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Mortality differentials in continuous time

Releasing the power of mortality laws

Presentation at Longevity 20 Conference in Singapore

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Introducing the speaker

Kai Kaufhold – Life Reinsurer & Actuarial Consultant

- Life Reinsurance
  - 1996 – Zurich Re (Cologne),
  - 2000 – Manulife Reinsurance, Toronto, from 2003: Cologne
- Since 2011: Ad Res
  - Quantitative analysis: mortality, persistency, disability, morbidity risks
  - CIA mortality projection scale, CIA pensioner tables, SAS life table project
  - Trainer for experts, researcher, author/presenter

Agenda

- Motivation
- Recap of continuous-time model
- Portfolio-specific trend assumptions
- Practical considerations (a.k.a. complications)
- Discussion

Why study mortality differentials?

Understand drivers

- Financial
  Wealth & income
- Demographic
  Education & occupation
- Behaviour
  Prevention vs. risk taking
- Genetic
  Incidence & prevalence of illness

Avoid mispricing – Risk management

- Life insurance & annuities
- Occupational pensions
- Understand differences between sub-populations / portfolios

LONGEVITY BASIS RISK

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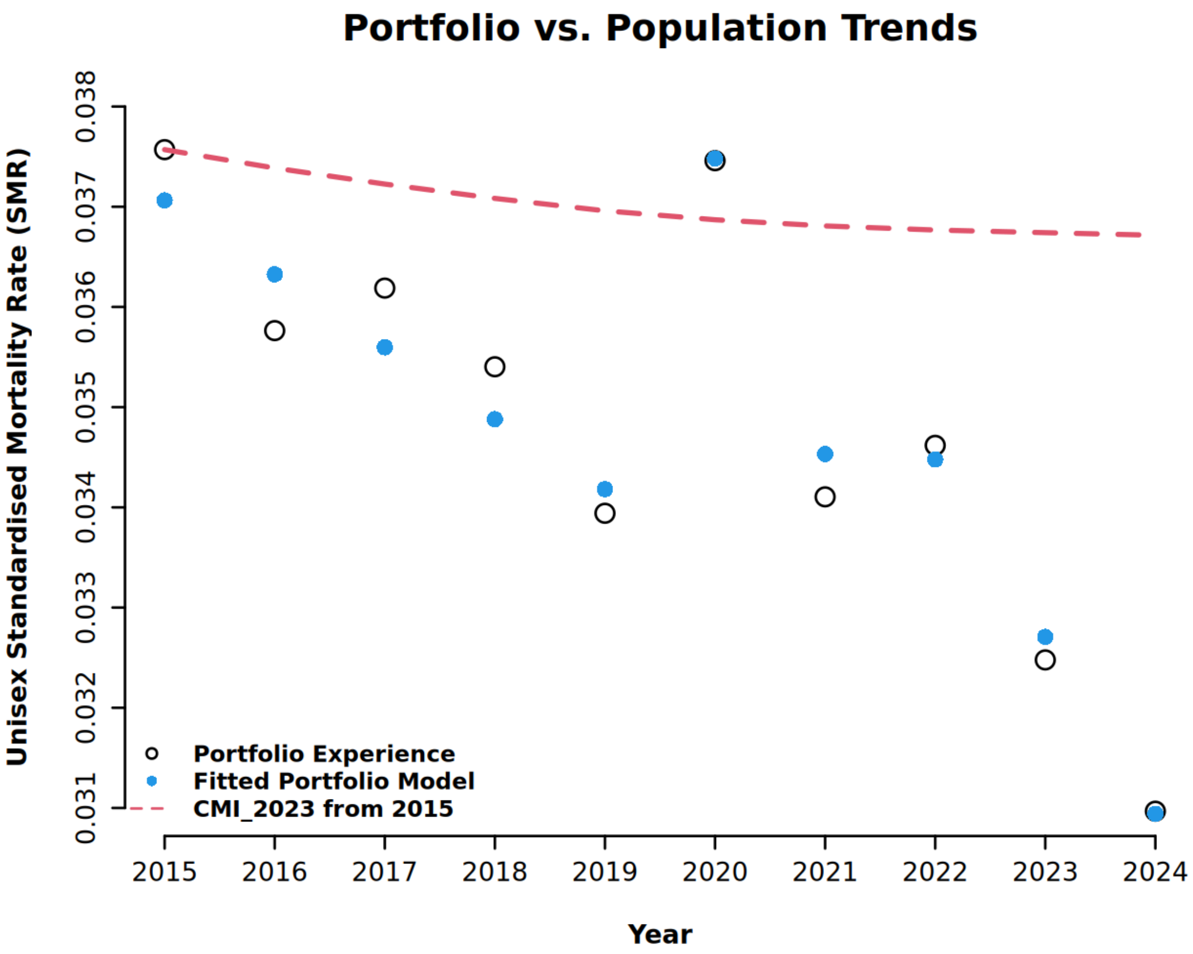
Longevity Basis Risk

Portfolio-specific Mortality Models

- Portfolio mortality
  $\neq$ Population mortality
  - Industry tables
  - Portfolio-specific tables
- Population trend $\neq$ portfolio trend
  - IFoA & LLMA Basis Risk Project
  - CMI Model based on population data

Portfolio-specific CMI Model?

Population trends miss the mark



A particular pensioner portfolio experiences greater improvements than England & Wales population

Implication: should model portfolio trends explicitly

Source: Data from a large U.K. pension scheme; own calculations. CMI_2023 Model from CMI working paper 189 using 1.5% long-term mortality improvement rate.

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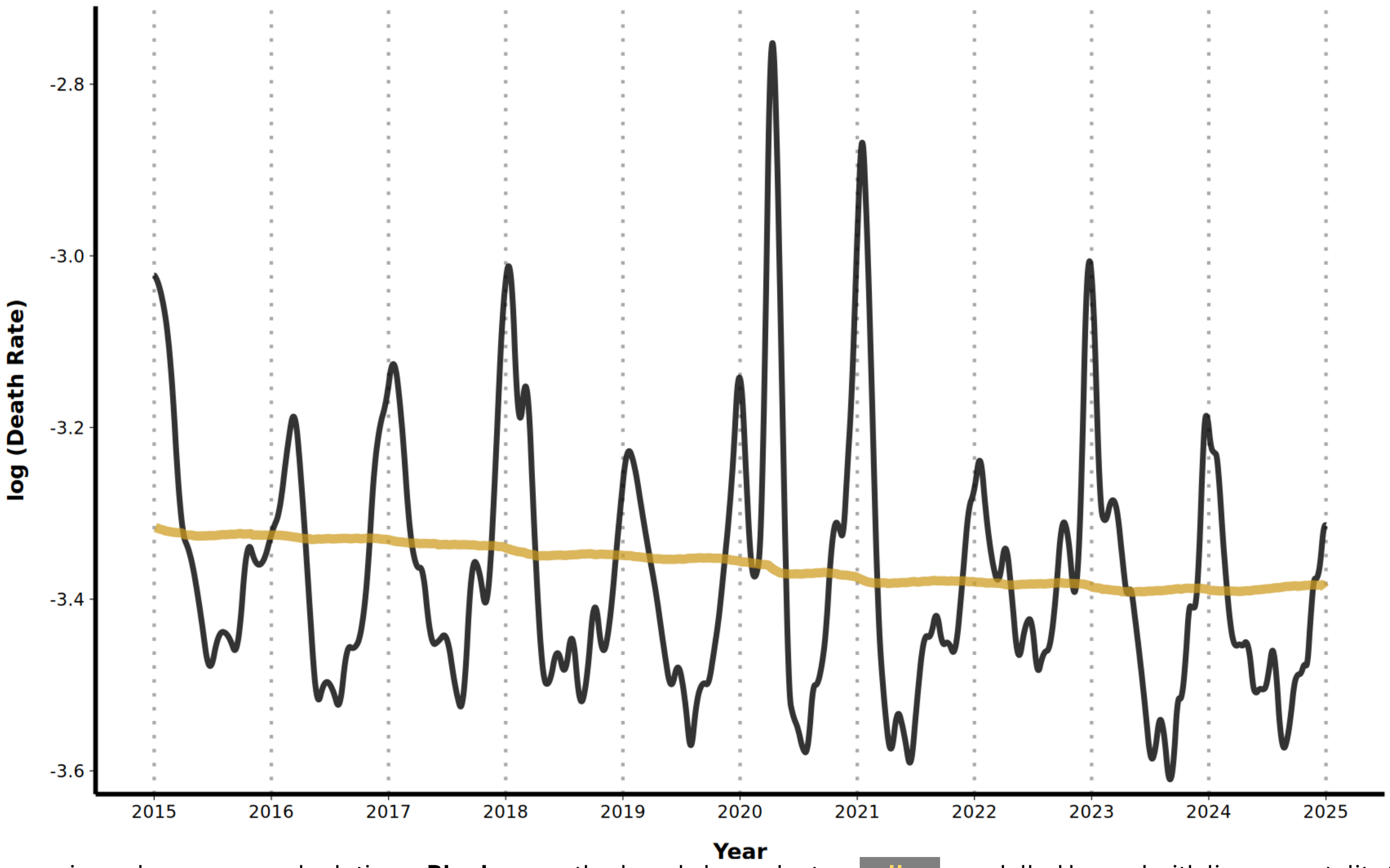
Pension case study

