Pension Risk Management with Funding and Buyout Options

Samuel H. Cox, Yijia Lin and Tianxiang Shi

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Outline

Pension De-risking

- DB pensions introduce significant risks
  - Market downturns
  - Low interest rates
  - New pension accounting standards
  - Improved life expectancy of retirees

- In January 2015, Milliman 100 Pension Funding Index (PFI) decreased to 79.6%, down from 83.5% in December 2014 (Milliman, 2015)

- In recent years, there has been a surge of interest from defined benefit (DB) pension plan sponsors in de-risking their plans.

Pension De-risking Tools

- **Buyouts** transfer a proportion of the entire pension liabilities.
- **Longevity hedges** transfer the high-end longevity risk.
- Buyouts are more effective in improving firm value in the enterprise risk management framework (Lin et al. 2015).

Challenges of the Buyout Market

- Buyouts generally have had lower business volume than longevity swaps in recent years.
  - Capital intensive and relatively expensive: In December 2014, on average, the price of a buyout annuity transaction across US, UK, Ireland and Canada was 14% higher than the equivalent accounting liability (Mercer LLC, 2014).
  - Expensive for firms with underfunded plans: DB firms have to satisfy a minimum funded status.

Contributions

- We propose two "gap" type options that provide buyout financing.
  - Pension funding option and pension buyout option
  - Under-funded plans vs. fully funded plans
- We create a transparent pension funding index based on market indices and publicly available mortality tables.
- We show how to price these new pension de-risking securities considering investment risk, longevity risk, and interest rate risk.

Pension Funding Index

- We use a market-based pension funding index to increase liquidity and reduce moral hazard and adverse selection.
- Pension funding index at time t, PFI_t,

$$PFI_t = \frac{PAI_t}{PLI_t}$$

  - PLI_t: pension liability index based on $N(0)$ retired life cohort aged x_0 at time 0,

$$PLI_t = N(t) \cdot Pa_{x_0+t} \qquad t = 1, 2, \cdots,$$

    where $N(t)$ is the number of survivors at t, P is the annual payment, and a_{x_0+t} is the immediate life annuity factor.
  - PAI_t: pension asset index

Pension Asset Index

- PAI_t is determined by the value of a market portfolio composed of I indices at time t.

$$PAI_t = \sum_{i=1}^{I} A_{i,t-1}(1 + r_{i,t}), \qquad i = 1, 2, \cdots, I; \quad t = 1, 2, \cdots$$

  - $A_{i,t-1}$: amount invested in index i at time $t-1$
  - $r_{i,t}$: return of index i in period t
  - PAI_0: predetermined pension asset with investment weights w_i
- Periodic portfolio adjustment (when $t = 1, 2, \cdots$)

$$PA_t = \sum_{i=1}^{I} A_{i,t} = PAI_t + k \cdot UL_t \cdot 1_{\{UL_t > 0\}} - N(t)P$$

  - UL_t : funding deficit $UL_t = PLI_t - PAI_t + N(t)P$
  - k: amortization factor
- Portfolio rebalance can be easily incorporated.

Pension Options for Fully Funded Plans

- Pension funding option: cover a future funding deficit and satisfy a minimum funding requirement for a future buyout.
- Pension funding option payoff:

$$F_t^w = \frac{NA}{PLI_0} \times \begin{cases} PLI_t\,(K - PFI_t) & \text{if} \quad PFI_t < z \\ 0 & \text{if} \quad PFI_t \geq z \end{cases},$$

for $t = 1, 2, \ldots, n$.

  - NA: notional amount
  - n: option period/term
  - z: trigger funding index level
  - K: strike funding level

Pension Options for Fully Funded Plans (Cont'd)

- Pension buyout option: not only fill the funding gap (up to K), but also cover the required buyout risk premium.
- Pension buyout option payoff:

$$B_t^w = \frac{NA}{PLI_0} \times \begin{cases} PLI_t\left(K - PFI_t + R_t\right) & \text{if} \quad PFI_t < z \\ 0 & \text{if} \quad PFI_t \geq z \end{cases},$$

for $t = 1, 2, \ldots, n$.

