

Time-Varying Temporal Reconciliation for Mortality Forecasts

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Issues with Using A Rich Hierarchy

- Available frequencies for death counts data: Annual, semiannual, four-monthly, quarterly, bimonthly and monthly.
- The model fitted on each frequency gives its own forecast of the same underlying mortality process.
- $\hat{D}_{\text{annual}} \neq \sum_{j=1}^{12} \hat{D}_{\text{month } j}$ can happen before reconciliation.

Reconciliation in Mortality Studies

The core idea: to ensure the sum of disaggregated-level forecasts aligns with forecasts at a more aggregated level. (Hyndman, Ahmed, et al. (2011), Hyndman, Lee, et al. (2016), and Wickramasuriya et al. (2019))

- Common practice in mortality studies: cross-sectional structure.
 - Australian **state-level** mortality rates (Shang and Yang (2021))
 - **Sex-specific** mortality rates from five countries (Rusyda et al. (2026))
- Time-series forecasting: temporal dimension (Athanasopoulos et al. (2017))
- We consider temporal reconciliation for mortality forecasting
 - death counts for the **same population**, measured at **different frequencies**:

$$\hat{\mathbf{y}} = (\hat{D}_{\text{annual}}, \hat{D}_{\text{half},1}, \hat{D}_{\text{half},2}, \dots, \hat{D}_{\text{month},12})^{\top}.$$

Same benefits:

- **Improve the accuracy** of reconciled forecasts across the hierarchy.
- **Ensure forecast coherence** across aggregation levels.

Approaches in Mortality Modelling

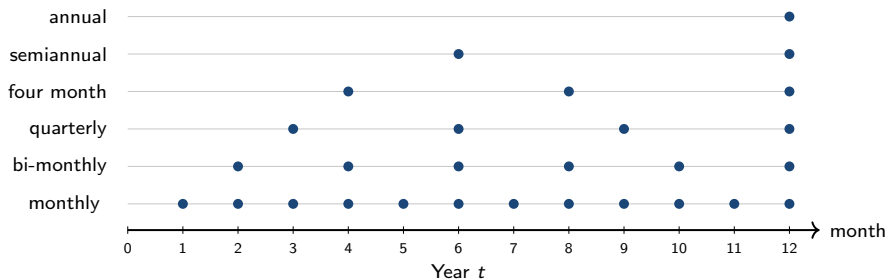
- Li et al. (2026a) develop Mixed Data Sampling (MIDAS) models for mortality forecasting.
 - Monthly and annual information incorporated simultaneously; Forecast annual mortality only
- Li et al. (2026b) integrate annual and monthly mortality models within a unified state-space framework.
 - A common monthly latent factor to drive mortality at both frequencies.
- Both methods use mixed-frequency relationship in the modelling stage → require a specific model form
- The proposed reconciliation approach
 - is a post-forecast technique
 - relaxes the constraints on models used to produce forecasts.

Contributions

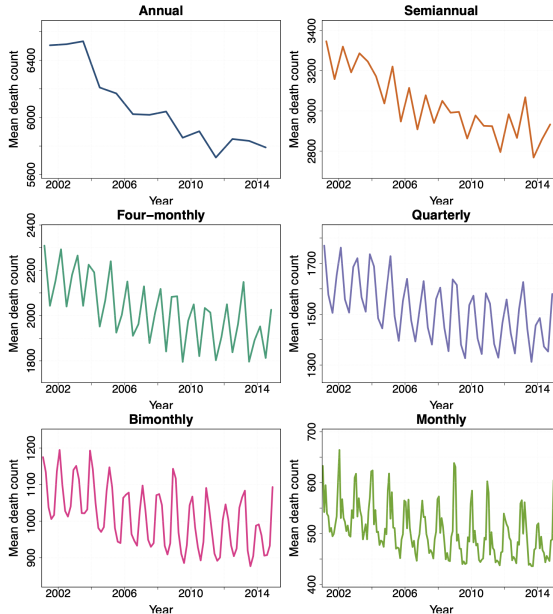
- First to introduce temporal reconciliation to mortality forecasting.
- Extend temporal reconciliation to a time-varying setting which updates mortality forecasts as new information emerges.
- Develop a model-agnostic framework to use mortality data observed at different frequencies.

