Longevity 18, London

Robust mortality forecasting in the presence of outliers

Stephen J. Richards

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Overview

1. Motivation

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2. Univariate forecasts

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3. Multivariate forecasts

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4. 2D P-spline model

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1. Motivation
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4. 2D P-spline model
5. Conclusions

About this presentation

Fast introduction to robust mortality forecasting.

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Further details in Richards [2023], freely available at:

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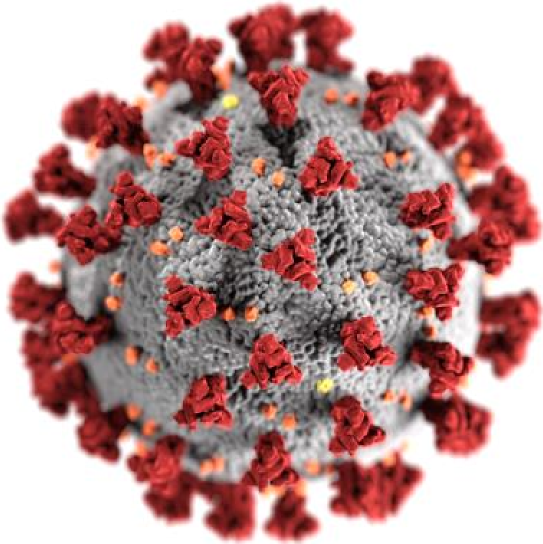
www.longevitas.co.uk/robust-forecasting

1 Motivation



1 Three problems for actuaries

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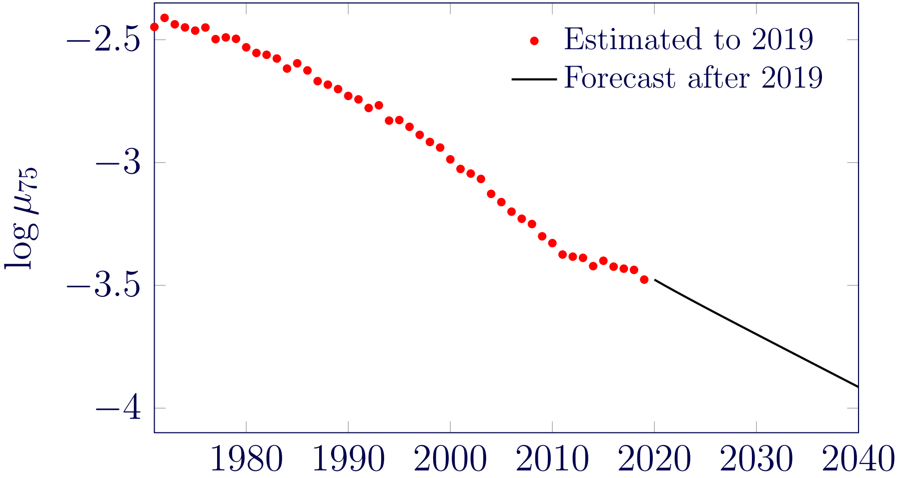
3. Inflated variance.

1 Broken forecasts

Covid-19 distorts central projections…

Using data to 2019

ARIMA forecast of time index in Lee-Carter model:



Source: Data for males in England & Wales, ages 50–105, 1971–2019.

Using data to 2020

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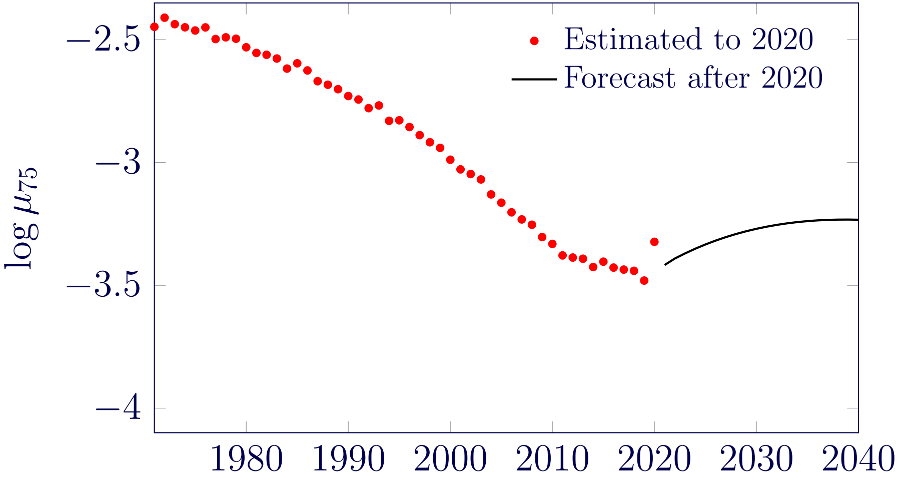