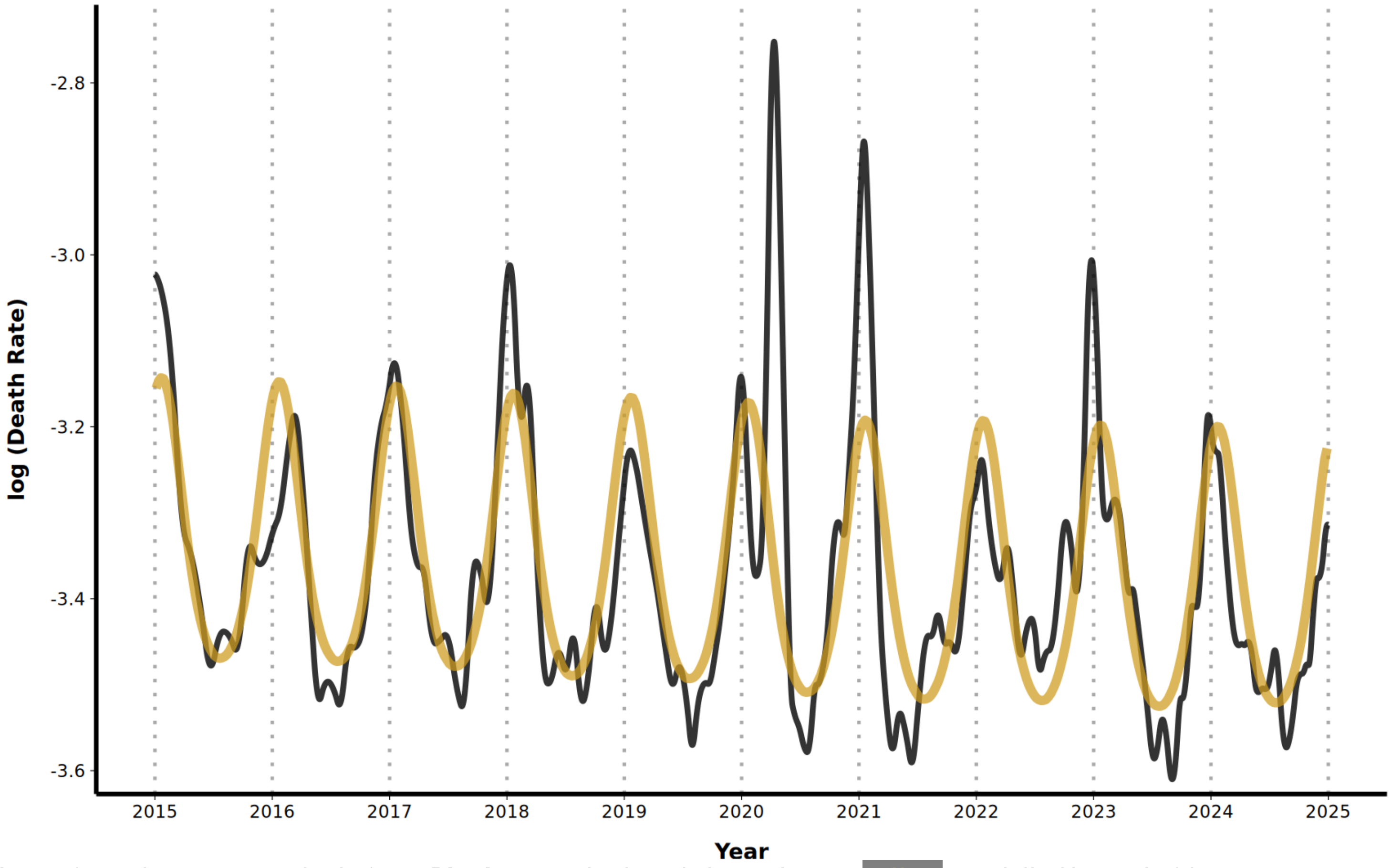
Modelling mortality trends based on individual-level data

Mortality hazard continuous in time



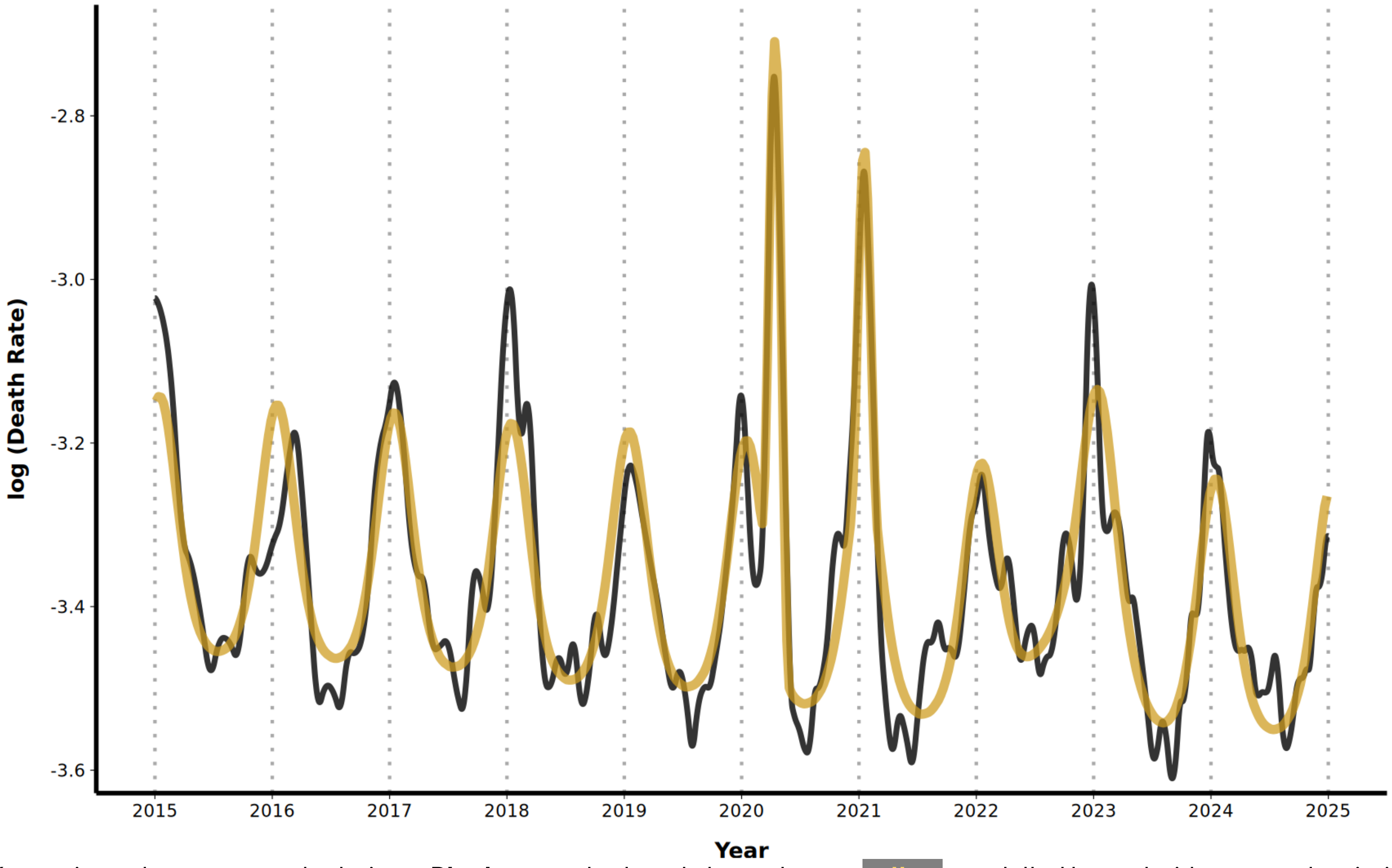
Source: Data from a large U.K. pension scheme; own calculations. **Black:** smoothed crude hazard rates, yellow: modelled hazard with linear mortality trend

Mortality hazard with seasonal variation



Source: Data from a large U.K. pension scheme; own calculations. **Black:** smoothed crude hazard rates, yellow: modelled hazard with average seasonal variation

Trend including COVID-19 experience



Source: Data from a large U.K. pension scheme; own calculations. **Black:** smoothed crude hazard rates, yellow: modelled hazard with seasonal variation and three mortality spikes

