

Longevity 21, Rome

COVID-19 in mortality forecasts

Stephen J. Richards

8th September 2026



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1. The COVID-19 problem

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2. Forecasting models

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3. Outliers

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4. Conclusions

ON THE NATURE
AND HANDLING OF
COVID-19
IN ARIMA
MORTALITY
FORECASTS

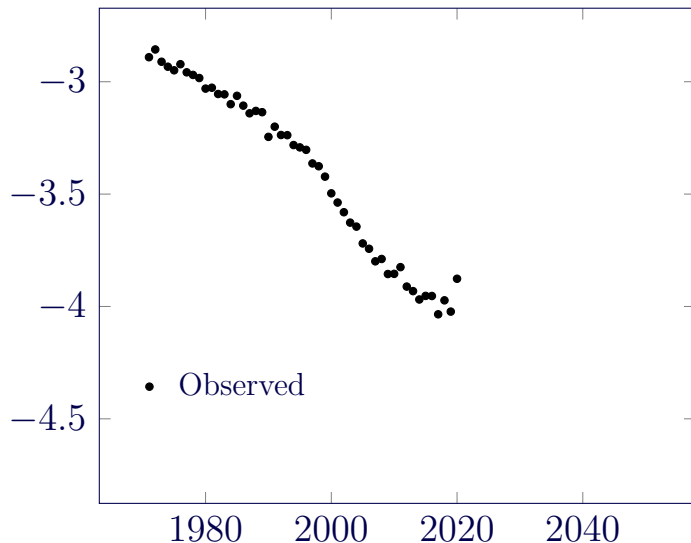
By S. J. Richards



PDF available at: <https://www.longevity.co.uk/published-paper/nature-and-handling-covid-19-arima-mortality-forecasts>

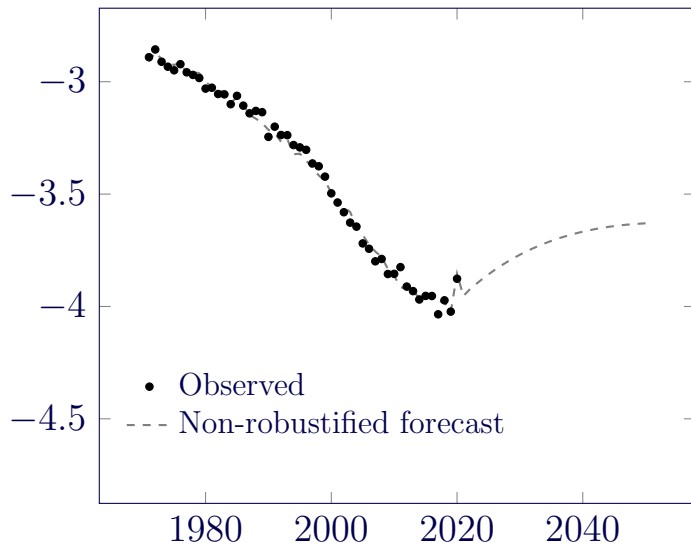
1 The COVID-19 problem

COVID-19 forecasting problem LONGEVITAS



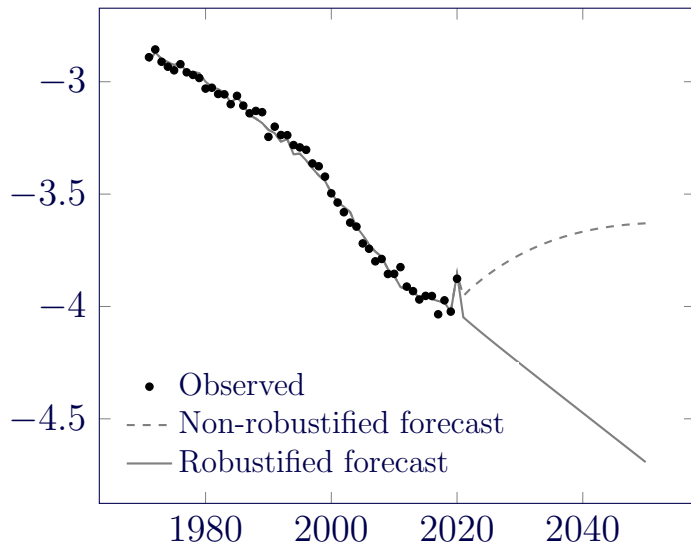
Source: Richards [2024, Figure 4(b)]. Data for males in England & Wales, 1971-2020.

