Longevity Risk in China and its Financial Impact: Evidence from Model Test

Xiao WEI
Joint work with Chenzhe LIU

China Institute for Actuarial Science (CIAS)
Central University of Finance and Economics (CUFE)
Beijing, China

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Outline of Topics

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- Literature Review
- Motivation

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- Mortality trend of Chinese case and other countries
- GLM of mortality of two countries

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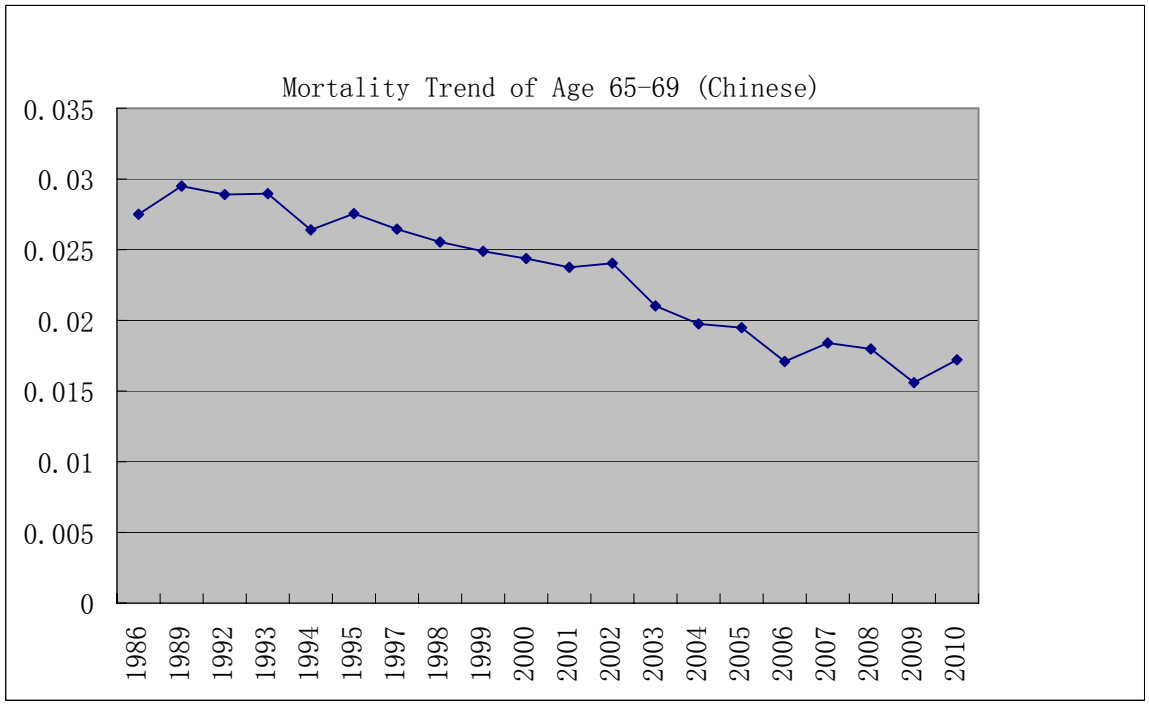
- Error comparison of Lee-Carter Model with two data set

Longevity Risk of China
- Longevity Risk of China

Chinese Population Data

- Missing Data: Chinese population Data is available since 1986, missing data of year 1987, 1988, 1990, 1991, 1996
- Inconsistent structure:
  - Some provide for individual age, some only available for grouped-age format (range of five years)
  - Some provide mortality data of longer age (range at 100+), some only available for 85+
  - Some are original census data, some based on sample data

Chinese Mortality Trend



Literature Review

- Chinese Case:
  - Wang & Cai (2008) concluded that the Lee-Carter model is the preferable model for Chinese data based on the analysis and comparison of different mortality models.
  - Li & Liu (2010) used Chinese data from 1992-2007 to fit the Lee-Carter Model with SVD, OLS, WLS methods and found that the WLS outperforms the other two methods.
- International Case:
  - According to data of POPIN, the expectancy of life of Chinese, Indian, Japanese, Brazilians and American increased 20.5 years from 1995 to 2005
  - Amato et al (2012) detected longevity common trends of multiple population

Motivation

- How to make up for the deficiency of Chinese data
- How to unify the inconsistent structure of Chinese data
- Improve the forecast ability of Lee-Carter model for Chinese Case

The Lee-Carter Model

Lee-Carter(1992) proposed a log-bilinear model for the force of mortality:

$$m_{x,t} = \exp\left(\alpha_x + \beta_x \kappa_t + \epsilon_{x,t}\right)$$

$$\log(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t} \tag{1}$$

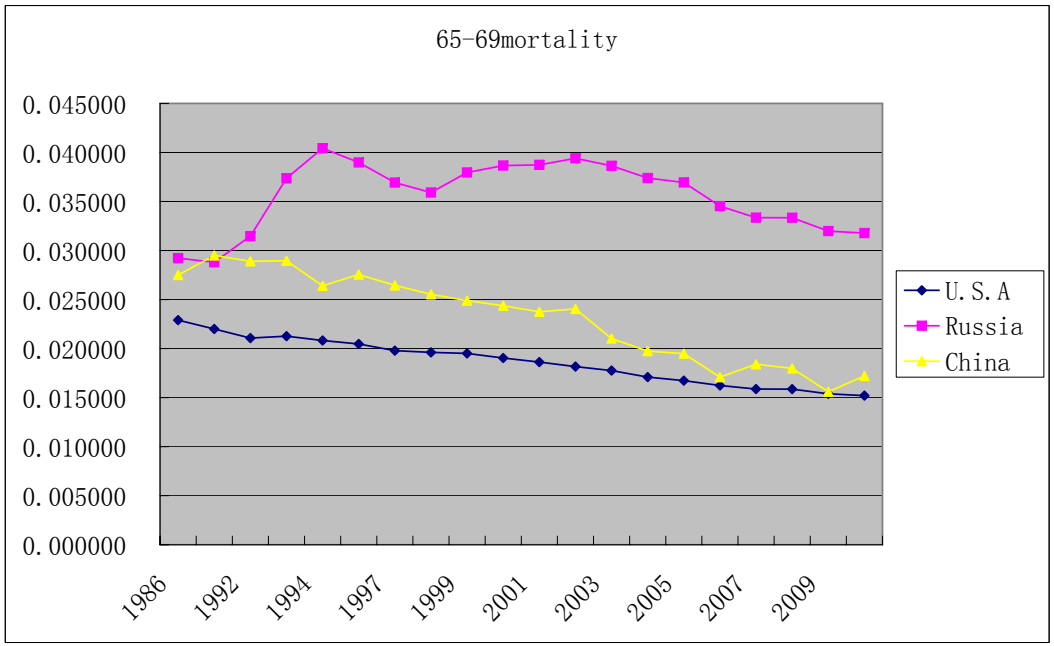
where $m_{x,t}$ is the force of mortality of population of age x at time t

$$\sum_t \kappa_t = 0, \quad \sum_x \beta_x = 1.$$

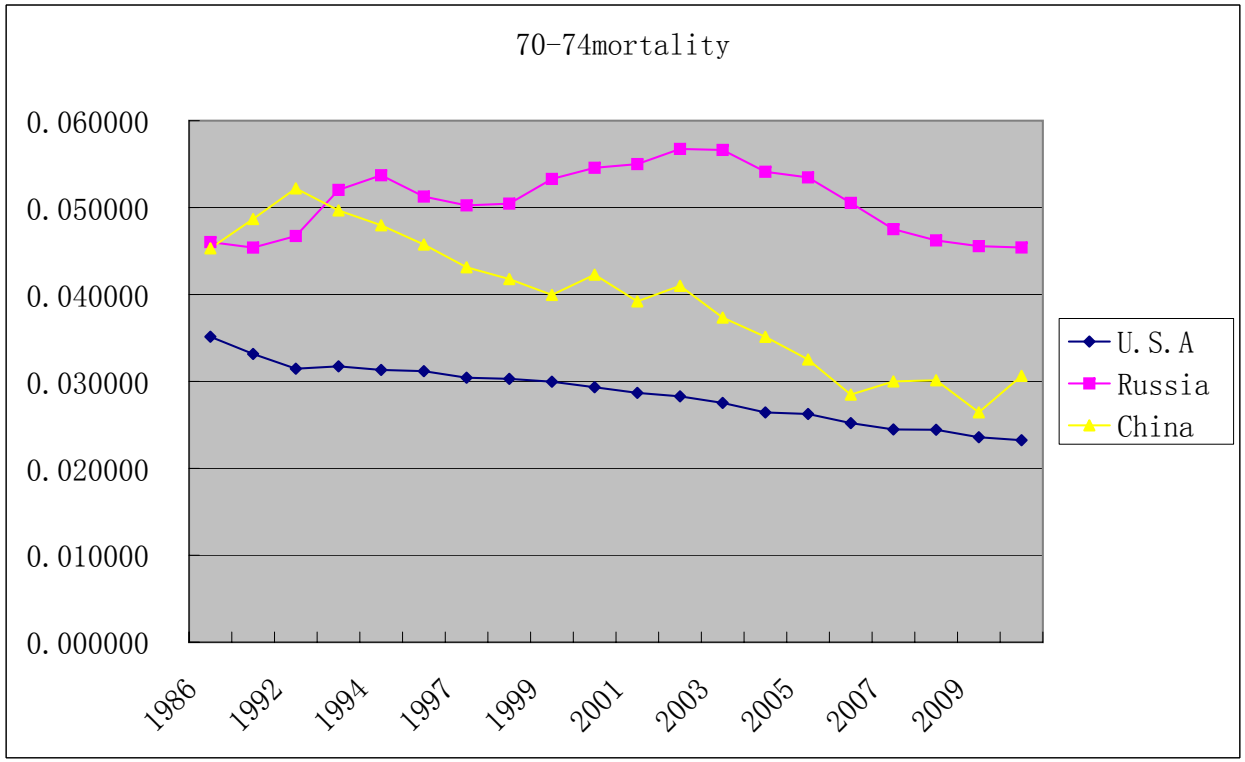
Mortality trend of Chinese case and other countries

Which country can be used to project the Chinese missing data?