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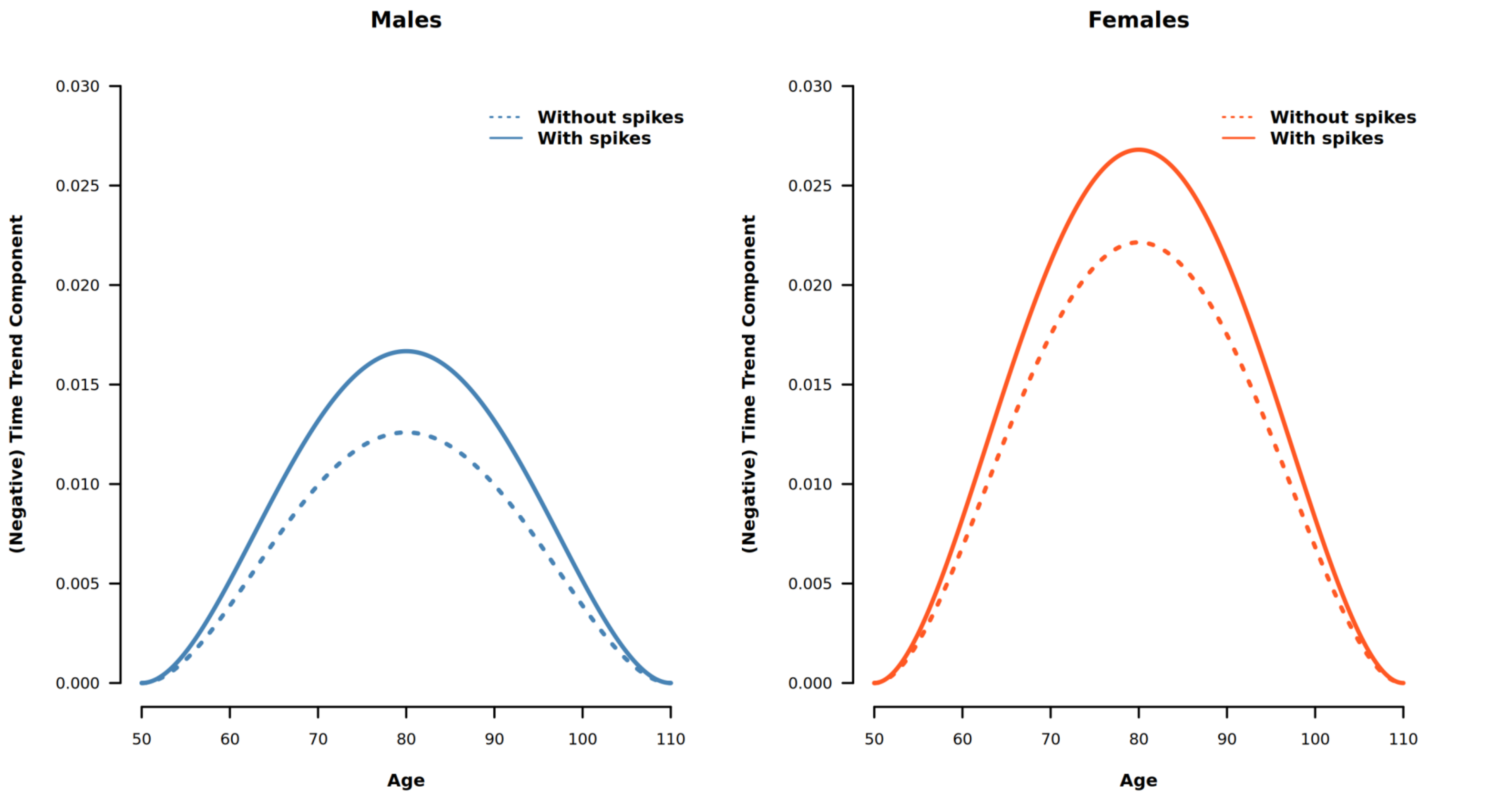
Resulting portfolio-specific trend



Source: Data from a large U.K. pension scheme; own calculations. Trends modelled as time-dependent Hermite spline coefficients.

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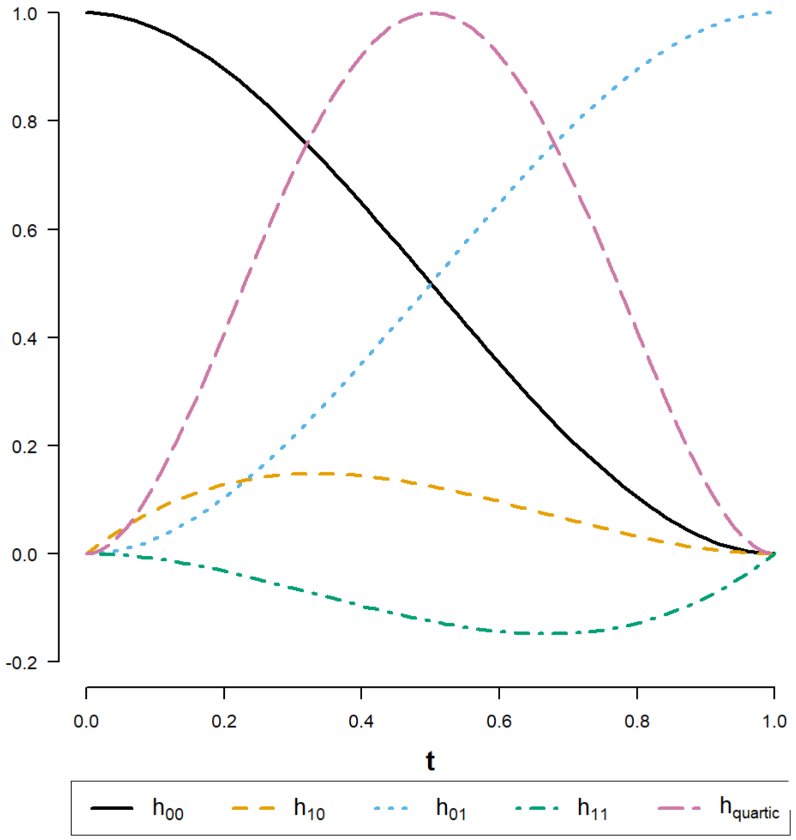
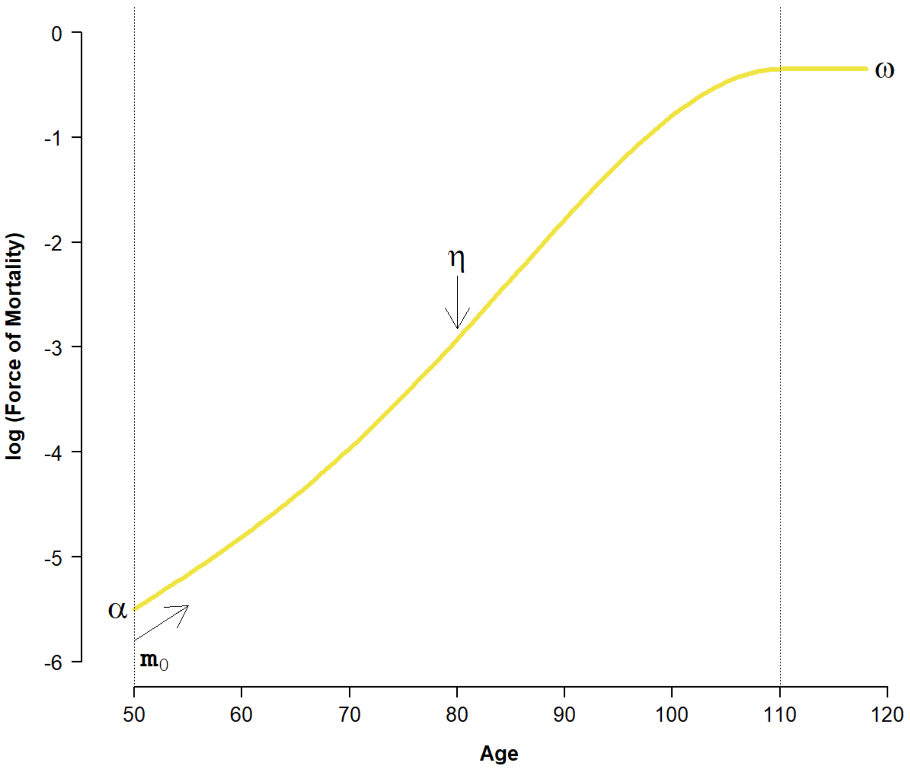
Continuous-time model

Hermite-spline approach for modelling mortality trends

Hermite splines for pensioner mortality

$$\log(\mu_x) = \alpha\, h_{00}(t) + \omega\, h_{01}(t) + m_0\, h_{10}(t) + \eta\, h_{quartic}(t)$$





Source: Own calculations, illustrative parameters

$\boldsymbol{\alpha}$	$\log(\mu_x)$ at min age (50)
$\boldsymbol{\omega}$	$\log(\mu_x)$ at max age (110)
$\boldsymbol{m_0}$	Initial slope at min age
$\boldsymbol{\eta}$	Quartic shape parameter

- Scale age $x \longrightarrow t = \frac{x - x_0}{x_1 - x_0}$
- Omit $\boldsymbol{h_{11}}$: mortality plateau
- CMI: generic polynomials

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Age-dependent mortality trends

$$\log(\mu_{x,y}) = \begin{aligned} &(\alpha_{base} + y\,\alpha_{time})\, h_{00}(t(x)) && \rightarrow \text{Trend highest at age 50} \\ &+ (\omega_{base} + y\,\omega_{time})\, h_{01}(t(x)) && \rightarrow \text{Trend highest at age 110} \\ &+ ({m_0}_{base} + y\,{m_0}_{time})\, h_{10}(t(x)) && \rightarrow \text{Slope change, highest at age 70} \\ &+ (\eta_{base} + y\,\eta_{time})\, h_q(t(x)) && \rightarrow \text{Trend around age 80} \end{aligned}$$