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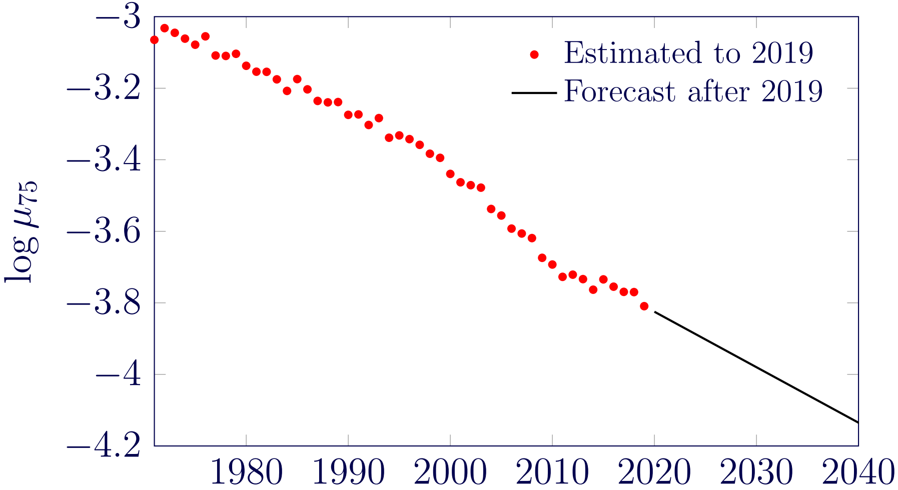
1 Biased starting points

Covid-19 leads to biased starting points...

Using data to 2019

Bivariate random-walk forecast under M5 model:



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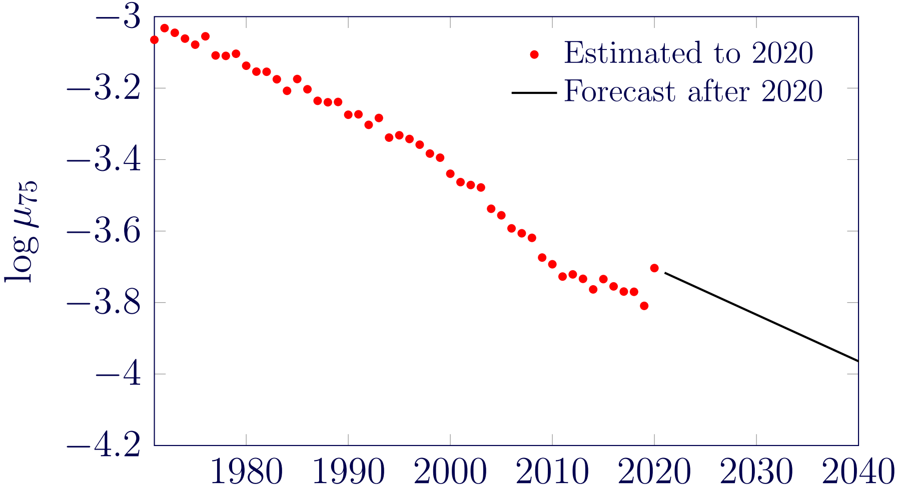

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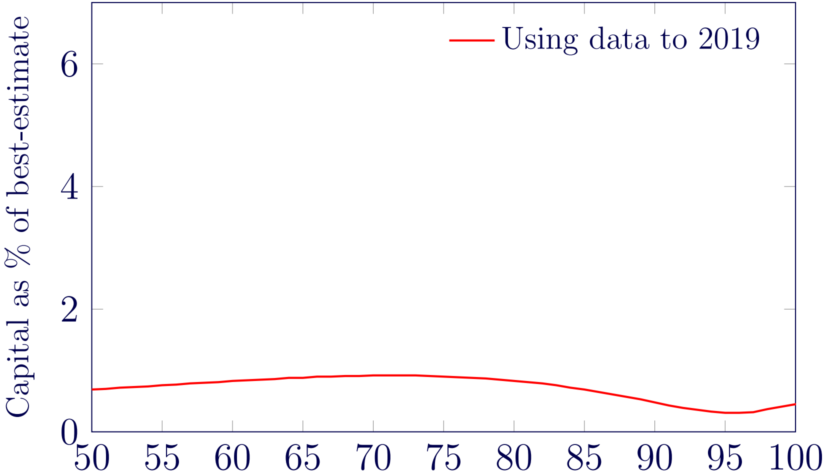
1 Inflated variance

Outliers increase VaR capital requirements...