- According to data of POPIN, the expectancy of life of Chinese, Indian, Japanese, Brazilians and American increased 20.5 years from 1995 to 2005



Mortality Trend



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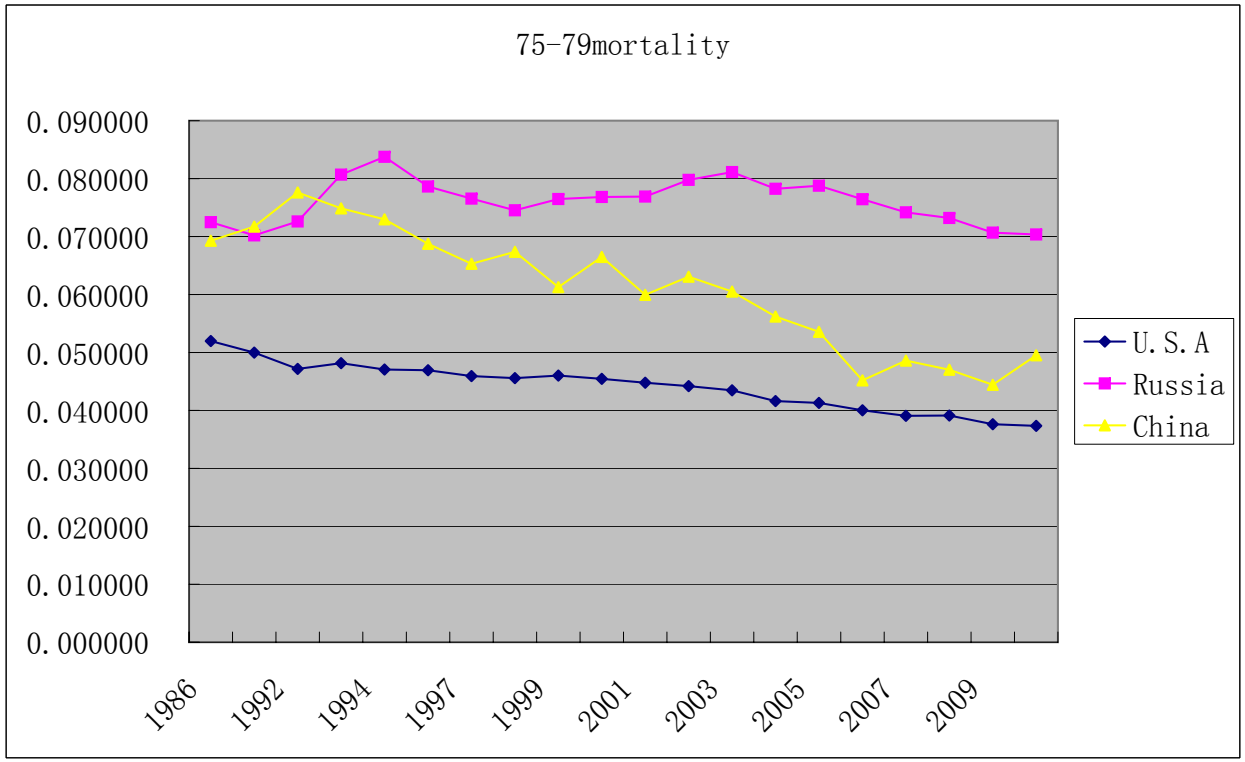
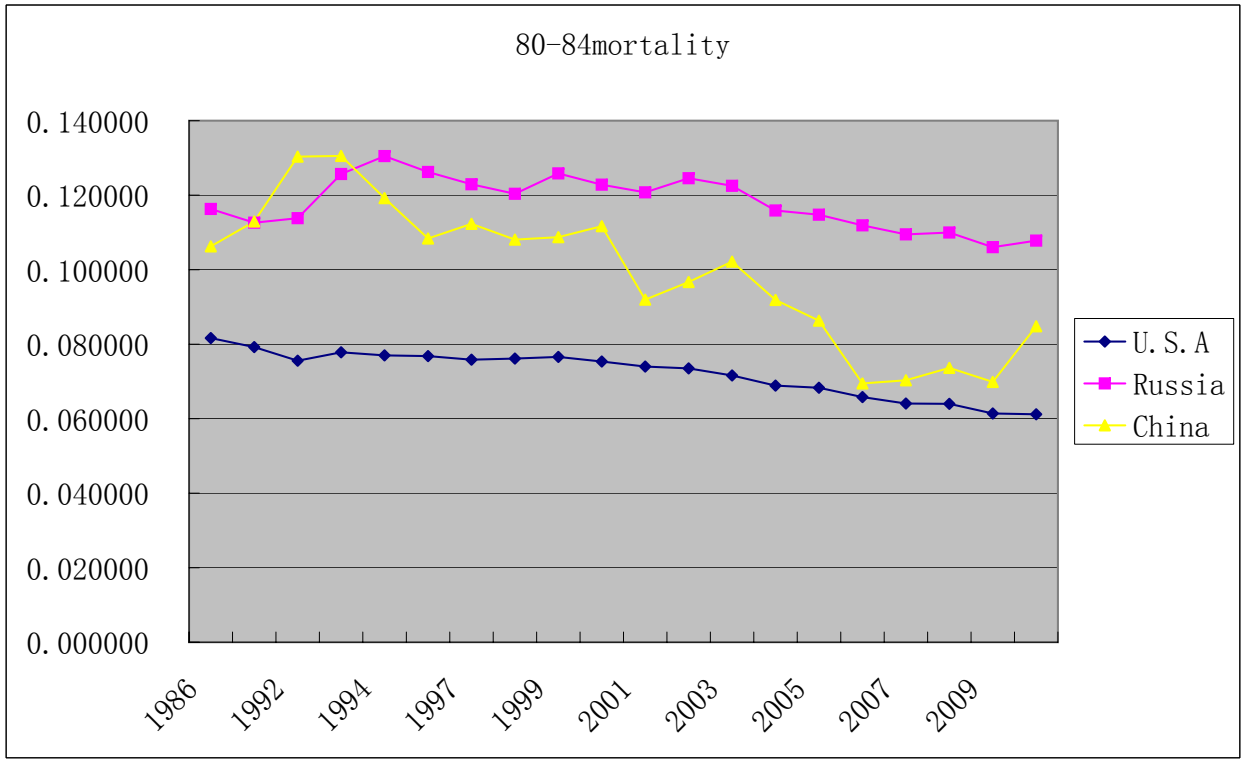