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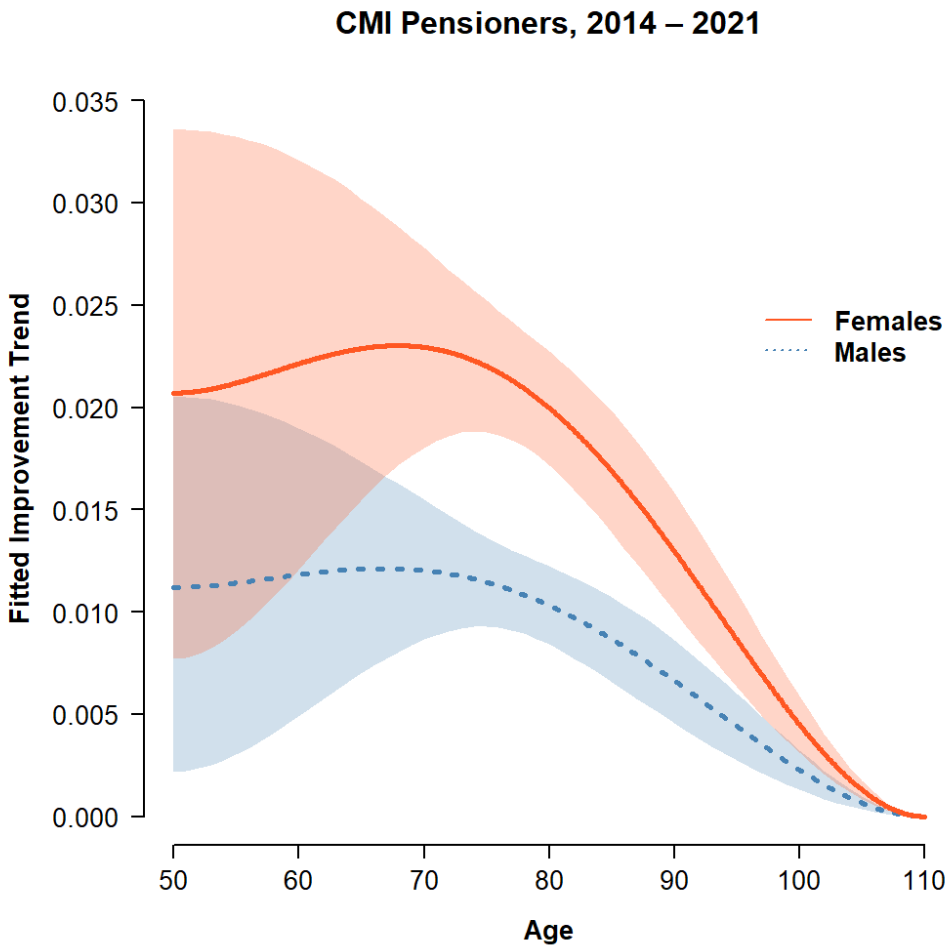
Interpreting model results (CMI example)



Parameter	Value
α_{time}	-0.0112 ± 0.0047
$\alpha_{time:female}$	-0.0095 ± 0.0081
η_{time}	-0.0047 ± 0.0026
$\eta_{time:female}$	-0.0046 ± 0.0045
ω_{time}	0

Source: Data underlying CMI Working Paper 169; Human Mortality Database (HMD); own calculations.

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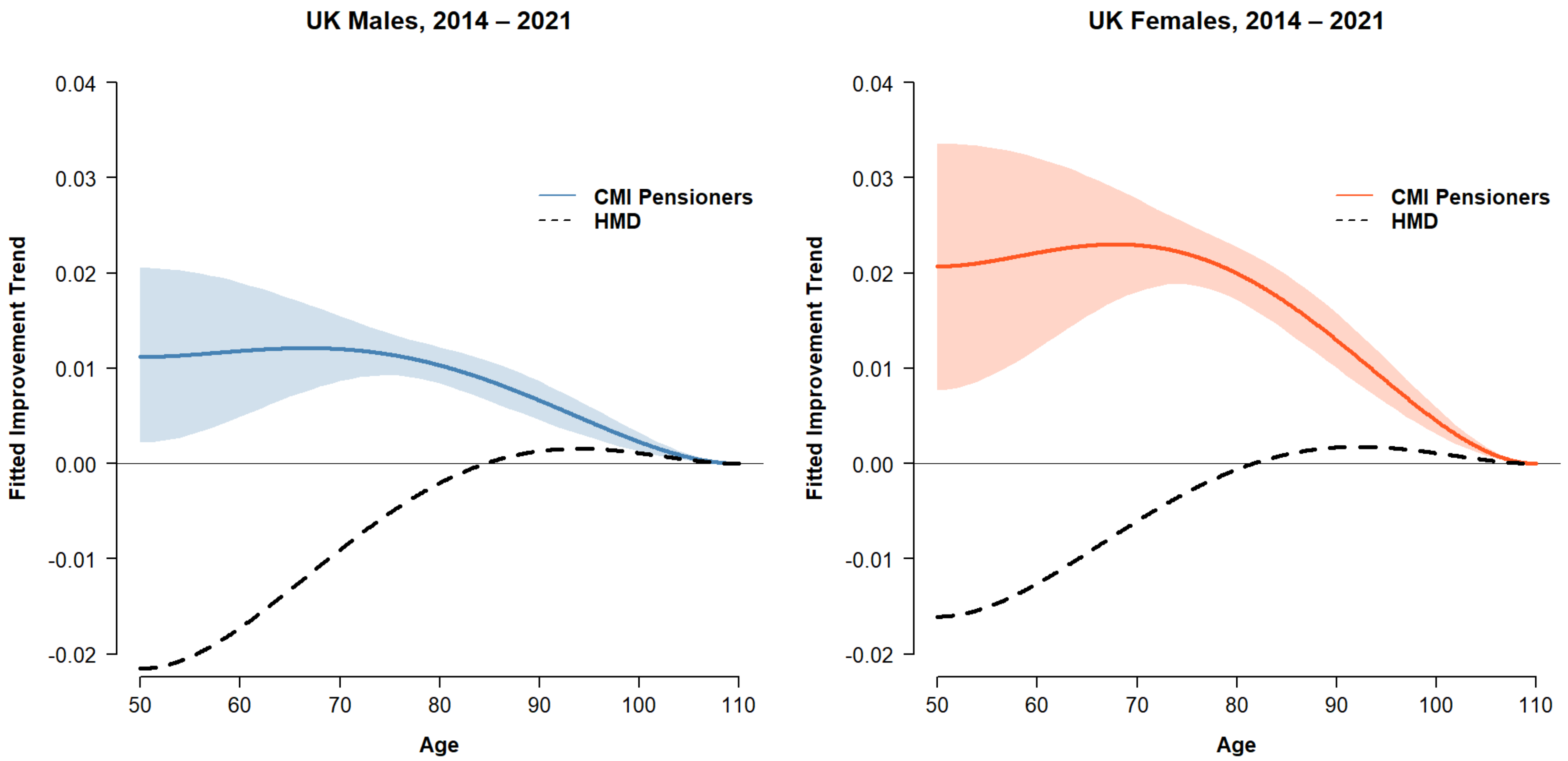
Comparing industry trends to population