Value-at-risk capital at 99.5%



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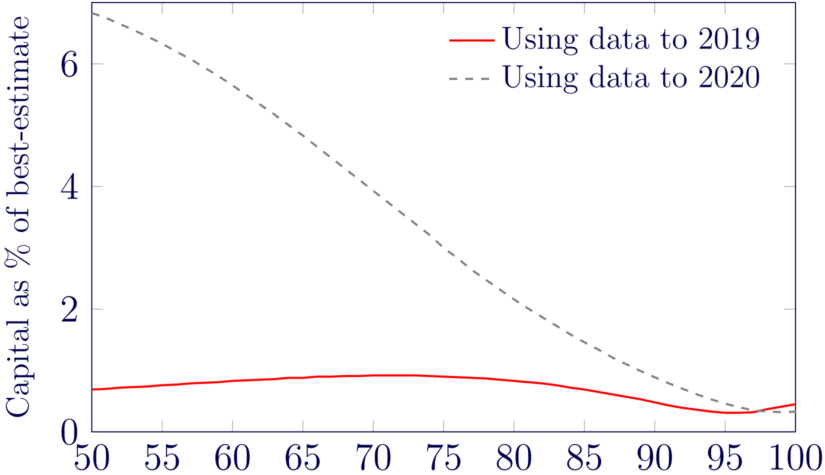

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- How can we robustify forecasts for actuarial tasks?

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1. Remove distortion in parameter estimates.
2. Calculate "clean" starting points for forecasts.
3. Estimate variance robustly.
4. Need objective methodology for (1)-(3) to allow repeated recalibration under VaR-style simulations.

1 Solution

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- Co-estimate outlier effects with other parameters.

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- $\hat{\alpha}_x$ and $\hat{\beta}_x$ are held constant in the forecast.
- An ARIMA model is fitted to the $\hat{\kappa}_y$ time index to forecast the trend.

2 Outlier definition

Outlier

An observation that is further from the one-year-ahead forecast than is consistent with the noise variance.

2 Univariate outliers

To robustify an ARIMA model, Chen and Liu [1993]:

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2. Proposed further test statistics to *classify* outliers.

IO Innovation outlier

2 Four outlier types

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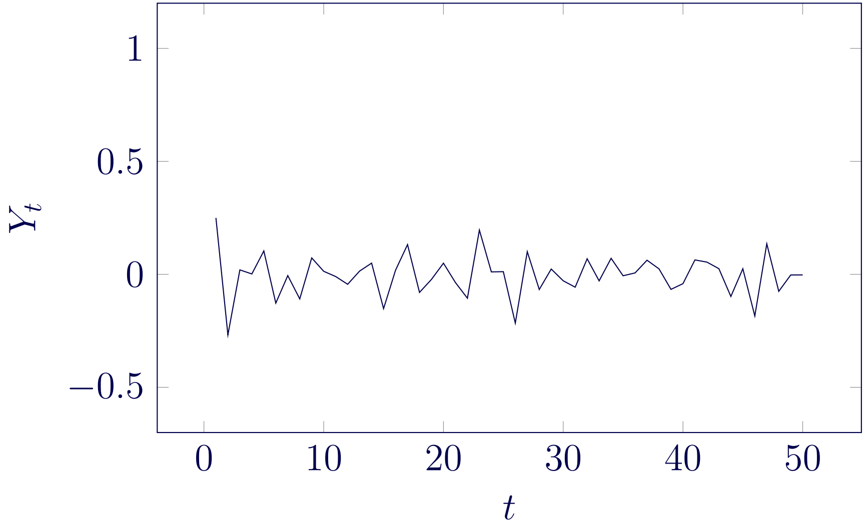
IO	Innovation outlier
AO	Additive outlier
TC	Temporary change
LS	Level shift

2 Illustrating outlier types

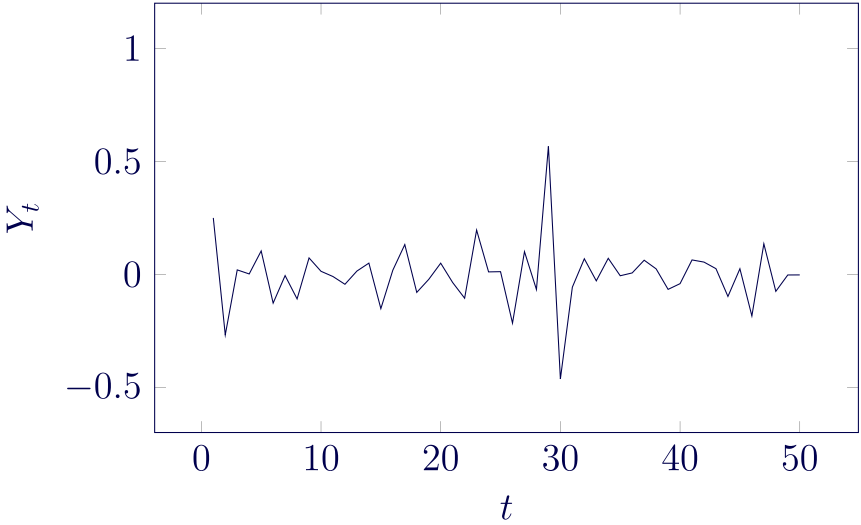
Consider a moving-average (MA) process:

$$Y_t = \epsilon_t - 0.8\epsilon_{t-1} \tag{1}$$



Source: Richards [2023, Figure 3].

