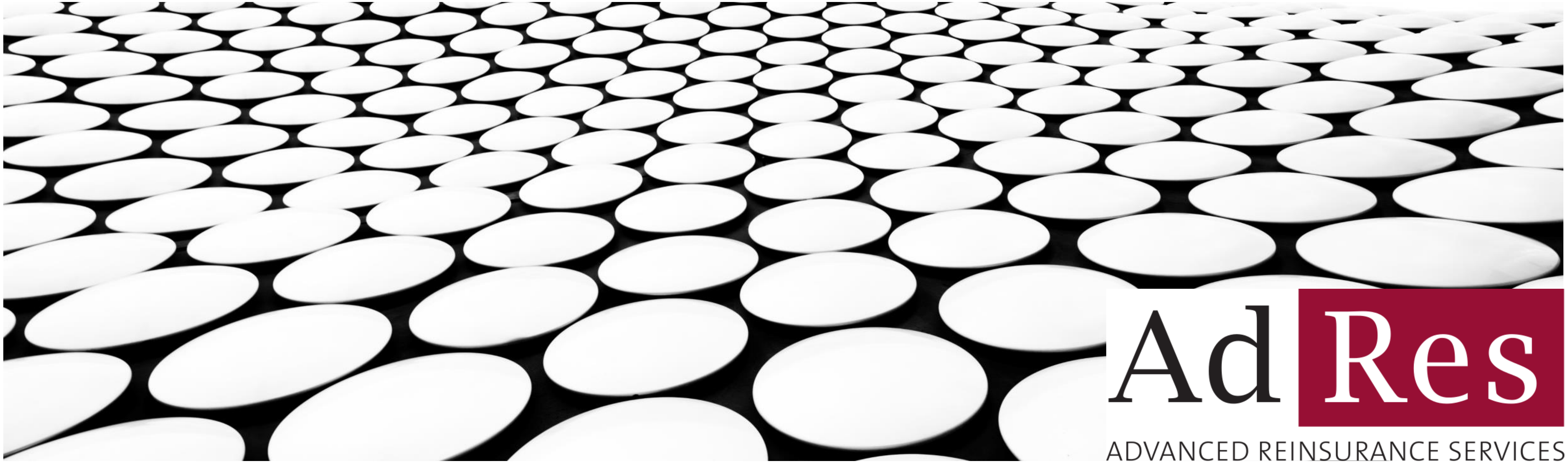
RECONCILING STOCHASTIC AND DETERMINISTIC MORTALITY FORECASTING METHODS

PRESENTATION AT LONGEVITY 17 WATERLOO – 13TH SEPTEMBER 2022, MARTINA BRÜCK, KAI KAUFHOLD, CLAUS NEIDHARDT



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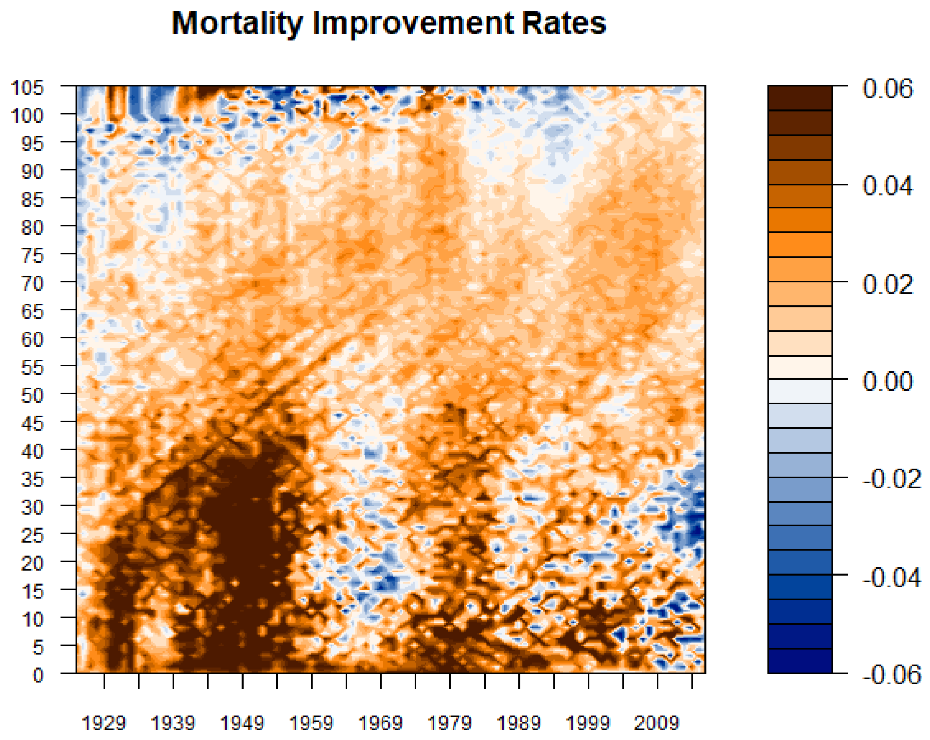
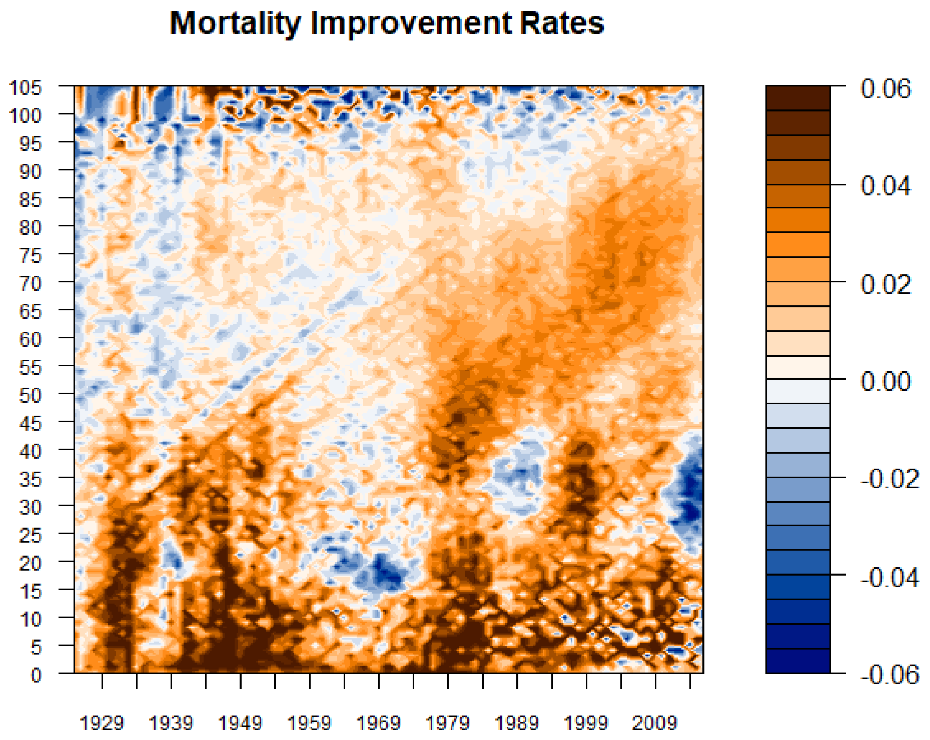
ADVANCED REINSURANCE SERVICES

AGENDA

1. Model Selection
2. Impact of Model Selection
3. Managing Model Risk: Model Choice Criteria
4. Managing Model Risk: Expert Input
5. Reconciling Expert Input to Stochastic Model Output
6. Outlook

ALL MODELS ARE WRONG – WHICH MODELS ARE USEFUL?





