Optimal Hedging of Longevity Risks for Group Self-Annuity Portfolios

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Research motivation

Background

- Risk-pooling products[1] are increasingly popular:
  - No risk premium is charged.
  - No capital is required.
  - Reduce adverse selection and increase post-retirement utility (Valdez et al., 2006; Hanewald et al., 2013).
  - Existing products: QSuper Lifetime Pension[2]; GuardPath Modern Tontine[3].
- The **systematic longevity risk** undermines the effectiveness of the risk-pooling products:
  - Participants prefer a stable and high level of survival benefit.
  - **Benefits reduce** when members live longer than expected.
  - **Benefits are volatile** due to the uncertainty of the level of longevity.
  - Undiversifiable.

[1] For example, group self-annuity (Piggott et al., 2005), pooled annuity fund (Stamos, 2008; Donnelly et al., 2014), tontine (Milevsky and Salisbury, 2015) among others.

[2] `https://qsuper.qld.gov.au/our-products/superannuation/lifetime-pension`.

[3] `https://www.guardiancapital.com/investmentsolutions/guardpath-modern-tontine-trust/`.

Longevity risk transfer solutions

- Annuity providers and defined benefit pension plans.
- Hedge the adverse financial effect of longevity risk.

	Customised	**Index-based**
Counterparty	A third party	Capital market investors
Trading frequency	Static	Dynamic
Underlying	The book population	A whole population
Population basis risk	No	Yes
Effectiveness	More effective	Less effective
Cost	More expensive	Cheaper

Table 1: A comparison of longevity risk transfer solutions.

Longevity swap trading volume

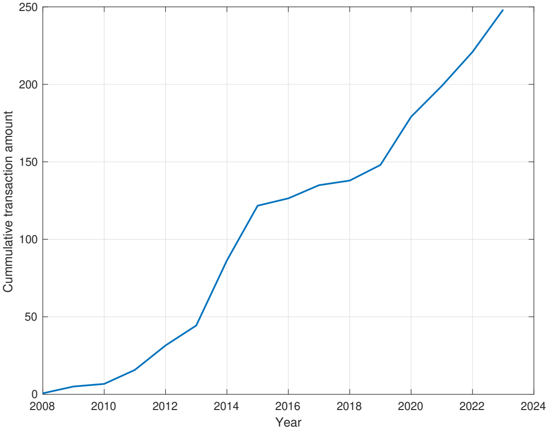


Figure 1: Cumulative amount of longevity swap transactions in US$ billion since the transaction of the q-forward in January 2008 between J.P.Morgan and Lucida. Data source: `https://www.artemis.bm/longevity-swaps-and-longevity-risk-transfers/`.

Research objective

Increase the **effectiveness** of risk-pooling products with **index-based dynamic** longevity risk hedging solutions.

Methodology

Mortality model

Two populations:

- Population F: The GSA fund population.
- Population R: The reference population of the longevity securities.

Augmented Common Factor (ACF) mortality model (Li and Lee, 2005):

$$\log\left(m_{x,t}^{(i)}\right) = a_x^{(i)} + \underbrace{G_x K_t}_{\text{common factor}} + \underbrace{g_x^{(i)} k_t^{(i)}}_{\text{population-specific factor}} + \epsilon_{x,t}^{(i)}, \text{ for } i \in \{F, R\}, \tag{1}$$

where K_t follows a random walk with drift and $k_t^{(i)}$ follows an AR(1) process.

Notations: Death and survival probabilities

- One-year death probability:

$$q_{x,t}^{(i)} \approx 1 - \exp(-m_{x,t}^{(i)}). \tag{2}$$

- T-year survival probability:

$$S_{x,t}^{(i)}(T) := \prod_{s=1}^{T} \left(1 - q_{x+s-1,t+s}^{(i)}\right). \tag{3}$$

- Best estimate of T-year survival probability given current information:

$$p_{x,u}^{(i)}(T, K_t, k_t^{(i)}) := \mathbb{E}\left[S_{x,u}^{(i)}(T) | \mathcal{F}_t\right] = \mathbb{E}\left[S_{x,u}^{(i)}(T) | K_t, k_t^{(i)}\right]. \tag{4}$$