2 IO — Innovation Outlier



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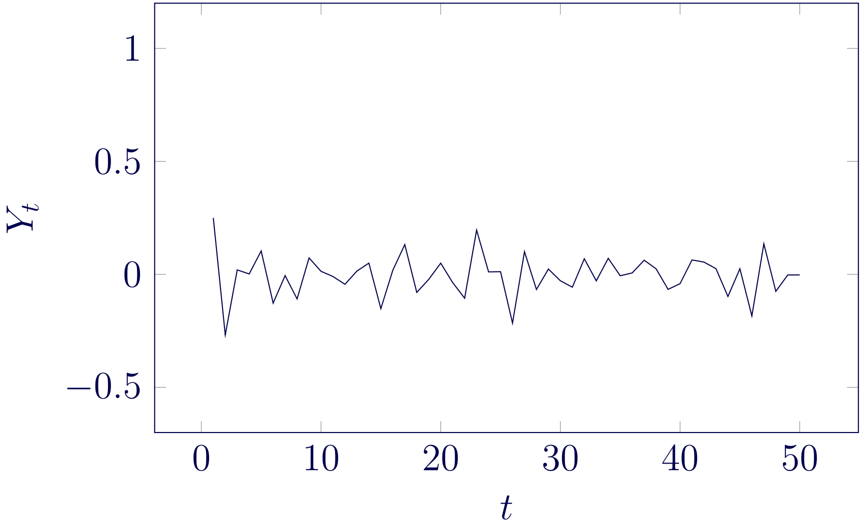
Handling: leave alone.

2 Uncontaminated MA process



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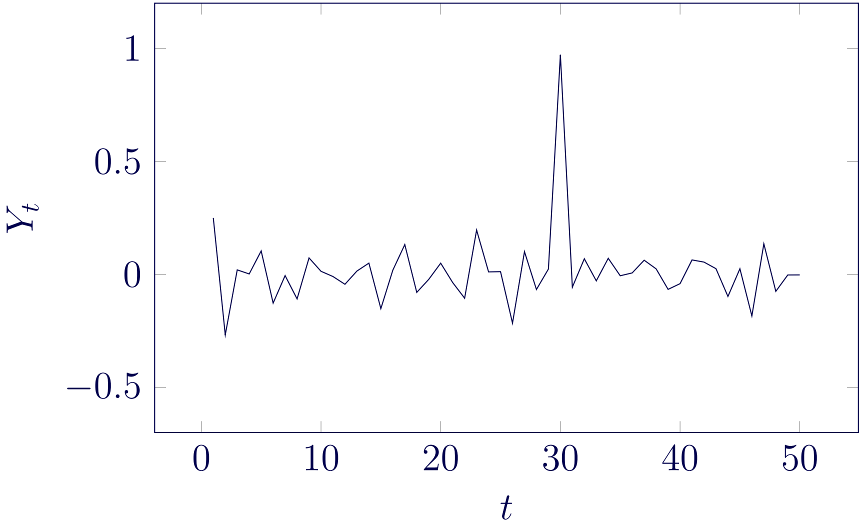
2 AO — Additive Outlier



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2 Additive outlier

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Example

War or pandemic in a single year.

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Handling: co-estimate the outlier effect to remove bias in other parameters.

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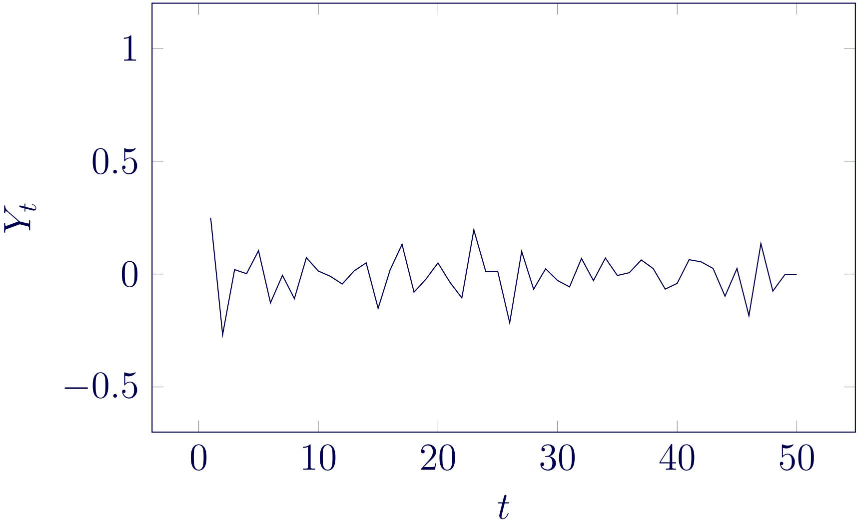