COVID-19 forecasting problem LONGEVITAS



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- A COVID-19 biases parameter estimates, wherever it sits in the data.

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Source: $\hat{\sigma}^2$ estimates from Richards [2024, Figures B.5 and B.6].

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$\Rightarrow$ allowing for outlier halves the parameter estimate that drives VaR capital requirements.

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2 Forecasting models

Let $\mu_{x,y}$ denote the mortality hazard at age x in year y .

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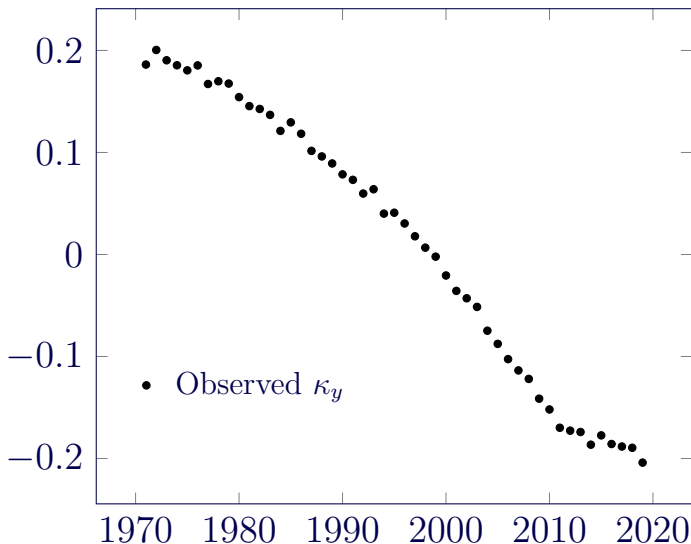
$$\log \mu_{x,y} = \alpha_x + \beta_x \kappa_y$$

with two identifiability constraints.

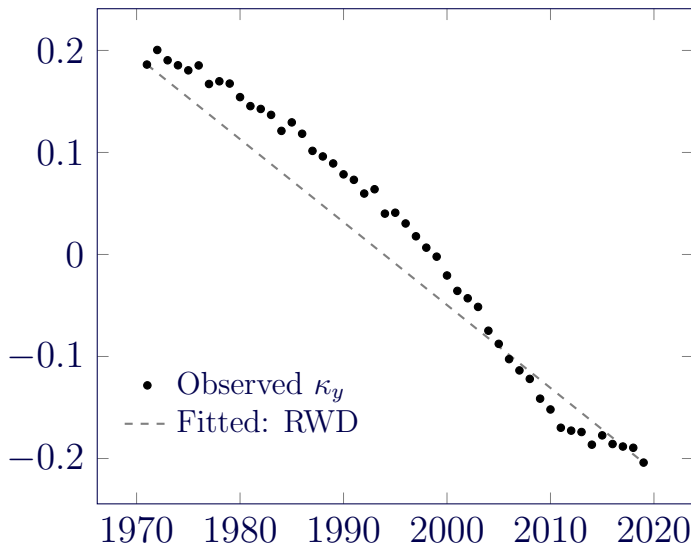
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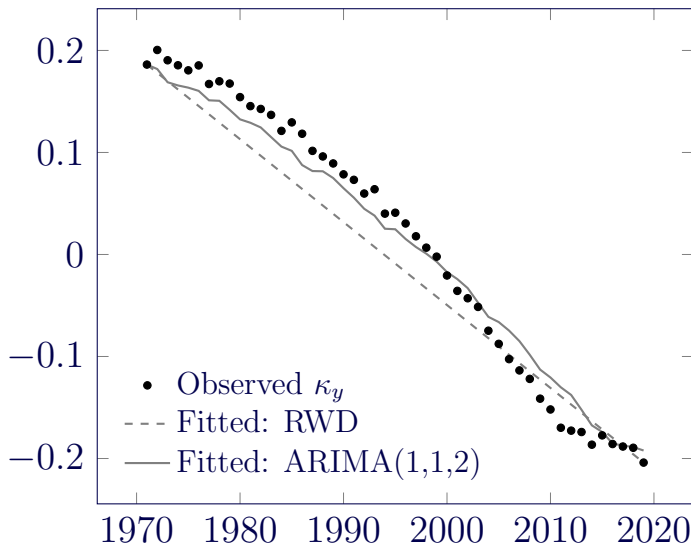
- Forecast mortality by projecting κ_y .
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...but ARIMA models often fit better.