The GSA fund process

The GSA fund evolves as follows:

$$\text{Number of survivors: } N_t = \underbrace{N_{t-1} S^{(F)}_{x+t-1,t-1}(1)}_{\text{Ignore small sample risk}}, \tag{5}$$

$$\text{Investment return: } F_t^- = F_{t-1}^+(1+r), \tag{6}$$

$$\text{Survival benefit: } B_t = \frac{F_t^-}{\ddot{a}^e_{x+t,t} N_t}, \tag{7}$$

$$\text{Death benefit: } D_t = \beta \frac{F_t^-}{N_{t-1}}, \quad 0 \le \beta < 1, \tag{8}$$

$$\text{Fund value after payment: } F_t^+ = F_t^- - B_t N_t - D_t \Delta N_t, \tag{9}$$

where

$$\ddot{a}^e_{x+t,t} = \sum_{s=0}^{+\infty} (1+r)^{-s} p^{(F)}_{x+t,t}\left(s, K_t, k_t^{(F)}\right), \quad \Delta N_t = N_{t-1} - N_t. \tag{10}$$

S-forwards

- Settlement at maturity:

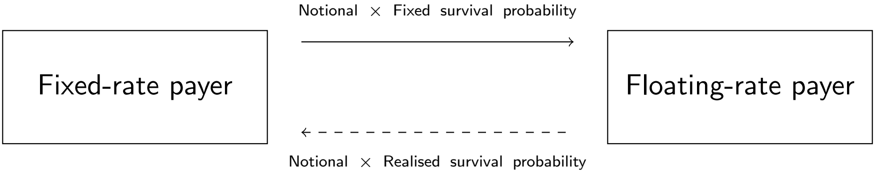


Figure 2: Settlement of an S-forward contract at maturity.

- The fixed (forward) survival probability at inception:

$$p_t^f := (1+\lambda)\mathbb{E}\left[S_{x^f,t}^{(R)}(T^*)|K_t, k_t^{(R)}\right],$$

where $\lambda \geq 0$ is the risk loading parameter.

A yearly rolling hedging strategy

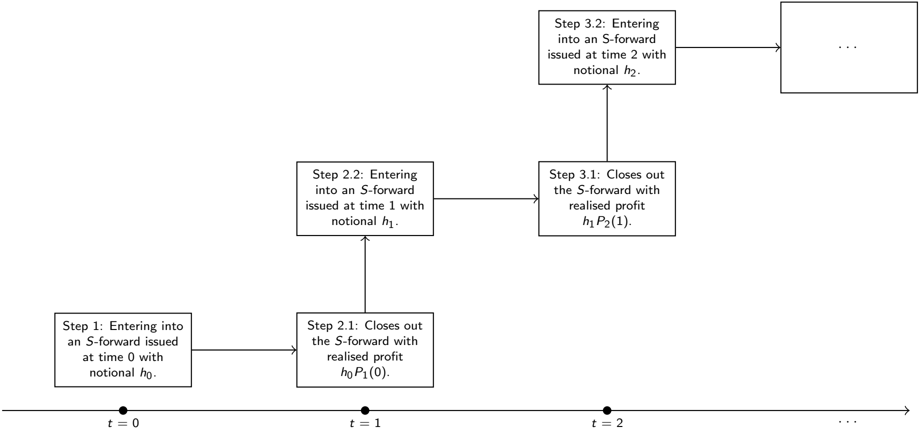


Figure 3: The yearly rolling hedging strategy (Zhou and Li, 2017; Tan et al., 2022). **The GSA fund is the fixed-rate payer.** $P_{t+1}(t)$ is the hedging profit or loss per \$1 notional.

Hedging mechanism

The total benefit $B_{t+1}^{(H)}$ is determined as follows:

$$B_{t+1}^{(H)} := \underbrace{B_{t+1}}_{\text{unhedged benefit}} + \underbrace{\frac{h_t P_{t+1}(t)}{N_t S_{x+t,t}^{(F)}(1)}}_{\text{hedging profit or loss}} . \tag{11}$$

At time $t+1$, the hedging profit (or loss) of the **fixed rate payer** is:

$$P_{t+1}(t) = (1+r)^{-(T^*-1)} \left[\underbrace{\mathbb{E}_{t+1}\left[S_{x^f,t}^{(R)}(T^*)\right]}_{\text{Expectation at } t+1} - (1+\lambda) \underbrace{p_{x^f,t}^{(R)}\left(T^*, K_t, k_t^{(R)}\right)}_{\text{Expectation at } t} \right] \tag{12}$$

$$N_{t+1} \nearrow \Rightarrow B_{t+1} \searrow ; \mathbb{E}_{t+1}\left[S_{x^f,t}^{(R)}(T^*)\right] \nearrow \Rightarrow \text{Hedging profit} \nearrow$$

