Inequality in the Length of Life: Moving Beyond Conventional Lifespan Measures and Approaches

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Introduction & Motivation

- Significant and persistent inequality in the length of life across the members of a population are a critical issue in the design and reform of retirement income schemes and health policy (Ayuso et al., 2021)
- Longevity heterogeneity perverts the redistributive objectives of pension schemes and risks invalidating reform approaches establishing a closer contribution-benefit link and linking pension benefits to uniform life expectancy developments (Bravo et al., 2021)
- Reforms that are either inter-generationally and/or intra-generationally unfair are likely to be rejected by voters
- Lifespan inequality has been classified as the "mother" of all inequalities (van Raalte et al., 2018)
    - measures both population-level heterogeneity and individual-level uncertainty in the timing of death
    - it is an ultimate manifestation of health and living conditions disparities
    - studies suggest that the most deprived groups experience the lowest life expectancy and highest amount of variation in age at death, with inequalities still growing

Introduction & Motivation

- Classical health transition theories suggest that longevity increases go in tandem with the so-called "mortality compression" or (outer) "rectangularization hypothesis" (Kannisto, 1980; Fries, 1980)
- Empirical studies in high-income countries show that period life expectancy increases are strongly inversely correlated with lifespan variation when one considers the entire human lifespan (Wilmoth & Horiuchi, 1999; Aburto et al., 2021)
- Lifespan disparity has decreased mainly as a result of survival improvements at premature ages, which shifted deaths toward the end of the lifespan
- Yet, when
  - observed from a birth cohort perspective, and
  - across adult (retirement) and more advanced age ranges only

  the compression in period life tables may hide mortality shifting and expansion and stagnant or increasing lifespan inequality in birth cohorts due to heterogeneous mortality regimes

This paper...

- Empirically investigates the relationship between life expectancy developments and lifespan inequality at retirement ages using a birth cohort approach
- Contrary to most previous studies, we study conditional age-at-death distributions
- To measure lifespan variation, we consider both
  - traditional measures such as life disparity, the life table entropy, the Gini coefficient or the coefficient of variation,
  - new measures such as
    - the probability of attaining the birth cohort mean lifespan equality potential
    - the Maximum Shared Lifespan
  - social welfare-based approaches incorporating distributive considerations such as
    - the Equivalent Length of Life (distributionally adjusted life expectancy)
    - the Atkinson inequality index
- To forecast mortality, we use a BME of heterogeneous stochastic mortality models (Bravo et al., 2021)

Outline

1. Introduction & Motivation
2. Materials and methods
3. Results
4. Discussion and final remarks
5. Selected references

Notation

$\dot{e}_x$: complete (period, cohort) life expectancy at age x
d_x : life table deaths at age x
l_x : survivorship at age x
ω : highest age at death
z_x : average age at death over the interval $[x, x+1)$, $z_x = x + a_x$
a_x : average number of years lived within the age interval $[x, x+1)$

Traditional lifespan inequality measures

Measure	Notation	Formulae
Life disparity	$e^\dagger_x$	$e^\dagger_x = \sum_{y=x}^{\omega-x} d_y \dot{e}_y$
Life table entropy	H_x	$H_x = \frac{e^\dagger_x}{\dot{e}_x}$
Coefficient of Variation	CV_x	$CV_x = \frac{1}{\dot{e}_x}\sqrt{\sum_{y=x}^{\omega-x} d_y \left(z_y - \dot{e}_y\right)^2}$
Gini coefficient	G_x	$G_x = 1 - \frac{1}{\dot{e}_x}\sum_{y=x}^{\omega-x} l^2_{y+1}$
Theil index	T_x	$T_x = \sum_{y=x}^{\omega-x} d_y \left(\frac{z_y}{\dot{e}_y} \ln\left(\frac{z_y}{\dot{e}_y}\right)\right)$
Mean Log Deviation	MLD_x	$MLD_x = \sum_{y=x}^{\omega-x} d_y \ln\left(\frac{z_y}{\dot{e}_y}\right)$
Probability to survive up to $x + \dot{e}_x$	π_x	$\pi_x = \frac{l_{\dot{e}_x}}{l_x}$