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- Δ is the backward difference operator,
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- $\{\epsilon_y\}$ is a collection of uncorrelated innovations with mean zero and constant variance σ^2 .

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- $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$, and
- $\theta(B) = 1 + \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$.

Random walk with drift:

$$\kappa_{y+t} = \underbrace{\kappa_y + t\mu}_{\substack{\text{linear} \\ \text{trend}}} + \sum_{i=1}^t \epsilon_{y+i}$$

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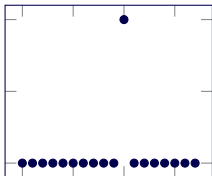
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ARIMA($p, 1, q$) model:

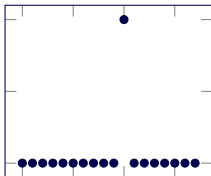
$$\kappa_{y+t} = \underbrace{\kappa_y + t\mu}_{\text{linear trend}} + \underbrace{\sum_{i=1}^t \frac{\theta(B)}{\phi(B)} \epsilon_{y+i}}_{\text{cumulative stochastic deviation}}$$

3 Outliers

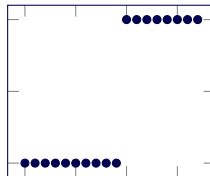
Additive Outlier (AO)



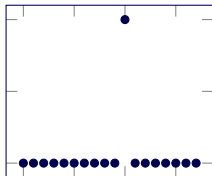
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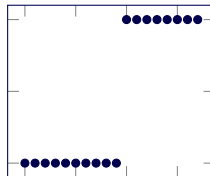
Level Shift (LS)



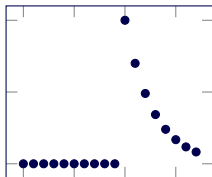
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Temporary Change (TC)



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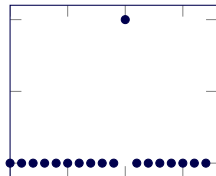
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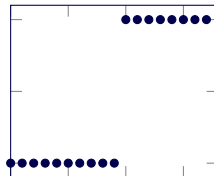
1. Identify location of outliers, and
2. Classify type of outlier.

Note: The type cannot be determined for an outlier at the end of a series.

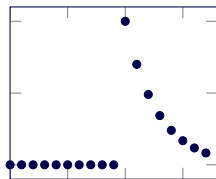
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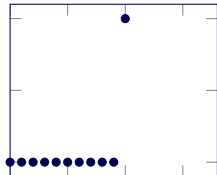
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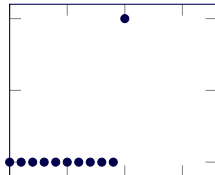
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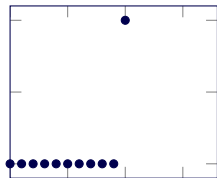
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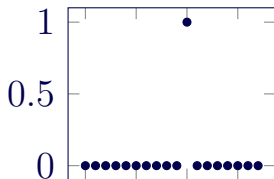
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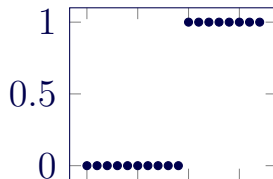
- ω_j is the size of outlier j , and
- $I_j(y+t)$ is the relative effect of outlier j at $y+t$.

