


Mortality Compression and Its Impact on Managing Longevity Risk

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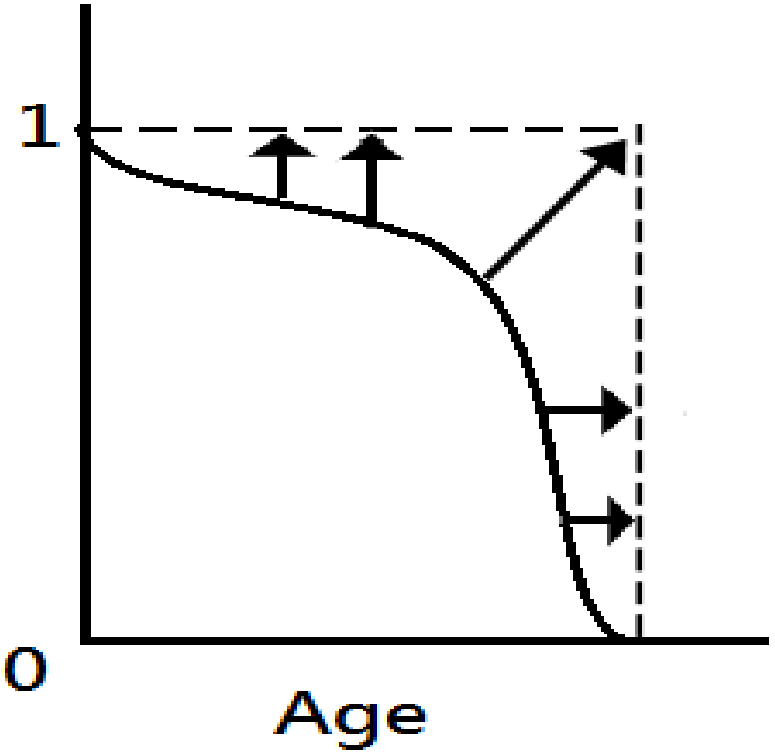
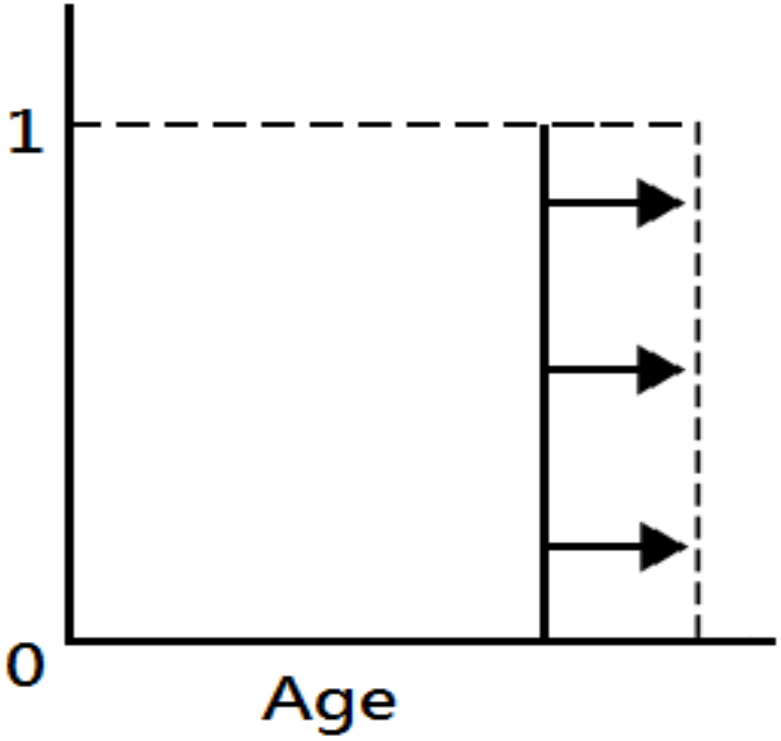
About the Human Longevity

- Life with a limit!



- Life without a limit!



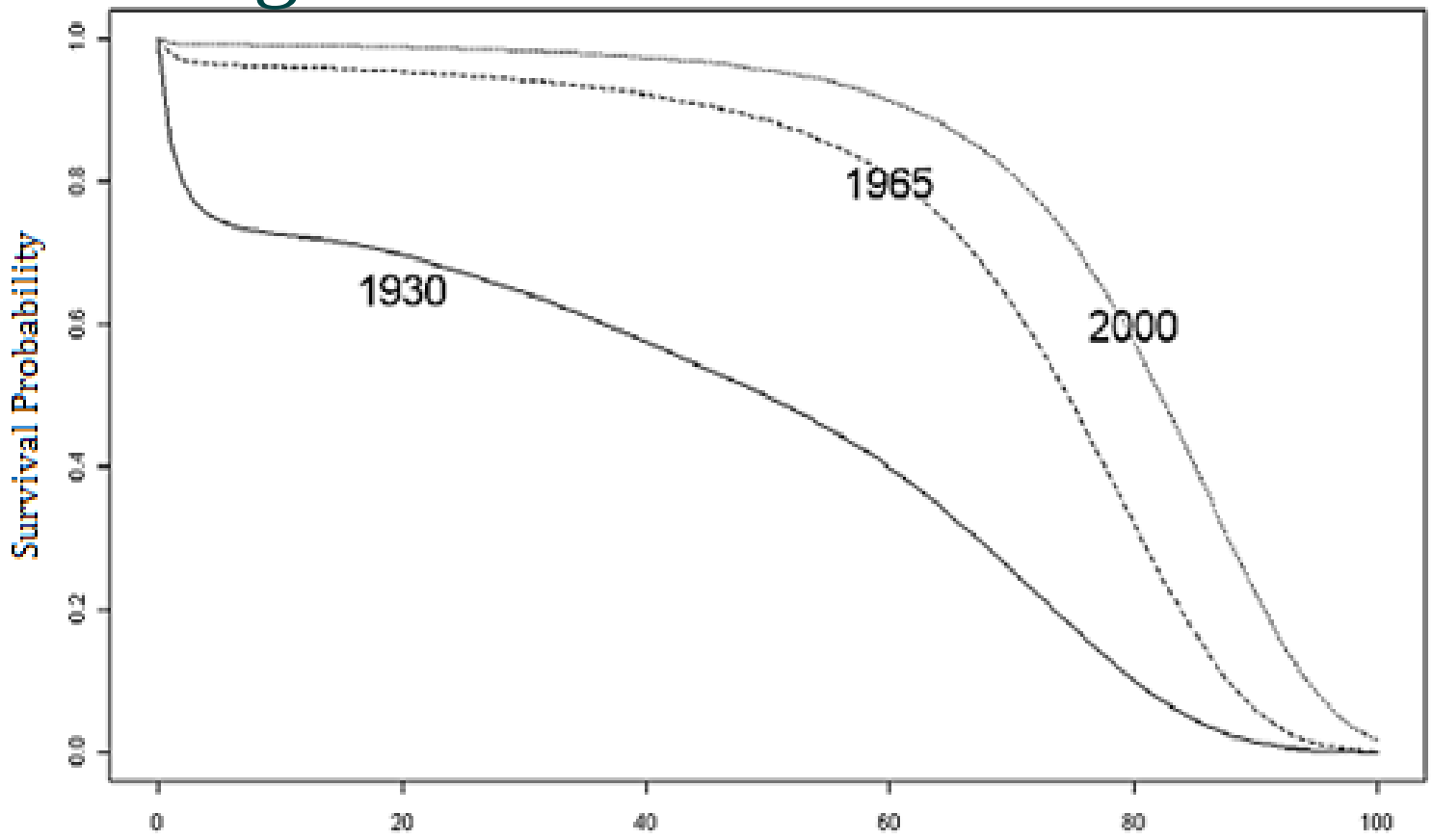
Rectangularization and Lifespan

- Regarding the theory of lifespan, there are two opinions: life with or without a limit.

→The rectangularization is a consensus.

→Premature deaths (including infants) will gradually decrease and some postulates that the distribution of death number will behave like a normal curve.