Data and Hierarchy Construction

- Bottom level: monthly death data sourced from CDC for US mortality and from ONS for UK mortality
- A 6-level hierarchy of death count time series

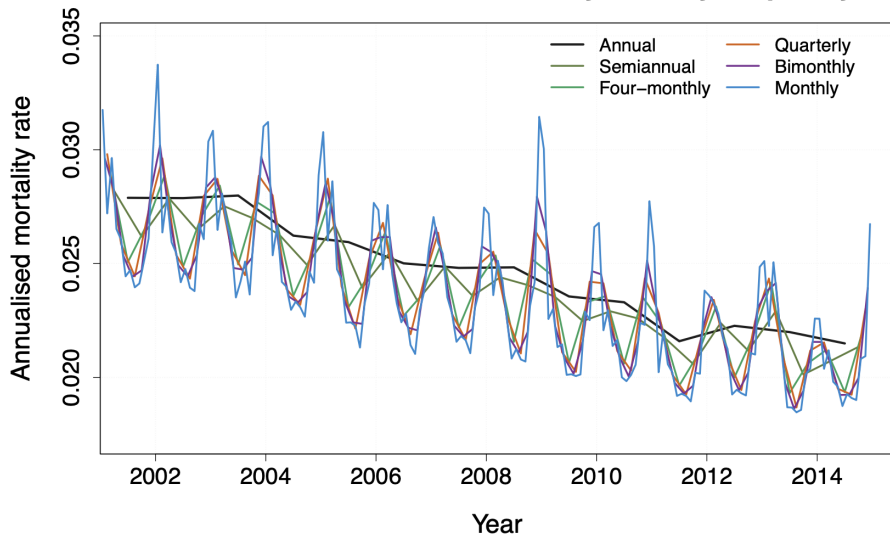


Data Visualisation: UK



Data Visualisation: UK

UK: observed annualised mortality rates by frequency



Basic Approach: Bottom-Up Reconciliation

Suppose we are interested in forecasting annual mortality

Approach 1: **Direct annual forecast**, $\hat{D}_A$

Approach 2: **Bottom-up reconciled forecast**, $\tilde{D}_A^{BU}$

$$\tilde{D}_A^{BU} = \sum_{j=1}^{12} \hat{D}_{mj}$$

- Coherent
- More accurate?

$$\tilde{D}_A^{BU} \text{ or } \hat{D}_A$$

Optimal Combination

Optimal combination approach reconciles forecasts in two steps:

$$\underbrace{\begin{pmatrix} \hat{D}_A \\ \hat{D}_{m1} \\ \hat{D}_{m2} \\ \vdots \\ \hat{D}_{m12} \end{pmatrix}}_{\hat{\mathbf{D}}} = \underbrace{\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}}_S \underbrace{\begin{pmatrix} \beta_{m1} \\ \beta_{m2} \\ \vdots \\ \beta_{m12} \end{pmatrix}}_{\beta_W} + \text{reconciliation error}.$$

Optimal Combination

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Step 1: Estimate β , the adjusted monthly forecasts.

$$\hat{\beta}_W = (\mathbf{S}^\top \mathbf{W}^{-1} \mathbf{S})^{-1} \mathbf{S}^\top \mathbf{W}^{-1} \hat{\mathbf{D}}.$$

$\mathbf{W}$: the covariance matrix for base forecast errors.

Optimal Combination

Optimal combination approach reconciles forecasts in two steps:

$$\underbrace{\begin{pmatrix} \hat{D}_A \\ \hat{D}_{m1} \\ \hat{D}_{m2} \\ \vdots \\ \hat{D}_{m12} \end{pmatrix}}_{\substack{\hat{\mathbf{D}} \\ \text{base forecast}}} = \underbrace{\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}}_{\substack{\mathbf{S} \\ \text{reconciled forecast, } \tilde{\mathbf{D}}}} \underbrace{\begin{pmatrix} \beta_{m1} \\ \beta_{m2} \\ \vdots \\ \beta_{m12} \end{pmatrix}}_{\beta_W} + \text{reconciliation error.}$$

Step 1: Estimate β , the adjusted monthly forecasts.

$$\hat{\beta}_W = (S^\top W^{-1} S)^{-1} S^\top W^{-1} \hat{\mathbf{D}}.$$

W : the covariance matrix for base forecast errors.

