Group



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Discussion

Summary

- Continuous pension amount model with Gaussian transform and Hermite IV amounts effect function
- Transforms (and amounts effects) vary by geo-demographic Lifestyle Group
- Predict pension amounts by Lifestyle Group using inverse of transform

Future Research

1. Apply predictive model for different pension schemes
2. Optimise lifestyle classification for mortality modelling

References

- Ramonat, S. J. (**2024**). Extending the Hermite-spline basis for mortality modelling. *Working Paper*.
- Richards, S. J. (**2019**). A Hermite-spline model of post-retirement mortality. *Scandinavian Actuarial Journal*, 2020(2), 110–127. https://doi.org/10.1080/03461238.2019.1642239
- Richards, S. J. (**2022**). Modelling mortality by continuous benefit amount. *Scandinavian Actuarial Journal*, 2022(8), 695–717. https://doi.org/10.1080/03461238.2022.2025891

That’s all folks!

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