Rectangularization of Survival Curve



Survival Curves of Taiwan Female

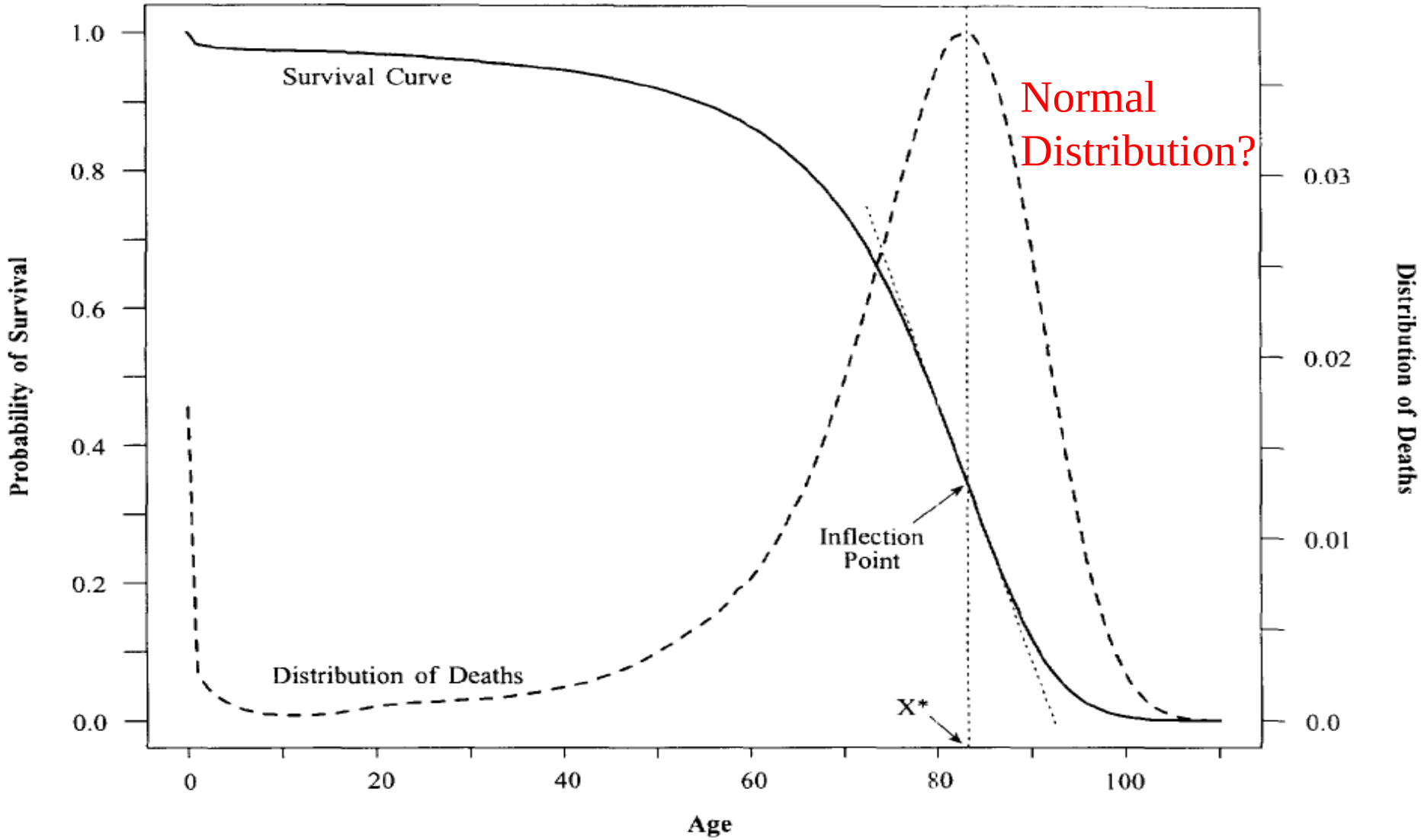
What is Mortality Compression?

- Mortality Compression is (Fries, 1980)

→ Rectangularization of the survival curve

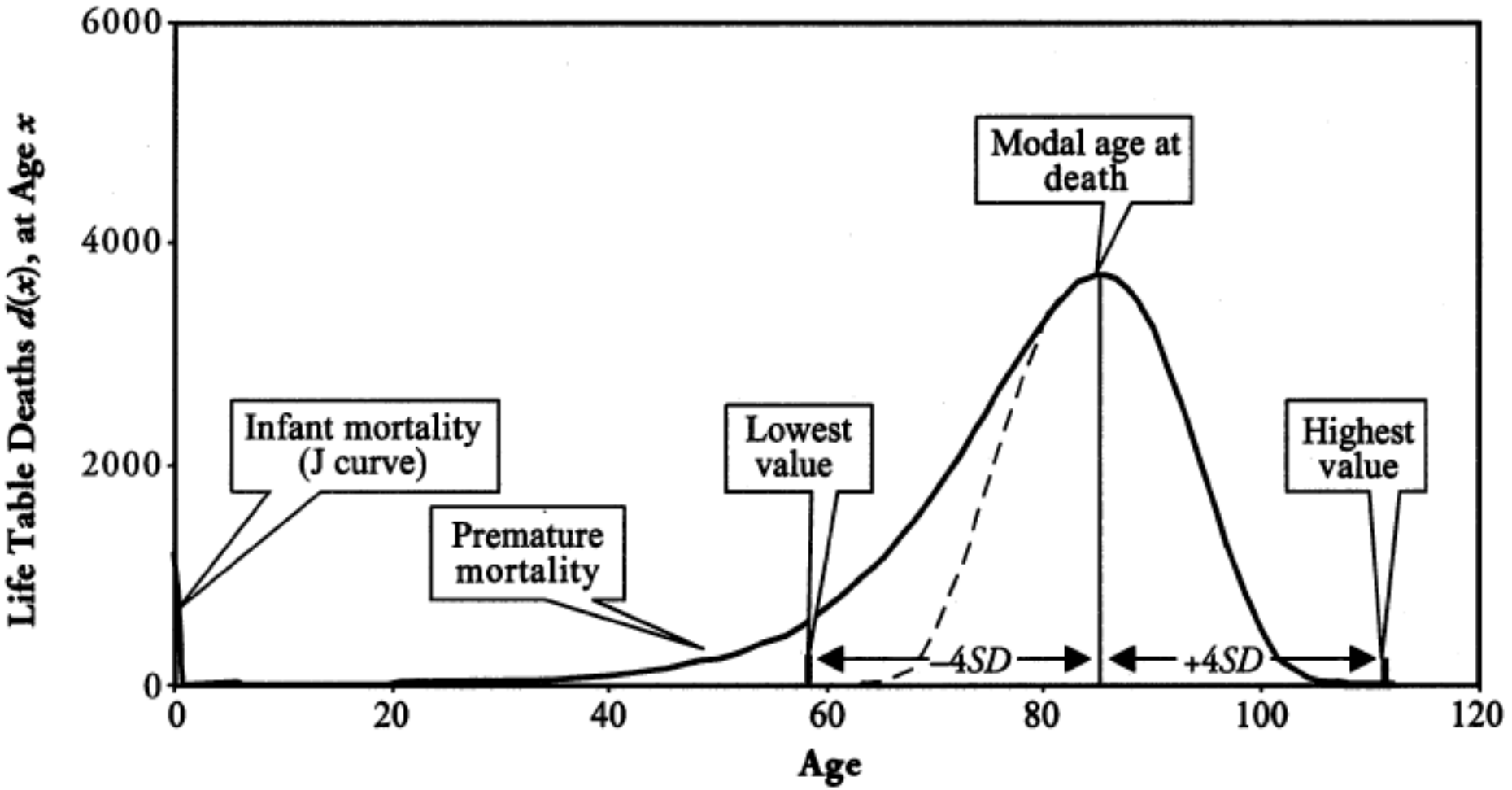
→ A state in which mortality from exogenous causes is eliminated and the remaining variability in the age at death is caused by genetic factors.

- Mortality compression is linked with morbidity compression.



Mortality Compression (Wilmoth and Horiuchi, 1999)

Horizontalization, Longevity Extension, Verticalization

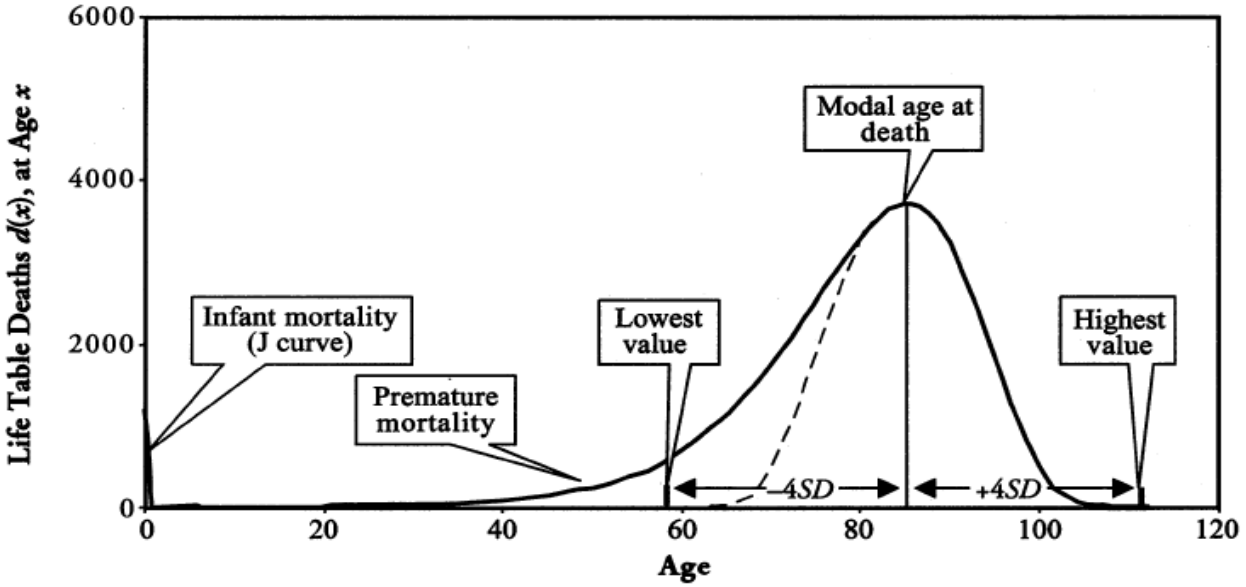


Mortality Compression (Cheung et al., 2005)

Measuring Compression

- Wilmoth and Horiuchi (1999) proposed 10 measurements and they recommended the Interquartile (IQR).
- Kannisto (2000, 2001) calculated percentiles, IQR, shortest age interval (e.g., C50) on numbers of deaths from 22 countries.
- Cheung et al. (2005) computed SD(M+) for Hong Kong data.
- Thatcher et al. (2010) computed SD(M+) for 6 countries from HMD.