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SOURCE: HMD DATA FOR CANADA, FEMALES (LEFT) & MALES (RIGHT), 1921 – 2019, AGES 0 – 105, 7-YEAR MOVING AVERAGE MI RATES

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1. **Model Selection**
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STOCHASTIC MORTALITY MODELS – USUAL SUSPECTS

Mortality Rate Models	Mortality Improvement Rate Models
M1: Lee-Carter for $\log(m_{x,y})$	M1: Lee-Carter for $MI_{x,y}$
M2: Renshaw-Haberman for $\log(m_{x,y})$	
M3: Age-Period-Cohort for $\log(m_{x,y})$	M3: Age-Period-Cohort for $MI_{x,y}$
M5: Cairns-Blake-Dowd for $\log(m_{x,y})$	M5: Cairns-Blake-Dowd for $MI_{x,y}$
M6: CBD with cohort for $\log(m_{x,y})$	
M7: CBD with cohort and quadratic age for $\log(m_{x,y})$	
M8: CBD with age-dependent cohort for $\log(m_{x,y})$	
Simplified Plat for $\log(m_{x,y})$	Simplified Plat for $MI_{x,y}$
Heat-wave model for $\log(m_{x,y})$	
APCI model for $\log(m_{x,y})$	

SOURCES: CAIRNS ET AL. (2009), VILLEGAS ET AL. (2018), LI ET AL. (2020)

CANDIDATES FOR STOCHASTIC PROJECTION METHODS

Model Type	Time Series	Process Candidates
Lee-Carter	κ_y	ARIMA (difference stationary), or broken-trend stationary
Renshaw-Haberman / APC	κ_y ; γ_{y-x}	ARIMA (difference stationary)
Cairns-Blake-Dowd (M5-M8)	$\kappa_y^{(1)}; \kappa_y^{(2)}; \kappa_y^{(3)}; \gamma_{y-x}$	Multivariate ARIMA or broken-trend stationary
Plat	$\kappa_y^{(1)}; \kappa_y^{(2)}; \kappa_y^{(3)}; \gamma_{y-x}$	Multivariate ARIMA

See also

- Li, Chan & Cheung (2011) Structural Changes in the Lee-Carter Mortality Indexes, Detection and Implications, *North American Actuarial Journal*
- Sweeting (2011) A Trend-Change Extension to the Cairns-Blake-Dowd Model, *Annals of Actuarial Science*

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CMI METHOD FOR DETERMINISTIC FORECASTING

CMI Working Paper 98 “CMI Mortality Projections Model: Methods”(2017)

1. Mortality improvements (MI) are split into age-period and cohort effects
2. Deterministic projection based on expert judgement:
   - Long-term MI-rates (split by age-period and cohort effects)
   - Convergence period (time until reaching the long-term rates)
   - “Direction of travel” (initial slope of interpolation curves)
3. Historical MI and deterministic Long-Term MI-Rates are interpolated using cubic splines

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DETERMINISTIC VERSUS STOCHASTIC FORECASTING

Stochastic Approach

- Well-established estimation procedures (based on explicit assumptions)
- Ability to show confidence regions, not just best estimate forecasts
- Dependence on model choice
- Basic assumption: Past is representative of the future

Deterministic Approach

- Include expert judgement in a practical way
- Make quite clear, that future structures can be different from past ones
- Risk of arbitrary choices inconsistent with model framework
- Cannot measure projection uncertainty

