

# Quantum-Compatible Vitagion Mortality Modelling

## A Quantum Representation of Age–Time Non-Separability

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Longevity Conference

# From Lee–Carter to Lee–Carter 2

## Classical Lee–Carter

$$D_{x,t} = \log m_{x,t} - a_x$$

$$D \simeq b_1 k_1^T$$

### Rank-one approximation

One dominant age profile with time-varying intensity.

### Insight from PRICONA

Numerical and experimental evidence shows that

**PC1 captures the common underlying structure**

while

**PC2 contains information on effects acting on that structure.**

## Lee–Carter 2

$$D \simeq b_1 k_1^T + b_2 k_2^T$$

### Minimal spectral extension

Mode 1: dominant common mortality structure.

Mode 2: systematic information remaining beyond that structure.

## Why retain the second component?

Not merely to explain more variance, but to recover information that may be qualitatively different from the dominant common structure.

# Quantum computing I: from bit strings to quantum states

## The same $2^n$ bit strings define the computational basis

**Classical bit strings** 000, 001, 010, ..., 111  $\longleftrightarrow$  **Quantum basis states**  $|000\rangle, |001\rangle, |010\rangle, \dots, |111\rangle$

Each basis state  $|i\rangle$  corresponds to one  $n$ -bit string;  $i = 0, \dots, 2^n - 1$  labels that string.

## Classical register: $n$ bits

At any given time, the register occupies **one** of the  $2^n$  possible bit strings.

$i \equiv$  one  $n$ -bit string

Example for  $n = 3$ :  $i = 5 \leftrightarrow 101$ .

## Quantum register: $n$ qubits

A pure state may be a coherent superposition of those same basis states:

$$|\psi\rangle = \sum_{i=0}^{2^n-1} c_i |i\rangle, \quad \sum_i |c_i|^2 = 1$$

The amplitudes  $c_i$  weight the basis states  $|i\rangle$ .

## Readout / measurement

**Classical:**  $i \xrightarrow{\text{readout}} i$

**Quantum:**  $|\psi\rangle \xrightarrow{\text{readout / measurement}} \underbrace{i}_{\text{one } n\text{-bit string}},$

$$P_i = |c_i|^2$$

# Quantum computing II: classical vs quantum evolution

## Classical computation

A classical operation transforms one register configuration into another:

$$x \xrightarrow{f} f(x).$$

For example,

$$101 \xrightarrow{f} 011 \xrightarrow{\text{readout}} 011.$$

The final state is directly readable as the result of the operation.

## Quantum computation

**Quantum gates** are the elementary operations of a quantum computer. A gate — or a sequence of gates — implements a unitary transformation  $U$ .

$$|\psi\rangle = \sum_i c_i |i\rangle \xrightarrow{U} U|\psi\rangle.$$

By linearity,

$$U|\psi\rangle = \sum_i c_i U|i\rangle.$$

## Where the quantum advantage is sought

Not simultaneous readout of all basis states, but the ability to **evolve the coherent superposition as a whole** before measurement.

# Quantum computing III: engineering the measurement probabilities

## After quantum evolution

$$U|\psi\rangle = \sum_i c_i^{(U)} |i\rangle.$$

A computational-basis measurement samples **one bit string**  $i$  according to

$$P_i = |c_i^{(U)}|^2.$$

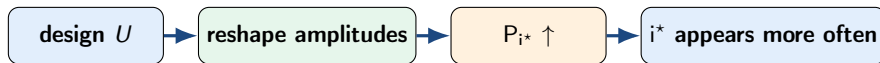
A single measurement need not return the most probable string.

## Making a desired outcome likely

Let  $i^*$  denote a desired bit string. To make it occur more frequently over repeated measurements, we must increase

$$P_{i^*} = |c_{i^*}^{(U)}|^2.$$

Thus  $U$  must be designed to reshape the amplitudes before measurement.



## Amplitude engineering

Quantum computation engineers amplitudes before measurement so that the **desired outcomes become more likely to be observed.**

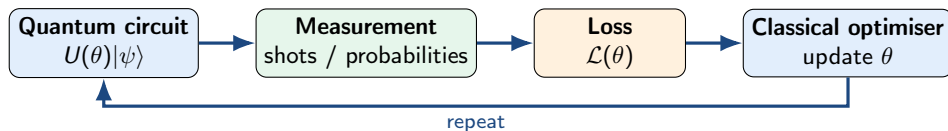
# Quantum computing IV: the variational quantum–classical loop

## What is fixed and what is learned?

We choose the **architecture** of a parametrised circuit in advance, but the gate parameters

$$\theta = (\theta_1, \dots, \theta_p)$$

are learned iteratively from measurement outcomes.



$$\theta^{(0)} \rightarrow \theta^{(1)} \rightarrow \theta^{(2)} \rightarrow \dots, \quad \theta^* \approx \arg \min_{\theta} \mathcal{L}(\theta).$$

## Why this matters for the QSVD

The objective is not simply to make one “solution state” maximally probable. In the variational QSVD, measurements define a problem-specific loss, and the classical optimiser learns circuit parameters that reveal the singular-vector structure encoded in the quantum state.