The mean-variance set

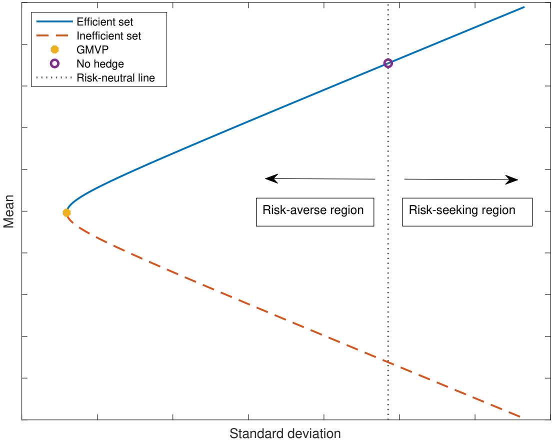


Figure 4: The mean-variance set of longevity hedge using S-forwards for the GSA fund members. The mean-variance set shows a parabolic shape. GMVP: The global minimum variance point.

The mean-variance optimisation problem

Given the information at time t, fund members aim to solve the following mean-variance optimisation problem (Wong et al., 2017):

$$\min_{h_t \in \mathbb{R}} \left\{ V_t(h_t|\mathcal{F}_t) := \underbrace{\mathrm{Var}_t\left[B_{t+1}^{(H)}\right]}_{\text{Minimise}} - 2\phi_t \left(\underbrace{\mathbb{E}_t\left[B_{t+1}^{(H)}\right]}_{\text{Maxmise}} - \mathbb{E}_t\left[B_{t+1}\right] \right) \right\}, \quad (13)$$

where $V_t(h_t)$ is the value function, the parameter $\phi_t(\geq 0)$ controls the mean-variance trade-off.

$\phi_t \nearrow \Rightarrow$ Put more weights on the mean $\Rightarrow$ Less risk-averse

The mean-variance trade-off

- **Hedging benefit:** Variance reduction ratio (VRR):

$$\mathrm{VRR}_t(\phi_t) := 1 - \frac{\mathrm{Var}_t\left[B_{t+1}^{(H)}\right]}{\mathrm{Var}_t\left[B_{t+1}\right]}, \quad \text{for } \phi_t \leq \phi_t^{(RN)}, \tag{14}$$

where $\phi_t^{(RN)}$ is the risk-neutral (mean-variance) trade-off parameter.

- **Hedging cost:** Mean reduction ratio (MRR):

$$\mathrm{MRR}_t(\phi_t) := 1 - \frac{\mathbb{E}_t\left[B_{t+1}^{(H)}\right]}{\mathbb{E}_t\left[B_{t+1}\right]}, \quad \text{for } \phi_t \leq \phi_t^{(RN)}. \tag{15}$$

Decision rule: The value function

- For two hedging strategies, h_t^A and h_t^B, participants prefer strategy h_t^A, if

$$V_t\left(h_t^A|\mathcal{F}_t\right) < V_t\left(h_t^B|\mathcal{F}_t\right). \tag{16}$$

- For a risk-mitigating strategy h_t, participants gain from the longevity hedge, if

$$V_t\left(h_t|\mathcal{F}_t\right) < V_t\left(0|\mathcal{F}_t\right). \tag{17}$$

Numerical results

Data and assumption

Age of the GSA members at the inception of the fund	x	65
Initial contribution per member	c	\$10,000
GSA payment frequency		Yearly
Risk-free interest rate	r	1% per annum
Fund population		EW population[4]
Reference population		UK total population
Reference age of the S-forwards	x_f	75
Time-to-maturity of the S-forwards	T^*	10 years
Risk loading parameter	λ	0.3%[5]

Table 2: Baseline assumptions in the numerical study.

[4] England and Wales population.

[5] The risk loading parameter is equivalent to an annual risk premium of 0.03% reported in Tang and Li (2021).