- Cheung et al. (2005)
  - Horizontalization
  - Longevity Extension
  - Verticalization



Distribution on

統計方法	• M • σ • P95
分配假設	
資料品質	• 非修勻資料

Proposed Approaches

- Three estimation methods: (Yue, 2002)

→Maximal Likelihood Estimation (MLE), Non-linear Maximization (NM), and Weighted Least Squares (WLS).

→The MLE is expected to produce the most reliable estimates (smallest mean squared error), and the WLS is easy to use.

→We choose the NM method since it has the best overall performance.

Evaluating the Proposed Approaches

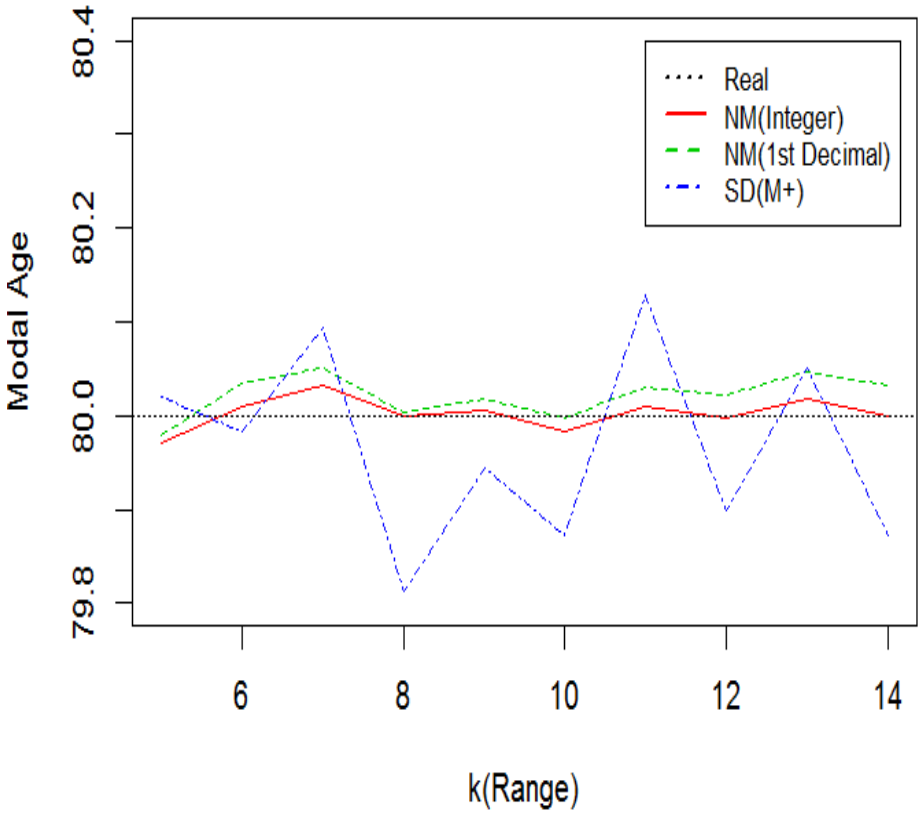
- Computer simulation:

→The modal age M is 80 and the standard deviation σ is 10. Randomly generate 100,000 deaths from normal or logistic distribution.

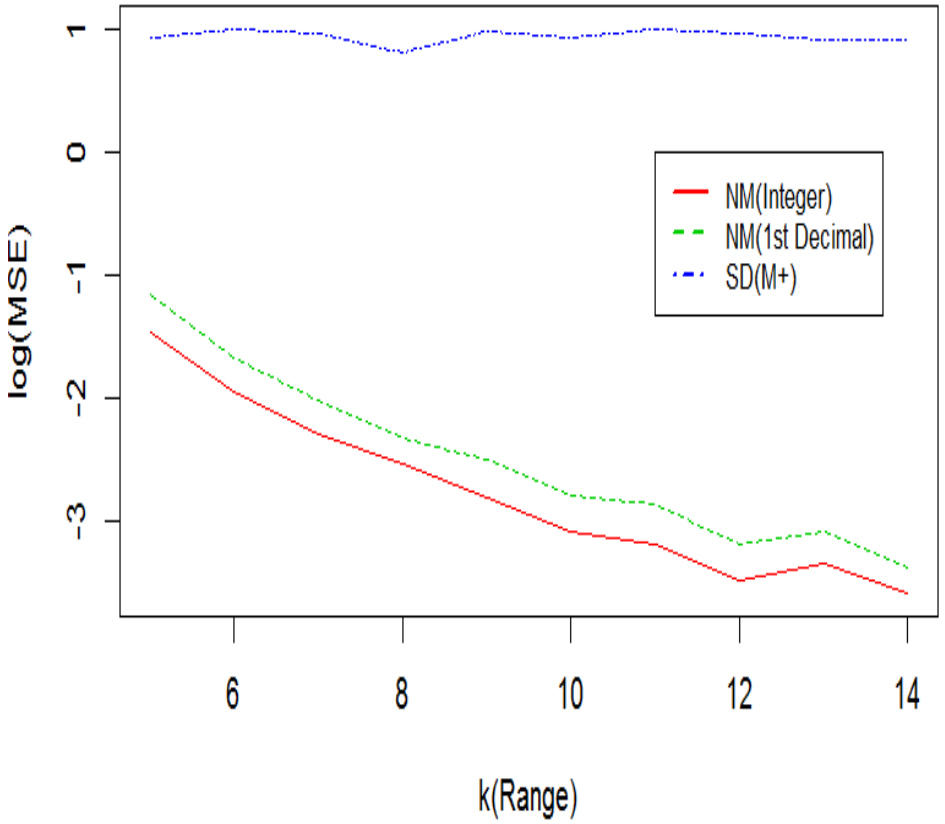
→Comparison criteria: Mean Squares Error (MSE), *Loss function (MSE)* = $Bias^2$ + *Variance*. and the probability of confidence interval covering true parameter (Coverage probability).

Modal Age of Normal Dist. (M=80)

Bias



MSE



Coverage Probability of Normal Dist.

	Estimation Method			
K	WLS	NM	MLE	SD(M+)
6	0.961	0.951	0.953	0.954
8	0.941	0.947	0.937	0.951
10	0.957	0.952	0.940	0.960
12	0.963	0.955	0.943	0.967

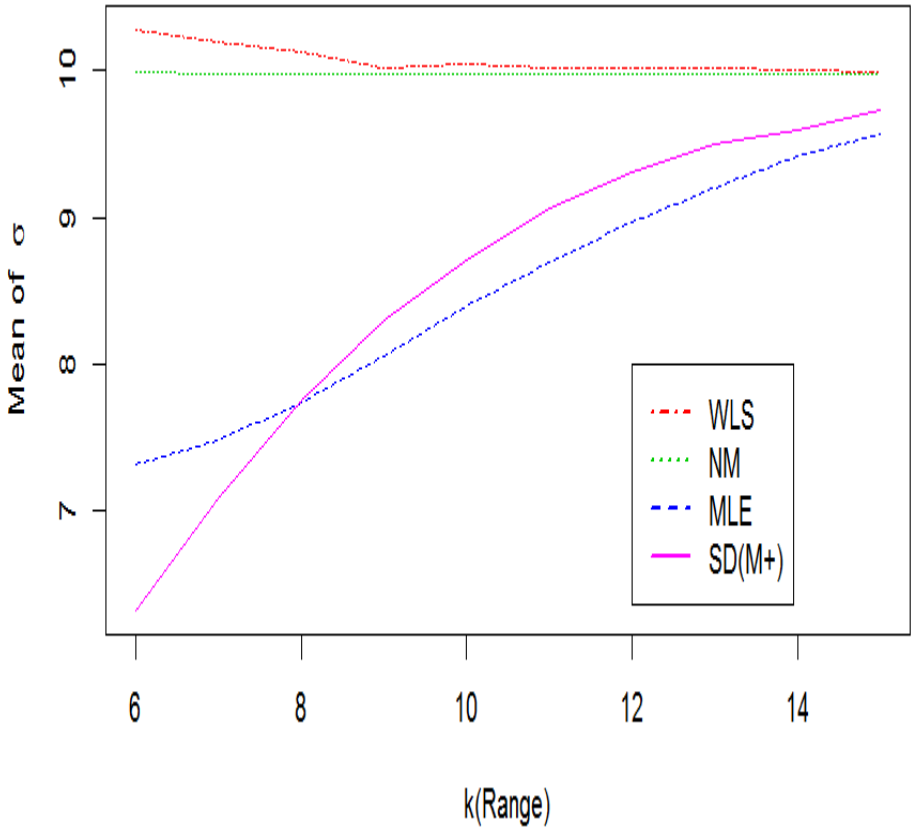
Note: M = 80 and σ = 10

Standard Deviation of Normal Dist. (σ=10)

