Multi-state Health-contingent Mortality Pooling: A Heterogeneous, Actuarially Fair, and Self-sustaining Product

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Background: Long-term Care (LTC) Cost

Individuals:

- Functional disability leads to demand for care and thus LTC costs
- Higher LTC costs can be hedged by increased retirement income payments

Public:

- Increasing LTC cost globally
- Rising concerns about future affordability
- Need for private market solutions

Background: LTC Products

LTC risk-hedging products include:

- **LTC insurance**
  - Provides benefit payments if functionally disabled
  - High loading: 18% – 51% (Brown and Finkelstein, 2007; Brown, 2009)
- **LTC annuity**
  - Murtaugh et al. (2001), Brown and Warshawsky (2013), and Pitacco (2016)
  - Avoids underwriting issues and lower capital requirements
- **LTC pooling products** – a recent line of work.

Why a Pooling Product for LTC Risk?

Pooling products:

- Tontines (Milevsky and Salisbury, 2015; Chen et al., 2019)
- Pooled Annuities (Piggott et al., 2005; Qiao and Sherris, 2013; Bernhardt and Donnelly, 2021)
- Risk-sharing rules (Sabin, 2010; Donnelly and Young, 2017; Denuit and Robert, 2021; Fullmer and Sabin, 2018; Zhou et al., 2025)

Advantages:

- Provide retirement income to reduce idiosyncratic mortality risks
- Self-sustaining, not promising lifetime payments, thus no capital requirements and needs a lower premium loading

We extend **risk-sharing rules** to **health-contingent** settings:

- Actuarial fairness at an individual level, thus allows heterogeneous member profiles

Challenges in Existing LTC Pooling Products

Existing LTC pooling products:

- Chen et al. (2022)
- Hieber and Lucas (2022)
- Kabuche et al. (2024)

Challenges: No existing product has all the following features

1. Recovery from functional disability
2. Actuarial fairness
3. Self-sustaining
4. Heterogeneous members and open pooling

The **research objective** is to propose a framework for LTC pooling products with **all the features** above.

Our Contributions

Research question:

- How to offer an adaptive solution for LTC risk management and enhance retirement income adequacy?

Contributions:

- A novel health-contingent pooling product
- Extend the literature of risk sharing and pooling products
- Actuarial fairness and self-sustainability while having multiple health states with recovery

Insights:

- Improves affordability
- Avoids over-subsidising and reduces unfairness
- Reduces underwriting barriers
- Offers insights for policymakers on complementary private LTC product solutions

Multi-state Health Model

- We use the three-state model in Fu et al. (2021) calibrated with US Health and Retirement Study (HRS) data.
- For an individual i, we denote $p_{j,k}^{(i)}(t)$ the transition probability from state j at time t to state k at time $t+1$, where $j,k = H,\ F,\ D$.

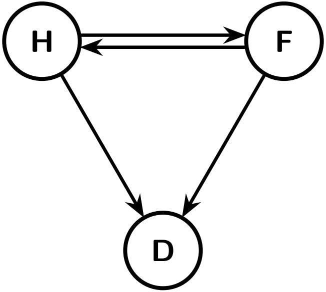
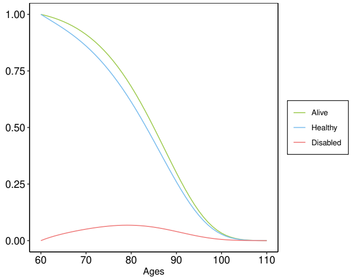
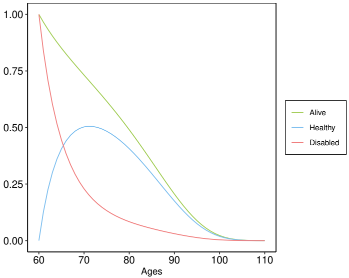


Figure 1: A three-state transition model in Fu et al. (2021).

Outputs of Multi-state Health Model



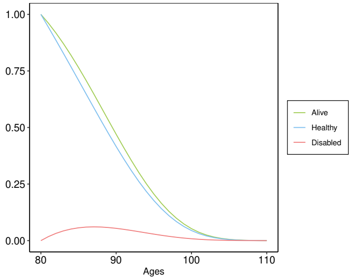
(a) H state



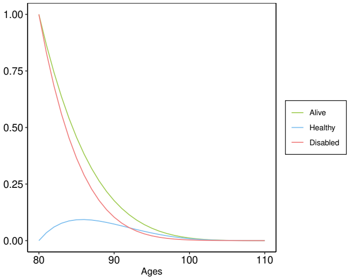
(b) F state

Figure 2: Transition probabilities to different health states for the aged 60 cohort.