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2. **Impact of Model Selection**
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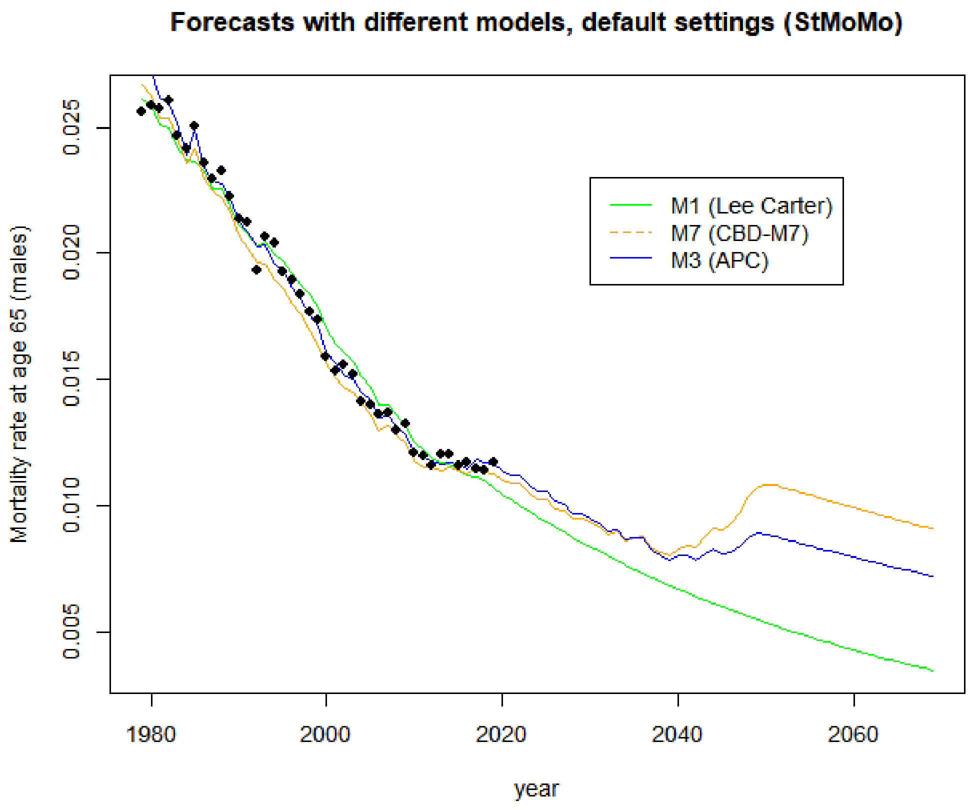
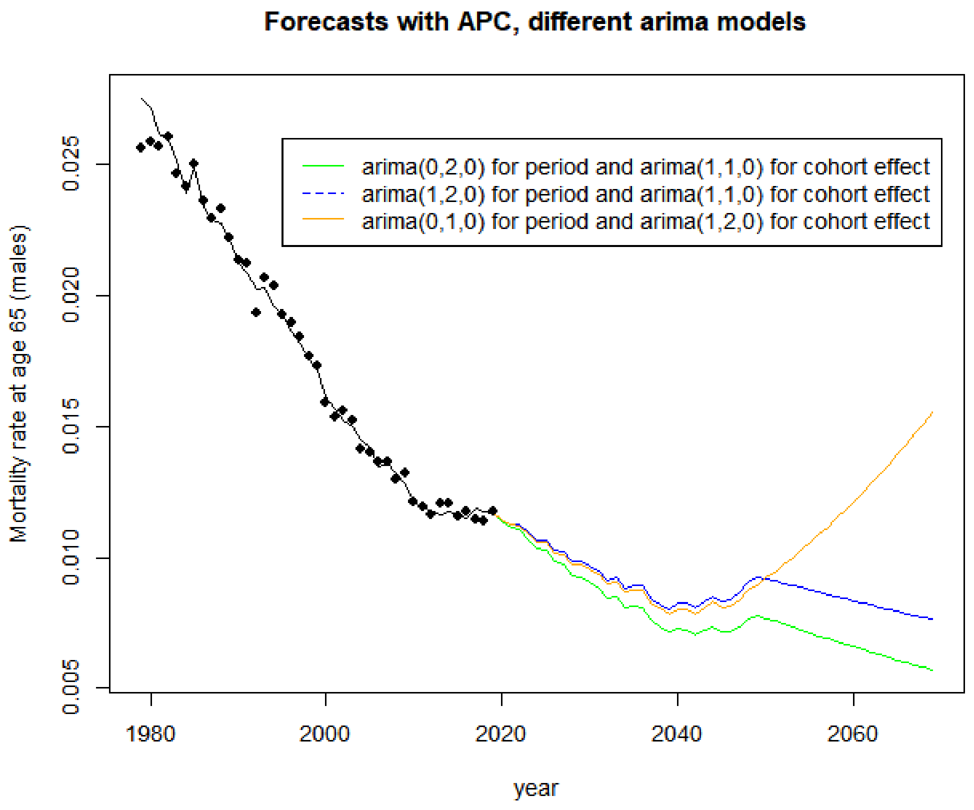
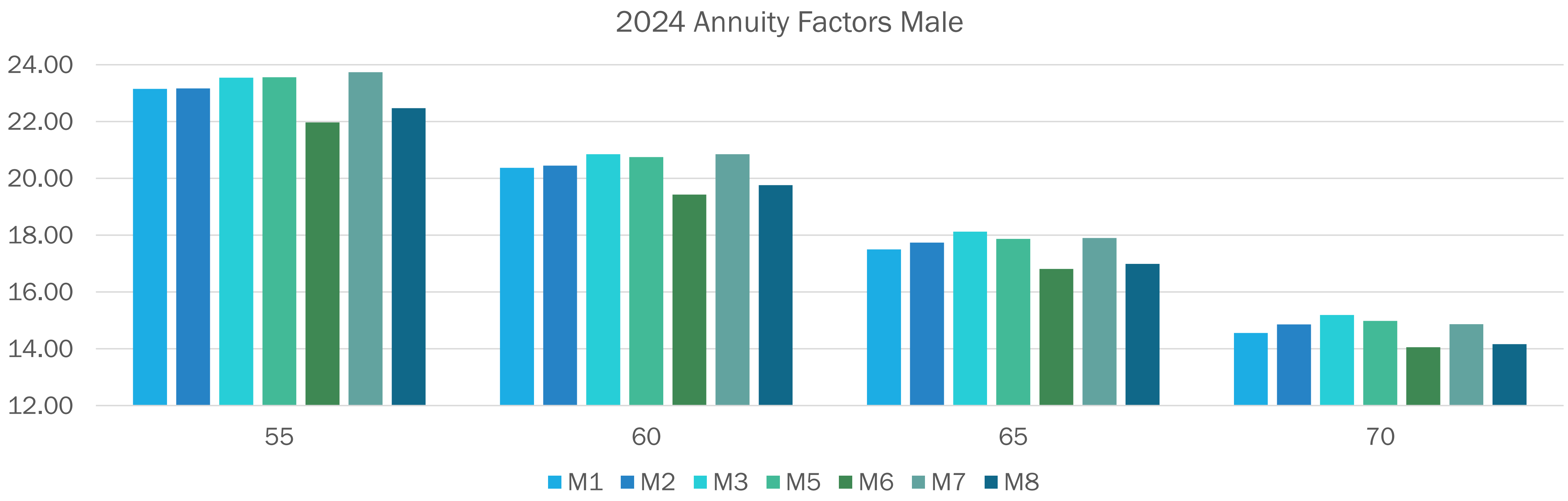
IMPACT OF SELECTING MODEL AND PROJECTION METHOD