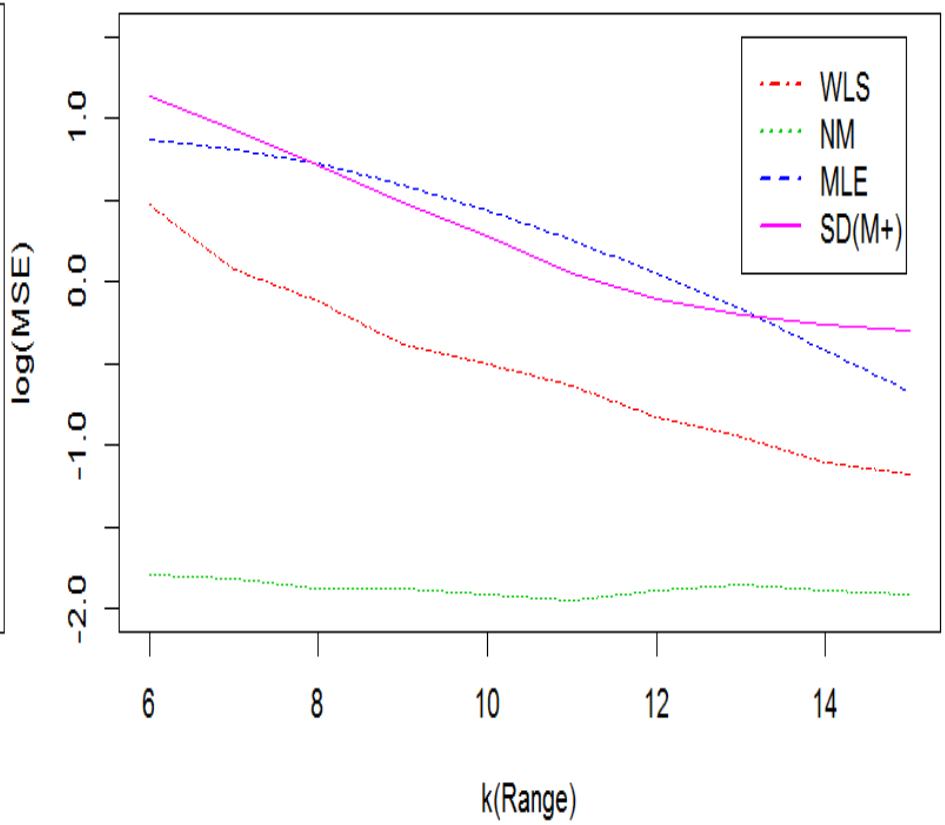
涵蓋機率	方法			
k	WLS	NM	MLE	SD(M+)
6	0.951	0.933	0.000	0.000
8	0.950	0.955	0.001	0.000
10	0.956	0.939	0.003	0.234
12	0.956	0.951	0.018	0.735

Standard Deviation of Normal Dist. (σ=10)

Bias

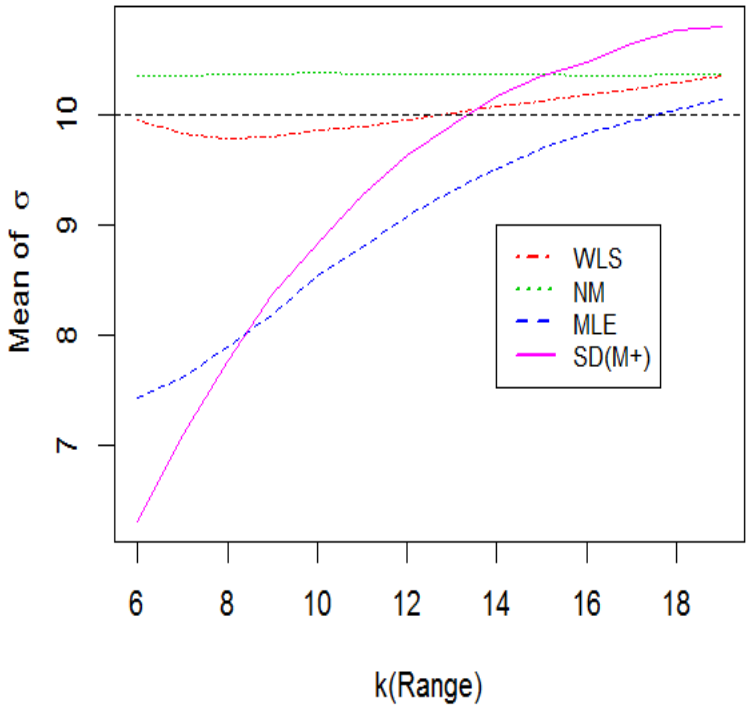


MSE

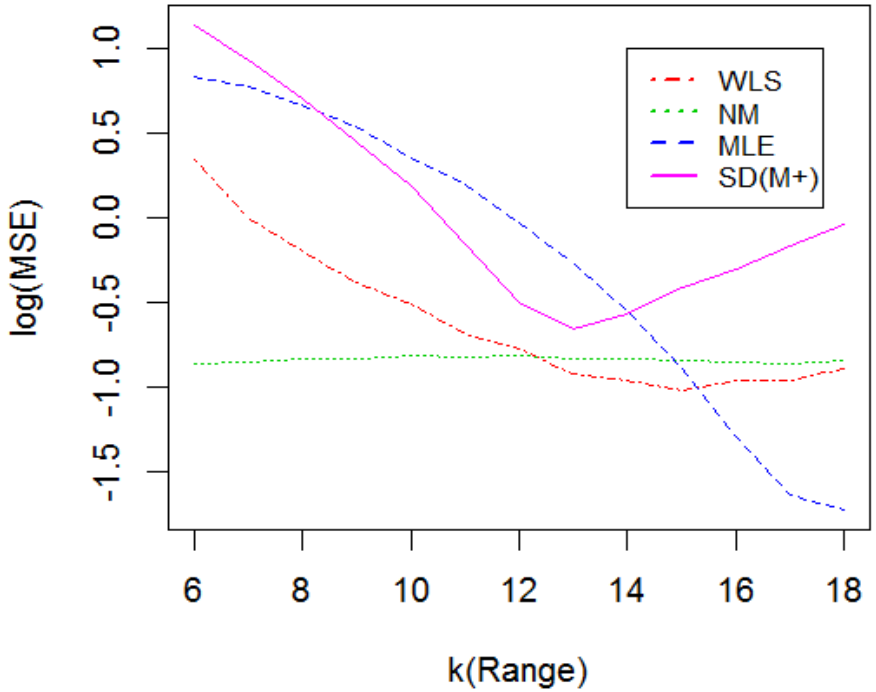


Standard Deviation of t Dist. (σ=10)