Outputs of Multi-state Health Model



(a) H state



(b) F state

Figure 3: Transition probabilities to different health states for the aged 80 cohort.

Fund Operation

- The product allows each member i to have their **own account**
- Member i can join the fund in either H or F state at time t with a fund balance of $F^{(i)}(t)$ equal to their initial contribution

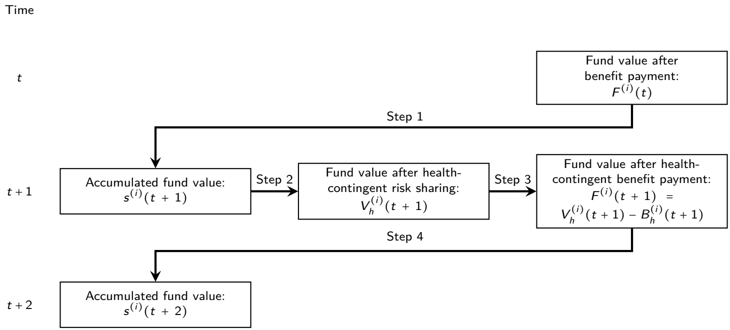


Figure 4: Health-contingent fund operation between time t and $t+2$.

Step 1: Accumulation

- Denote $F^{(i)}(t)$ as the fund balance of individual i at time t, and $ROR^{(i)}(t)$ the deterministic rate of return of individual i between time t and $t+1$
- The accumulated fund balance is $s^{(i)}(t) = F^{(i)}(t)(1 + ROR^{(i)}(t))$

Total Mortality Credit

- Individual i will loss the accumulated fund balance $s^{(i)}(t+1)$ in the event of death
- Total mortality credits $S(t+1)$ is the sum of accumulated fund balances of members who pass away between time t and $t+1$:

$$S(t+1) = \sum_{i \in D(t+1)} s^{(i)}(t+1),$$

  where $D(t+1)$ is the set of members die during the period $[t, t+1]$.
- The total mortality credit $S(t+1)$ is distributed to members by a risk-sharing rule

Numerical Illustration

Imagine three participants in the pool.

Table 1: Three members in the pool.

Members	Initial Health State	Fund balance	$p_{h,H}$	$p_{h,F}$	q_h
1	H	1	0.90	0.09	0.01
2	H	1	0.90	0.09	0.01
3	F	1	0.08	0.88	0.04

If only member 2 dies, the total mortality credit will be $S = 1$.

Step 2: Health-contingent Risk sharing

- The health-contingent fund value after risk sharing is:

$$\begin{aligned} V_h^{(i)}(t+1) =& s^{(i)}(t+1)\left(1+q_h^{(i)}(t)+\gamma_h^{(i)}(t+1)\right) \\ &+ w^{(i)}(t+1)\left[S(t+1)-E_t[S(t+1)]-\delta(t+1)\right], \end{aligned} \quad (1)$$

  where $\gamma_h^{(i)}(t+1)$ for $h(t+1) = H, F, D$ are the health-contingent additional returns, $w^{(i)}$ follows the proportional rule in Donnelly (2017), and $\delta(t+1)$ is an adjustment term for deviation.
  - $E_t\left[w^{(i)}(t+1)\left[S(t+1)-E_t[S(t+1)]-\delta(t+1)\right]\right]=0$
- In the context of a mortality pooling product, we set $\gamma_D^{(i)}(t+1) = -1$.

Numerical Illustration

Table 2: Three members in the pool.

Members	Initial Health State	Fund balance	$p_{h,H}$	$p_{h,F}$	q_h
1	H	1	0.90	0.09	0.01
2	H	1	0.90	0.09	0.01
3	F	1	0.08	0.88	0.04

- Member 1: $1 \times (1 + 0.01 + \gamma_h^{(1)})$
- Member 2: $1 \times (1 + 0.01 + \gamma_h^{(2)})$
- Member 3: $1 \times (1 + 0.04 + \gamma_h^{(3)})$

Property: Fairness

Proposition 1

When $\dfrac{\gamma_H^{(i)}(t+1)}{\gamma_F^{(i)}(t+1)} = -\dfrac{p_{h,F}^{(i)}(t)}{p_{h,H}^{(i)}(t)}$ *and* $\gamma_D^{(i)}(t+1) = -1$*, the health-contingent risk-sharing rule in Equation (1) is actuarially fair:*

$$E[V_i^h(t+1)|\mathcal{F}(t)] = s_i(t+1),$$

where $\mathcal{F}(t) = \sigma\{(H(s), F(s)) : 0 \le s \le t\}$ *is the* σ*-algebra generated by* $H(t)$ *the set of health states of members at time* t *and* $F(t)$ *the set of fund balances of members at time* t.