  - R_t: buyout risk premium at time t (Lin et al, 2015)

Pension Options for Under-Funded Plans

- Pension funding option payoff:

$$F_t^u = \frac{NA}{PLI_0} \times \begin{cases} 0 & \text{if} \quad PFI_t \geq K \\ PLI_t\,(K - PFI_t) & \text{if} \quad z < PFI_t < K \\ 0 & \text{if} \quad PFI_t \leq z \end{cases}$$

- Pension buyout option payoff:

$$B_t^u = \frac{NA}{PLI_0} \times \begin{cases} 0 & \text{if} \quad PFI_t \geq K \\ PLI_t\,(K - PFI_t + R_t) & \text{if} \quad z < PFI_t < K \\ 0 & \text{if} \quad PFI_t \leq z \end{cases}$$

Pension Asset Index model

- Pension fund assets: S&P 500 index $A_{1,t}$, Merrill Lynch corporate bond index $A_{2,t}$ and 3-month T-bill $A_{3,t}$.
- S&P 500 index $A_{1,t}$: Merton's jump-diffusion process (Merton, 1976)

$$A_{1,t} = A_{1,0} \exp\left[(\alpha_1 - \frac{1}{2}\sigma_1^2 - \lambda_1 k_1)t + \sigma_1 W_{1t}^P + \sum_{j=1}^{N_{1t}} Y_{1j}\right]$$

  - α_1: instantaneous expected return
  - σ_1: instantaneous volatility
  - W_{1t}^P: standard Brownian motion with mean 0 and variance t
  - N_{1t}: Poisson process with arrival of λ_1 per unit of time
  - Y_{1j}: standard normal with mean m_1 and s.d. s_1
  - k_1: expected percentage change in the S&P 500 index

Pension Asset Index model (Cont'd)

- Processes of $A_{i,t}$, $i = 2, 3$, as a geometric Brownian motion:

$$\frac{\mathrm{d}A_{i,t}}{A_{i,t}} = \alpha_i \,\mathrm{d}t + \sigma_i \,\mathrm{d}W_{it}^P, \qquad i = 2, 3$$

  - α_i: instantaneous expected return
  - σ_i: instantaneous volatility of asset i
  - W_{it}^P: standard Brownian motion with mean 0 and variance t
- S&P 500 index $A_{1,t}$ and Merrill Lynch corporate bond index $A_{2,t}$ are correlated with

$$\mathrm{Cov}(W_{1t}^P, W_{2t}^P) = \rho\sigma_1\sigma_2 \; t,$$

- Monthly data from 1988 to 2010 are used to estimate parameters

Pension Valuation Rate

- Interest rate risk in pension risk management should be carefully considered.
- Incorporate the dynamics of pension valuation rate into pension options pricing.
- Cox-Ingersoll-Ross (CIR) model (Cox et al., 1985)

$$\mathrm{d}r_{p,t} = \nu\left(\theta - r_{p,t}\right)\mathrm{d}t + \sigma_p\sqrt{r_{p,t}}\,\mathrm{d}W^P_{p,t},$$

  - ν: mean-reversion rate
  - θ and σ_p: long-term mean and instantaneous volatility
  - $W^P_{p,t}$: standard Brownian motion
- Equally weighted average of US funding yield curve segment rates from August, 2008 to March, 2015 are used to estimate parameters.

Lee and Carter (1992)'s Mortality Model

- One-year death rate $q_{x,t}$ for age x ($x = 0, 1, 2, \cdots$) in year t ($t = 1, 2, \cdots, K$)

$$\ln q_{x,t} = \kappa_x + b_x \gamma_t + \epsilon_{x,t},$$

$$\gamma_t = \gamma_{t-1} + g + e_t, \qquad e_t \sim N(0, \sigma_\gamma)$$

  - κ_x and b_x: age-specific parameters
  - g: drift rate
  - $\epsilon_{x,t}$ and e_t: normal errors with mean zero
- Data: U.K. male population mortality tables from 1950 to 2003

Option Pricing Formulas For Fully Funded Plans

- Funding option for n-year term, in terms of the percentage of the nominal amount:

$$PF_w = \frac{1}{PLI_0}\mathrm{E}^Q\left[e^{-r\tau_w}N^*(\tau_w)\cdot Pa_{x_0+\tau_w}\left(K-PFI_{\tau_w}\right)^+\right]$$

  - Risk neutral Esscher measure is selected (Gerber and Shiu, 1994)
  - $N^*(t)$: survival evolution based on transformed mortality rates
  - $\tau_w = \inf\{t : PFI_t < z,\ t \in \{1,2,...,n\}\}$ (∞ if the option is not triggered)
- Buyout option for n-year term:

$$\begin{aligned} PB_w &= \frac{1}{PLI_0}\mathrm{E}^Q\left[e^{-r\tau_w}N^*(\tau_w)\cdot Pa_{x_0+\tau_w}\left(\left(K-PFI_{\tau_w}\right)^+ + R_{\tau_w}\right)\right] \\ &= PF_w + PR_w \end{aligned}$$

Option Pricing Formulas For Under-Funded Plans

- Funding option for n-year term:

$$PF_u = \frac{1}{PLI_0}\mathrm{E}^Q\left[e^{-r\tau_u}N^*(\tau_u)\cdot Pa_{x_0+\tau_u}\left(K-PFI_{\tau_u}\right)^+\right]$$

where $\tau_u = \inf\{t : PFI_t > z,\ t \in \{1,2,...,n\}\}$ (∞ if the option is not triggered)