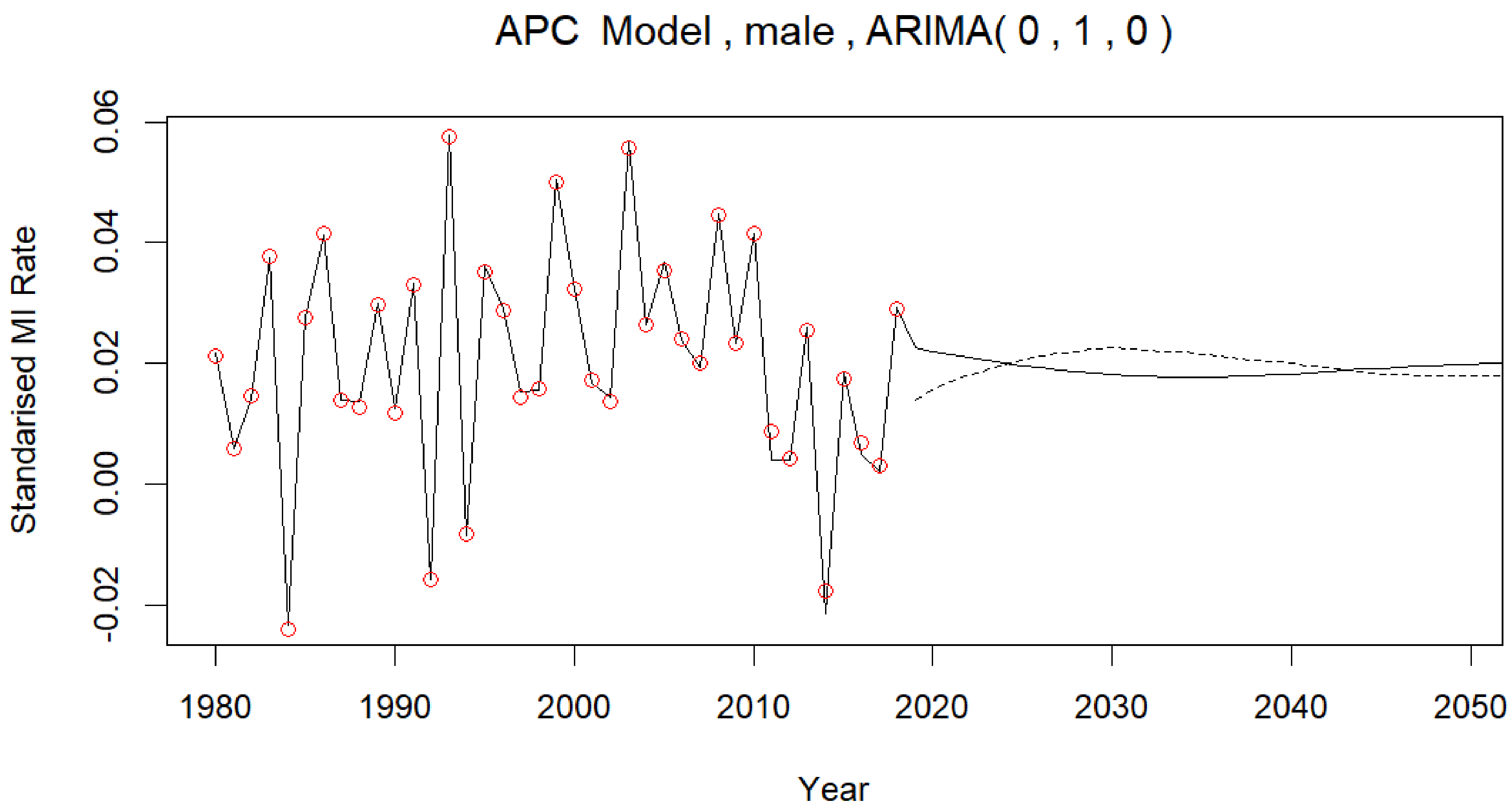


Source: Data underlying CMI Working Paper 169; Human Mortality Database (HMD); own calculations.

Applying portfolio-trends to mortality projections

Taking the idea of the CMI Model a step further

CMI Model Illustration – England & Wales



Source: Data on population and deaths in England and Wales from Human Mortality Database (HMD); own calculations.

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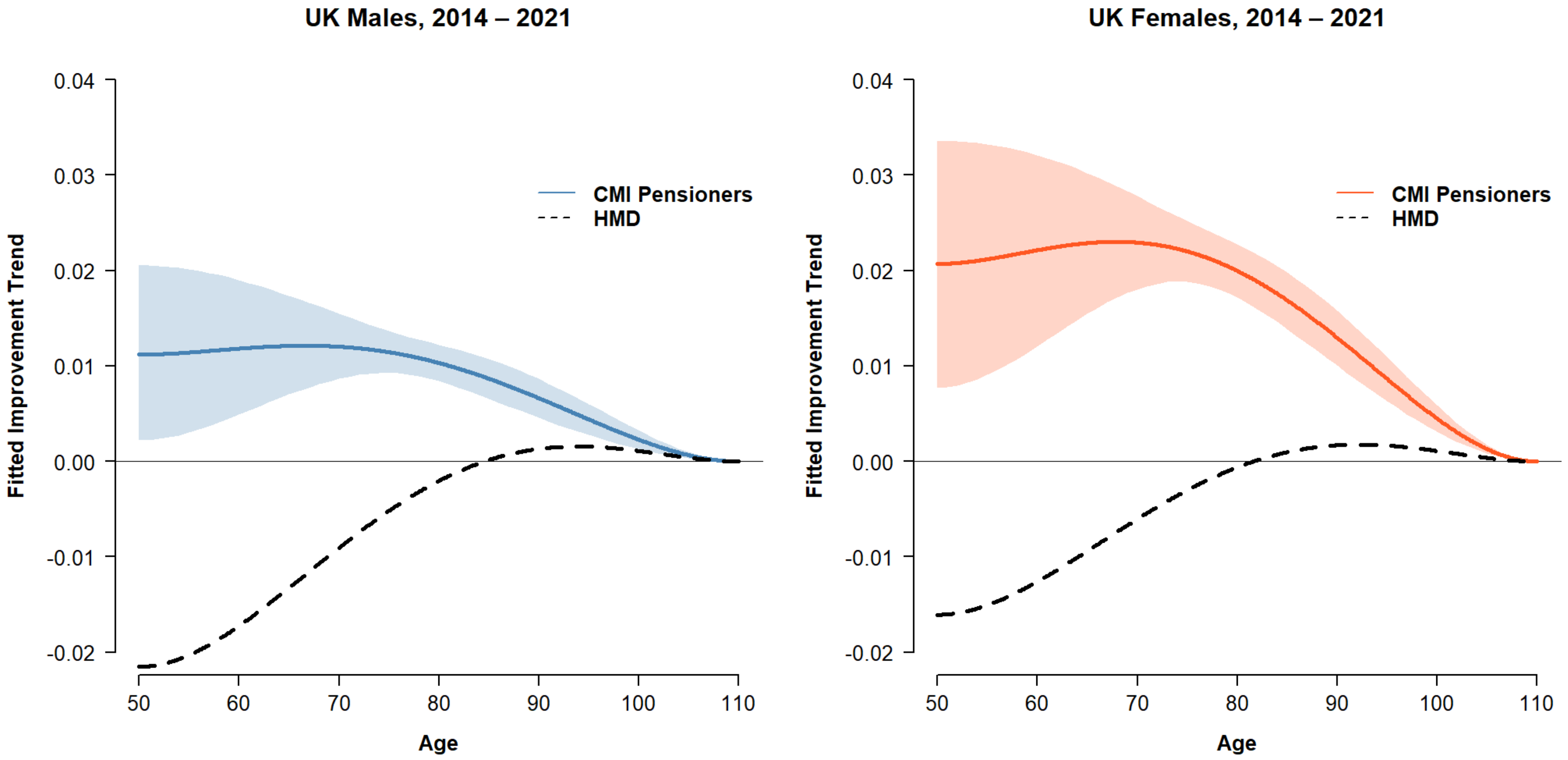
Estimation error due to limited data



Source: Data underlying CMI Working Paper 169; Human Mortality Database (HMD); own calculations.
Shaded areas represent 95% confidence intervals (assuming independence by age)

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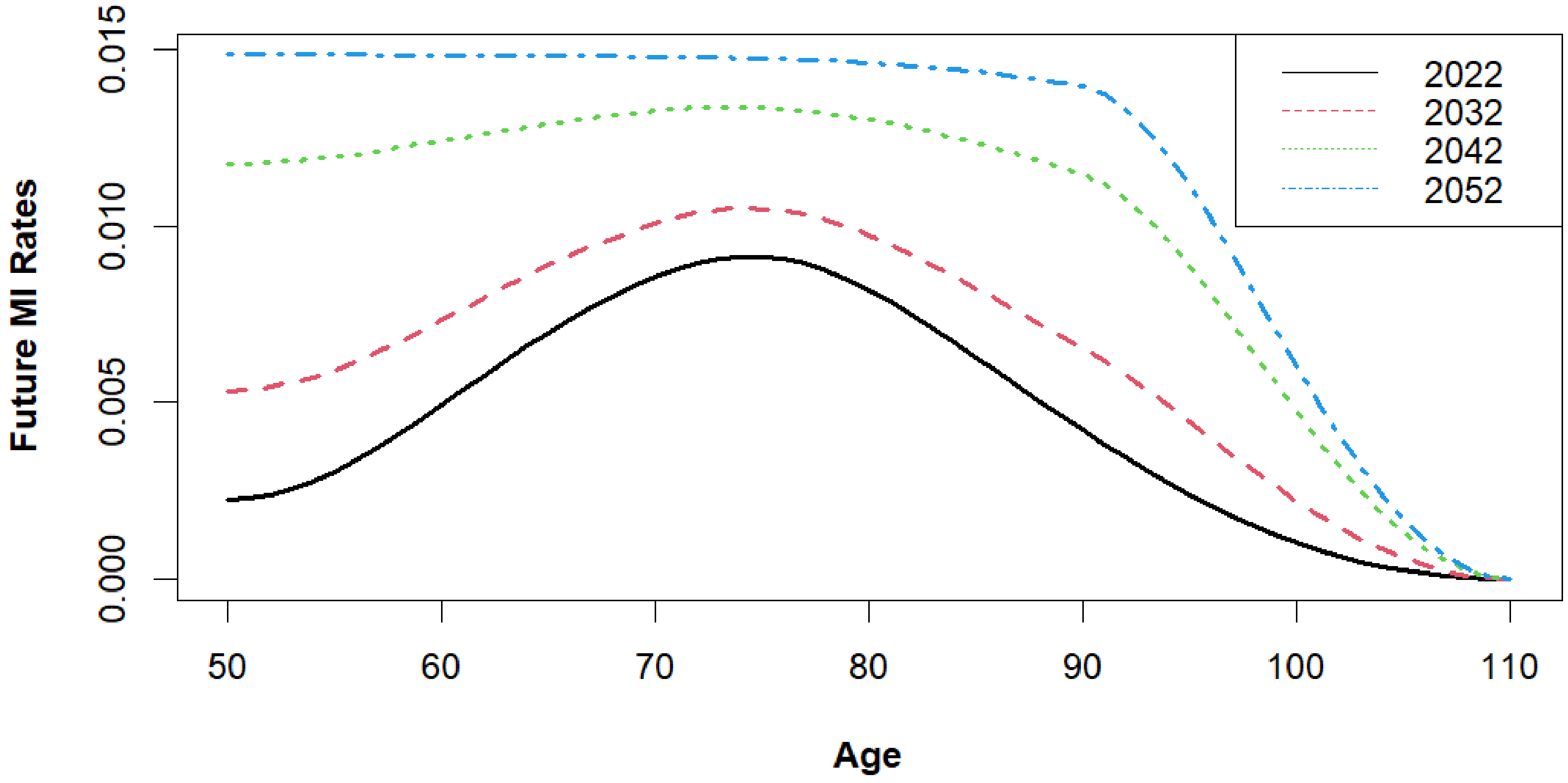
Portfolio-specific mortality improvements



Source: Own calculations based on data underlying CMI Working Paper 169 and population estimates and death counts from the Human Mortality Database (www.mortality.org).