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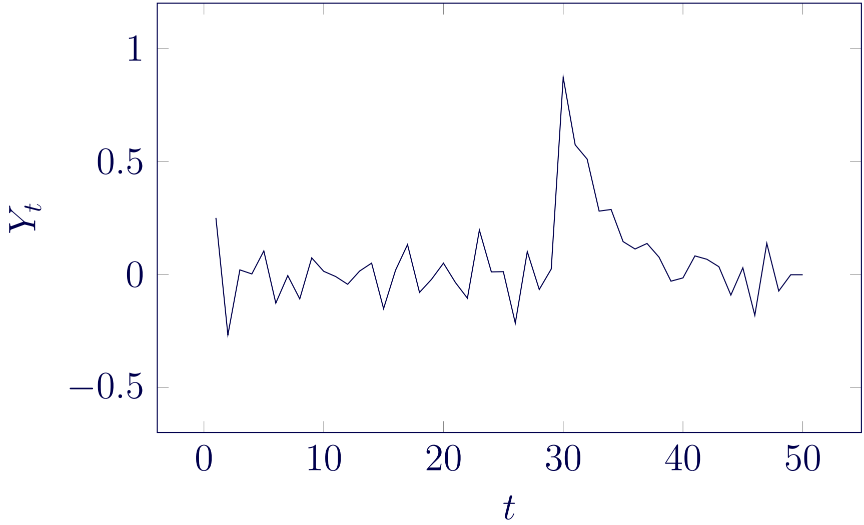
2 TC — Temporary Change



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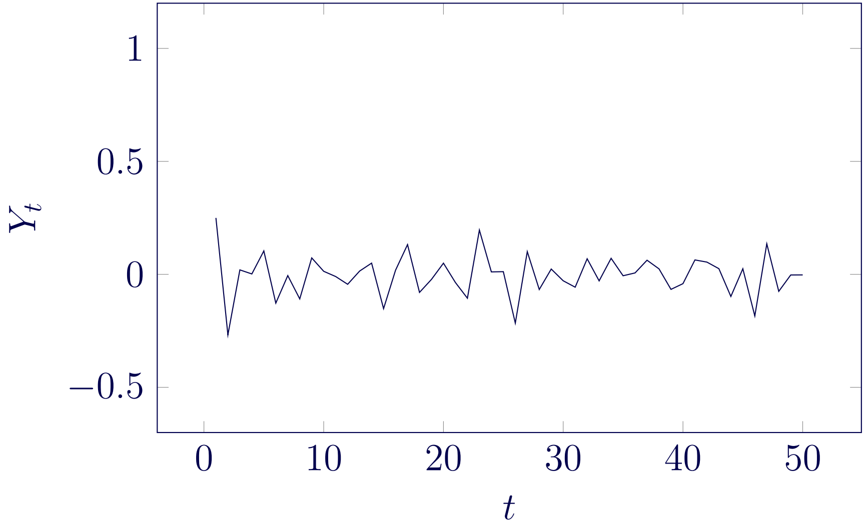

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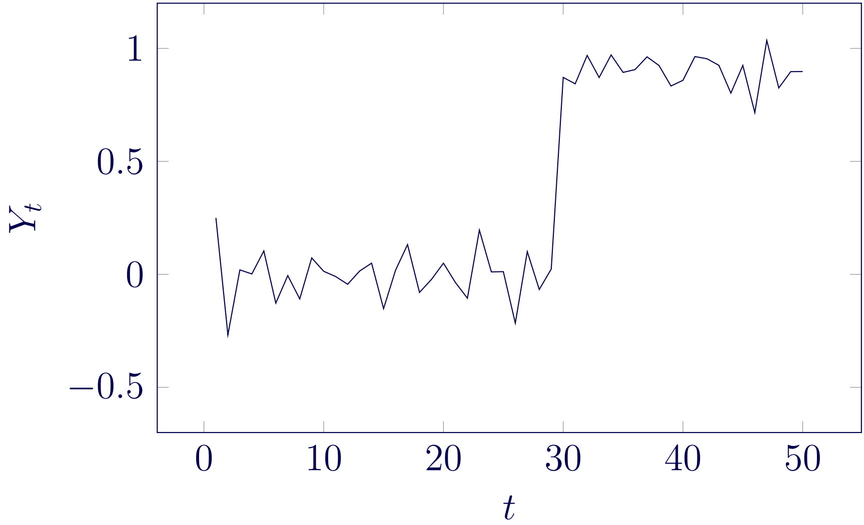
2 LS — Level Shift