Mortality Trend



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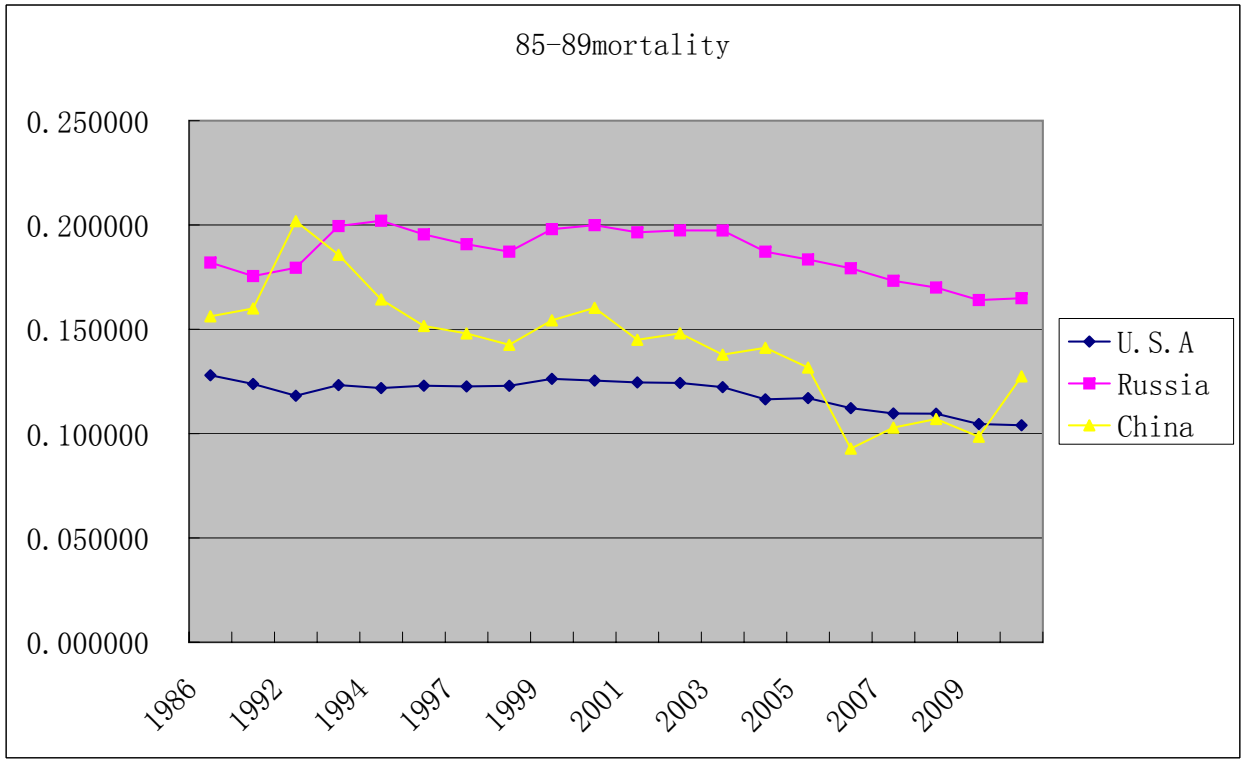

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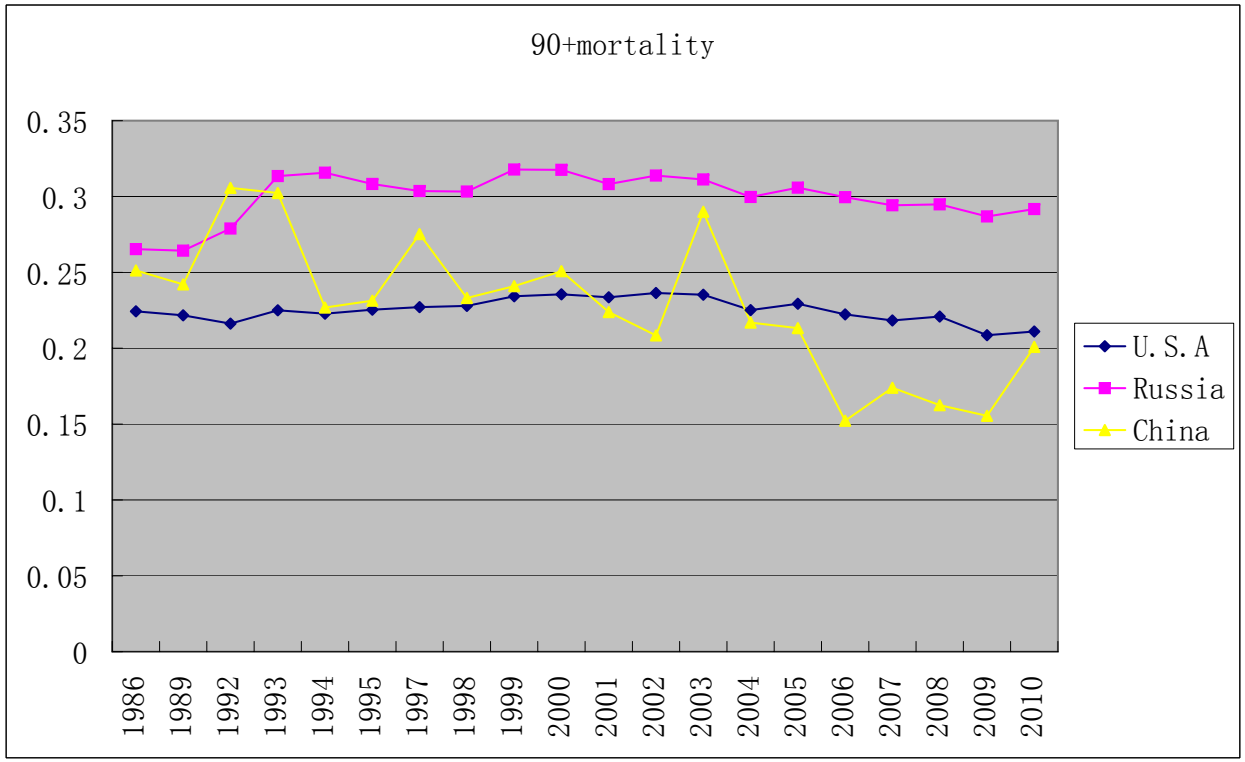