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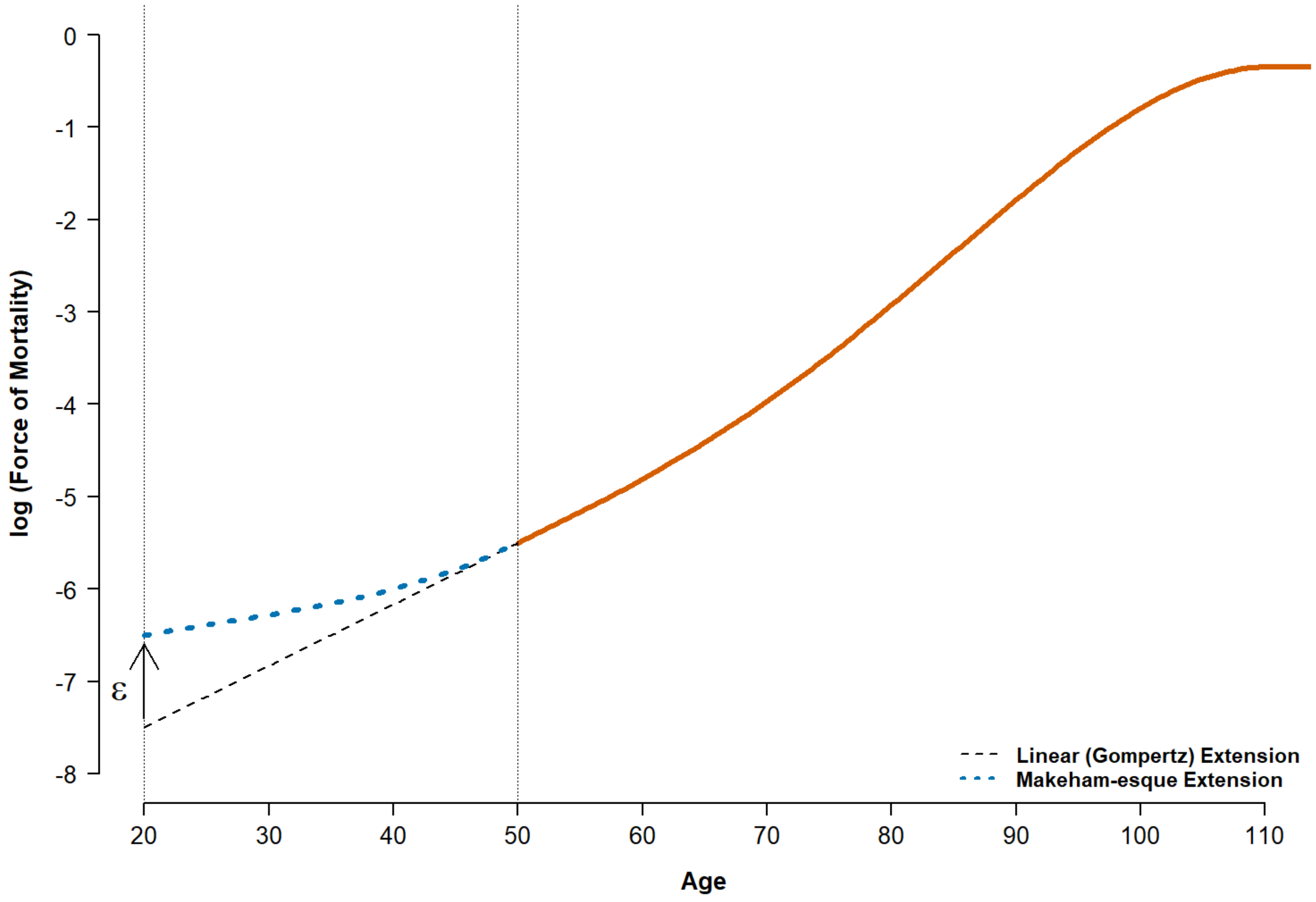
Complications

Insurance applications require pre-retirement ages and selection.

Trends can also vary by risk group and be volatile.

Extension to younger and older ages

- Young age extension based on initial slope (à la Gompertz)
- Additive constant (à la Makeham)
- Old age extrapolation based on plateau hypothesis (at ~110 years)



Source: Own calculations in *R* using illustrative parameters.

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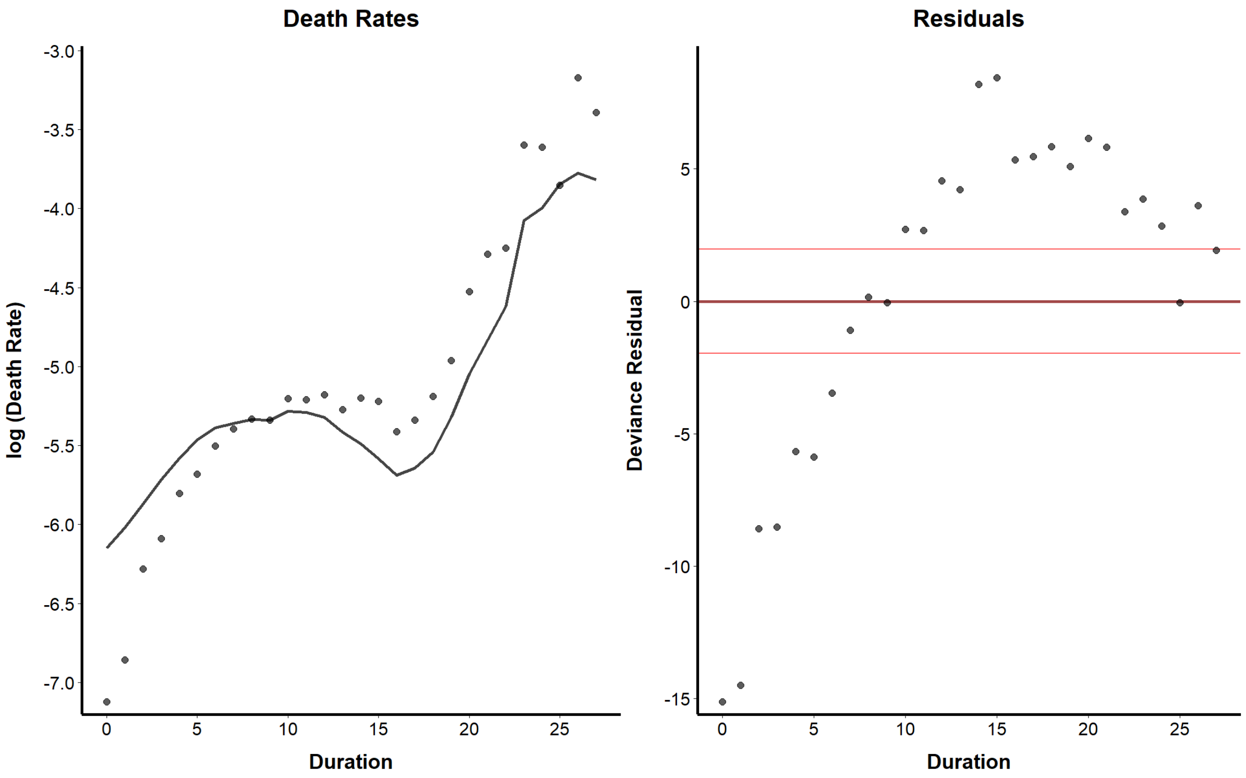
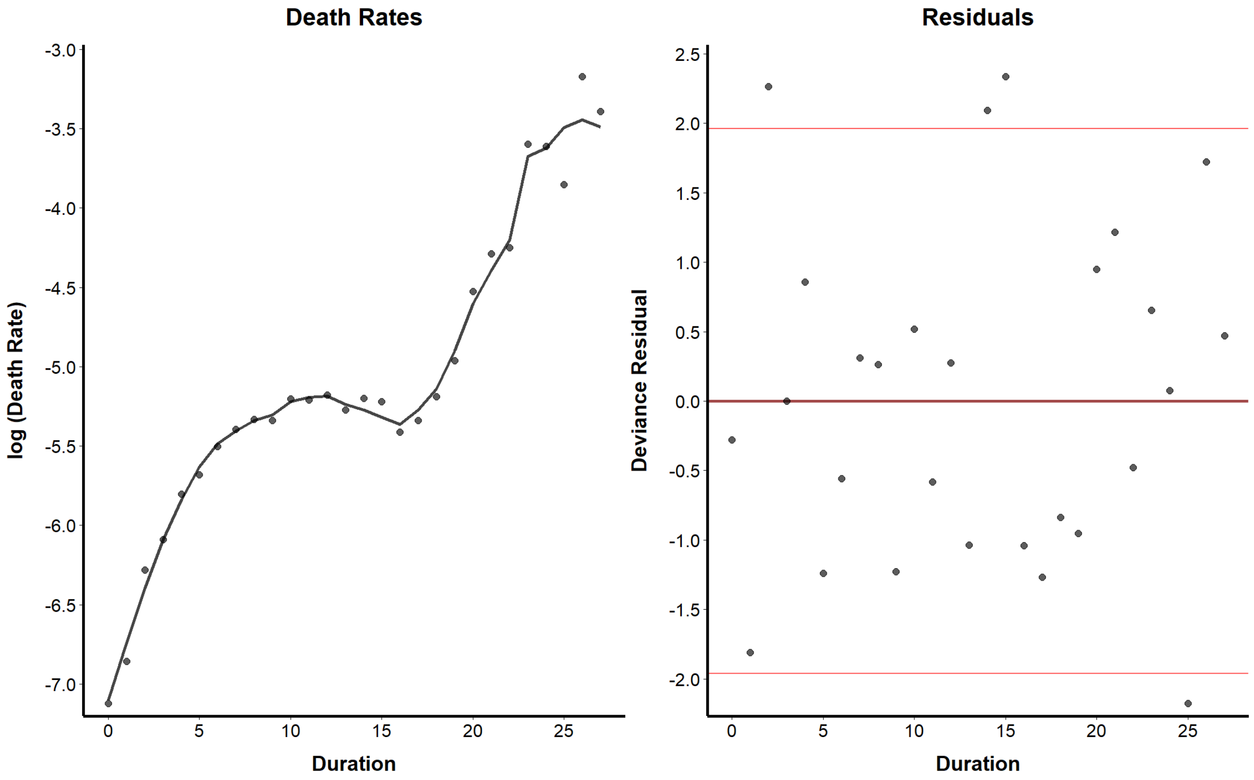
Selection effects in life insurance