SOURCE: HMD DATA FOR CANADA, MALES, 1979 – 2019, AGES 30 – 100, MODELS BASED ON STMOMO

COMPARING MALE ANNUITY FACTORS BY MODEL - 2024



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AGENDA

1. Case Study
2. Model Selection
3. Impact of Model Selection
4. **Managing Model Risk: Model Choice Criteria**
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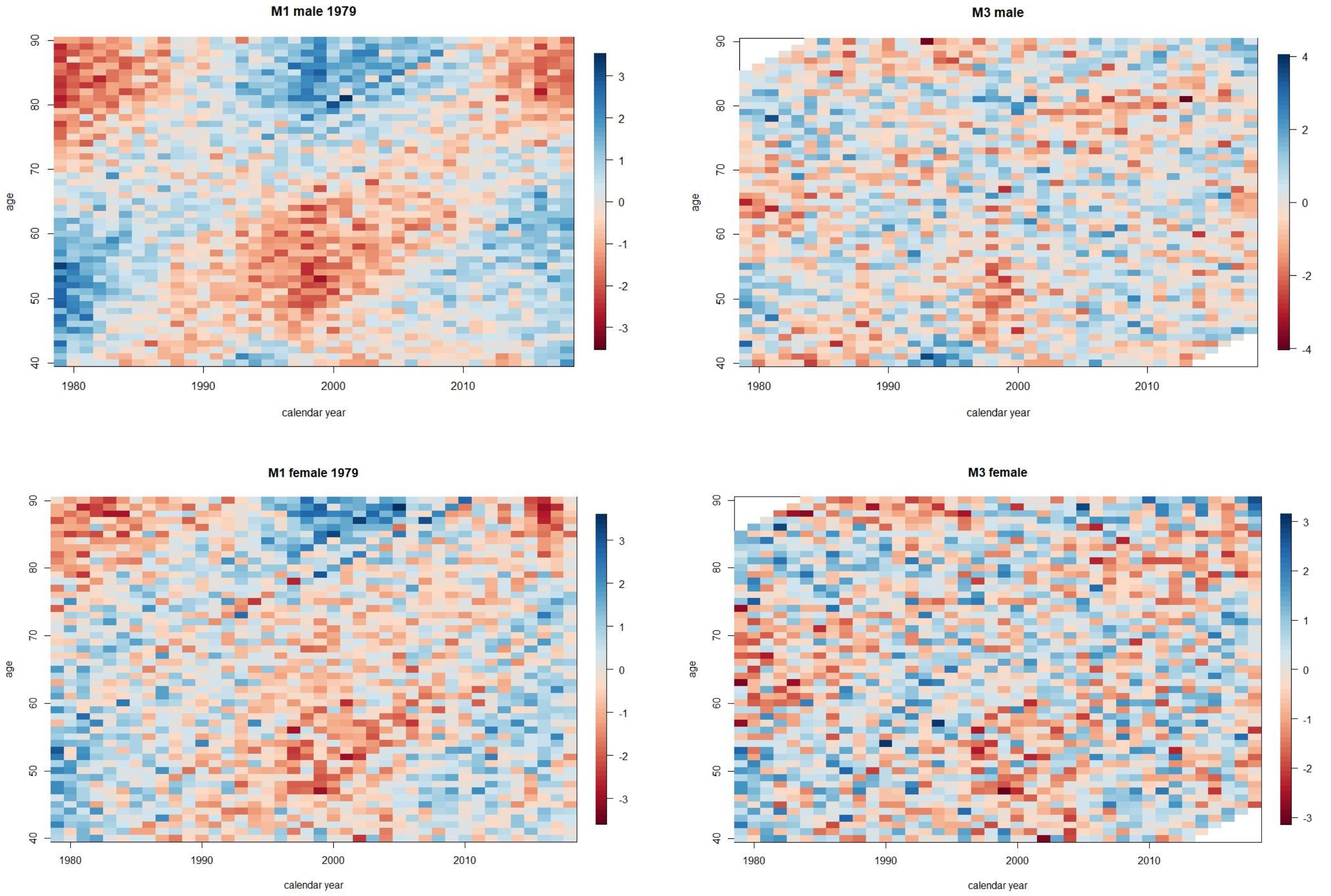
MODEL CHOICE CRITERIA: GOODNESS OF FIT

Comparing residuals qualitatively

- Structure should be captured by model
- Residuals should show none

Quantitative criteria for goodness of fit: AIC, BIC



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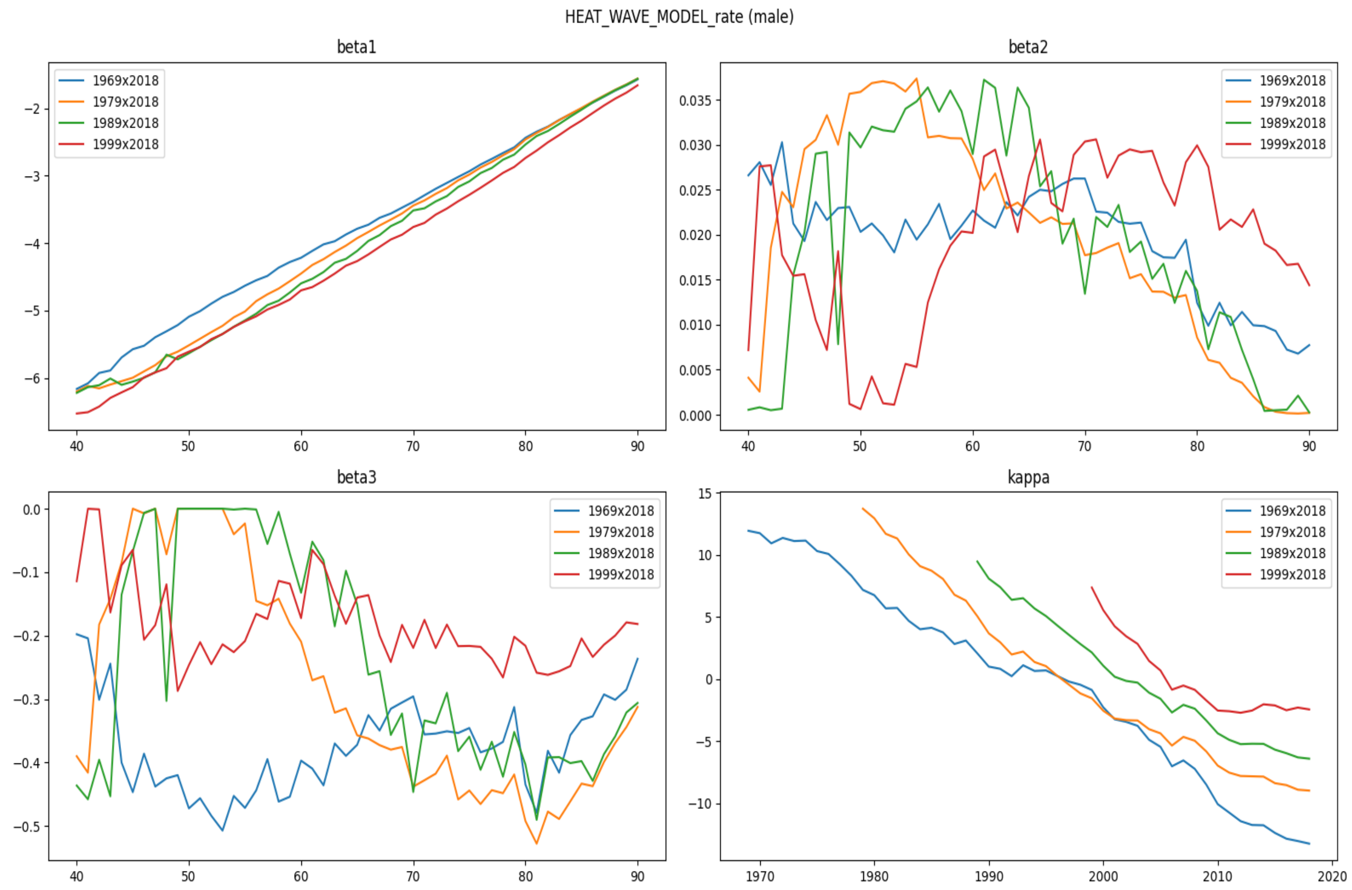
MODEL CHOICE CRITERIA: PARAMETER ROBUSTNESS

HEAT WAVE MODEL

- Data Ranges: 1969:2018, 1979:2018, 1989:2018, 1999:2018
- Robustness failure for multiple parameters
- Signs of over-fitting and lack of identifiability
- ‘beta3’ hitting $cap = 0$