The mean-variance trade-off: VRR and MRR

Age	Mean	Minimum	Maximum	95% confidence interval
Hedging benefit: Variance reduction ratio (VRR)				
65	95.89%	95.88%	95.90%	(95.89%, 95.90%)
70	95.91%	95.88%	95.93%	(95.90%, 95.92%)
75	95.95%	95.94%	95.96%	(95.95%, 95.96%)
80	95.93%	95.92%	95.94%	(95.93%, 95.94%)
85	94.78%	93.71%	95.41%	(94.34%, 95.14%)
90	84.17%	81.76%	85.60%	(83.24%, 84.93%)
95	50.09%	45.19%	53.71%	(48.35%, 51.75%)
Hedging cost: Mean reduction ratio (MRR)				
65	0.1729%	0.1726%	0.1731%	(0.1728%, 0.1730%)
70	0.2037%	0.1918%	0.2124%	(0.1989%, 0.2083%)
75	0.2306%	0.1980%	0.2664%	(0.2146%, 0.2485%)
80	0.2617%	0.1905%	0.3678%	(0.2247%, 0.3059%)
85	0.2913%	0.1934%	0.4402%	(0.2391%, 0.3530%)
90	0.2943%	0.1630%	0.5773%	(0.2213%, 0.3894%)
95	0.2678%	0.1461%	0.3992%	(0.2136%, 0.3251%)

Table 3: The variance and mean reduction ratios of the hedge at selected ages ($\alpha = 0.8$ and $\beta = 0$).

Overall hedging gain: Value function

Age	Mean	Minimum	Maximum	95% confidence interval
Optimal longevity hedge $V_t\left(h_t^*\middle|\mathcal{F}_t\right)$				
65	8.92	8.41	9.49	(8.67, 9.17)
70	9.40	5.15	18.77	(6.88, 12.55)
75	9.50	4.21	26.68	(6.09, 14.24)
80	9.58	4.10	24.37	(6.26, 14.13)
85	9.76	5.45	19.63	(7.18, 13.16)
90	12.63	3.89	43.67	(7.20, 21.05)
95	27.31	9.76	98.35	(16.12, 44.58)
No longevity hedge $V_t\left(0\middle|\mathcal{F}_t\right)$				
65	24.73	23.32	26.32	(24.05, 25.42)
70	26.06	14.28	52.04	(19.09, 34.81)
75	26.38	11.67	74.02	(16.91, 39.52)
80	26.59	11.37	67.59	(17.37, 39.21)
85	26.49	14.98	52.32	(19.63, 35.47)
90	28.74	8.95	96.75	(16.52, 47.43)
95	40.98	14.21	140.76	(24.19, 66.36)

Table 4: The impact of longevity hedge on the optimal value function at selected ages ($\alpha = 0.8$ and $\beta = 0$).

Population basis risk index

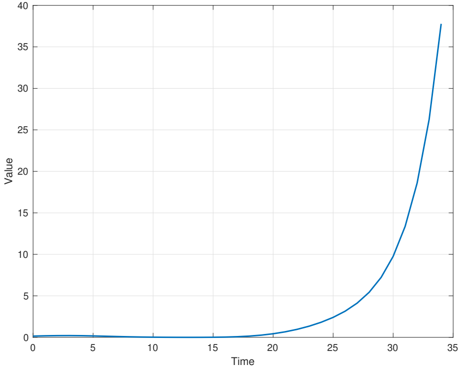
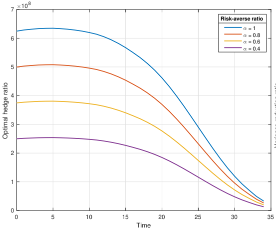


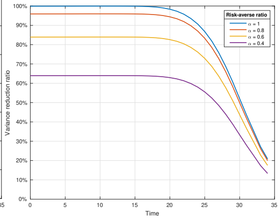
Figure 5: The value of the population basis risk index $I_t := \frac{\mathrm{Var}_t\left(g_{x+t}^{(F)} k_{t+1}^{(F)}\right)}{\mathrm{Var}_t(G_{x+t} K_{t+1})}$ for $t = 0, 1, \cdots, 34$.