# From a mortality matrix to a bipartite quantum state

## 1. Two indices $\rightarrow$ two quantum registers

$$D = (D_{xt}), \quad x = \text{age}, \quad t = \text{time}$$

$$x \longleftrightarrow |x\rangle_A$$

$$t \longleftrightarrow |t\rangle_B$$

A: age register    B: time register

$$(x, t) \longleftrightarrow |x\rangle_A |t\rangle_B$$

## 2. Matrix entries $\rightarrow$ quantum amplitudes

$$|\psi_D\rangle = \frac{1}{\|D\|_F} \sum_{x,t} D_{xt} |x\rangle_A |t\rangle_B$$

Each normalized entry  $D_{xt}/\|D\|_F$  is the amplitude of the basis state associated with  $(x, t)$ .

Example:  $D_{2,3} \rightarrow (D_{2,3}/\|D\|_F) |2\rangle_A |3\rangle_B$ .

## 3. The SVD becomes the Schmidt decomposition of the same state

$$D = \sum_j \sigma_j u_j v_j^\dagger \implies$$

$$|\psi_D\rangle = \sum_j \underbrace{\frac{\sigma_j}{\|D\|_F}}_{\lambda_j} |u_j\rangle_A |v_j\rangle_B$$

$$|u_j\rangle_A = \sum_x (u_j)_x |x\rangle_A, \quad |v_j\rangle_B = \sum_t (v_j)_t |t\rangle_B.$$

# Entanglement: quantum correlations between two qubits

## Separable state

Two qubits  $A$  and  $B$  are separable if

$$|\psi\rangle_{AB} = |\psi_A\rangle \otimes |\psi_B\rangle$$

For example,

$$|0\rangle_A \otimes \frac{|0\rangle_B + |1\rangle_B}{\sqrt{2}}.$$

**Each qubit has its own quantum state.**

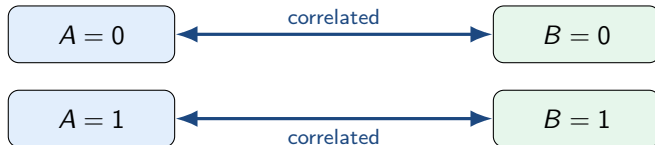
## Entangled state

A Bell state cannot be factorized:

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$P(00) = P(11) = \frac{1}{2}, \quad P(01) = P(10) = 0.$$

**Individual outcomes are uncertain,  
but the two outcomes are perfectly correlated.**



# Variational QSVD: learning the singular bases

## Encoded matrix in Schmidt form

$$|\psi_D\rangle = \sum_i \lambda_i |u_i\rangle_A |v_i\rangle_B, \quad \lambda_i = \sigma_i / \|D\|_F.$$

Register A  $|u_i\rangle_A \rightarrow U_A(\theta_1) \rightarrow e^{ip_i}|i\rangle_A$

Register B  $|v_i\rangle_B \rightarrow V_B(\theta_2) \rightarrow e^{iq_i}|i\rangle_B$

$$(U_A \otimes V_B)|\psi_D\rangle = \sum_i \lambda_i e^{i(p_i+q_i)} |i\rangle_A |i\rangle_B$$

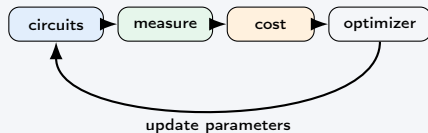
## Ideal measurement pattern

(00,00) (01,01) (10,10)

**matching outcomes**

The same basis index is observed in both registers.

## Variational learning loop



The circuits learn rotations from the unknown singular bases to a common measurable basis.

# From measurements to the variational objective

## Measurement shots

| shot | A  | B  |   |
|------|----|----|---|
| 1    | 00 | 00 | ✓ |
| 2    | 01 | 10 | × |
| 3    | 10 | 10 | ✓ |
| 4    | 01 | 01 | ✓ |

Matching score:

$$C_{\text{match}} = \frac{1}{N_m} \sum_{x=1}^{N_m} \delta(p_x, q_x)$$

$$\delta(a, b) = \begin{cases} 1, & a = b, \\ 0, & a \neq b. \end{cases}$$

For the example:  $C_{\text{match}} = 3/4$ .

## Optimization target

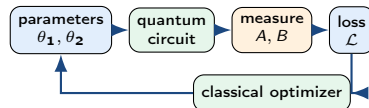
Correct alignment implies

$$C_{\text{match}} \rightarrow 1,$$

$$\mathcal{L}(\theta_1, \theta_2) = 1 - C_{\text{match}}(\theta_1, \theta_2)$$

and hence

$$(\theta_1^*, \theta_2^*) = \arg \min_{\theta_1, \theta_2} \mathcal{L}.$$



## Interpretation

$$C_{\text{match}} \uparrow \iff \mathcal{L} = 1 - C_{\text{match}} \downarrow.$$

Liu et al. define the matching quantity through coincident bit strings; here  $1 - C_{\text{match}}$  is used explicitly as the minimization loss.