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MODEL CHOICE CRITERIA: MODEL ROBUSTNESS

$$\max\left(|\Delta(E_{x,y}\widehat{m}_{x,y})|/\sqrt{E_{x,y}\widehat{m}_{x,y}}\right)$$

Observation Periods starting in 1969, 1979, 1989, 1999

Model	1969	1989	1999
M2	1.37	2.25	1.82
M3	2.14	1.01	1.49
M7	0.86	0.91	1.34
APCI	1.68	1.73	1.74
SIMPLIFIED PLAT	1.41	1.02	1.47
APC_MI	**4.54**	**3.27**	**4.35**
SIMPLIFIED PLAT (MI)	**4.11**	**2.88**	**4.35**
SIMPLIFIED PLAT (MI) pen.	**3.92**	**4.01**	**4.51**

Age ranges: 30:90, 40:90, 50:90, 40:100

Model	30:90	50:90	40:100
M2	1.25	0.50	0.57
M3	0.85	0.68	0.49
M7	**4.74**	**1.67**	**2.38**
APCI	2.36	0.76	0.61
SIMPLIFIED PLAT	2.60	0.89	1.46
APC_MI	**3.39**	**2.32**	**2.37**
SIMPLIFIED PLAT (MI)	**4.34**	**2.51**	**2.61**
SIMPLIFIED PLAT (MI) pen.	**3.91**	**2.07**	**2.91**

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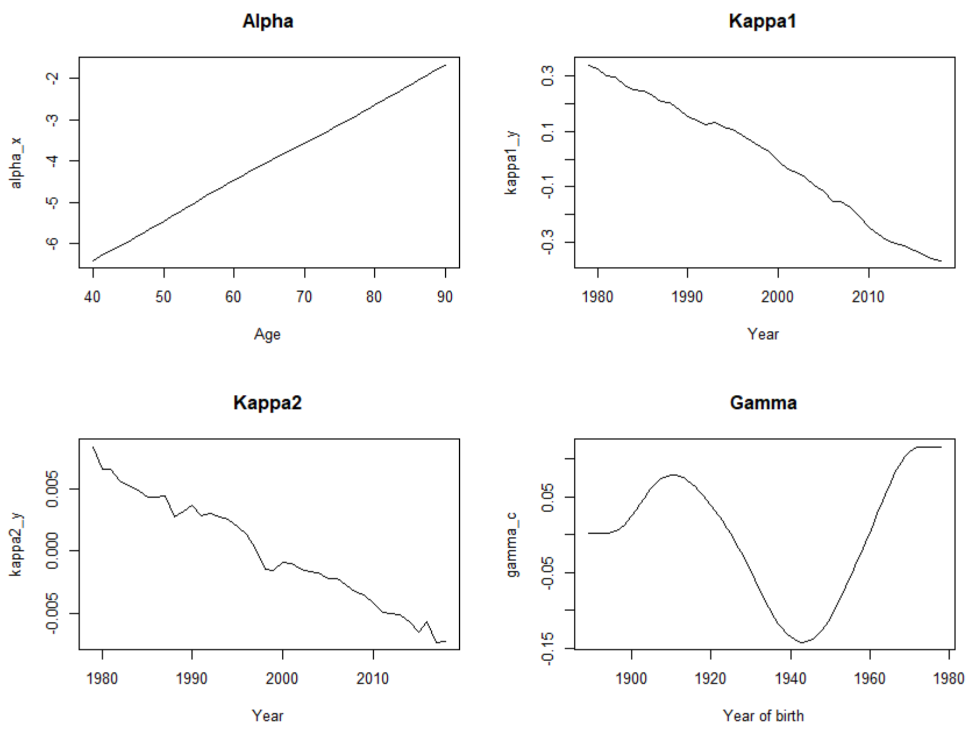
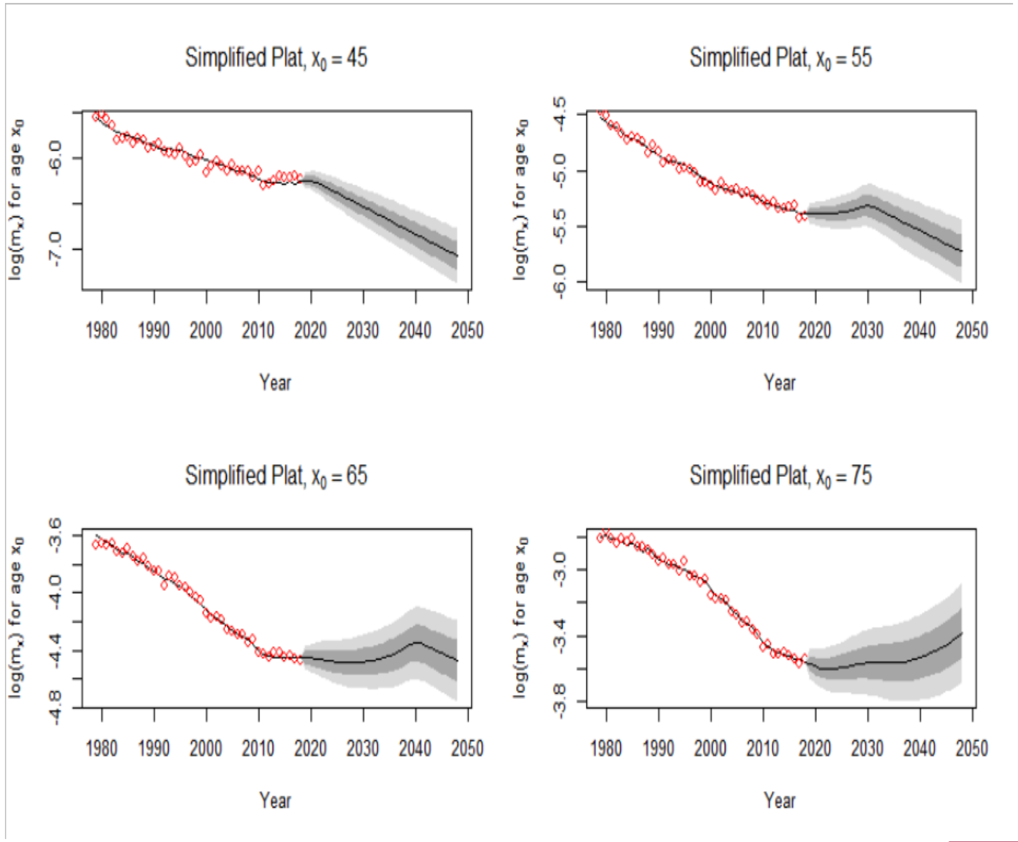
OWN CALCULATIONS USING STMOMO IN R AS WELL AS PYTHON

MODEL CHOICE CRITERIA: REASONABLE FORCASTS

SIMPLIFIED PLAT: $\log(m_{x,y}) = \alpha_x + \kappa_y^{(1)} + \kappa_y^{(2)}(\bar{x} - x) + \gamma_{y-x}$





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MODELS WHICH PERFORM BEST: APCI AND APC

APCI model most suitable for forecasting Canadian population mortality experience based on combination of quantitative and qualitative selection criteria. Next most suitable: APC

APCI model structure

$$\log(m_{x,y}) = \alpha_x + \beta_x \cdot (y - \bar{y}) + \kappa_y + \gamma_{y-x}$$

- Model structure implies existence of an age-dependent mortality improvement rate β_x which is constant over time.
- Model for the mortality rate which contains an explicit parameter for the mortality improvement rate at age x.