The ACF model: $\log\left(m_{x,t}^{(i)}\right) = a_x^{(i)} + \underbrace{G_x K_t}_{\text{common factor}} + \underbrace{g_x^{(i)} k_t^{(i)}}_{\text{population-specific factor}} + \epsilon_{x,t}^{(i)}.$

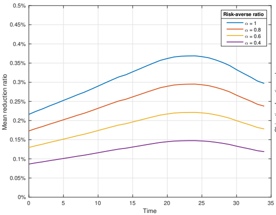
Hedge effectiveness: Risk-averse ratio



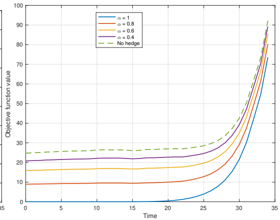
(a) Mean of h_t^*



(b) Mean of VRR_t



(c) Mean of MRR_t



(d) Mean of $V_t(h_t^*)$

Figure 6: The impact of risk-averse ratio α on h_t^*, VRR, MRR, and $V_t(h_t^*)$.

$\alpha \nearrow \Rightarrow$ More risk-averse

Conclusion

Conclusion

We propose a dynamic systematic longevity risk hedging framework for the GSA in the presence of the population basis risk:

- The mean-variance set serves as a tool for hedging strategy selection.
- The framework is practical:
  - Provides a semi-closed-form solution of the optimal hedge ratio.
  - In a discrete-time setting.
- The framework is robust (see the appendix):
  - The S-forwards' time-to-maturity and reference age.
  - The hedger's population.
  - Interest rate risk.
  - The size of the GSA pool.
  - Model risk.

Thank you!

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References I

Donnelly, C., Guillén, M., Nielsen, J.P., 2014. Bringing cost transparency to the life annuity market. *Insurance: Mathematics and Economics*, 56:14—27.

Hanewald, K., Piggott, J., Sherris, M., 2013. Individual post-retirement longevity risk management under systematic mortality risk. *Insurance: Mathematics and Economics*, 52(1):87—97.

Li, N., Lee, R., 2005. Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method. *Demography*, 42:575—594.

Milevsky, M.A., Salisbury T.S., 2015. Optimal retirement income tontines. *Insurance: Mathematics and Economics*, 64:91—105.

Piggott, J., Valdez, E.A., Detzel, B., 2005. The simple analytics of a pooled annuity fund. *Journal of Risk and Insurance*, 72(3):497—520.

Stamos, M.Z., 2008. Optimal consumption and portfolio choice for pooled annuity funds. *Insurance: Mathematics and Economics*, 43(1):56—68.

Tang, S., Li, J., 2021. Market pricing of longevity-linked securities. *Scandinavian Actuarial Journal*, 5:408—436.

Tan, K.S., Weng, C., Zhang, J., 2022. Optimal dynamic longevity hedge with basis risk. *European Journal of Operational Research*, 297:325—337.

References II

Valdez, E.A., Piggott, J., Wang, L., 2006. Demand and adverse selection in a pooled annuity fund. *Insurance: Mathematics and Economics*, 39(2):251—266.

Wong, T.W., Chiu, M.C., Wong, H.Y., 2017. Managing mortality risk with longevity bonds when mortality rates are cointegrated. *Journal of Risk and Insurance*, 84(3):987—1023.

Zhou, K.Q., Li, J.S., 2017. Dynamic longevity hedging in the presence of population basis risk: A feasibility analysis from technical and economic perspectives. *Journal of Risk and Insurance*, 2017(84):417—437.

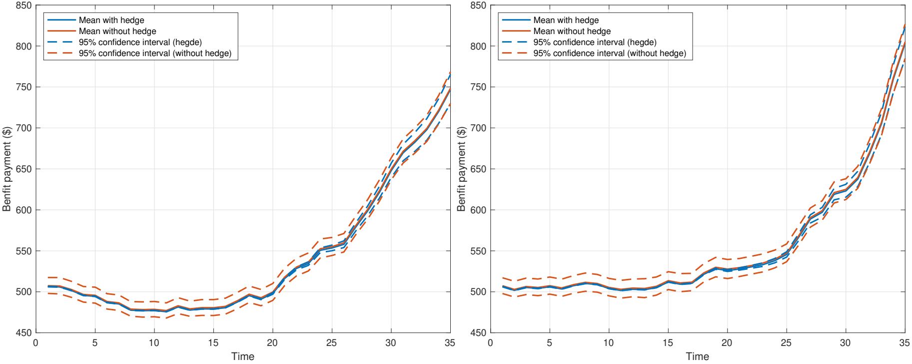
Appendix

Integrated risk management framework

Age	75			80			85		
Multiplier	Holistic	Dynamic	Static	Holistic	Dynamic	Static	Holistic	Dynamic	Static
0.1	16.9801	16.9292	16.7084	13.4847	13.4442	13.2699	11.2608	11.2270	11.1032
0.3	6.4686	6.4492	6.3675	5.1337	5.1183	5.0661	4.3880	4.3748	4.3305
0.5	3.9130	3.9013	3.8403	3.1088	3.0994	3.0633	2.6227	2.6148	2.5932
0.7	2.8781	2.8694	2.7460	2.2578	2.2511	2.1577	1.9015	1.8958	1.8180
0.9	2.3677	2.3606	2.0907	1.8494	1.8438	1.6168	1.5479	1.5432	1.3488