Bias

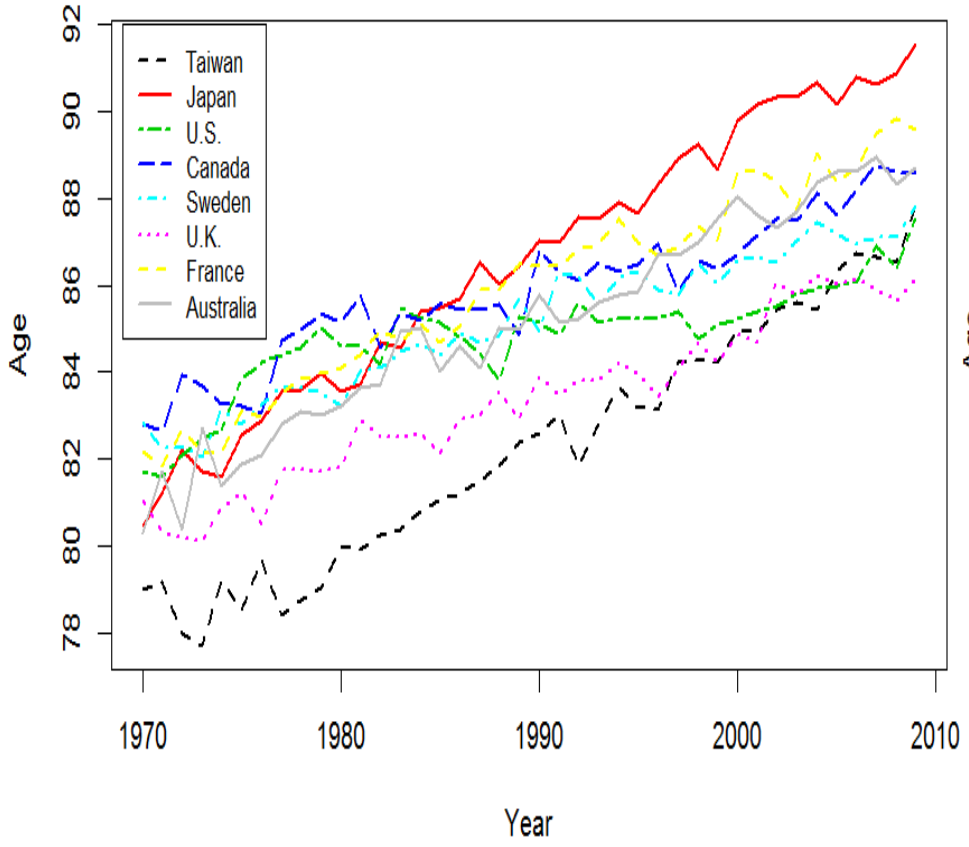


MSE

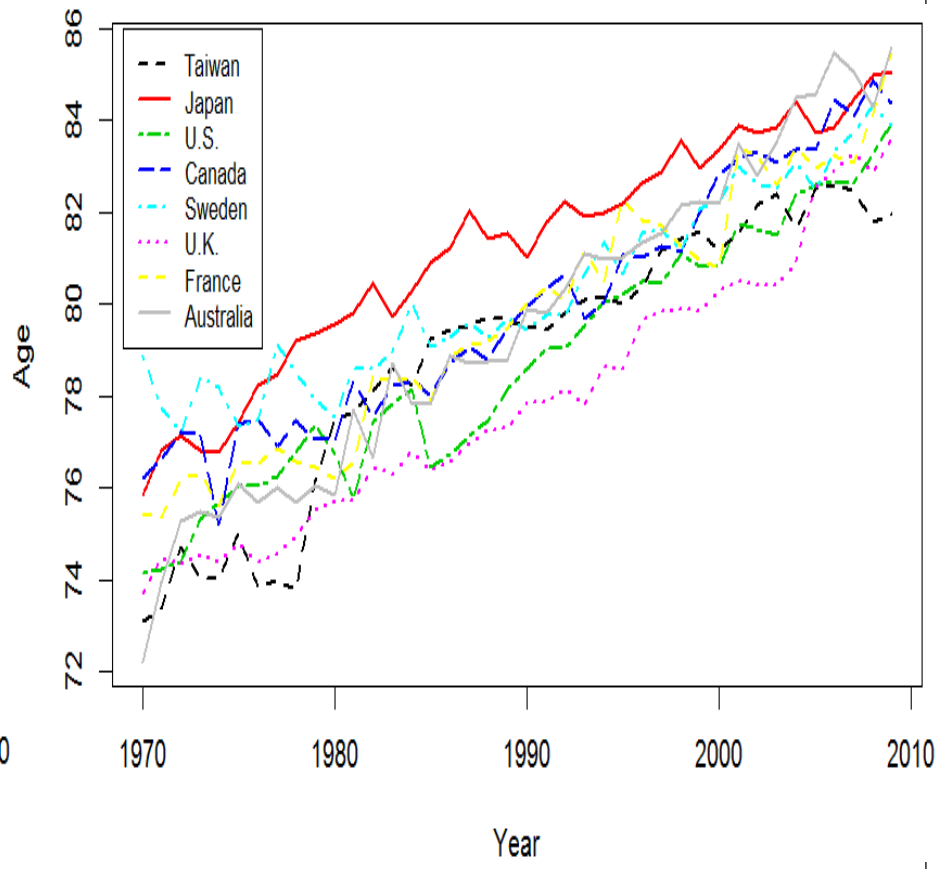


Empirical Analysis - M

- Female

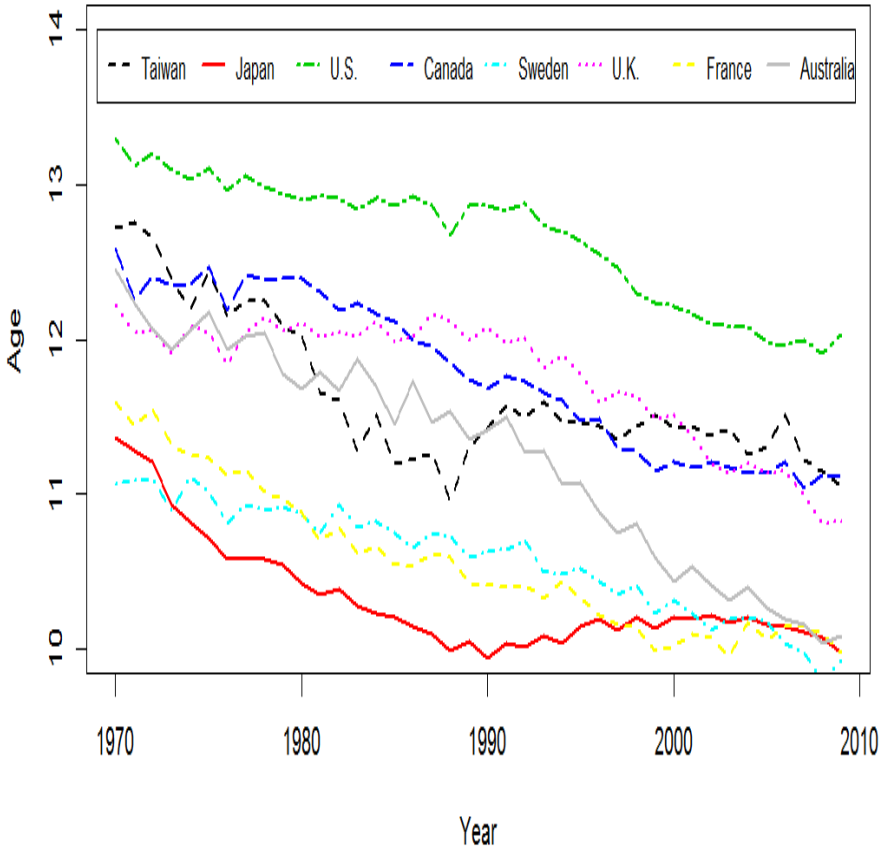


- Male

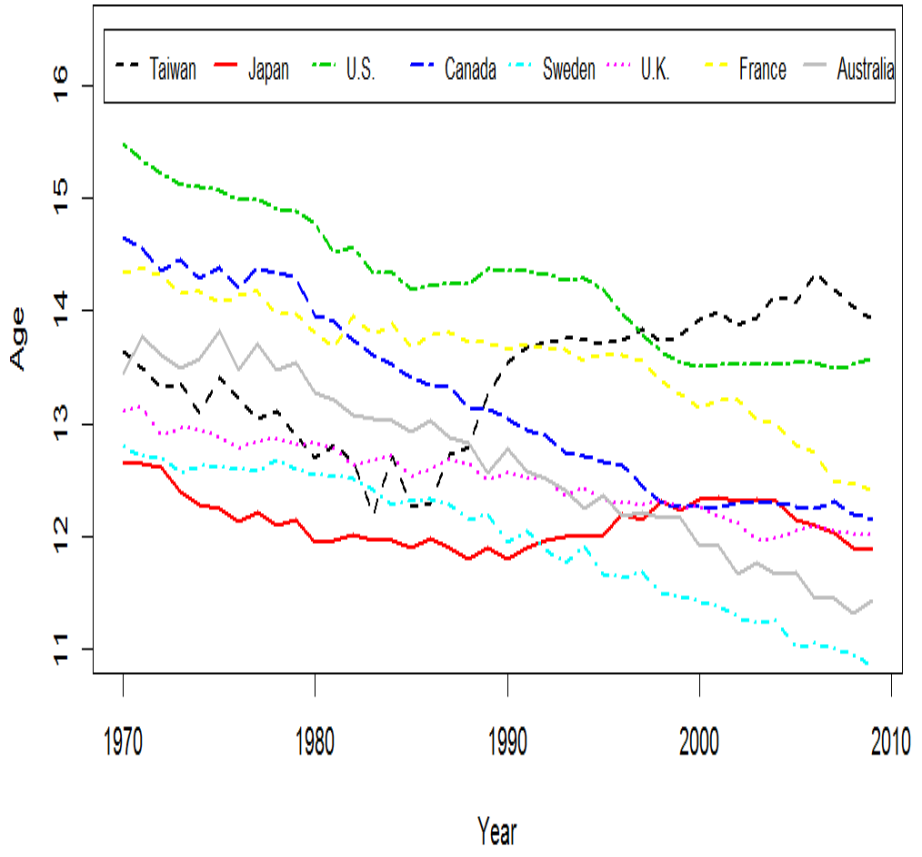


Empirical Analysis - σ (NM)