The Equivalent Length of Life & Atkinson's inequality index

- Life expectancy is indifferent to how the total lifetime is distributed among the generation
- The concept of Equivalent Length of Life (ELL) refers to the "*. . . length of life which, if being identical for all individuals, would give the same social welfare as the actual distribution of deaths by age.*" (Silber 1983, p.21)
- Introduced by Silber (1983) as a development indicator, it is based on the concept of Equally Distributed Equivalent (EDE) income by Atkinson (1970) and Kolm (1976)
- Summarizes mortality distributions in a single marker taking into account two moments: mean and lifespan variation
- Social welfare is derived from the number of years lived by the population
- It assumes that society has a preference for equality in the distribution of length of life

Equivalent Length of Life & Atkinson's inequality index

- Consider the following additively separable and symmetric utility function of the individual lengths of life z_x

$$U\left(z,\eta,\epsilon,x\right)=\left(\sum_{y=x}^{\omega-x}\eta_y z_y^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}},\quad \epsilon\geq 0$$

where

$$\eta_x=\frac{d_x}{\sum_{y=x}^{\omega-x}d_y},\qquad \sum_{y=x}^{\omega-x}\eta_y=1$$

- Parameter ϵ can be interpreted as a coefficient of aversion to inequality
  - if: $\epsilon=0: U\left(z,\eta,\epsilon,x\right)=\dot{e}_x$
  - if: $\epsilon>0$, $U\left(z,\eta,\epsilon,x\right)$ expresses a preference for equality and $ELL_x<\dot{e}_x$
  - In the empirical study we assume $\epsilon=1$

Equivalent length of life & Atkinson inequality index

- The social welfare function can be expressed as ("rightist" approach):

$$ELL_x := U(\bar{z}, \eta, \epsilon, x) = \dot{e}_x (1 - A_\epsilon) \tag{1}$$

where A_ϵ is the relative Atkinson inequality (CRRA) index

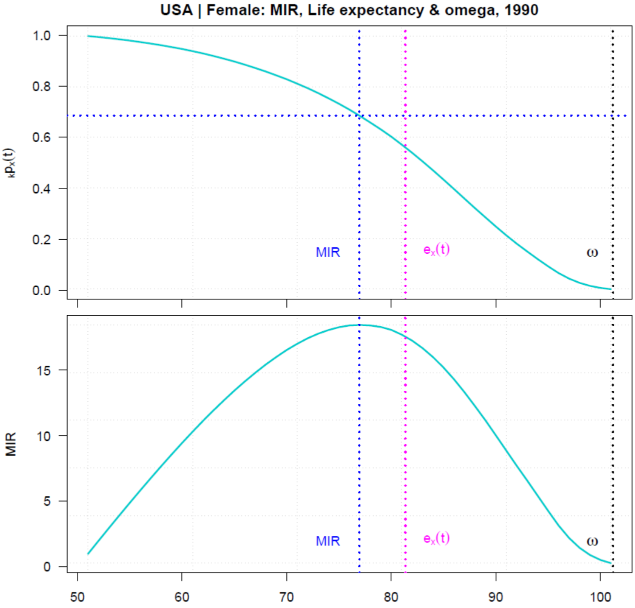
$$A_\epsilon = 1 - \left[\sum_{y=x}^{\omega-x} \eta_y \left(\frac{z_y}{\dot{e}_y} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \quad \epsilon > 0 \tag{2}$$

or as ("leftist" approach):

$$ELL_x = \dot{e}_x - K_\epsilon \tag{3}$$

with K_ϵ the Kolm's (CARA) inequality index

Maximum Inner Rectangle Approach (MIRA)



Maximum Inner Rectangle Approach (MIRA)