Source: Richards [2026, Section 6].

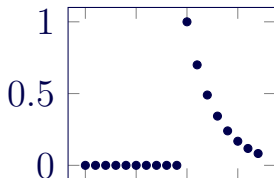
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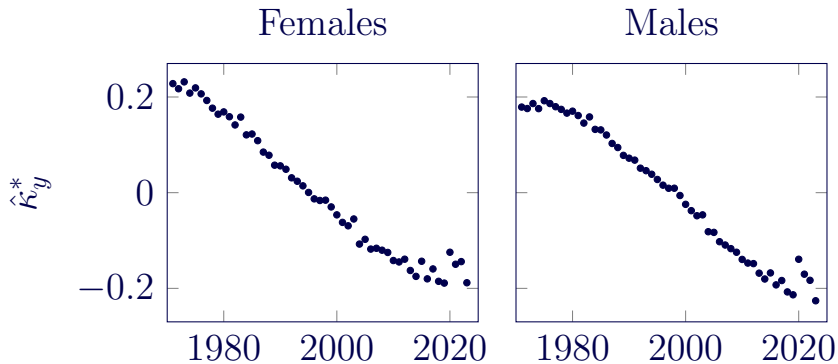


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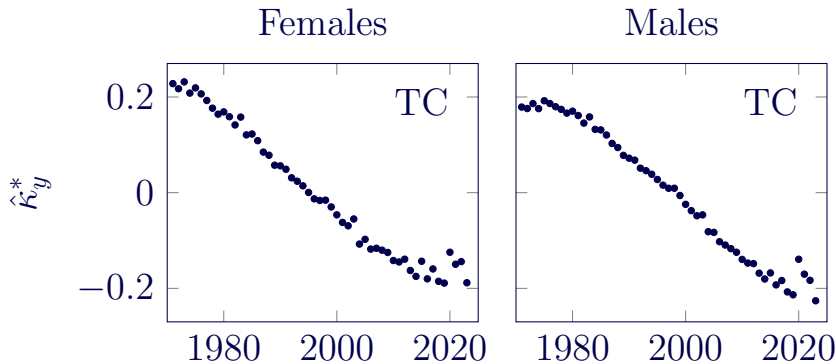
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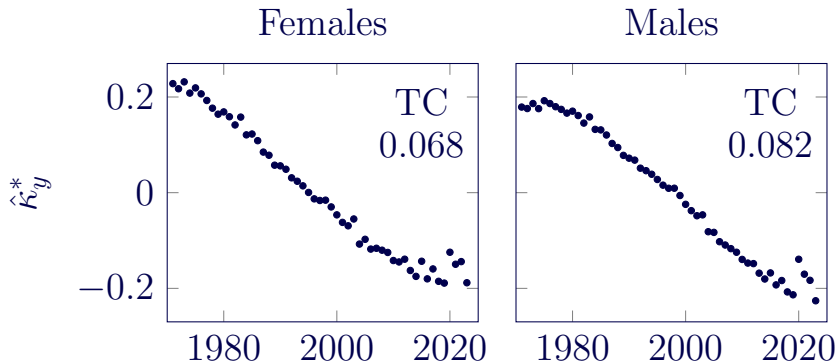
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2020 outliers: Italy