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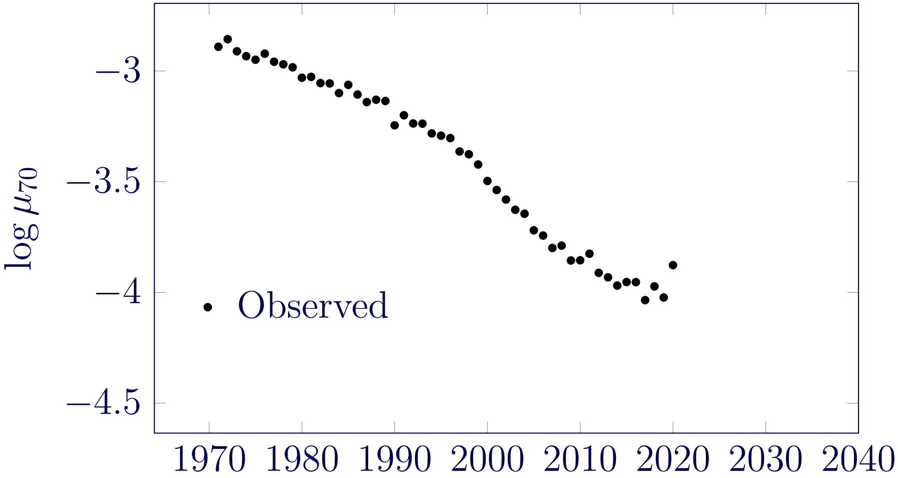
Handling: review model or data period.

2 Chen and Liu [1993]

Note

An outlier can be detected anywhere, but “it is impossible to empirically distinguish the type of an outlier at the very end of a series” [Chen and Liu, 1993, page 286].

2 Robust Lee-Carter forecast



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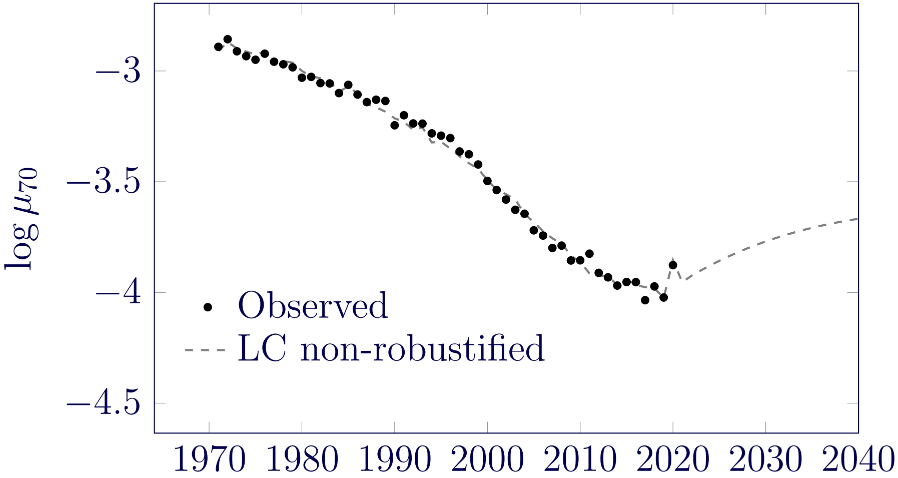

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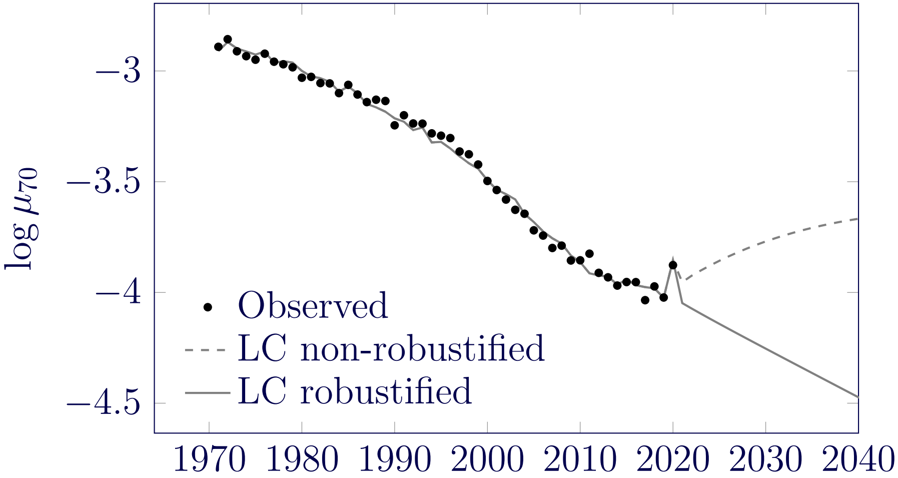

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$$\log \mu_{x,y} = \kappa_{0,y} + (x - \bar{x})\kappa_{1,y}$$

- $\hat{\kappa}_{0,y}$ and $\hat{\kappa}_{1,y}$ are forecast jointly as a bivariate random walk with drift.

3 Multivariate robustification

- Use approach of Galeano et al. [2006].