Step 2: Calculate reconciled forecasts

$$\tilde{\mathbf{D}}^{Op} = S \hat{\beta}_W = S (S^\top W^{-1} S)^{-1} S^\top W^{-1} \hat{\mathbf{D}}.$$

Reconciliation Covariance Matrix

$$\tilde{D}^{Op} = S \left(S^{\top} W^{-1} S \right)^{-1} S^{\top} W^{-1} \hat{D}.$$

- Estimate W from the in-sample n -step-ahead base forecast errors: $\widehat{W}_n$.
- W depends on n : uncertainty accumulates with longer horizons.
- Shrinkage controls how much cross-series forecast-error dependence is used.



High stability

$$W = \text{diag}(\widehat{W}_n)$$

- Uses variances only.
- Stable, but ignores covariance.

Balanced

$$W = \lambda_n \text{diag}(\widehat{W}_n) + (1 - \lambda_n) \widehat{W}_n$$

- Balances stability and flexibility.
- Tune λ_n for each horizon n .

High flexibility

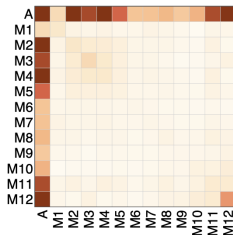
$$W = \widehat{W}_n$$

- Uses full covariance.
- Flexible, but may have high uncertainty

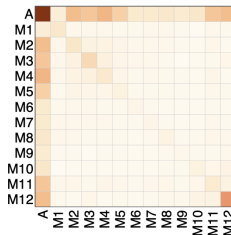
Illustration of The Covariance Matrix

full matrix

UK, $n = 1$ ($\lambda_1 = 0$)

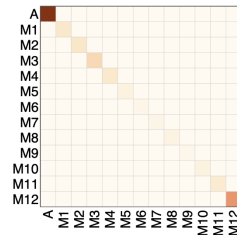


UK, $n = 1$ ($\lambda_1 = 0.5$)

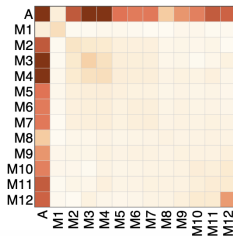


diagonal matrix

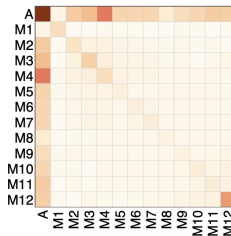
UK, $n = 1$ ($\lambda_1 = 1$)



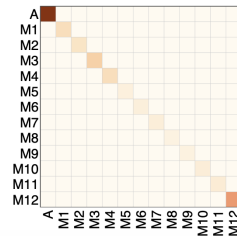
UK, $n = 2$ ($\lambda_2 = 0$)



UK, $n = 2$ ($\lambda_2 = 0.5$)



UK, $n = 2$ ($\lambda_2 = 1$)



Time-Varying Reconciliation

Consider T -year-ahead forecast at current month h

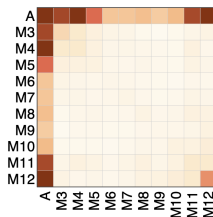
- h new monthly data points observed; remaining forecast steps reduce
- base forecast errors should change with h ,
 - case 1: $n=1, h=1$
 - case 2: $n=1, h=11$

We extend W_n to $W_{n,h}$, which accounts for the evolution of errors over month h , $h = \{0, 1, 2, \dots, 11\}$

$$\tilde{D}_{n,h}^{Op} = S_{n,h} \left(S_{n,h}^\top W_{n,h}^{-1} S_{n,h} \right)^{-1} S_{n,h}^\top W_{n,h}^{-1} \hat{D}_{n,h}.$$