GLM of mortality of two countries

- We propose to build mortality rate model between Chinese data and one other country to find out the common trend of the two countries, by this model, the missing mortality data of Chinese population will be projected.
- For population of age x,

$$PRC_{x,t} = c_{x,1} + c_{x,2}X_{x,t} + \epsilon_t \tag{2}$$

where $PRC_{x,t}$ and $X_{x,t}$ are the mortality of Chinese population of age x at time t and that of other country respectively.

- From the mortality trend, we take the US population as a reference population.

GLM fitted result

$$PRC65 = 1.854417 \times USA65 - 0.011407 \qquad R^2 = 0.927678$$

T Value	(15.19497)	(-4.969037)
Std Error	(0.122041)	(0.002296)

$$PRC70 = 2.119014 \times USA70 - 0.021242 \qquad R^2 = 0.846972$$

T Value	(9.981272)	(-3.474302)
Std Error	(0.212299)	(0.006114)

$$PRC75 = 2.289359 \times USA75 - 0.039864 \qquad R^2 = 0.824245$$

T Value	(9.187766)	(-3.609862)
Std Error	(0.249175)	(0.011043)

GLM fitted result

$$PRC80 = 2.630394 \times USA80 - 0.090740 \qquad R^2 = 0.729333$$

T Value	(6.964368)	(-3.314163)
Std Error	(0.377693)	(0.027380)

$$PRC85 = 2.608392 \times \mathrm{USA85} - 0.167495 \qquad R^2 = 0.467645$$

T Value	(3.976431)	(-2.142268)
Std Error	(0.655963)	(0.078186)

Model fitted of Lee-Carter Model

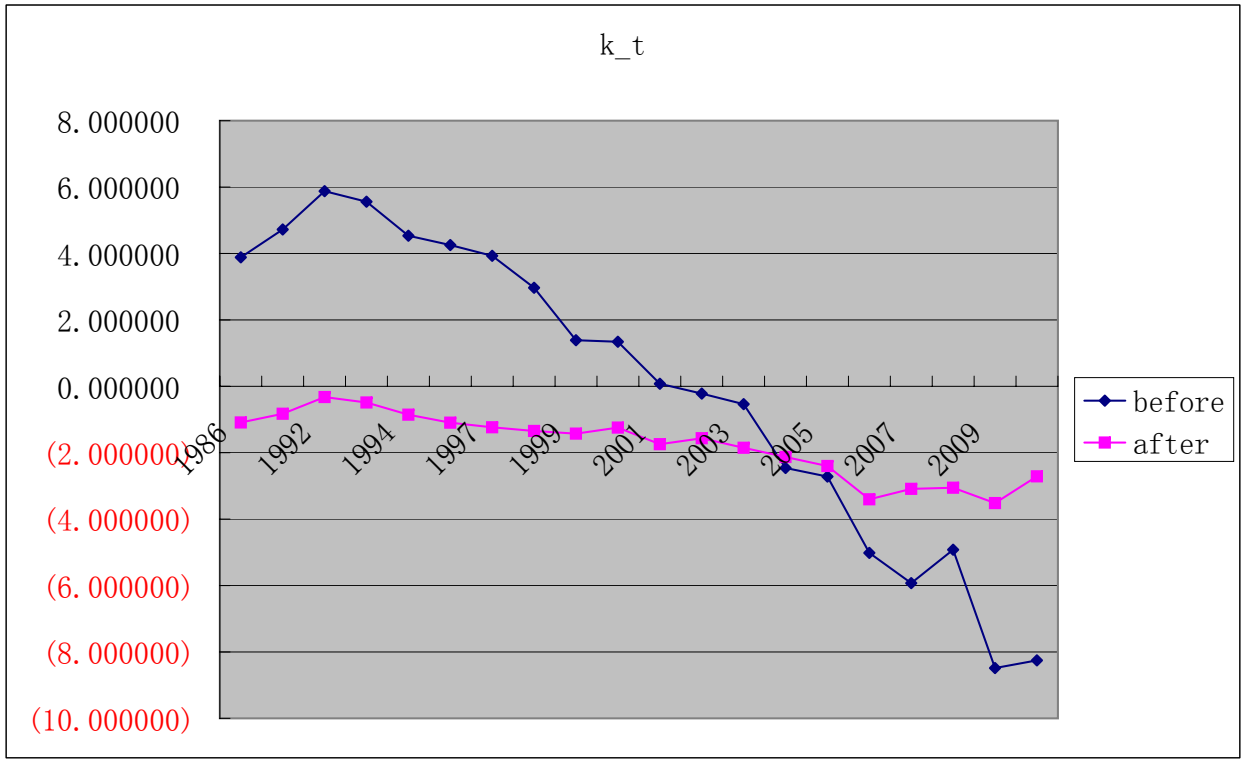
α

	Data 1	Data 2
65-69	-3.7818359	-3.40862
70-74	-3.25371062	-2.8889
75-79	-2.80815369	-2.44472
80-84	-2.32835919	-1.97534
85-89	-1.9648159	-1.64941