Numerical Illustration

Member	Initial Health State	Fund balance	$p_{h,H}$	$p_{h,F}$	q_h
1	H	1	0.90	0.09	0.01

Member 1: Let $\gamma_F^{(1)} = 0.2$, $\gamma_H^{(1)} = -\frac{p_{h,F}^{(1)}}{p_{h,H}^{(1)}}\gamma_F^{(i)} = -\frac{0.09}{0.9} \times 0.1 = -0.02$, and $\gamma_D^{(1)} = -1$

The **health contingent fund values** are:

- **H**: $1 \times (1 + 0.01 - 0.02) = 0.99$ with probability of 90%
- **F**: $1 \times (1 + 0.01 + 0.2) = 1.21$ with probability of 9%
- **D**: $1 \times (1 + 0.01 - 1) = 0.01$ with probability of 1%

In expectation, $0.99 \times 0.9 + 1.21 \times 0.09 + 0.01 \times 0.01 = 1$.

Property: Self-sustainability

Proposition 2

The health-contingent risk-sharing rule in Equation (1) is self-sustaining:

$$\sum_{i=1}^{N(t)} V_h^{(i)}(t+1) = \sum_{i=1}^{N(t)} s^{(i)}(t+1),$$

where $N(t)$ is the number of members in the pool at time t.

Step 3: Health-contingent Benefit Payments

- For individual i in health state $h = H, F$ at time $t+1$, denote $\ddot{a}_h^{(i)}(\theta, t+1)$ the actuarial notation of a **hybrid annuity due** that pays the following until death:
  - 1 in the **H state**,
  - $\theta^{(i)}(t+1)$ in the **F state**,

  where $\theta^{(i)}(t+1) \in [1.5, 2.5]$.

Step 3: Health-contingent Benefit Payments

- We extend the approach of group-self annuitisation (GSA) in Piggott et al. (2005) by dividing the fund balance over the hybrid annuity factor, and the annuity-like payments are here defined as:

$$B_h^{(i)}(t+1) = \begin{cases} \dfrac{V_H^{(i)}(t+1)}{\ddot{a}_H^{(i)}(\theta, t+1)} & \text{if } h^{(i)}(t+1) = H, \\ \theta^{(i)}(t+1)\dfrac{V_F^{(i)}(t+1)}{\ddot{a}_F^{(i)}(\theta, t+1)} & \text{if } h^{(i)}(t+1) = F, \\ V_D^{(i)}(t+1) & \text{if } h^{(i)}(t+1) = D. \end{cases} \tag{2}$$

- At time $t+1$, we want:

$$\theta^{(i)}(t+1)\frac{1+q_h^{(i)}(t)+\gamma_F^{(i)}(t+1)}{1+q_h^{(i)}(t)+\gamma_H^{(i)}(t+1)}\frac{\ddot{a}_H^{(i)}(\theta, t+1)}{\ddot{a}_F^{(i)}(\theta, t+1)} = \theta^{(i)}(t+1).$$

Step 3: Health-contingent Benefit Payments

Proposition 3

When $\dfrac{\gamma_H^{(i)}(t+1)}{\gamma_F^{(i)}(t+1)} = -\dfrac{p_{h,F}^{(i)}(t)}{p_{h,H}^{(i)}(t)}$ *and* $\gamma_D^{(i)}(t+1) = -1$ *hold, the unique solution to the equation* $\dfrac{1+q_h^{(i)}(t)+\gamma_F^{(i)}(t+1)}{1+q_h^{(i)}(t)+\gamma_H^{(i)}(t+1)}\eta^{(i)}(\theta,t+1) = 1$ *is:*

$$\gamma_F^{(i)}(t+1) = \frac{\left(1-\eta^{(i)}(\theta,t+1)\right)\left(1+q_h^{(i)}(t)\right)}{\eta^{(i)}(\theta,t+1)+\phi^{(i)}(t)}, \tag{3}$$

where $\eta^{(i)}(\theta,t+1) = \dfrac{\ddot{a}_H^{(i)}(\theta,t+1)}{\ddot{a}_F^{(i)}(\theta,t+1)}$ *and* $\phi^{(i)}(t) = \dfrac{p_{h,F}^{(i)}(t)}{p_{h,H}^{(i)}(t)}$.