Time-Varying Weight Matrix, $W_{n,h}$

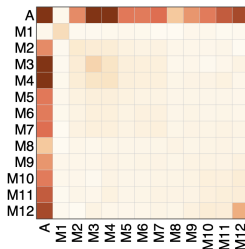
$W_{1,2}$, UK mortality



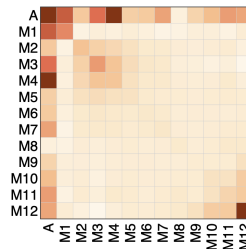
$W_{1,10}$, UK mortality



$W_{2,2}$, UK mortality



$W_{2,10}$, UK mortality



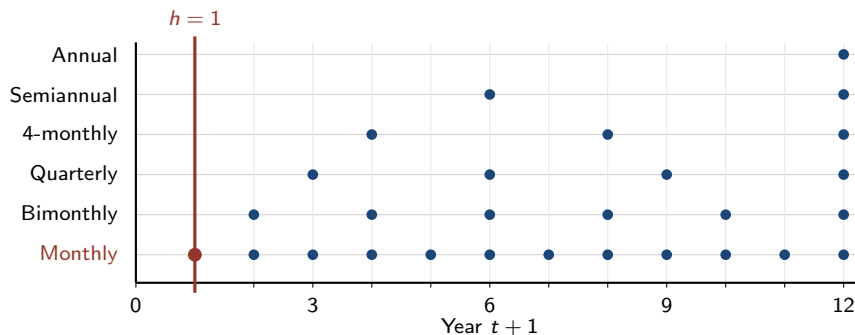
Base-Level Model: Linear-Interpolated APC Model

Linear-interpolated APC model (Bégin et al. (2025)).

$$\log m_{x,t,h}^{(f)} = \underbrace{\alpha_x^{(f)}}_{\text{age effect}} + \underbrace{\tilde{k}_1^{(f)}(t, h) + (x - \bar{x})\tilde{k}_2^{(f)}(t, h)}_{\text{period terms}} + \underbrace{\tilde{\gamma}^{(f)}(t - x, h)}_{\text{cohort term}} + \underbrace{w_x^{(f)} s_h^{(f)}}_{\text{seasonality term}} + \varepsilon_{x,t,h}^{(f)}.$$

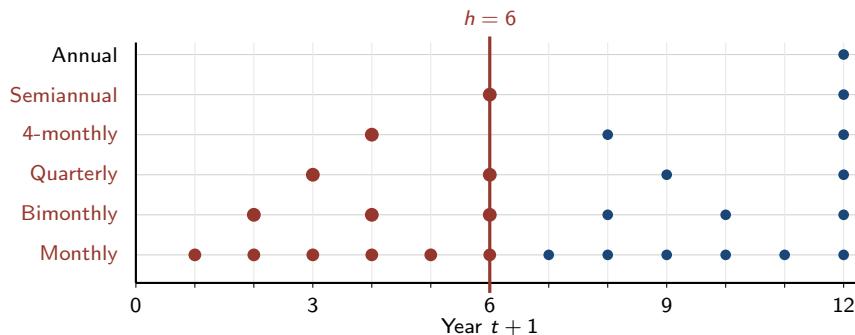
- (f) : frequency

Arrival of New Intra-Year Information



At Month 1. Update models by refitting up to Month 1. Only monthly model is updated

Arrival of New Intra-Year Information



At Month 6. Models on all higher frequencies are updated

Forecasting Experiment

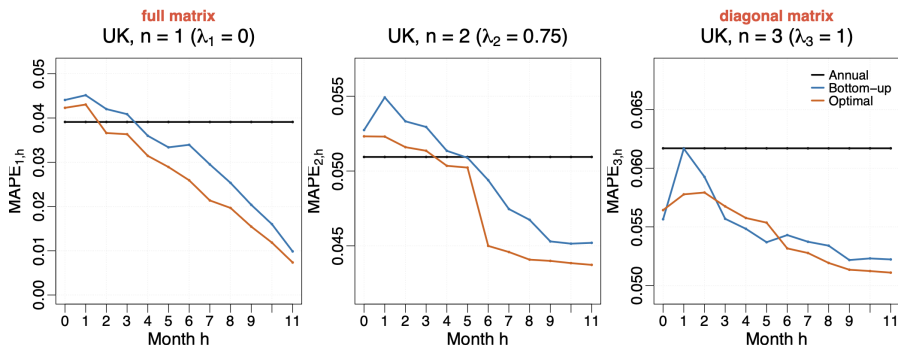
Rolling design

- Countries: UK and US.
- Forecast origin month: $h = 0, 1, \dots, 11$.
- Forecast horizons: $n = 1, 2, 3$ years ahead.
- Accuracy measure: MAPE of death-count forecasts.
- Full temporal hierarchy with 6 levels
- At each year horizon, we tune shrinkage parameter λ_n from five candidate values $\{0, 0.25, 0.5, 0.75, 1\}$