β

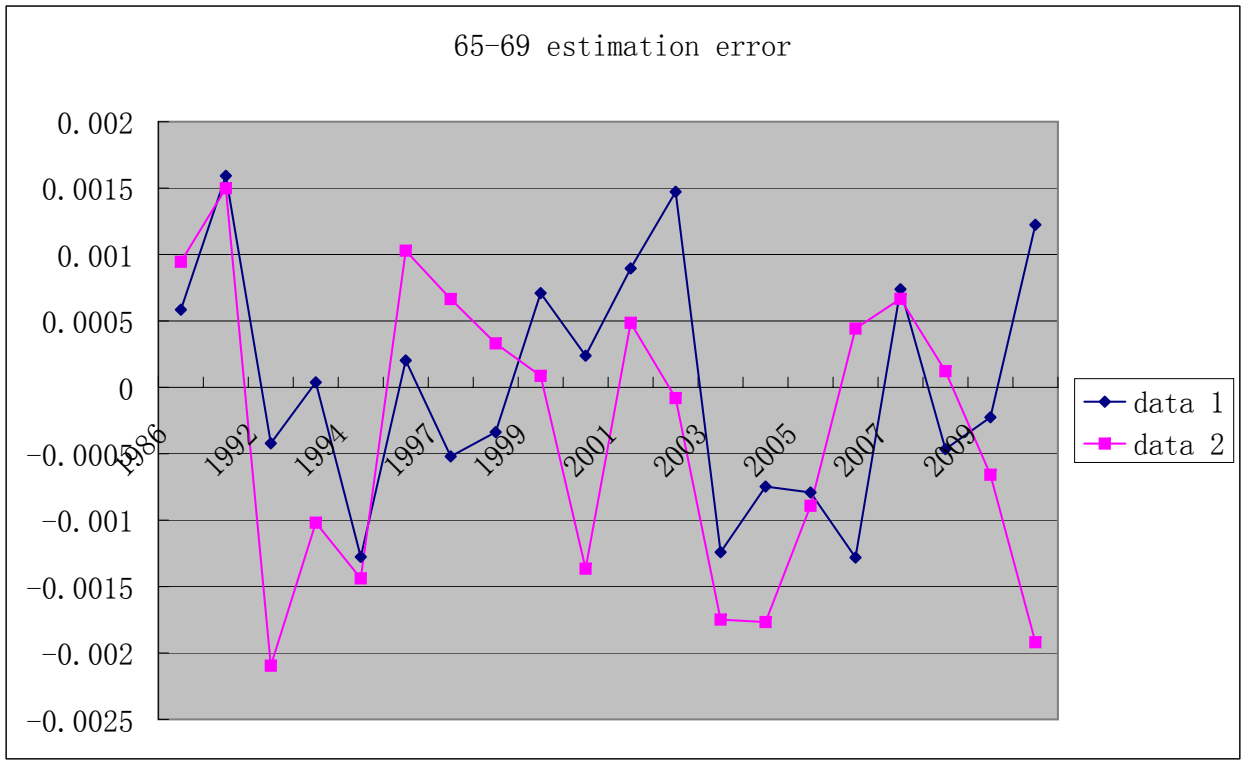
	Data 1	Data 2
65-69	0.042938	0.201965
70-74	0.042894	0.197779
75-79	0.036559	0.203496
80-84	0.040105	0.202383
85-89	0.038116	0.194377

Model fitted of Lee-Carter Model



Error comparison of Lee-Carter Model with two data set

Data 1: raw data, Data 2: raw data with generated data from GLM



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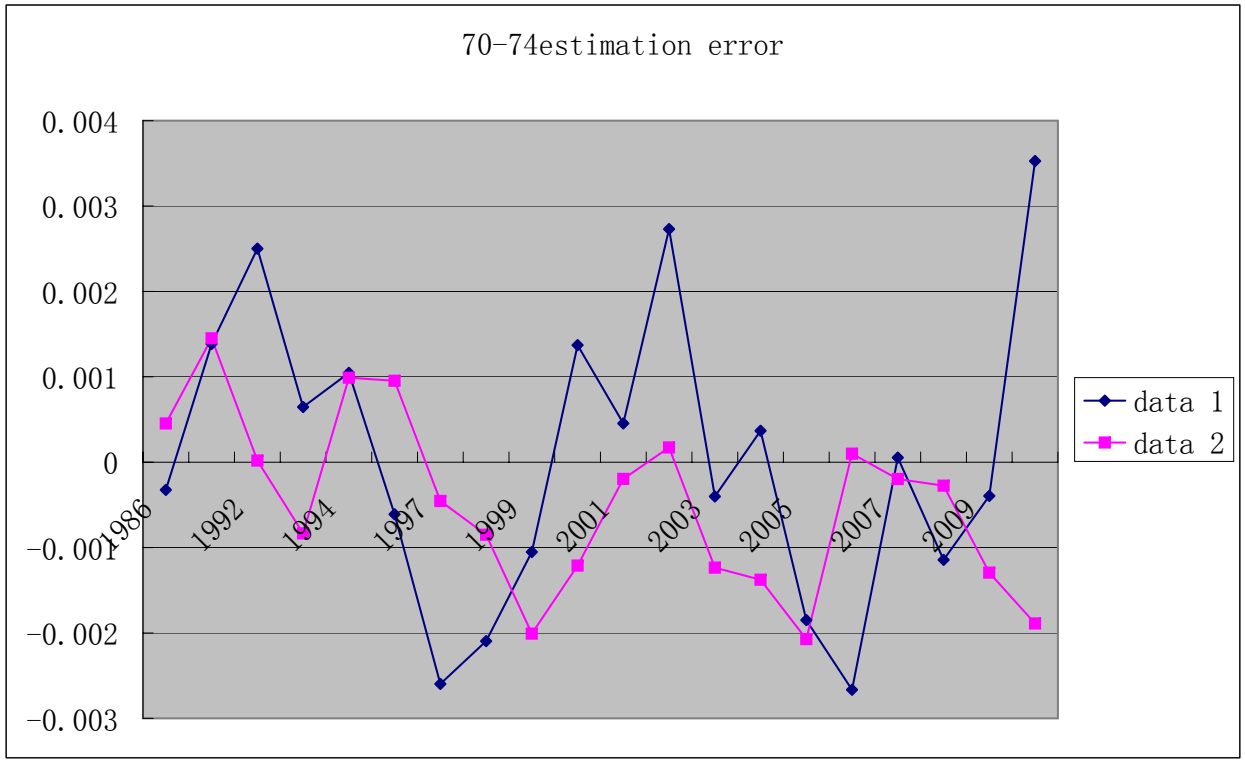