Numerical Illustration

Member	Initial Health State	Fund balance	$p_{h,H}$	$p_{h,F}$	q_h
1	H	1	0.90	0.09	0.01

- Assume we set $\theta^{(1)} = 2$, which gives $\ddot{a}_H^{(1)}(\theta) = 18$ and $\ddot{a}_F^{(1)}(\theta) = 20$
- Then, from Proposition 3, we obtain $\gamma_F^{(1)} = 0.101$ and $\gamma_H^{(1)} = -0.0101$, indicating a positive return for a transition into the **F state** of **10.1%**, and a negative return of **-1.01%** if remaining in the **H state**.
- The **health contingent payments** are:
  - **H**: $(1 + 0.01 - 0.0101)/18 = 5.555\%$ with probability of 90%
  - **F**: $2 \times (1 + 0.01 + 0.101)/20 = 11.11\%$ with probability of 9%

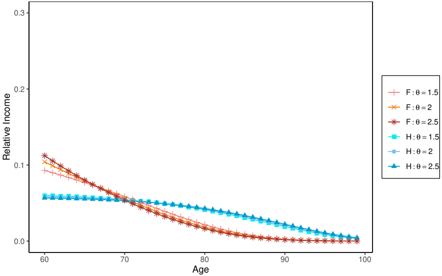
  It is easy to see that $11.11\% = 2 \times 5.55\%$.

Summary

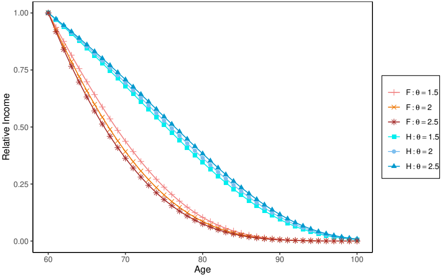
The following factors lead to higher payments in the F (functionally disabled) state:

1. Age, initial health state, ending health state, scaling factor θ
2. Health-contingent return
3. Higher probability of death if the initial health state is F
4. Higher payout ratio if the ending health state is F

Comparison of θ: No Risk Sharing, Size= 1



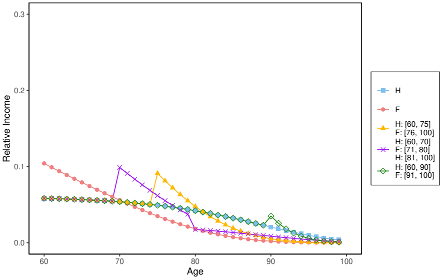
(a) Relative income payments



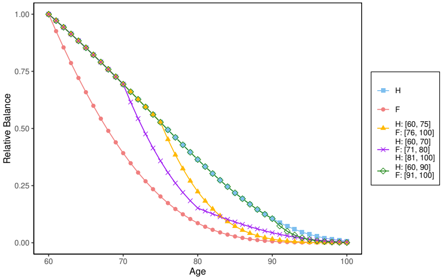
(b) Relative balances

Figure 5: Individuals joined at age 60 at time 0, pool size = 1 and $\theta = 1.5, 2, 2.5$.

Comparison of Health Paths: No Risk Sharing, Size= 1



(a) Relative income payments



(b) Relative balances

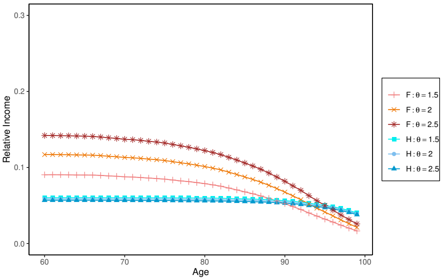
Figure 6: Individuals joined at age 60 at time 0 initially healthy with different health paths, pool size = 1 and $\theta = 2$.

Health-contingent Risk Sharing, Open Pool

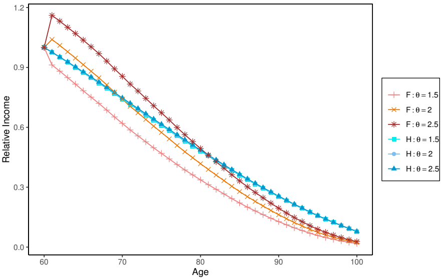
Table 3: Number of people in different health states and ages with age-dependent balances joining the pool at the beginning of every year.