# From the optimized QSVD to the singular components

$$|\psi_{\text{out}}\rangle \simeq \sum_i \lambda_i e^{i\phi_i} |i\rangle_A |i\rangle_B, \quad \lambda_i = \frac{\sigma_i}{\|D\|_F}.$$

## Measurements $\longrightarrow$ singular values

$$P(i_A = i, i_B = i) = |\lambda_i|^2 = \frac{\sigma_i^2}{\|D\|_F^2}$$

$$\boxed{\sigma_i = \|D\|_F \sqrt{P(i_A = i, i_B = i)}}$$

$N_m$  shots give

$$\hat{P}(i_A = i, i_B = i) = \frac{N_{ij}}{N_m}.$$

$(0, 0), (1, 1), (2, 2), \dots, (r-1, r-1), \quad r = \text{rank}(D).$

Here  $i_A$  and  $i_B$  are the outcomes measured on the two distinct registers; a matching event satisfies  $i_A = i_B = i$ .

## Optimized circuits $\longrightarrow$ singular vectors

From

$$U_A(\theta_1^*) |u_i\rangle_A = e^{ip_i} |i\rangle_A,$$

we obtain

$$\boxed{|u_i\rangle_A \simeq e^{-ip_i} U_A^\dagger(\theta_1^*) |i\rangle_A}$$

and similarly

$$\boxed{|v_i\rangle_B \simeq e^{-iq_i} V_B^\dagger(\theta_2^*) |i\rangle_B}.$$

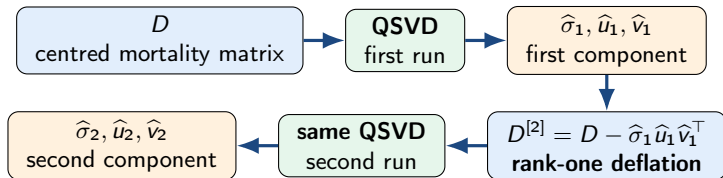
The learned inverse transformations encode the singular-vector bases.

## Reconstructing the dominant components

$$D = \sum_i \sigma_i u_i v_i^\dagger,$$

$$\boxed{D^{(2)} \simeq \sigma_1 u_1 v_1^\dagger + \sigma_2 u_2 v_2^\dagger}$$

# Sequential QSVD for two-component Lee–Carter



## 1. First QSVD component

$$D \xrightarrow{\text{QSVD}} (\hat{\sigma}_1, \hat{u}_1, \hat{v}_1)$$
$$\hat{D}_1 = \hat{\sigma}_1 \hat{u}_1 \hat{v}_1^\top$$
$$b^{(1)} \propto \hat{u}_1, \quad k^{(1)} \propto \hat{\sigma}_1 \hat{v}_1.$$

## 2. Deflate and apply QSVD again

$$D^{[2]} = D - \hat{D}_1$$
$$D^{[2]} \xrightarrow{\text{same QSVD}} (\hat{\sigma}_2, \hat{u}_2, \hat{v}_2)$$
$$b^{(2)} \propto \hat{u}_2, \quad k^{(2)} \propto \hat{\sigma}_2 \hat{v}_2.$$

## Two-component reconstruction

Rank-2 matrix reconstruction

$$\hat{D}^{(2)} = \hat{\sigma}_1 \hat{u}_1 \hat{v}_1^\top + \hat{\sigma}_2 \hat{u}_2 \hat{v}_2^\top$$

Two-component Lee–Carter

$$\log \hat{m}_{x,t} = a_x + b_x^{(1)} k_t^{(1)} + b_x^{(2)} k_t^{(2)}$$

The same QSVD is applied twice, with rank-one deflation in between.

## Conclusions: from LC2 to a new question

### From LC to LC2

$$D^{(1)} = \sigma_1 u_1 v_1^\top$$

$\Downarrow$

$$D^{(2)} = \sum_{j=1}^2 \sigma_j u_j v_j^\top$$

A second data-driven component moves mortality modelling beyond a single separable age–time structure.

### What the quantum representation reveals

$\text{rank}(D) = 1 \iff \text{Schmidt rank} = 1$   
**separable state**

$\text{rank}(D) > 1 \iff \text{Schmidt rank} > 1$   
**entangled state**

Entanglement makes explicit the **non-separability of the encoded age–time structure**; it does not imply that mortality is a physical quantum phenomenon.

### Quantum-compatible estimation

$\text{SVD}(D) \iff \text{Schmidt}(|\psi_D\rangle)$

$\text{QSVD}_1 \rightarrow \text{deflation} \rightarrow \text{QSVD}_2$

The same rank-one QSVD primitive can be reused sequentially, trading deflation error propagation against quantum-circuit complexity.

*The contribution is a structural quantum correspondence and a quantum-compatible architecture – not yet a claim of quantum computational advantage.*

## The question beyond computation

What does age–time non-separability tell us about mortality dynamics?

# Thank you for your attention

Questions and discussion

**Quantum-Compatible Vitagion Mortality Modelling**

Two-component Lee–Carter and variational QSVD