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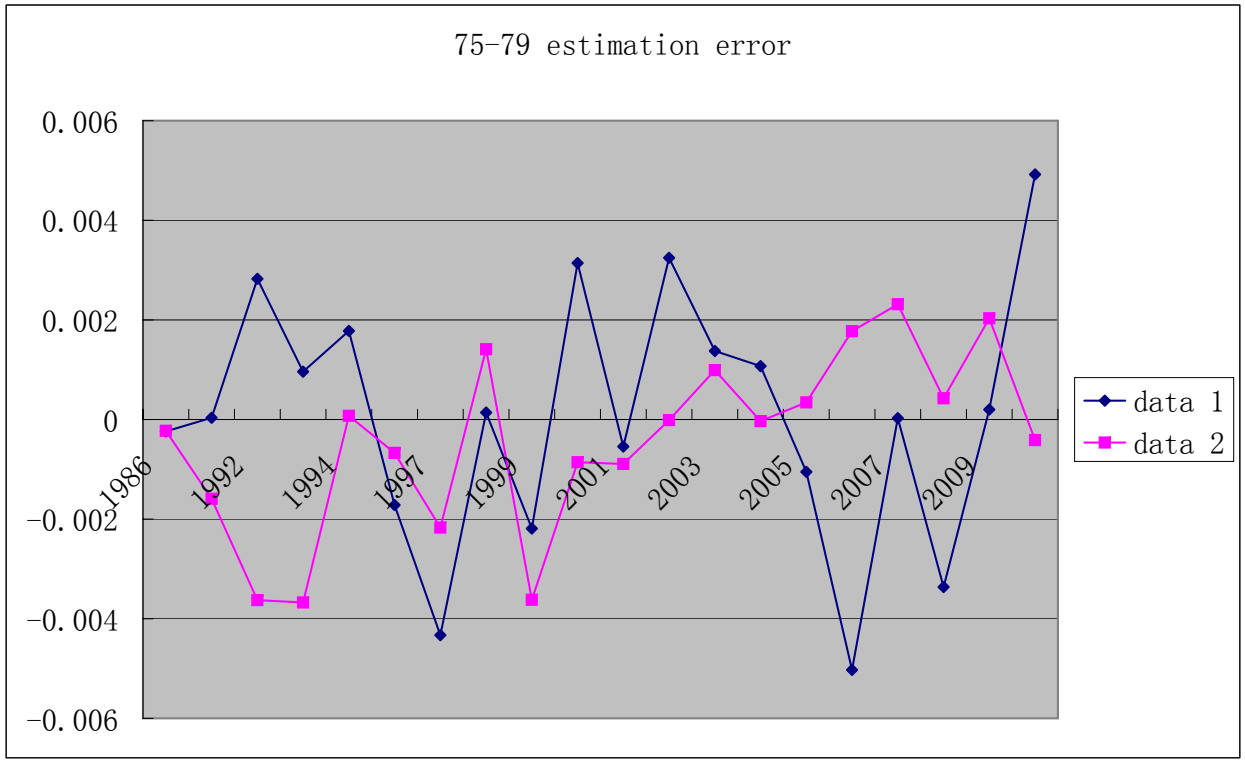