Name	Acronym	Expression	Interpretation
Inner rectangle	IR_x	$x \times l_x$	Age-specific uniformly shared person-years (PY)
Maximum shared lifespan	x^*	$\max[x \times l_x]$	Maximum number of uniformly shared life years by largest number of survivors
Maximum proportion	l_{x^*}	l_{x^*}	Largest proportion alive at the maximum shared lifespan
Maximum inner rectangle	MIR	$x^* \times l_{x^*}$	Population's current maximum number of uniformly shared PY
Life expectancy	e_0	$\int_0^\omega l_a da$	Population's current number of PY, i.e., mean lifespan
Outer rectangle	ω	$\omega \times l_0$	Maximum possible PY
Outer rectangle ratio	ORR	e_0/ω	Proportion of PY lived from maximum possible PY
Inner rectangle ratio	IRR	MIR/e_0	Proportion of uniformly shared PY from all PY lived
Total rectangle ratio	TRR	MIR/ω	Proportion of uniformly shared PY lived of maximum possible PY

Figure: Source: Ebeling et al. (2018).

Notes: To estimate MIRA quantities with higher precision, we first smoothed the inner rectangle $(x \cdot l_x)$ using cubic splines; the maximum age (ω) is set at the actual age at which 0.5% of the population is alive.

Life Expectancy Inequality Gap (LEIG)

- Let us introduce the concept of Life Expectancy Inequality Gap (LEIG) at age x and year t, $\dot{e}_x^{IG}(t)$
- The concept is defined as the difference between the actual mean (period or birth cohort) lifespan of the population and the distributionally adjusted Equivalent Length of Life

$$\dot{e}_x^{IG,ELL}(t) := \dot{e}_x(t) - ELL_x(t) \tag{4}$$

or as the difference between the actual mean lifespan of the population and the maximum shared lifespan (MIR)

$$\dot{e}_x^{IG,MIR}(t) := \dot{e}_x(t) - x^* \tag{5}$$

- The LEIG measures how far the actual distribution of deaths by age is from a distribution with perfectly uniform lifespans of length $\dot{e}_x(t)$
- In a hypothetical scenario in which all individuals in a population have a lifespan of length $\dot{e}_x(t)$, $\dot{e}_x^{IG}(t) = 0$

Bayesian Model Ensemble (BME) to mortality forecasting

- To forecast mortality, we adopt the BME approach developed in Bravo et al. (2021)
- Instead of pursuing a "winner-take-all" approach, BME aims at finding a composite model that better approximates the actual data generation process and its multiple sources of uncertainty, conditioning the statistical inference on the model confidence set
- Let M_k $(k = 1, ..., K)$ be each candidate model and Δ a quantity of interest present in all models (e.g., the future value of y)

$$p(\Delta|y) = \sum_{k=1}^{K} p\left(\Delta|y, M_k\right) p\left(M_k|\, y\right), \tag{6}$$

where $p\left(\Delta|y, M_k\right)$ denotes the forecast PDF based on model M_k alone, and $p\left(M_k|\, y\right)$ is the posterior probability of model M_k given data, with $\sum_{k=1}^{K} p\left(M_k|\, y\right) = 1$

- The BME PDF is a weighted average of the PDFs given the individual models, weighted by their posterior model probabilities

Bayesian Model Ensemble

- The BME Model weights $p\left(M_k \mid y\right)$ are estimated based on each model out-of-sample predictive accuracy
- We carry out a backtesting exercise considering a 5-year forecasting horizon for all models and populations
- Use the symmetric mean absolute percentage error (SMAPE) to assess the forecasting accuracy
- We compute $p\left(M_k \mid y\right)$ using the normalized exponential function

$$p\left(M_k \mid y\right) = \frac{\exp\left(-\left|\xi_k\right|\right)}{\sum_{l=1}^{K} \exp\left(-\left|\xi_l\right|\right)}, \quad k = 1, \ldots, K, \tag{7}$$

with $\xi_k = S_k / \max\left\{S_l\right\}_{l=1,\ldots,K}$ and $S_k = SMAPE_k$

- The normalized exponential function assigns larger weights to models with smaller forecasting error, with the weights decaying exponentially