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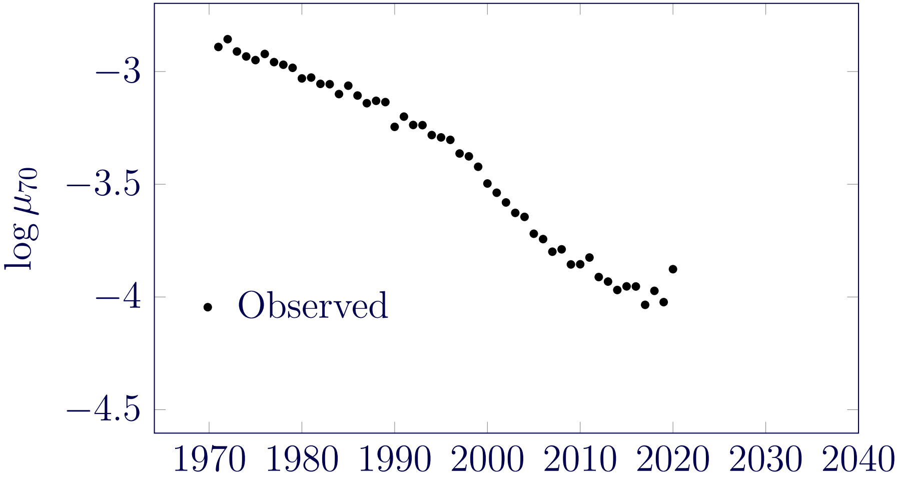
- Robustify first differences [Richards, 2023, Appendix C].

3 Robust M9 forecast



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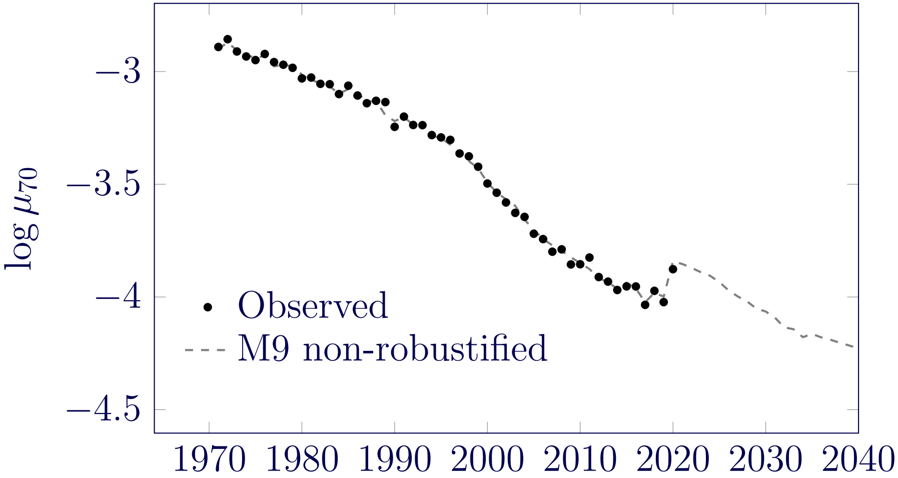

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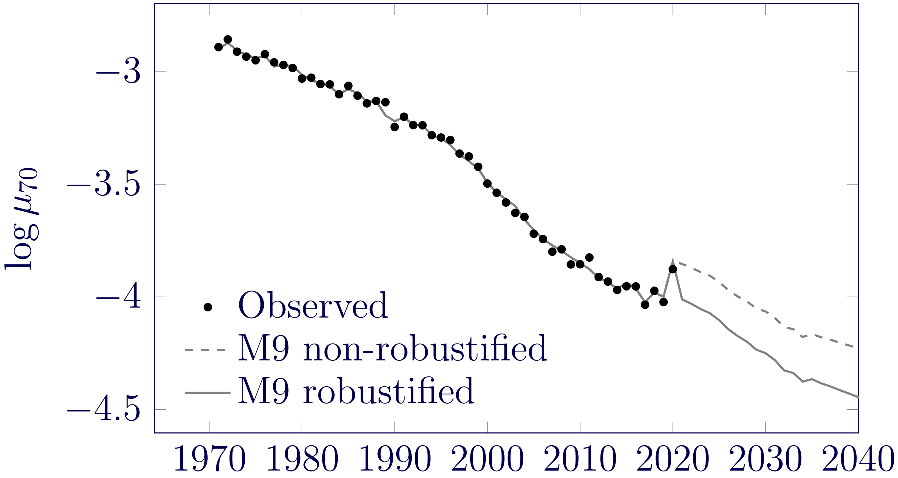

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3 Other multivariate options

Other approaches to identifying outliers in multivariate data, e.g. Hadi [1994].

4 2D P-spline model

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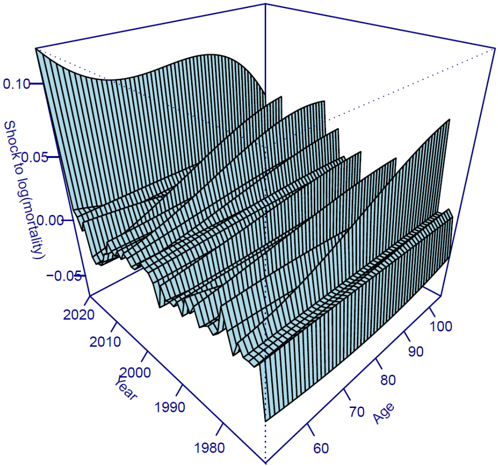
- Extended by Kirkby and Currie [2010] to estimate period shocks. . .