Table 5: Return per unit of risk ($\pi_t := \frac{\mathbb{E}_0[B_t]}{\mathrm{Std}_0[B_t]}$) at selected ages for different volatility targets with risk-averse ratio $\alpha = 1$ and death benefit ratio $\beta = 0$.

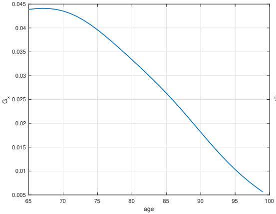
Benefit profile



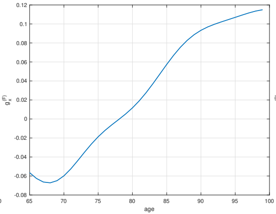
(a) Simulation 1 (b) Simulation 2

Figure 7: Survival benefit per member given two simulations of the mortality path. The initial contribution per member is $10,000.

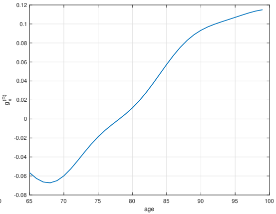
Estimates of parameters in the ACF model



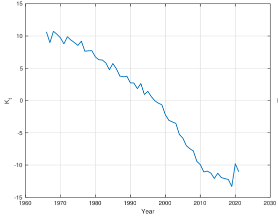
(a) G_x



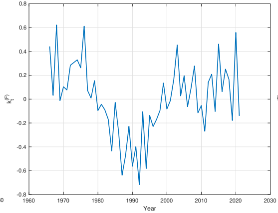
(b) $g_x^{(F)}$



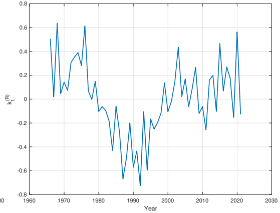
(c) $g_x^{(R)}$



(d) K_t



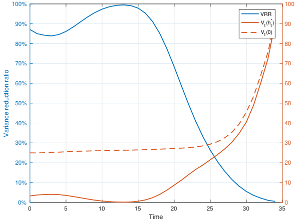
(e) $k_t^{(F)}$



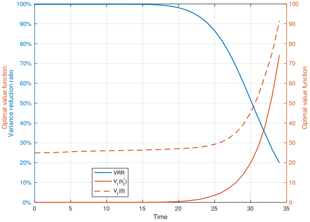
(f) $k_t^{(R)}$

Figure 8: Estimates of parameters in the ACF model. The ACF model parameters are calibrated to the mortality data of the population aged from 65 to 99 over the period from 1966 to 2019.

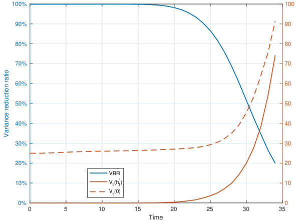
Robustness: The time-to-maturity of the S-forwards



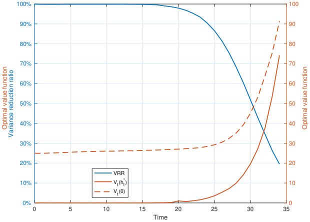
(a) $T^* = 1$



(b) $T^* = 4$



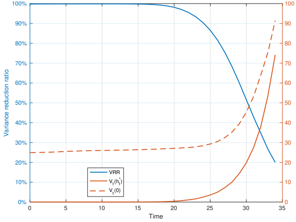
(c) $T^* = 8$



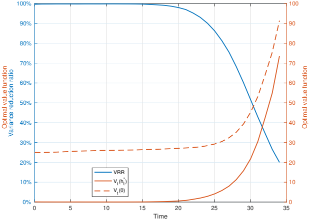
(d) $T^* = 12$

Figure 9: The impact of time-to-maturity on hedge effectiveness.