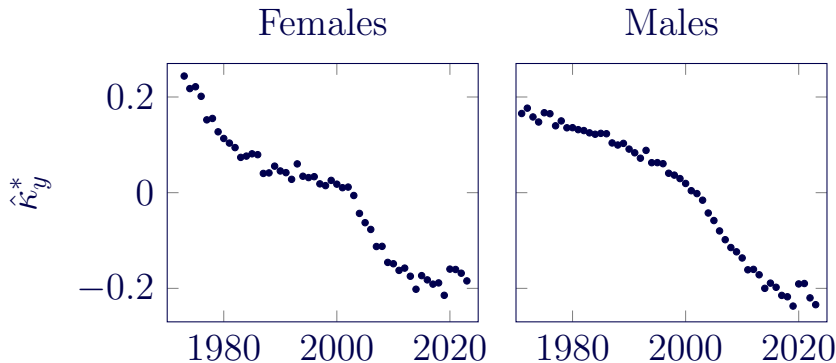


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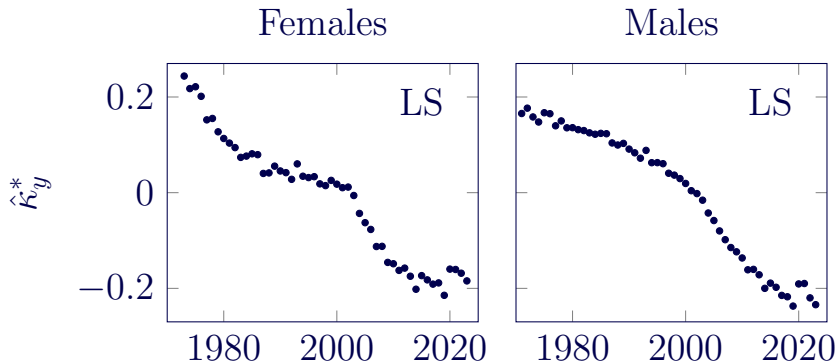
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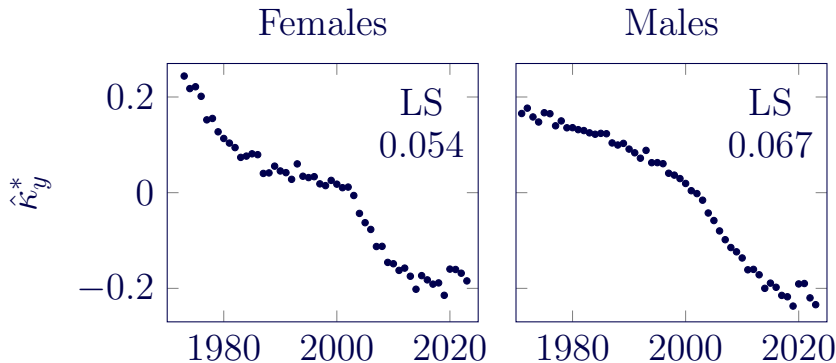
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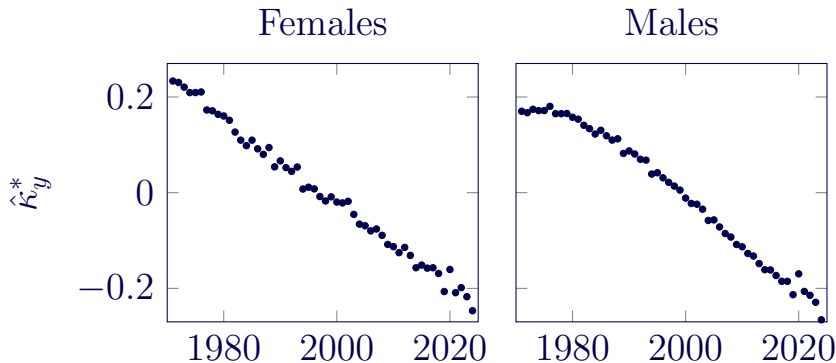