Candidate stochastic mortality models

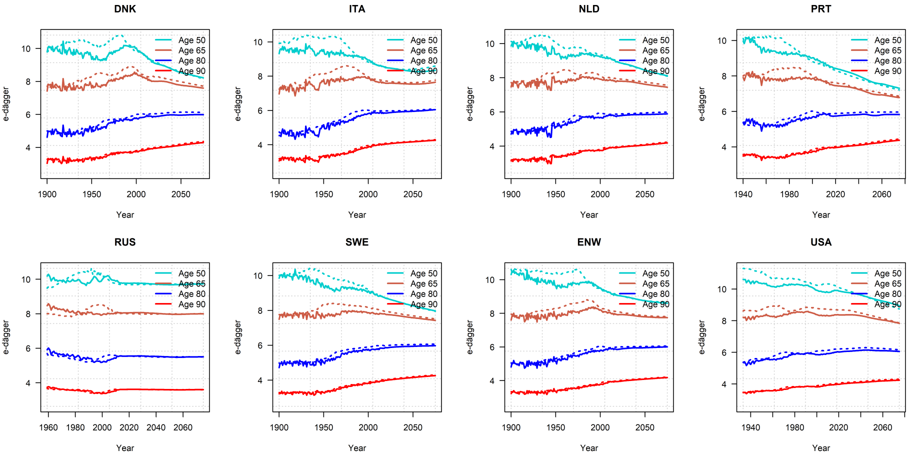
Model	Model structure
LC	$\eta_{x,t} = \alpha_x + \beta_x^{(1)} \kappa_t^{(1)}$
APC	$\eta_{x,t} = \alpha_x + \kappa_t^{(1)} + \gamma_{t-x}$
RH	$\eta_{x,t} = \alpha_x + \beta_x^{(1)} \kappa_t^{(1)} + \beta_x^{(0)} \gamma_{t-x}$
CBD	$\eta_{x,t} = \kappa_t^{(1)} + (x - \bar{x}) \kappa_t^{(2)}$
M7	$\eta_{x,t} = \kappa_t^{(1)} + (x - \bar{x}) \kappa_t^{(2)} + \left((x - \bar{x})^2 - \sigma \right) \kappa_t^{(3)} + \gamma_{t-x}$
Plat	$\eta_{x,t} = \alpha_x + \kappa_t^{(1)} + (x - \bar{x}) \kappa_t^{(2)} + (\bar{x} - x)^+ \kappa_t^{(3)} + \gamma_{t-x}$
HUw	$y_t(x_i) = f_t(x_i) + \sigma_t(x_i) \epsilon_{t,i}$
CPspl	$\eta = B\alpha$
RSVD	$m(x,t) = \sum_{j=1}^{q} d_j U_j(t) V_j(x) + \epsilon(x,t)$

Note: See Bravo et al. (2021) for details.

Data and BME calibration procedure

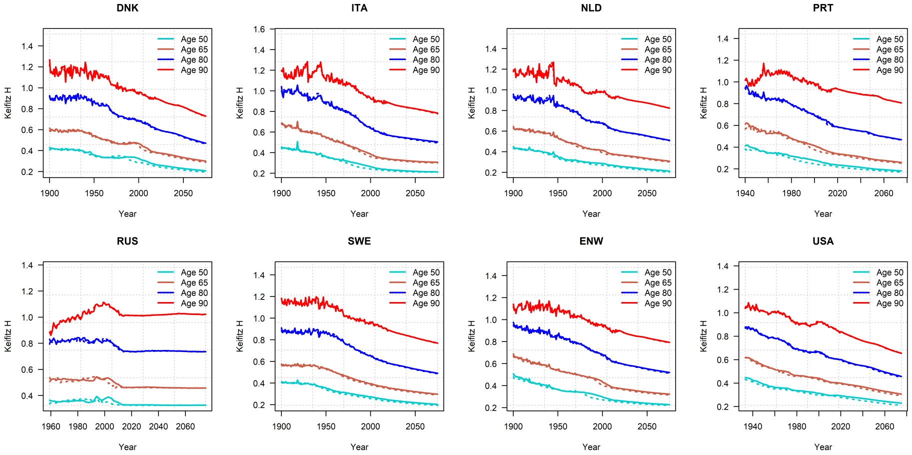
- We implement an adaptative BME procedure using a fixed-rule trimming scheme in which three out of six GAPC models are selected based on the model's out-of-sample forecasting performance
- We first calibrate the models using each country population data from 1900 to the most recent year available and for ages in the range 60-95
- The datasets used in this study consist of observed death counts and exposure-to-risk, classified by age, year and sex
- Data is from the Human Mortality Database
- Due to time and space constraints, we report results for the female population only