APC model structure

$$\log(m_{x,y}) = \alpha_x + \kappa_y + \gamma_{y-x}$$

- Model structure considers age-, period- and cohort effects, but no age-dependant mortality improvement rate.

$m_{x,y}$	Death rates
α_x	Average central log-mortality rate at age x over the calendar year range
β_x	Average yearly mortality improvement rate at age x over the calendar year range
κ_y	Calendar year effect in year y not captured by the average, age-dependent improvement rate
γ_{y-x}	Cohort effect of birth year $y - x$

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SMOOTHNESS OF PARAMETER ESTIMATES

- Smoothness conditions on parameters imposed in estimation where “biologically reasonable”
  - β_x , γ_{y-x} smoothed, κ_y unsmoothed

- Degree of smoothing mostly visually guided
  - Eliminate noise
  - Maintain characteristics of parameter series
  - Coherent smoothing of various parameter groups to avoid spilling effects

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TREATMENT OF COHORT EFFECTS

- Parameter estimation for marginal cohorts $c_{min}, \dots, c_{min} + k$ and $c_{max} - k, \dots, c_{max}$ based on very few observations, resulting in high variance of γ_c for these cohorts
- Alternative approach to omitting these observations: Combining marginal cohorts
- Resulting restrictions $\gamma_{c_{min}} = \cdots = \gamma_{c_{min}+k}$ and $\gamma_{c_{max}-k} = \cdots = \gamma_{c_{max}}$
- Serve as identifiability conditions eliminating the need for two conditions on $\gamma_{c_{min}}, \dots, \gamma_{c_{max}}$
- Condition $\sum_c (c - \bar{c})\gamma_c = 0$ additionally included to eliminate trends in the cohort effect

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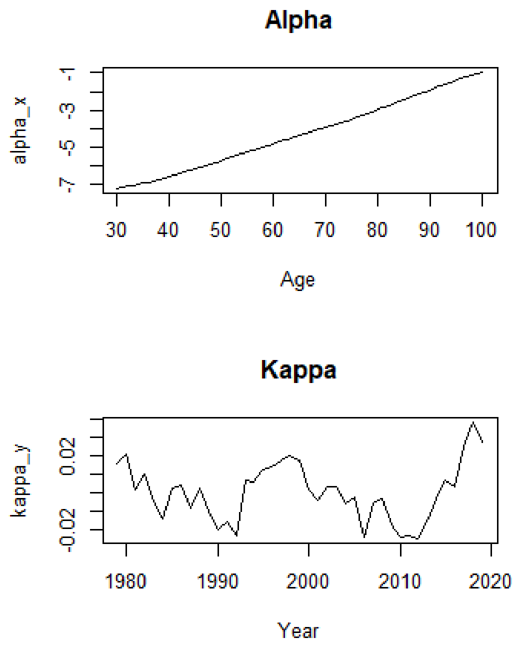
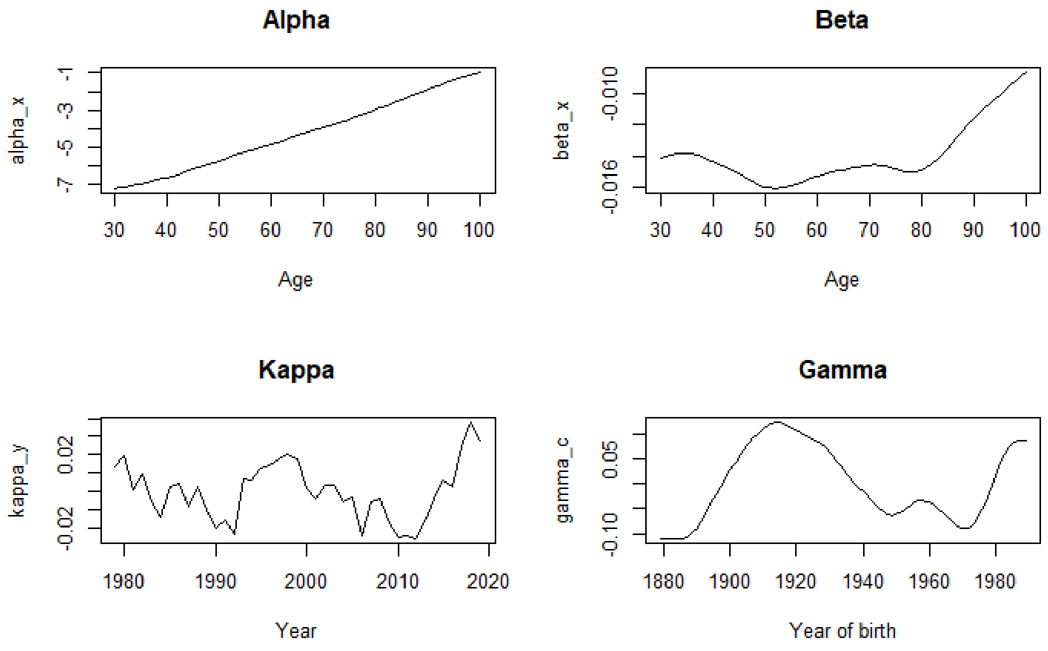
UNSMOOTHED AND SMOOTHED PARAMETER SERIES





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INITIAL RATE OF MORTALITY IMPROVEMENT

- Parameter series used to estimate initial rate $MI^{*}_{x,y_{max}} = \log\left(m_{x,y_{max}-1}\right) - \log\left(m_{x,y_{max}}\right)$
- Decomposition into age, cohort and period component:
  - Age component: $-\beta_x$
  - Cohort component: $\gamma_{c-1} - \gamma_c$
  - Period component: Weighted average of $\kappa_{y-1} - \kappa_y$ over recent period (e.g. exponentially decreasing weights over last 10 years)
- Projection to expert input of long-term improvement rates by third order polynomials

➢ Variations and extensions on the approach of “Continuous Mortality Investigation, Mortality Projections Committee (2017). WORKING PAPER 98 CMI Mortality Projections Model: Methods”