Base model without selection



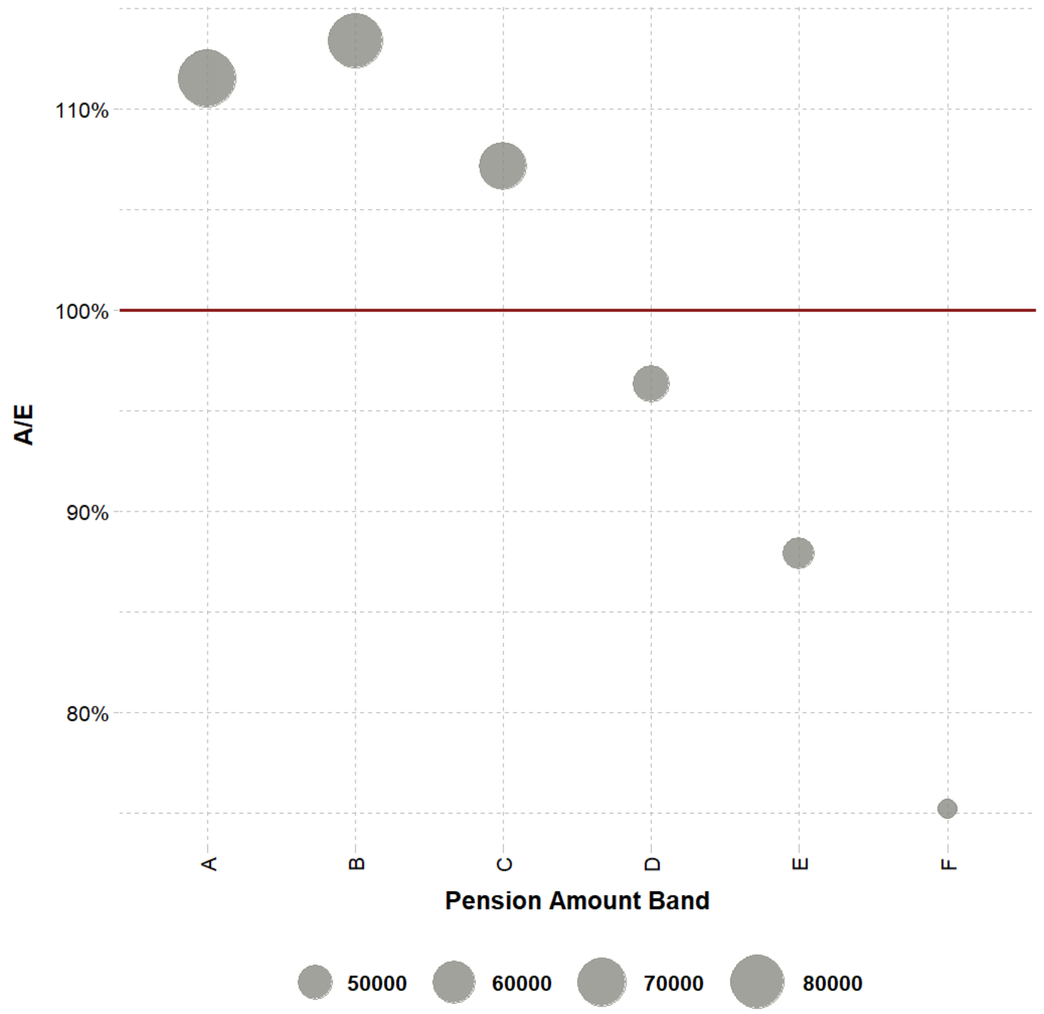
Selection effect explicitly modelled



Source: Data from a large US life insurance portfolio; own calculations. Selection modelled using splines in policy duration.

Mortality differentials mask trends

- Changing portfolio composition affects mortality over time
- Must control for all relevant risk factors:
  - Benefit amount
  - Underwriting class (if applicable)
  - Lifestyle (geo-demographic)
- Example: Pension amount



Source: Data underlying CMI Working Paper 169; own calculations.

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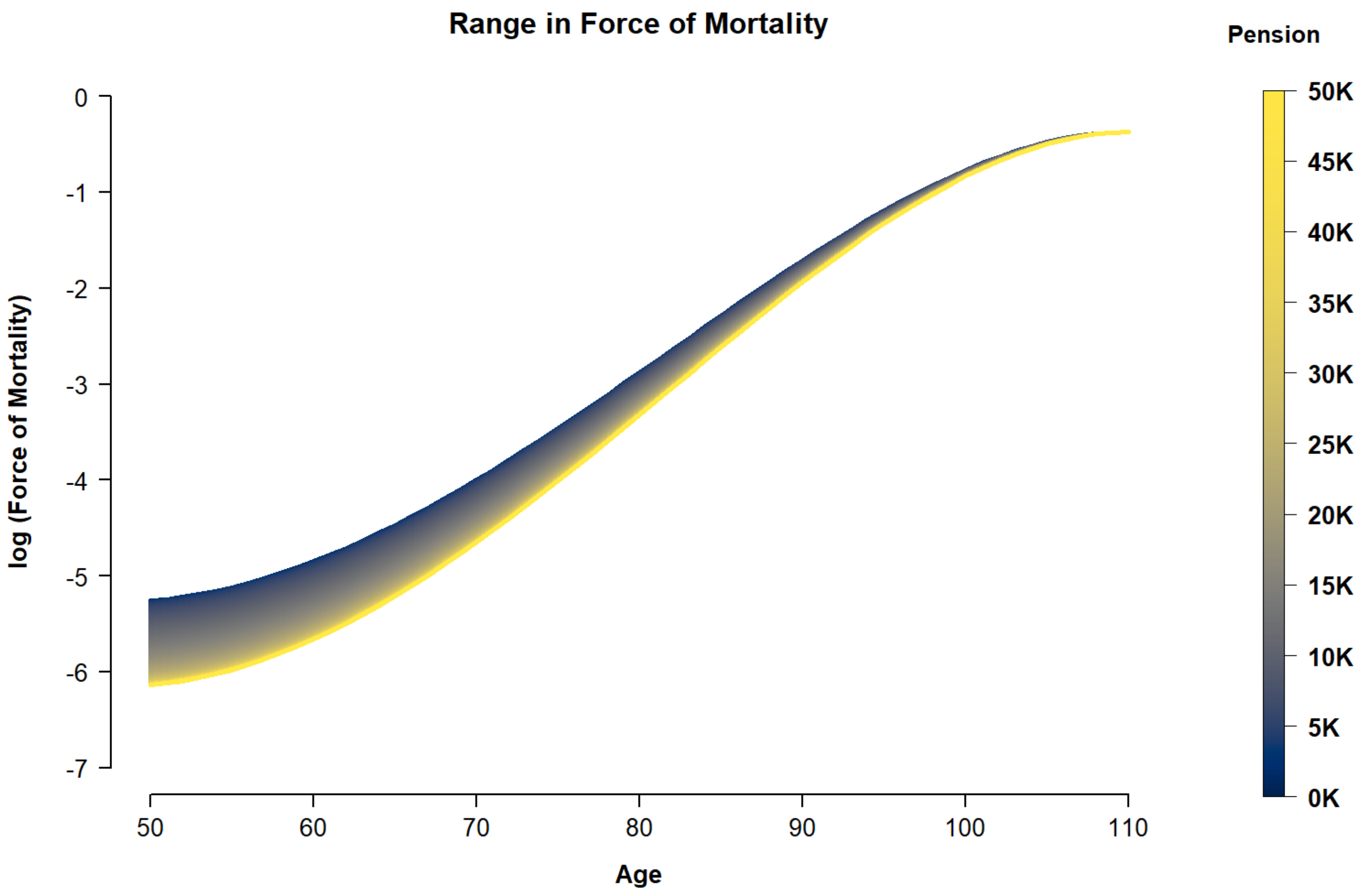
Continuous mortality differentials