Results: Life disparity (e-dägger)

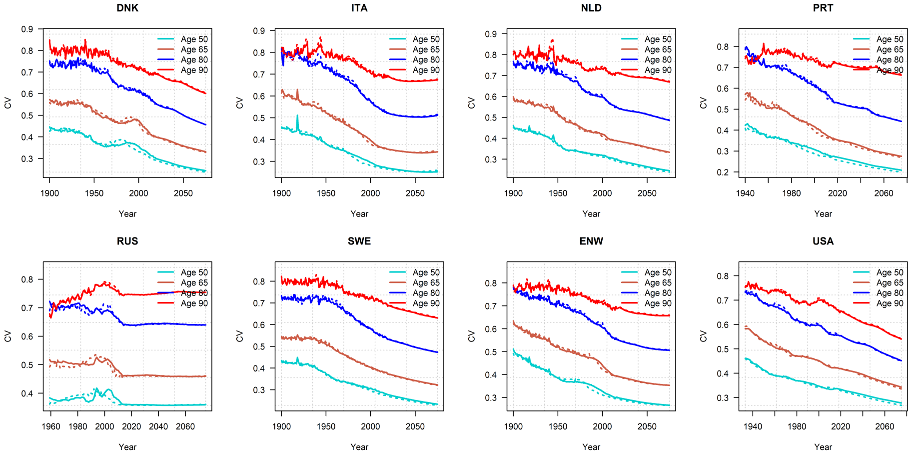


Note: Dotted lines represent values computed using a birth cohort approach.