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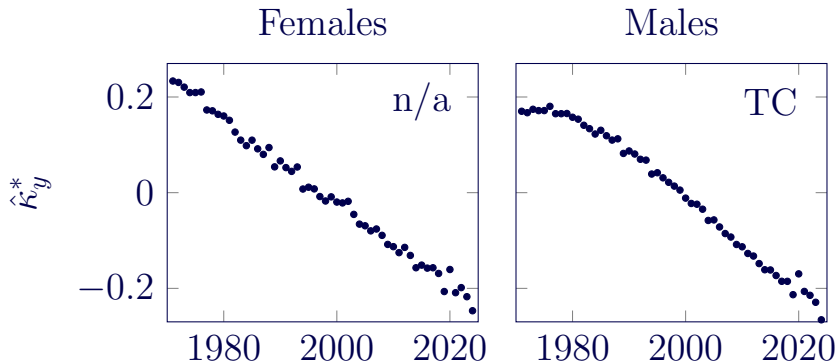
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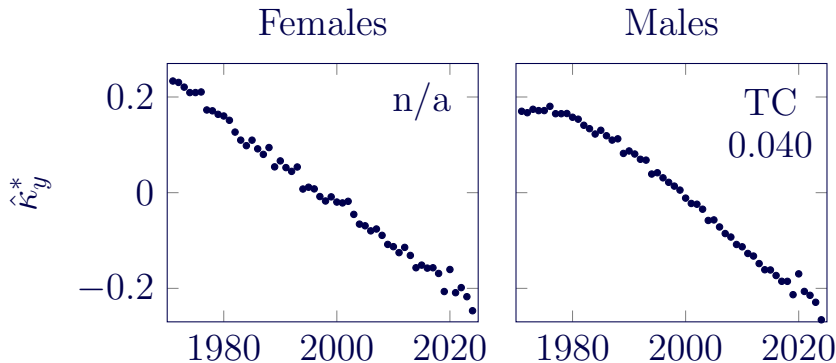
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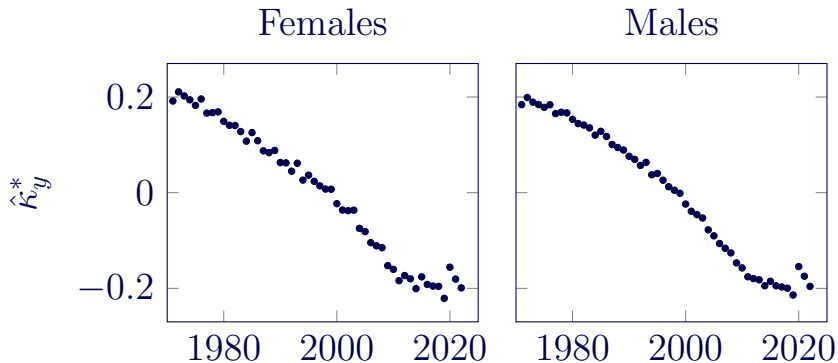
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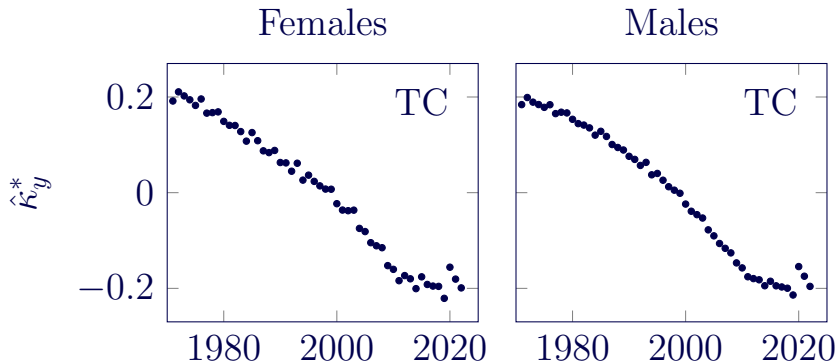