Main comparison

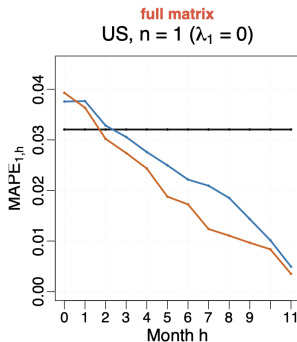
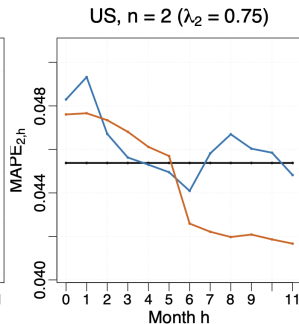
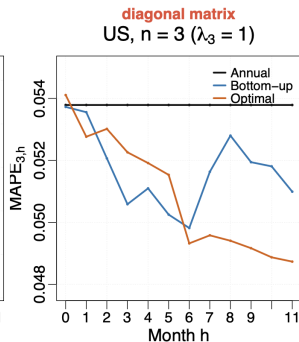
- Direct annual forecast
- Reconciled annual forecast by bottom-up
- Reconciled annual forecast by optimal combination

Performance for UK



- λ_n increase with forecast horizon $n \rightarrow$ more emphasis on stability
- Reconciliation methods reduce errors from annual direct forecast.
- Time-varying feature leads to gradual improvement in accuracy
- Optimal combination approach combines forecasts more effectively

Performance for US



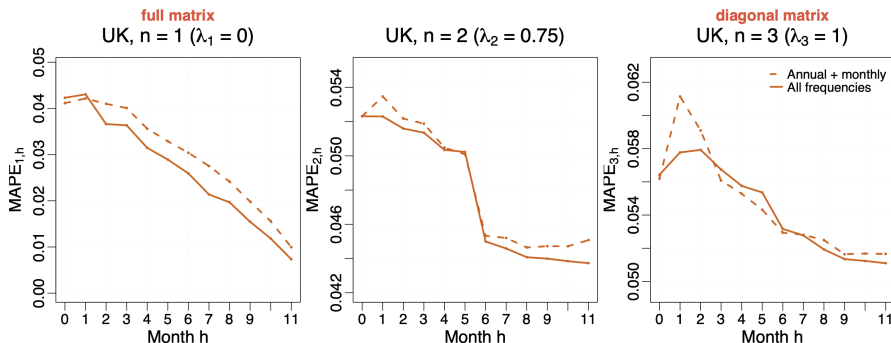
- Reconciliation is more effective in reducing errors, when base forecasts are subject to larger variance

Effect of Hierarchy Richness

We compare reconciliation with

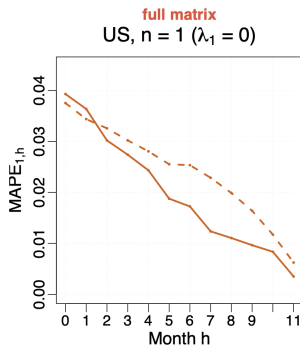
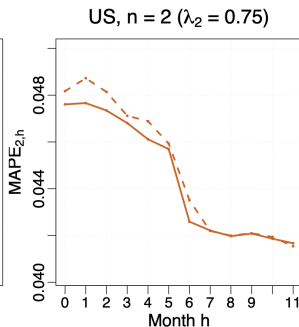
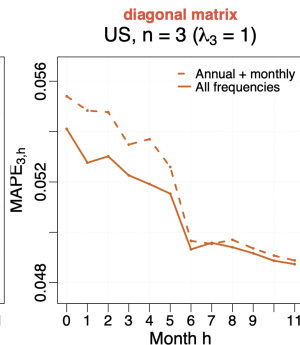
- Full hierarchy: All six frequencies
- Minimal hierarchy: annual and monthly frequencies

Effect of Hierarchy Richness: UK Case



- Reconciliation using 6-level hierarchy structure outperforms.
- Richer hierarchy provides more information
 - improves accuracy and stability

Effect of Hierarchy Richness: US Case



Conclusion

This project focuses on improving mortality forecasts when new information arrives at different frequencies.

- Introduce temporal reconciliation to mortality forecasting to improve accuracy and ensure coherence
- Enhance reconciliation performance by real-time update using time-varying covariance matrix
- Propose a model-agnostic framework that can be applied to forecasts produced from a range of mortality models

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