- Buyout option for n-year term:

$$\begin{aligned} PB_u &= \frac{1}{PLI_0}\mathrm{E}^Q\left[e^{-r\tau_u}N^*(\tau_u)\cdot Pa_{x_0+\tau_u}\left(\left(K-PFI_{\tau_u}\right)^+ + R_{\tau_u}\right)\right] \\ &= PF_u + PR_u \end{aligned}$$

Risk Parameters and Assumptions

- Pension Funding Index
  - The pension cohort has the same mortality experience as the U.K. male population.
  - At time 0, all plan participants reach the retirement age $x_0 = 65$.
  - Pension asset weights $(w_1, w_2, w_3) = (0.5, 0.45, 0.05)$, are rebalanced annually.
  - Amortization factor $k = 1/5.95$
- Pension valuation rate: $r_{p,0} = 4.8\%$, $\nu = 0.3713$, $\theta = 4.78\%$, $\sigma_p = 0.03$
- Risk-free interest rate: $r = 4\%$
- Market price of longevity risk (using Wang transform): $\lambda_{\text{EIB}} = 0.0666$, based on the European Investment Bank (EIB) bond issued in November 2004

Pension Option Premiums for Fully Funded Plans

Table 1: Life-Time Funding and Buyout Option Premiums for Fully Funded Plans

Initial Funding Ratio PFI_0	Trigger Level z	Strike Level K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	9.29%	3.23%
		0.95	7.43%	–
		0.90	5.56%	–
	0.70	1.00	7.09%	1.65%
		0.90	5.03%	–
		0.80	2.96%	–

Pension Option Premiums for Fully Funded Plans

Table 2: Life-Time Funding and Buyout Option Premiums for Fully Funded Plans

Initial Funding Ratio PFI_0	Trigger Level z	Strike Level K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	**0.80**	**1.00**	**9.29%**	3.23%
		0.95	7.43%	–
		0.90	5.56%	–
	0.70	1.00	7.09%	1.65%
		0.90	5.03%	–
		0.80	2.96%	–

Pension Option Premiums for Fully Funded Plans

Table 3: Life-Time Funding and Buyout Option Premiums for Fully Funded Plans

Initial Funding Ratio PFI_0	Trigger Level z	Strike Level K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	0.80	1.00	9.29%	3.23%
		0.95	7.43%	–
		0.90	5.56%	–
	0.70	1.00	7.09%	1.65%
		0.90	**5.03%**	–
		0.80	2.96%	–

Pension Option Premiums for Fully Funded Plans

Table 4: Life-Time Funding and Buyout Option Premiums for Fully Funded Plans

Initial Funding Ratio PFI_0	Trigger Level z	Strike Level K	Funding Option Premium PF_w	Buyout Add-on PR_w
100%	**0.80**	**1.00**	9.29%	**3.23%**
		0.95	7.43%	–
		0.90	5.56%	–
	0.70	1.00	7.09%	1.65%
		0.90	5.03%	–
		0.80	2.96%	–

Pension Option Premiums for Under-Funded Plans

Table 5: Life-Time Funding and Buyout Option Premiums for Under-Funded Plans

Initial Funding Ratio PFI_0	Trigger Level z	Strike Level K	Funding Option Premium PF_w	Buyout Add-on PR_w
		1.00	**5.29%**	**4.99%**
	0.85	0.95	2.81%	–
80%		0.90	0.84%	–
	0.90	1.00	2.21%	3.79%
		0.95	0.67%	–
		1.00	4.30%	3.77%
	0.85	0.95	2.30%	–
75%		0.90	0.70%	–
	0.90	1.00	1.74%	2.87%
		0.95	0.53%	–

Pension Option Premiums for Under-Funded Plans

Table 6: Life-Time Funding and Buyout Option Premiums for Under-Funded Plans

Initial Funding Ratio PFI_0	Trigger Level z	Strike Level K	Funding Option Premium PF_w	Buyout Add-on PR_w
80%	0.85	1.00	5.29%	4.99%
		0.95	2.81%	–
		0.90	0.84%	–
	0.90	**1.00**	**2.21%**	**3.79%**
		0.95	0.67%	–
75%	0.85	1.00	4.30%	3.77%
		0.95	2.30%	–
		0.90	0.70%	–
	0.90	1.00	1.74%	2.87%
		0.95	0.53%	–

Conclusion

- We propose pension funding and buyout options.
- We design a pension funding index based on market indices and publicly available mortality tables to increase market liquidity and reduce moral hazard and adverse selection problems.
- We study how to price pension options while recognizing investment risk, longevity risk, and interest rate risk.

Thank You!