- Female

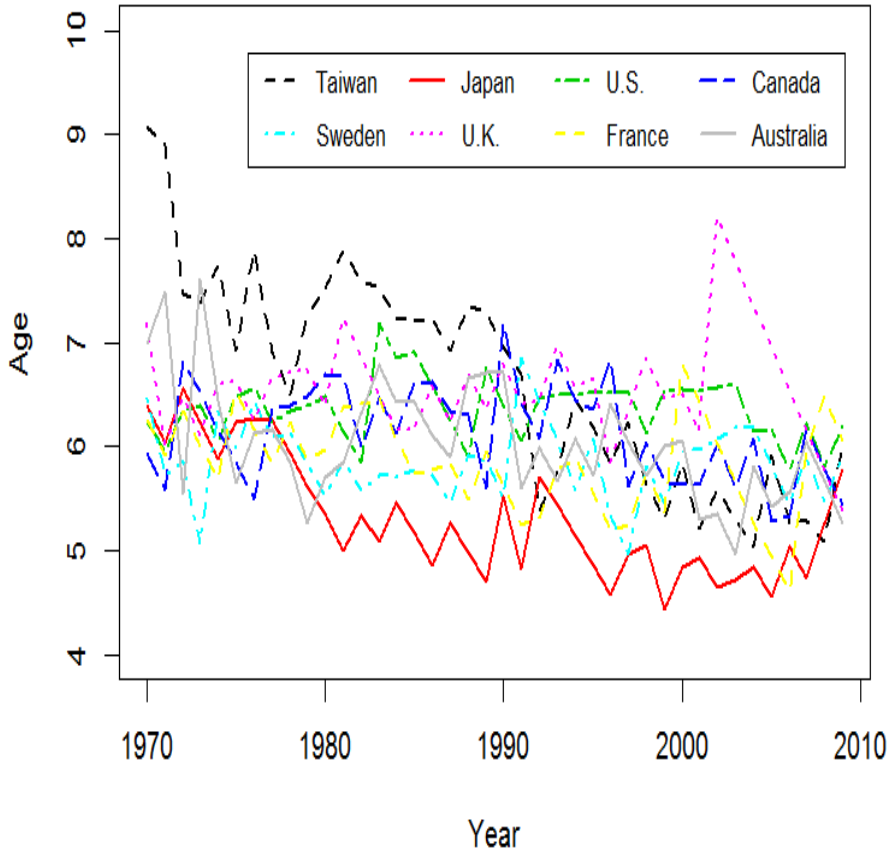


- Male

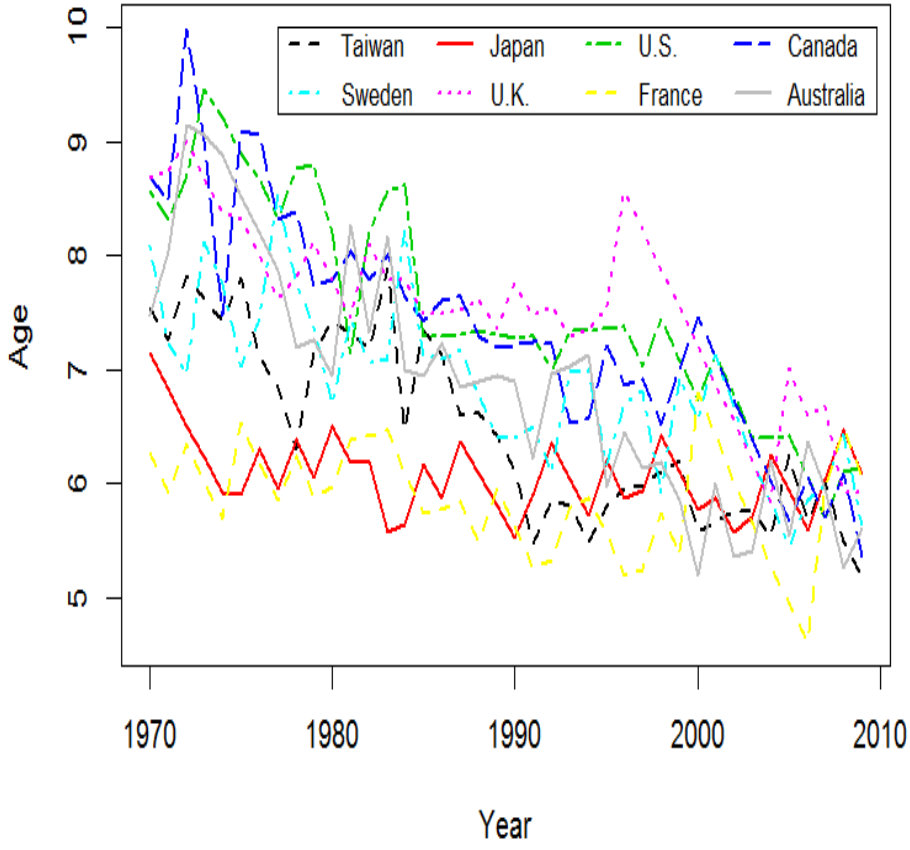


Empirical Analysis - σ (SD(M+))