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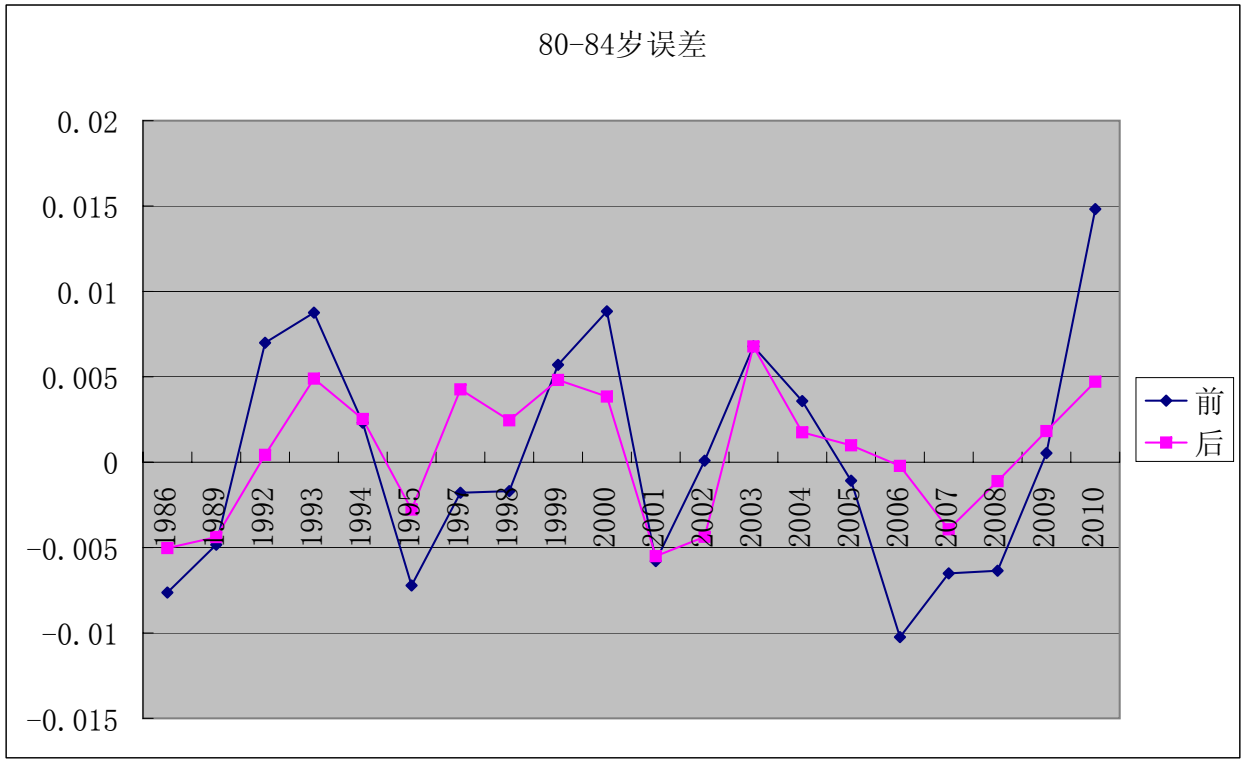

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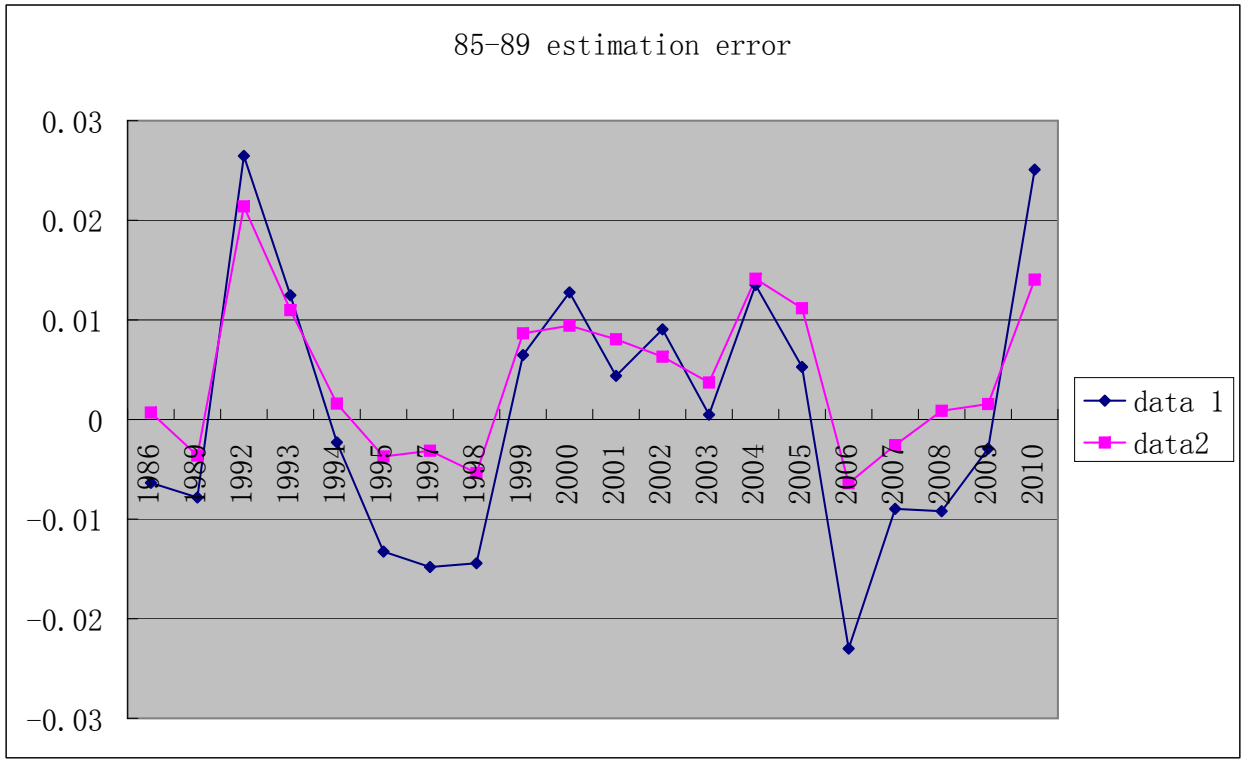

Longevity Risk of China

The longevity risk is proved to exist in China, the projected mortality rate is overestimated by Lee-Carter model with the raw data. With the enhance data generated by GLM using other country's more complete data will improved the forecast ability of Lee-Carter model.

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Thank you for your attention!