4 Period shocks (EW males)



Source: Richards [2023, Figure 12].

4 2D P-spline model

- Forecast using *penalty function*.

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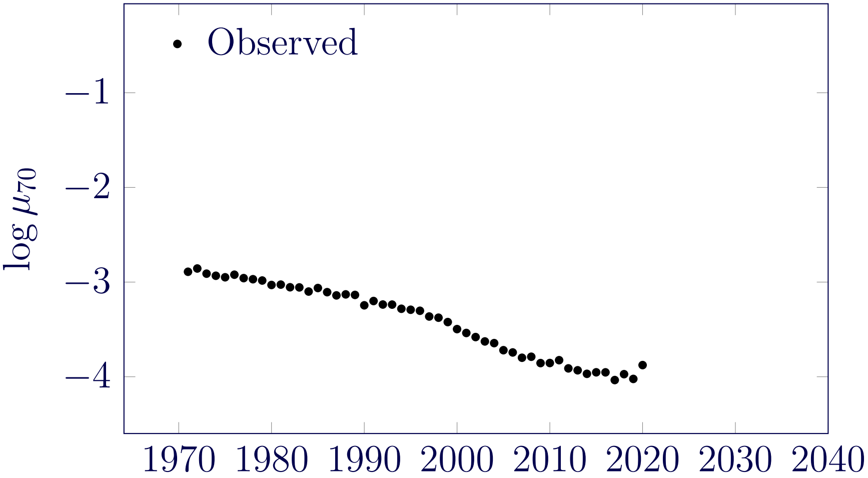
- Forecast using *penalty function*.
- Estimating period shocks robustifies penalty forecast...

4 Robust 2DAP forecast



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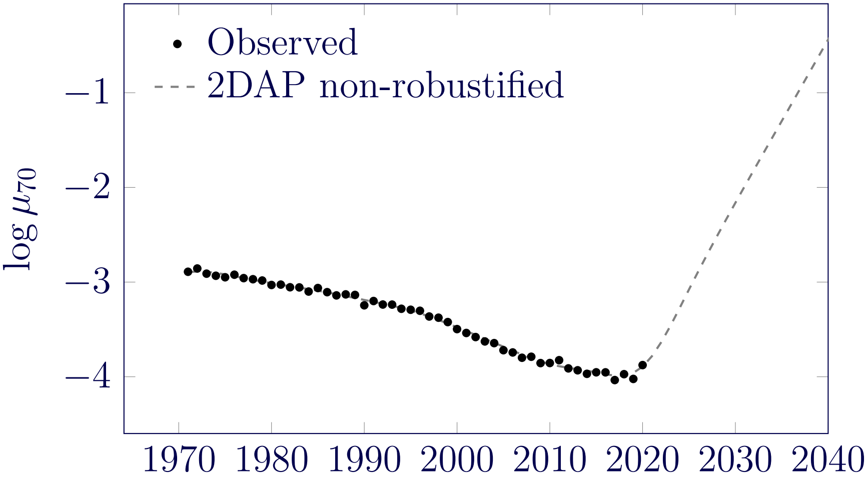

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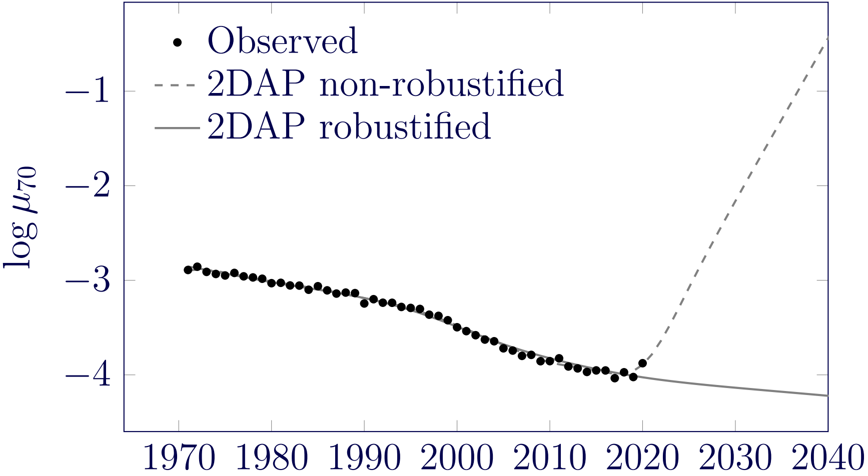

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S. J. Richards. Robust mortality forecasting in the presence of outliers. Longevitas working paper, 2023.

Coronavirus graphic  from CDC

More on robust forecasting at www.longevitas.co.uk/robust-forecasting

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