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RECONCILING EXPERT INPUT TO STOCHASTIC MODEL OUTPUT

- Two alternative methods for determining future mortality rates
  - **APCI model** best compromise between goodness of fit and robustness for recent mortality trends. Projections subject to increasing amount of uncertainty
  - **CMI Method** simplified view of future mortality improvements reducing age- and cohort effects, which are hard to predict anyway. Allows for expert input
- Indicative LTMI rate L^{CMI} minimizes sum of squared differences between projected $\log(m_{x,y})$-rates for reasonable projection horizon
  - Alternative indicators could be any actuarial present value of interest, e.g., (remaining) life-expectancy, annuity factor

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HISTORICAL AND PROJECTED LOG MORTALITY RATES FOR AGES 50-65

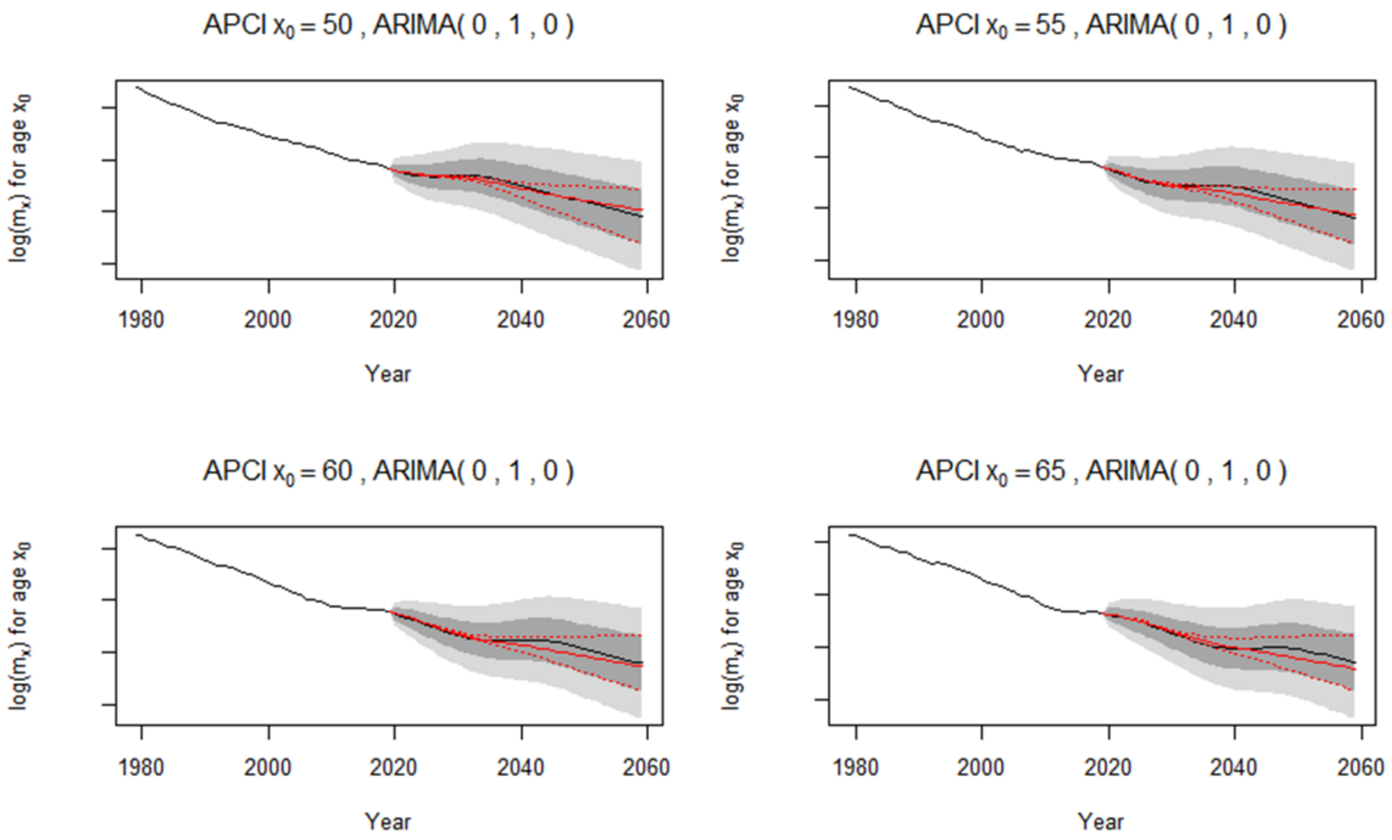
APCI model applied to males and females combined

Observation period 1979 – 2019

Age range 50 – 90 years for CMI fit

Shaded areas indicate 1σ and 95% confidence intervals

Solid red line corresponds to LTMI rate, dashed red lines indicate range of LTMI rates consistent with 1σ confidence band