- Female

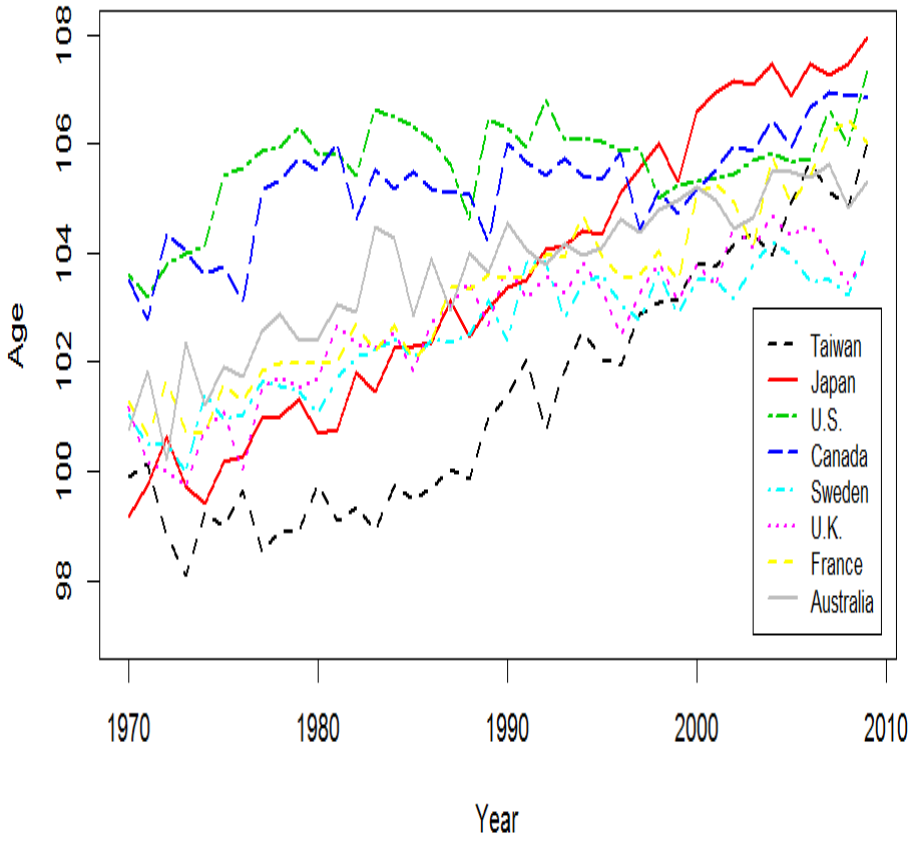


- Male

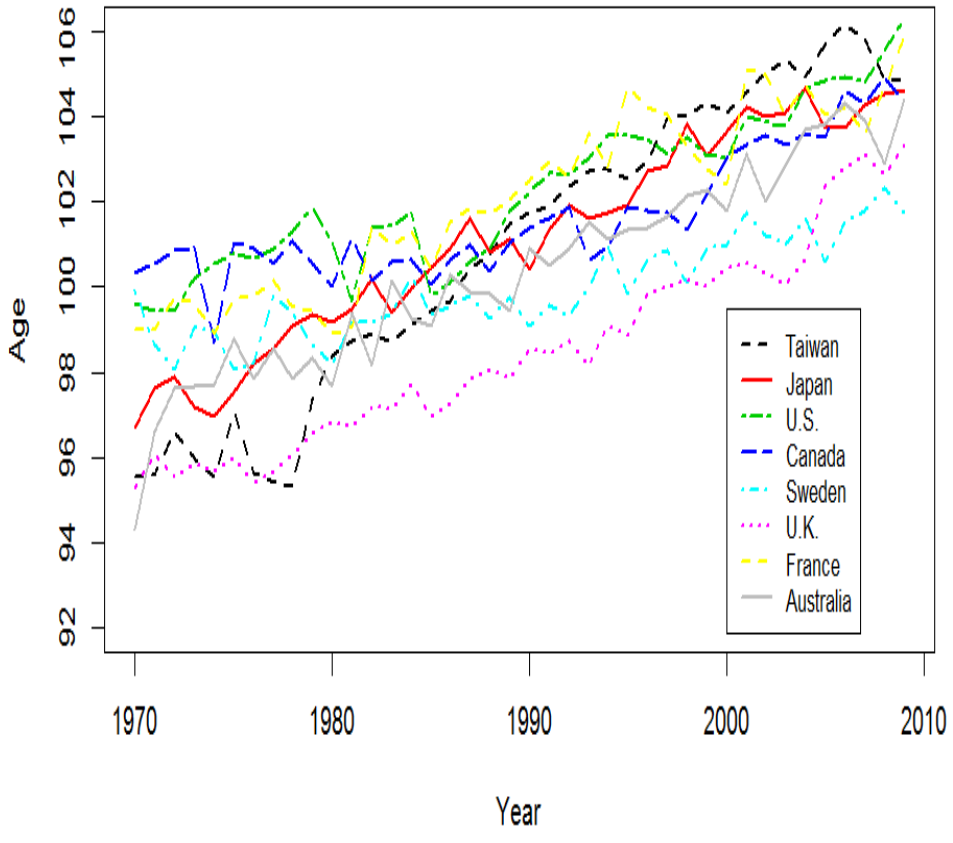


Empirical Analysis - P_{95}

- Female



- Male



Summary of Empirical Finding

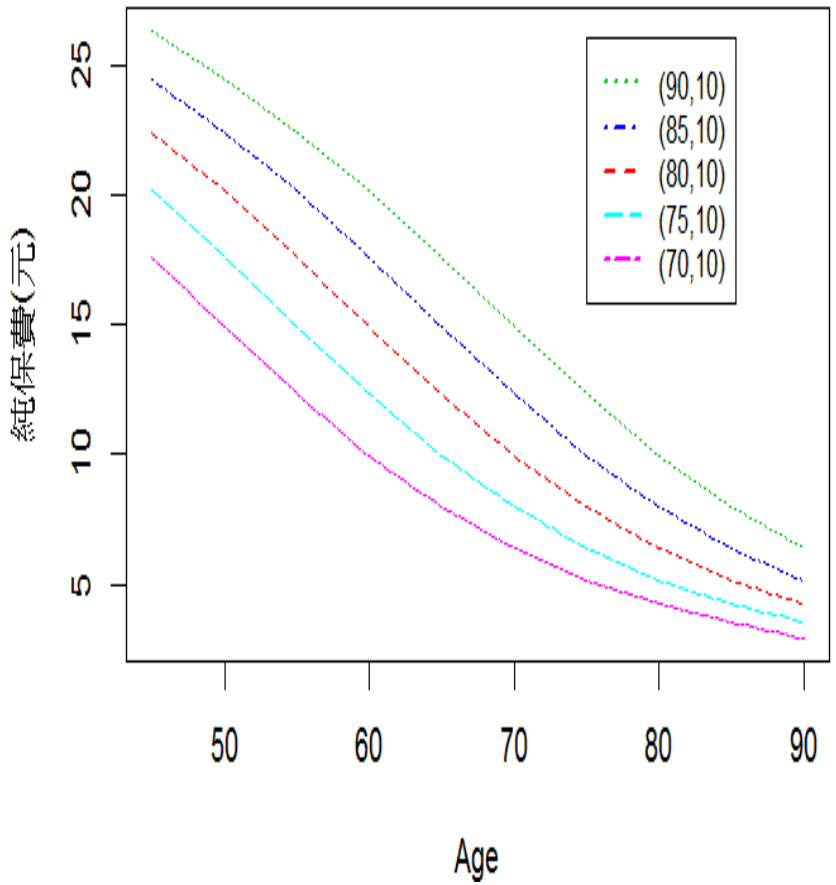
M is increasing → Life with a limit 

Σ is decreasing → Mortality Compression 

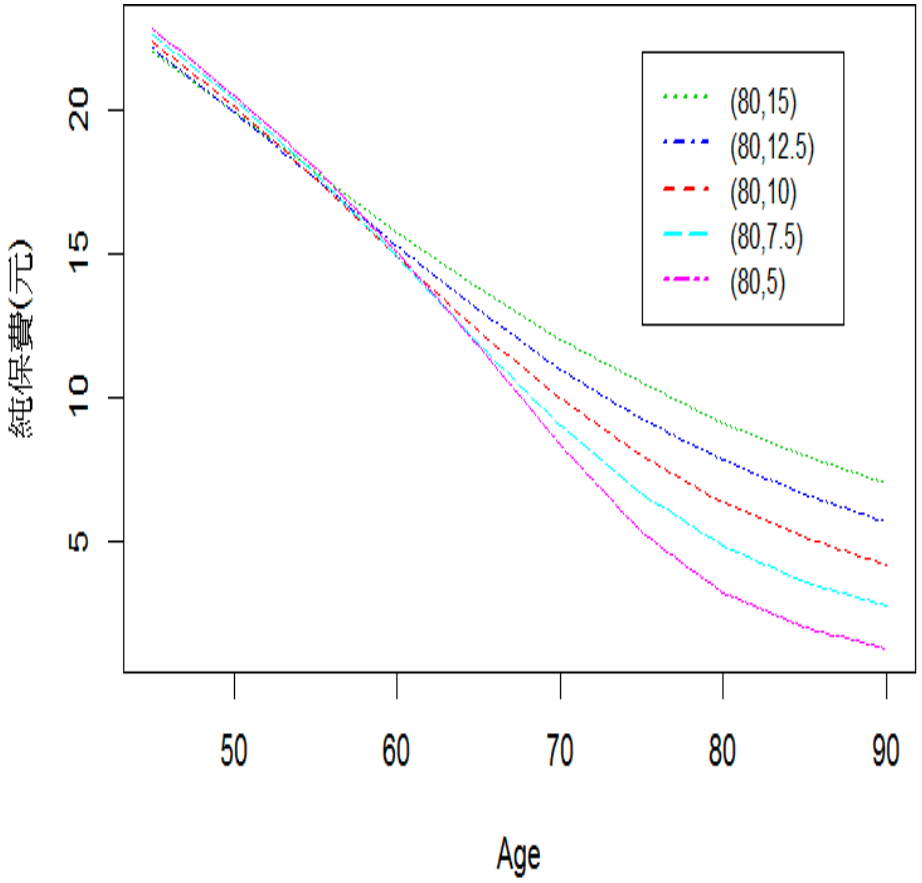
P_{95} is extended upward → Skewness 

Pricing Life Annuity

- M

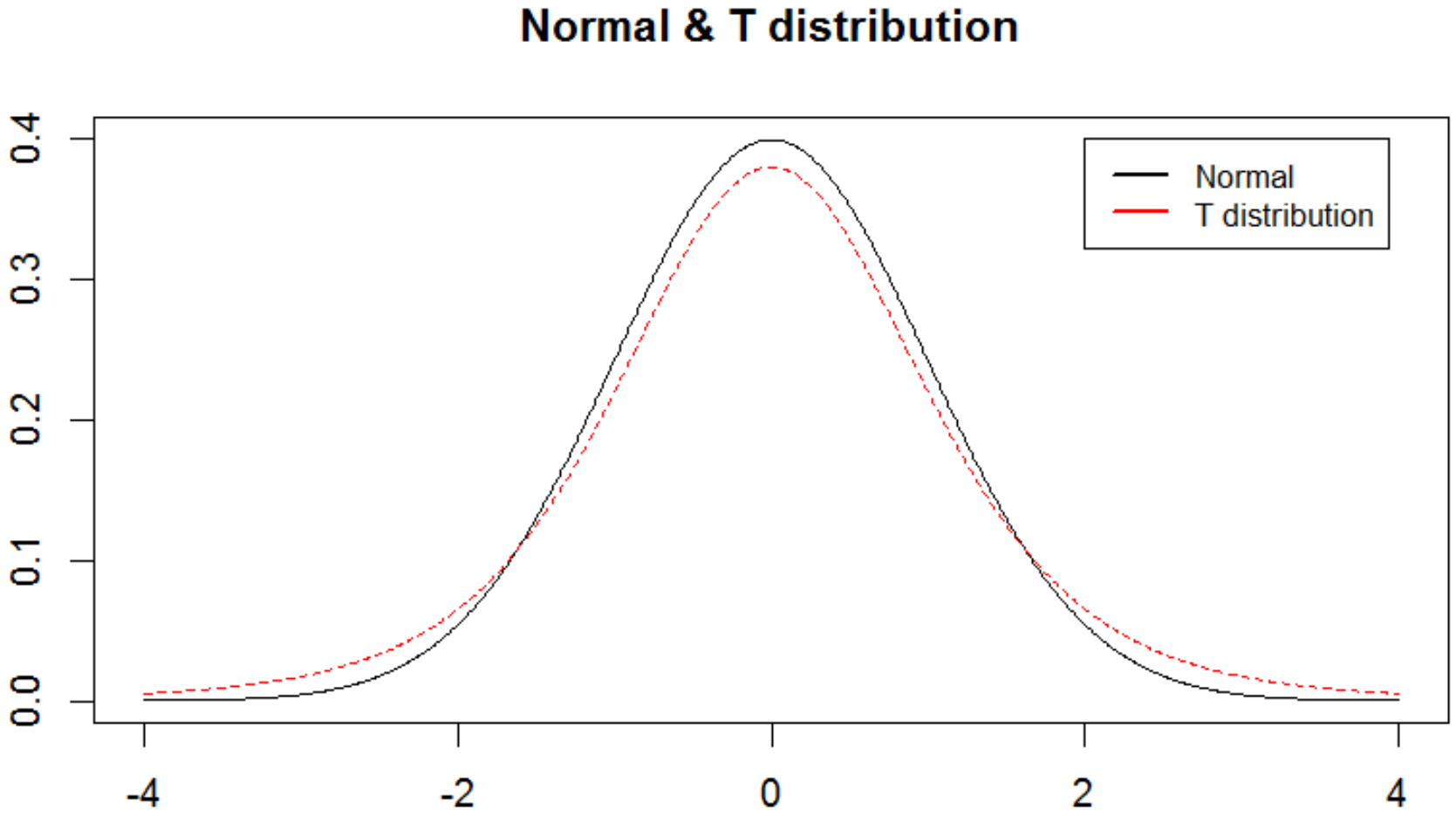


- σ



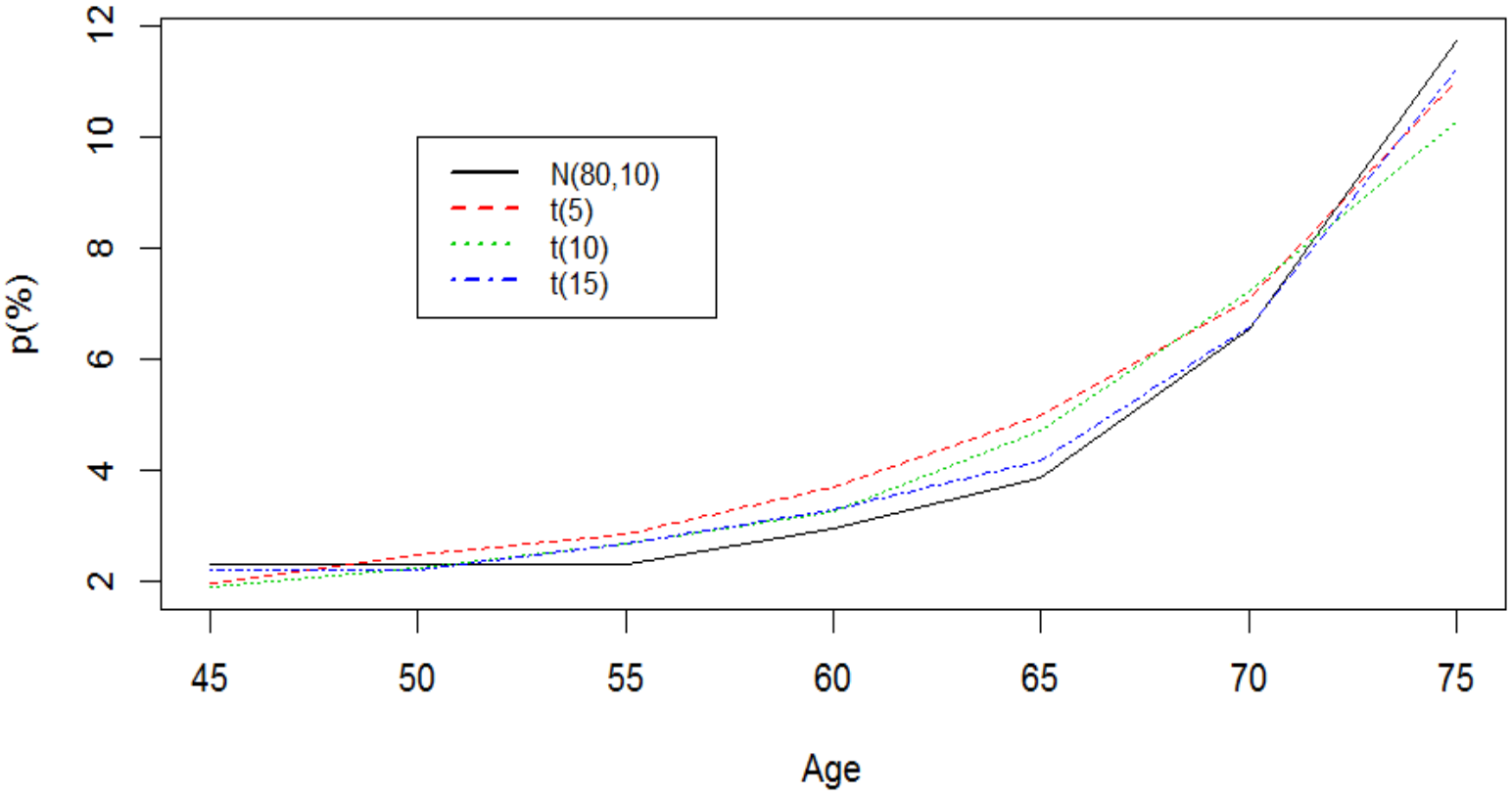