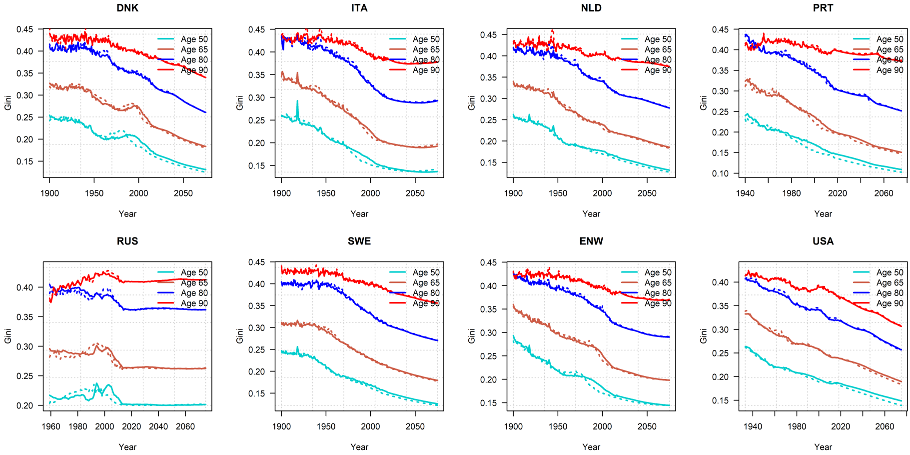
Results: Keyfitz' life table entropy



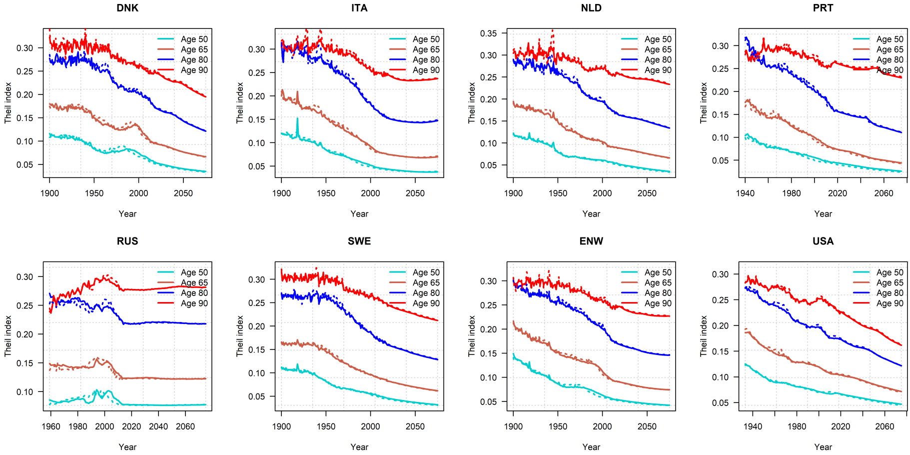
Results: Coefficient of variation



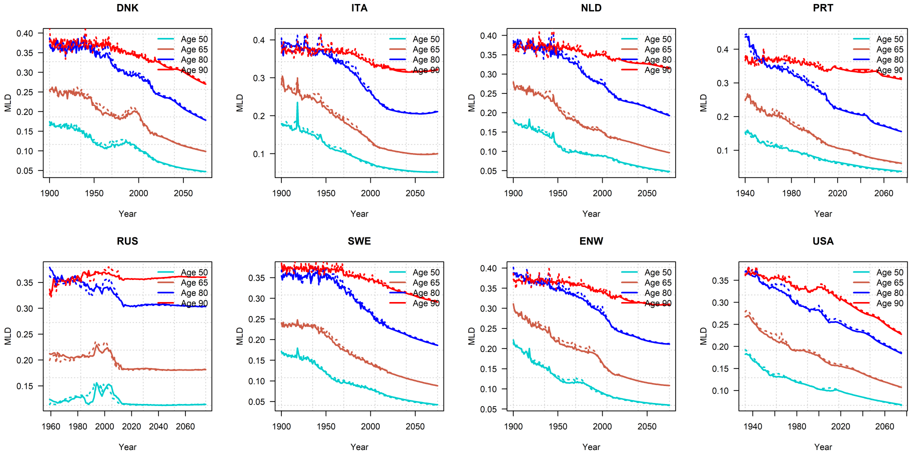
Results: Gini coefficient



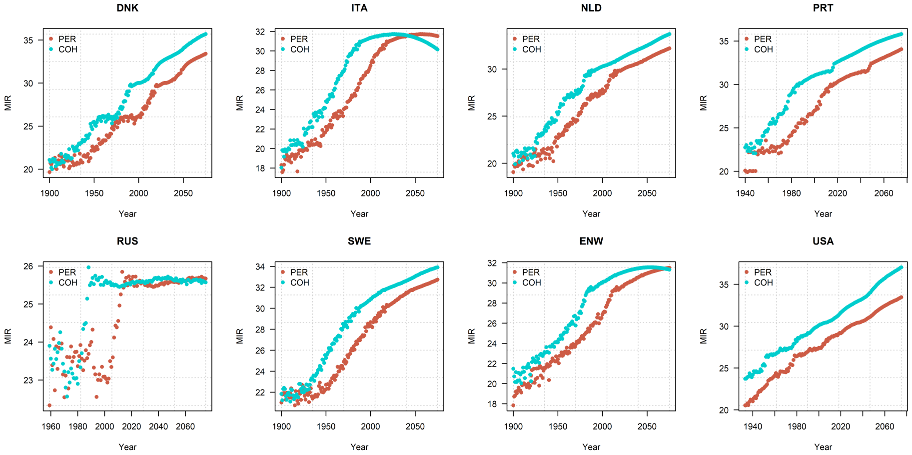
Results: Theil index



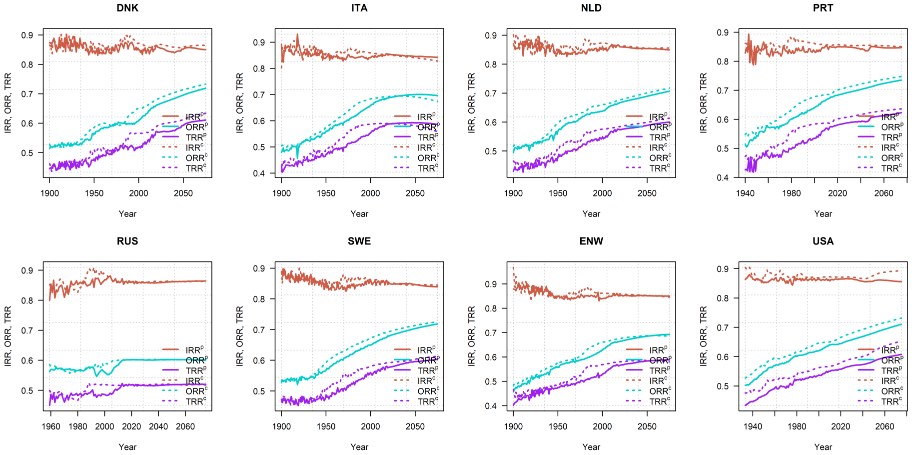
Results: Mean log deviation (MLD)