Age	H	F	Total	Balance
60	100	0	100	600,000
61	99	1	100	580,000
62	98	1	99	560,000
⋮	⋮	⋮	⋮	⋮
78	67	7	74	240,000
79	64	7	71	220,000
80	61	7	68	200,000
Subtotal	1771	90	1861	NA

Comparison of θ: Health-contingent Risk Sharing, Open Pool



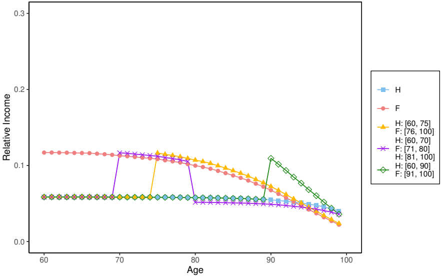
(a) Mean relative income payments



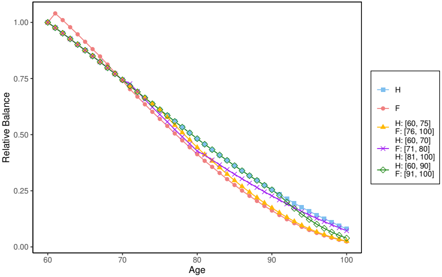
(b) Mean relative balances

Figure 7: Individuals joined at age 60 at time 0 with different health paths, open pool and $\theta = 1.5, 2, 2.5$.

Comparison of Health Paths: Health-contingent Risk Sharing, Open Pool



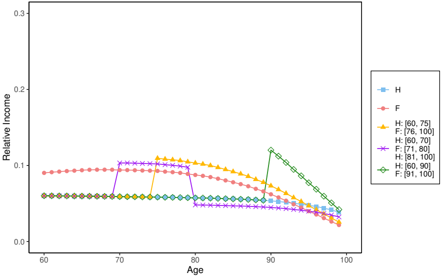
(a) Mean relative income payments



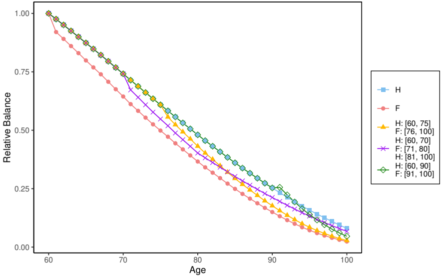
(b) Mean relative balances

Figure 8: Individuals joined at age 60 at time 0 initially healthy with different health paths, for open pool and $\theta = 2$.

Comparison of Health Paths: Health-contingent Risk Sharing, Open Pool, Age-dependent θ



(a) Mean relative income payments



(b) Mean relative balances

Figure 9: Individuals joined at age 60 at time 0 initially healthy with different health paths, for open pool and age-dependent θ.

Comparison of Expected Present Value (EPV)

Table 4: Sum of the EPV of income payments relative to initial contribution for members with different joining ages and health paths.

Age	Health Paths						
	H: dies at 85	F: dies at 85	H: dies at 100	F: dies at 100	H→F (middle)	H→F, F→H	H→F (old)
$\theta = 1.5$							
60	1.0448	1.4886	1.3502	1.7497	1.4569	1.4736	1.3741
70	0.9665	1.3642	1.5167	1.8462	1.6218	1.6509	1.5595
80	0.5709	0.8274	1.7159	1.9573	1.8321	1.8209	1.8011
$\theta = 2$							
60	1.0136	1.9214	1.3099	2.2579	1.6505	1.6066	1.3986
70	0.9326	1.7507	1.4624	2.3655	1.8615	1.8575	1.6206
80	0.5493	1.0586	1.6476	2.4957	2.1601	2.0304	1.9707
$\theta = 2.5$							
60	0.9843	2.3282	1.2719	2.7353	1.8324	1.7318	1.4218
70	0.9012	2.1102	1.4120	2.8475	2.0841	2.0502	1.6775
80	0.5293	1.2730	1.5849	2.9925	2.4630	2.2253	2.1272
Age-dependent θ							
60	1.0248	1.5934	1.3136	1.9038	1.6441	1.5178	1.4397
70	0.9367	1.6075	1.4555	2.2140	1.9058	1.7890	1.6814
80	0.5466	1.0670	1.6232	2.5798	2.2896	2.0303	2.0862

Conclusions

This paper has the following innovations:

- Allowing recovery from disability in a fair and self-sustaining pooling scheme
- Dynamic payments that suit the different needs of LTC in different health states
- Allowing heterogeneous membership and dynamic pooling
- Allowing a factor that directly controls the level of LTC protection
- Members with different levels of LTC protection can be mixed in the pool

In summary, the proposed product offers an adaptive, affordable, and fair private market solution for LTC risk management.

Thank You!

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