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HISTORICAL AND PROJECTED LOG MORTALITY RATES FOR AGES 70-85

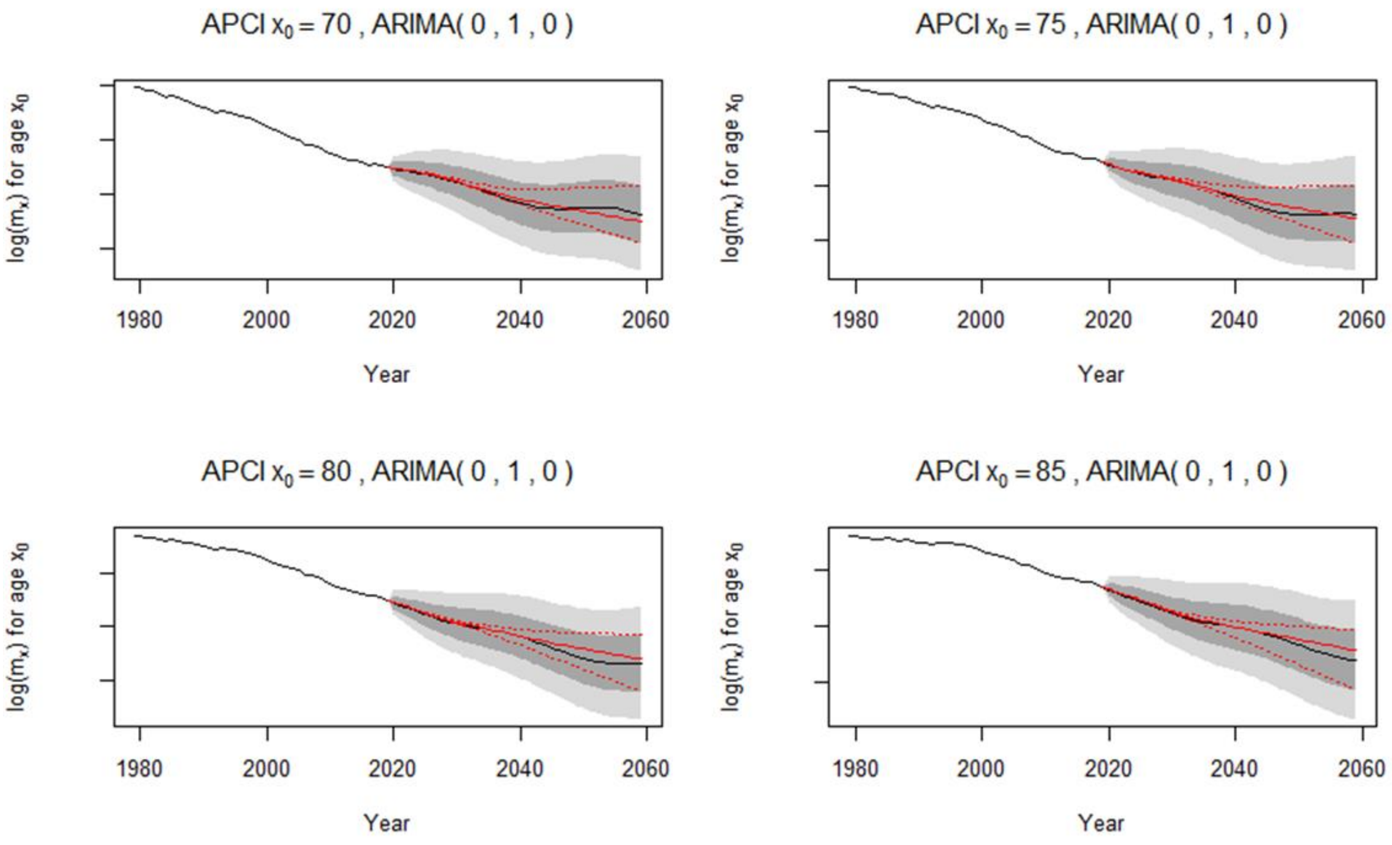
APCI model applied to males and females combined

Observation period 1979 – 2019

Age range 50 – 90 years for CMI fit

Shaded areas indicate 1σ and 95% confidence intervals

Solid red line corresponds to LTMI rate, dashed red lines indicate range of LTMI rates consistent with 1σ confidence band



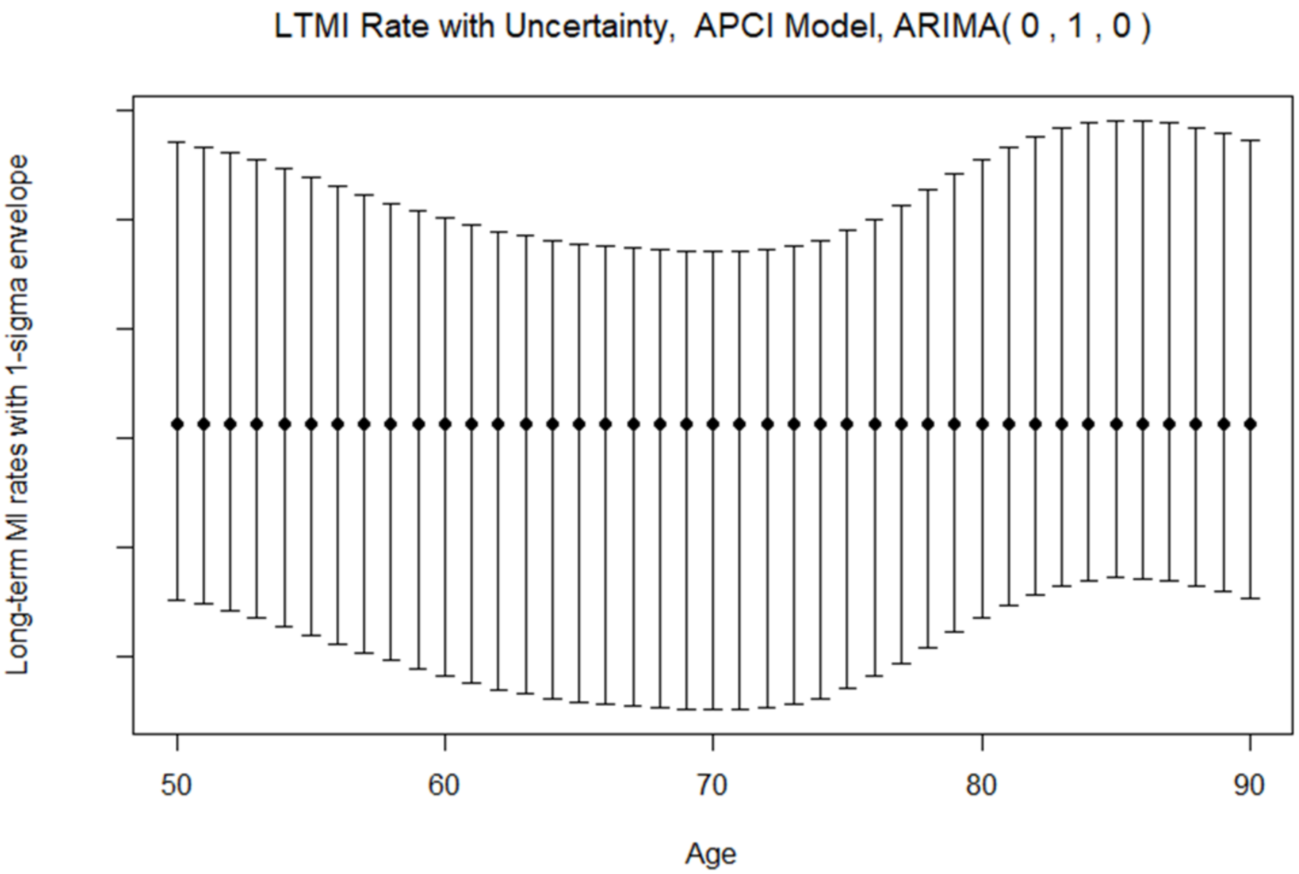
LTMI RATE FOR ALL AGES WITHIN CONFIDENCE BANDS

APCI model applied to males and females combined

Observation period 1979 – 2019

Age range 50 – 90 years for CMI fit

Error bars for each age correspond to the 1σ confidence intervals shown in previous charts as dashed red lines



IMPACT OF PRACTICAL DECISIONS

Practical decisions may have a large impact (ongoing discussion)

- Model choice
- Direction of travel
- Age range, year range
- Degree of smoothing
- Treatment of cohorts, particularly marginal cohorts
- Choice of method to determine equivalent LTMI (SSE, weighted SSE, life expectancy, annuity)

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OUTLOOK

Further Research

- Methodology to incorporate historical experience including pandemics or other outliers in mortality projection models
- Modelling of trends for different socio-economic groups
- Update procedure: How to decide whether new calibration / new model is necessary?

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