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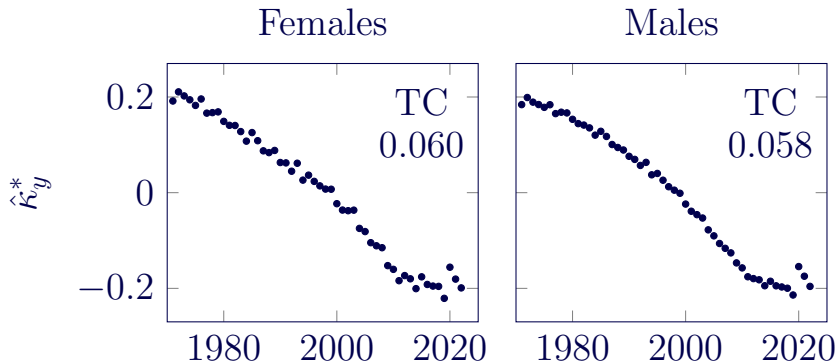
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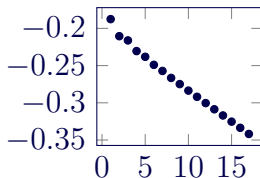
Forecasting t steps ahead:

$$\kappa_{y+t}^* = \underbrace{\kappa_y + t\mu}_{\text{linear trend}} + \underbrace{\sum_{j=1}^m \omega_j I_j(y+t)}_{\text{outlier component}} + \underbrace{\sum_{i=1}^t \frac{\theta(B)}{\phi(B)} \epsilon_{y+i}}_{\text{cumulative stochastic deviation}}$$

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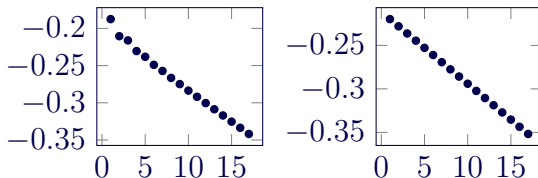
ARIMA(1,1,0) central forecast of κ_{2023+t}^* for females in Italy.

(a) κ_{2023+t}^* forecast



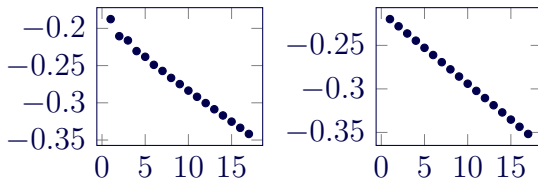
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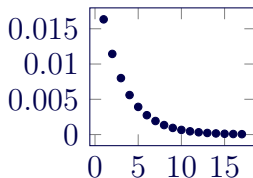


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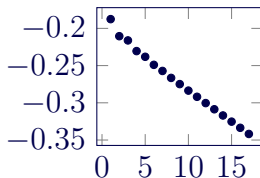
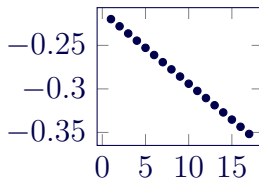


(c) TC outlier

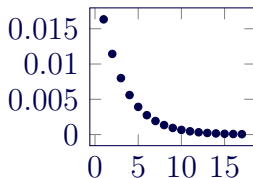
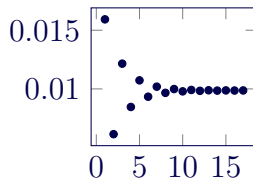


ARIMA(1,1,0) central forecast of κ_{2023+t}^* for females in Italy.

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4 Conclusions

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- C. Chen and L-M. Liu. Joint estimation of model parameters and outlier effects in time series. *Journal of the American Statistical Association*, 88(421): 284–297, 1993. doi: 10.2307/2290724.
- T. Kleinow and S. J. Richards. Parameter risk in time-series mortality forecasts. *Scandinavian Actuarial Journal*, 2016(10):1–25, 2016. doi: 10.1080/03461238.2016.1255655.
- R. D. Lee and L. Carter. Modeling and forecasting US mortality. *Journal of the American Statistical Association*, 87:659–671, 1992. doi: 10.1080/01621459.1992.10475265.

- S. J. Richards. Robust mortality forecasting in the presence of outliers. *British Actuarial Journal*, 29: e19, 2024. doi: 10.1017/S1357321724000175.
- S. J. Richards. On the nature and handling of COVID-19 in ARIMA mortality forecasts. *Longevity Ltd*, 2026.

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