Pricing Life Annuity

- Normal & T distribution(df=5)



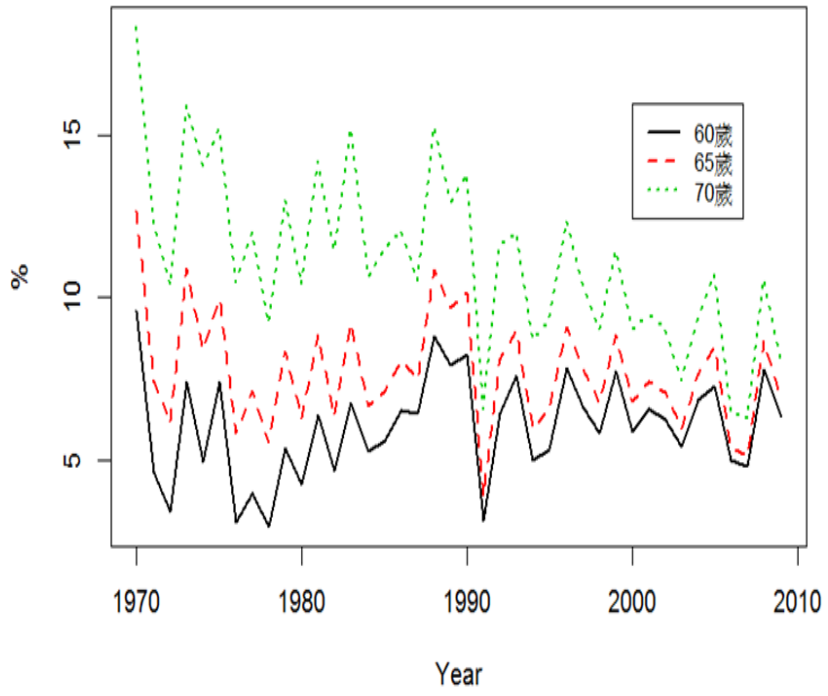
Pricing Life Annuity

- Ruin probability

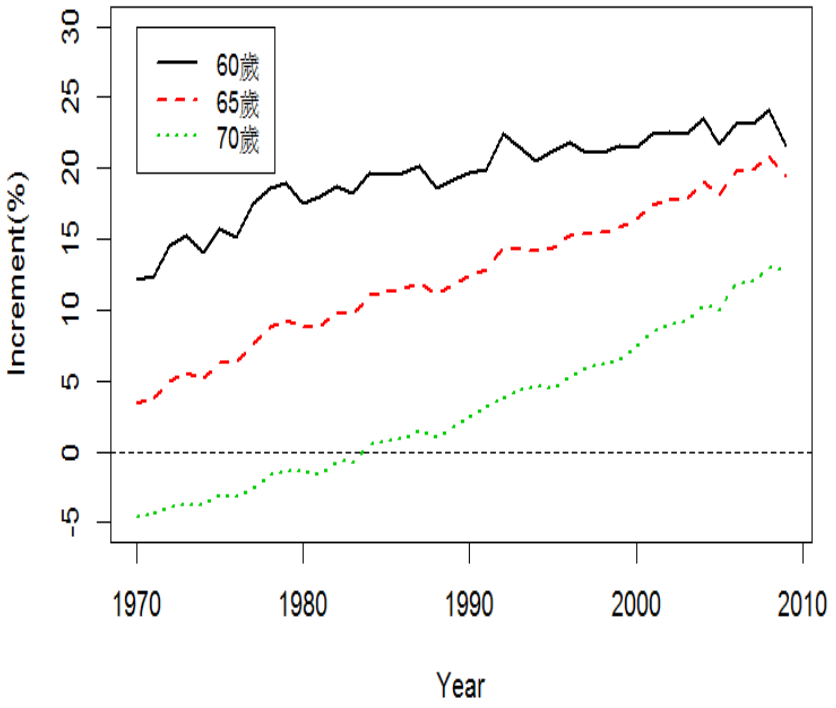


Normal Assumption vs. Mortality Data

- Mean(Taiwan, Female)



- S.d.(Taiwan, Female)



Conclusion

Estimation Method

→NM performs better.

Mortality Phenomenon

→(1) mortality compression is questionable.

(2) Life with a limit is questionable.

Pricing life annuity

→Shall consider the distribution of death.


Thank you!