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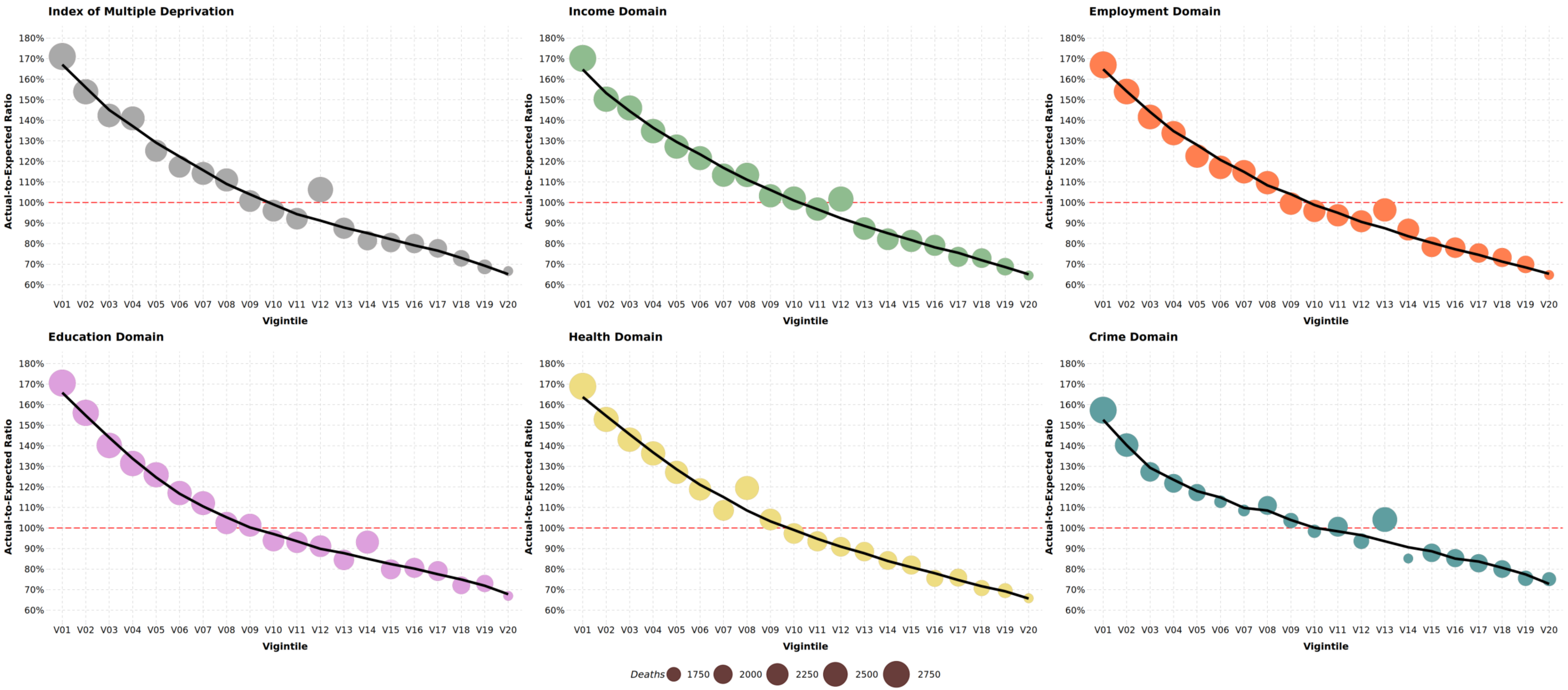
Mortality by selected deprivation domains



Source: Data from Office of National Statistics, U.K. by Lower Support Output Area; mortality data from LexisNexis® Risk Solutions; own calculations.

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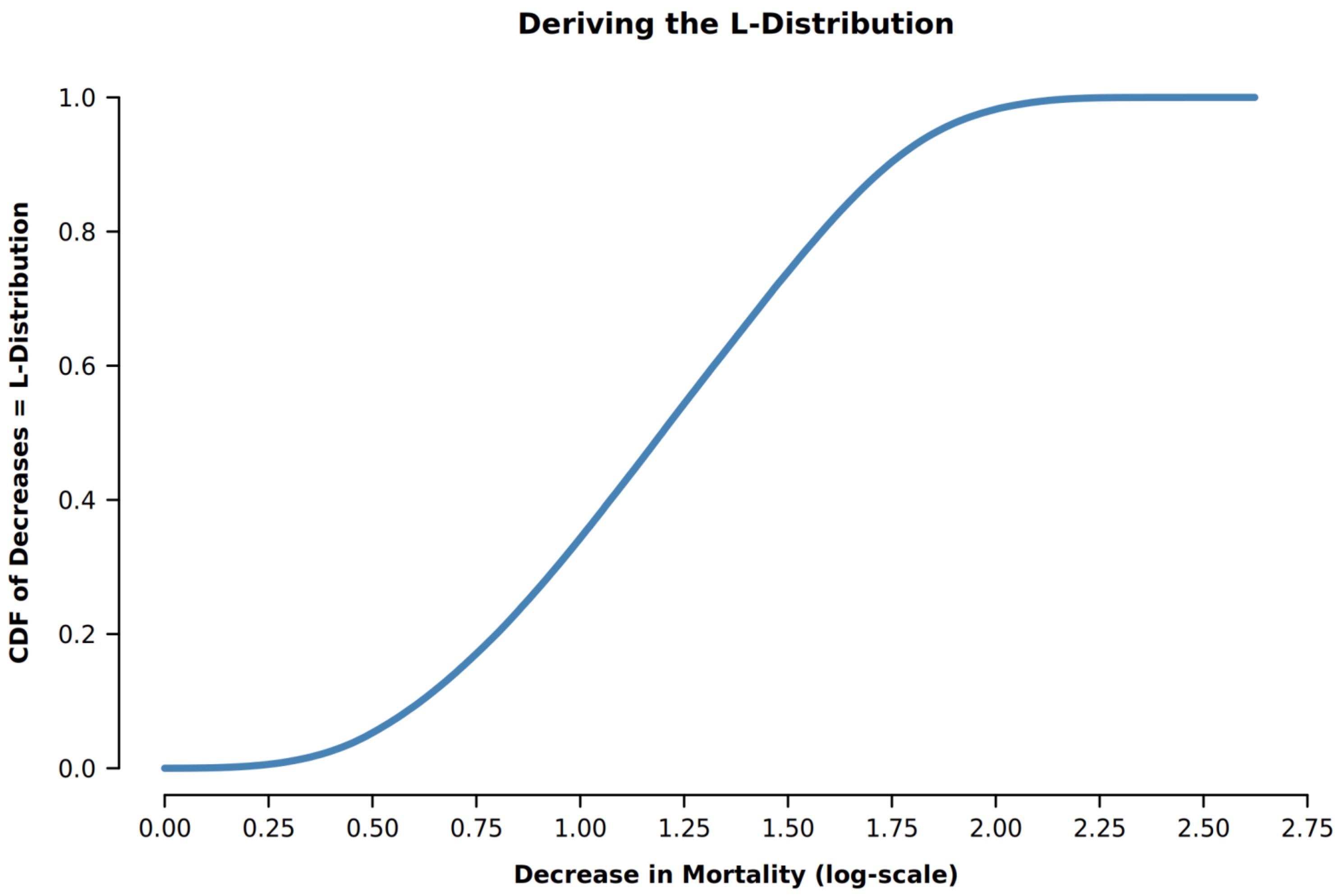
Geo-demographic lifestyle risk factor

LexisNexis L-Distribution

- ONS deprivation domain data
- Socio-economic indicators at the household level
- Individual-level mortality experience for entire U.K.



Source: Mortality and address data from LexisNexis® Risk Solutions; additional demographic data from Office of National Statistics; own calculations.

Discussion

Summary

- Hermite spline model framework captures age-dependent portfolio trends
- Portfolio-specific initial MI rates for CMI-style projection
- Must capture true improvements and not artefacts of selection and portfolio composition

Ongoing Research

1. Interactions between continuous risk factors as joint distribution functions
2. Time-dependence of parameters for continuous risk factors, e.g. amount & lifestyle

$\Rightarrow$ Socio-economic mortality trends

References

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- Richards, S. J. (2019). A Hermite-spline model of post-retirement mortality. *Scandinavian Actuarial Journal, 2020(2)*, 110–127. https://doi.org/10.1080/03461238.2019.1642239
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Great to see you!

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