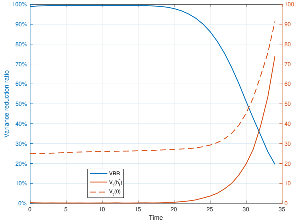
Robustness: The reference age of the S-forwards



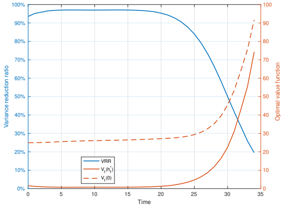
(a) $x_f = 65$



(b) $x_f = 70$



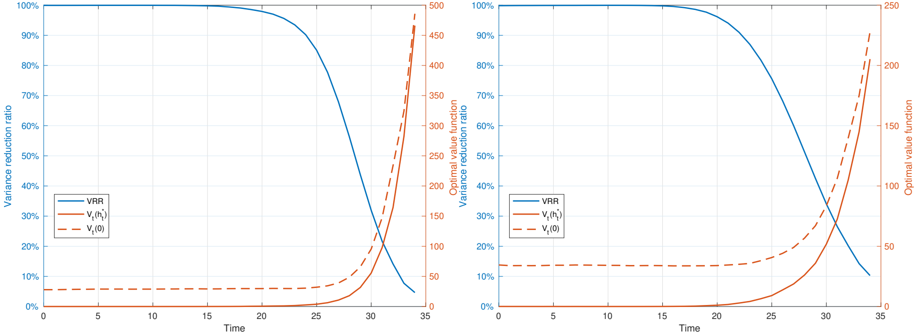
(c) $x_f = 85$



(d) $x_f = 90$

Figure 10: The impact of reference age on hedge effectiveness.

Robustness: The hedger's population

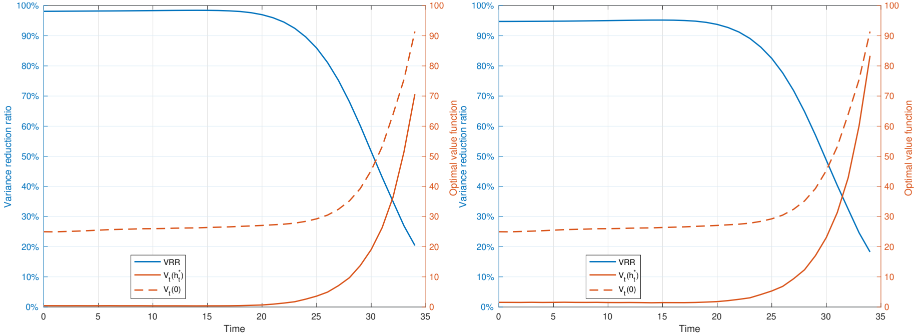


(a) Northern Ireland

(b) Scotland

Figure 11: The impact of hedger's population on hedge effectiveness.

Robustness: Interest rate risk



(a) Realised risk-free interest rate is 0.1%

(b) Realised risk-free interest rate is 3%

Figure 12: The impact of interest rate risk on hedge effectiveness.

Robustness: The GSA pool size

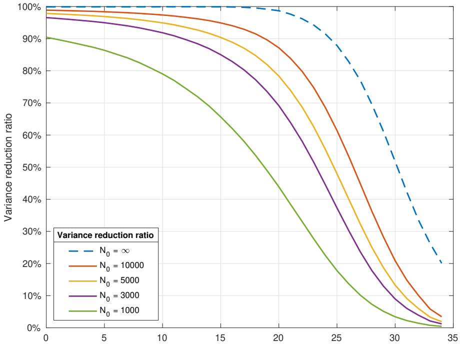


Figure 13: The impact of pool size on hedge effectiveness for a finite pool size with $N_0 =$ 10,000, 5,000, 3,000, and 1,000. The dashed line represents the case with no small sample risk.

Robustness: Model risk

Age	ACF	M-LC	M-CBD
65	99.88969%	99.88973%	99.88969%
70	99.90976%	99.91032%	99.90970%
75	99.94974%	99.95079%	99.94982%
80	99.92749%	99.92860%	99.92767%
85	98.75706%	98.72143%	98.74373%
90	88.00431%	87.66947%	87.95813%
95	53.11209%	52.18132%	53.03698%

Table 6: The impact of model risk on the VRR at selected ages. The time-to-maturity $T^* = 10$ years, the reference age $x_f = 75$, and the book population are kept fixed. All figures represent the mean value.