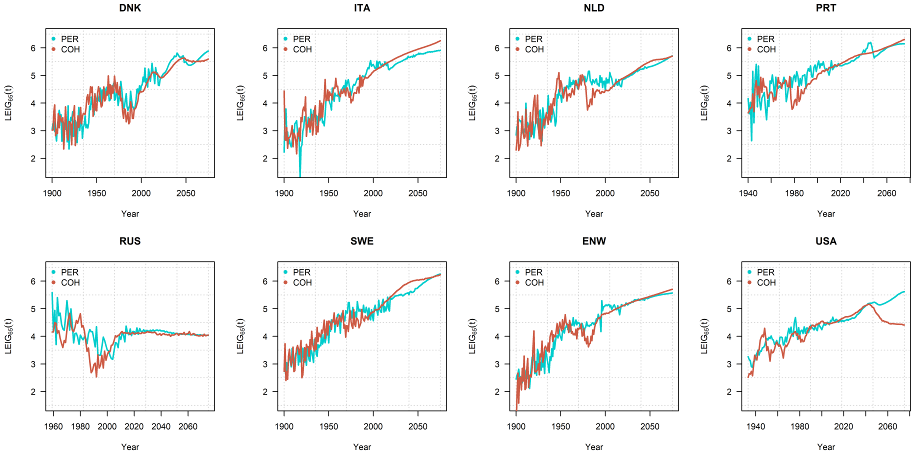
Results: Maximum shared lifespan



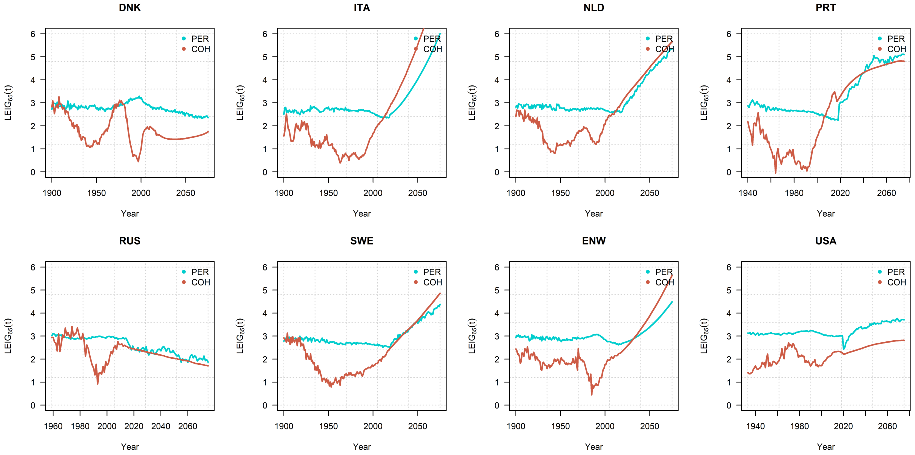
Results: IRR, ORR & TRR



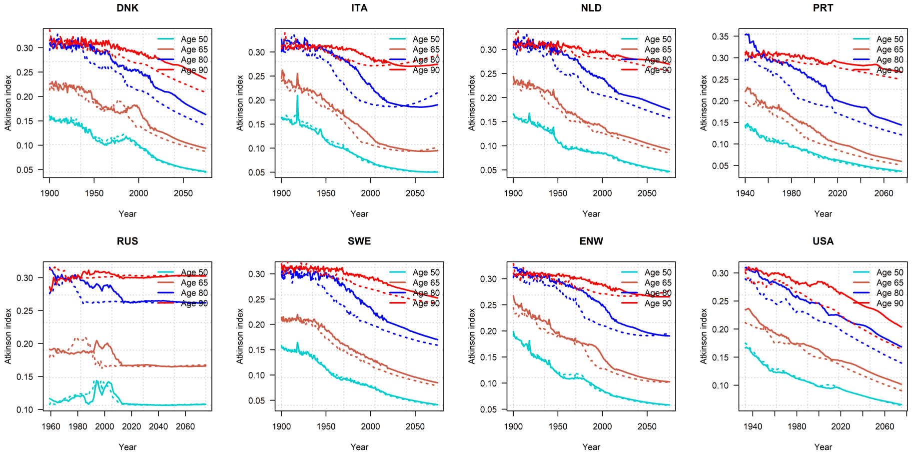
Life Expectancy Inequality Gap (LEIG): MIR approach



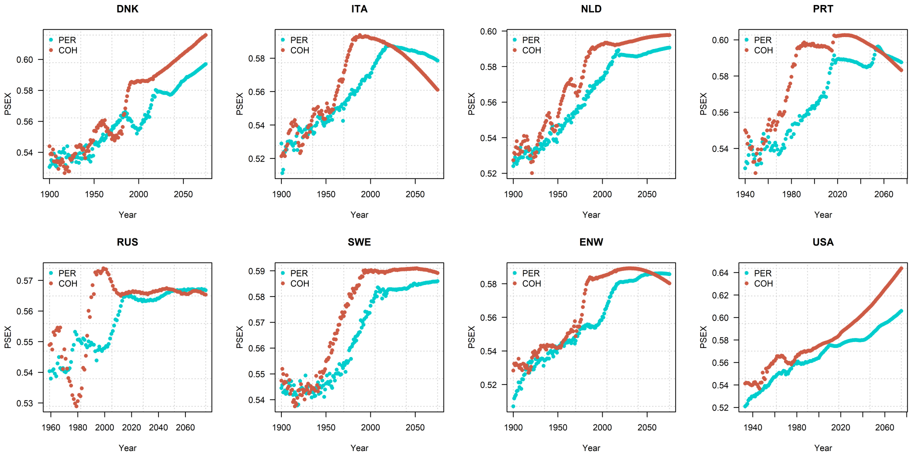
Life Expectancy Inequality Gap (LEIG): ELL approach



Results: Atkinson index



Conditional probability of surviving to mean lifespan



Note: Probability conditional to surviving to age 50.

Discussion & conclusion

- We observe generalized declines in adult lifespan variability at standard retirement ages when assessed through traditional measures
- This is true for both a period and a birth cohort approach
- This trend is more evident in relative lifespan variation measures (e.g., H, CV, Theil, Gini) than in absolute measures (e.g., e-dagger)
- The decline is at a much slower pace than declines in overall lifespan inequality found in previous studies considering the unconditional lifetime distribution
- We observe higher levels and stagnation and/or increase in lifespan variability at older ages

Discussion & conclusion

- The MIR approach results suggest that
  - Except for Russia and in part Italy, the conditional ORR and TRR trends are according to the classical rectangularization hypothesis
  - The inner rectangularization results suggest a different picture, with the IRR roughly stagnating (declining in many cases), despite the significant increases in the conditional maximum shared lifespan observed in most countries (the exceptions are RUS, ITA)
  - This is because the frame of reference for mortality progress and lifespan distribution analysis is different in the case of IRR
- The Life Expectancy Inequality Gap (LEIG) is increasing in most countries
- Trends in LEIG are more pronounced when considering the MIR approach than when using the ELL (distributionally adjusted life expectancy)
- The LEIG trends translate into pension wealth inequality gaps that need to be addressed at the accumulation, annuitization, or payout stages of pensions

Selected references

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THANK YOU for your attention

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