Basis Risk and Optimal longevity hedging framework for Insurance Company

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- Introduction
- **Modeling Mortality**
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- Numerical Result
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Introduction

- Longevity risk has become non-negligible and its influence is increasing gradually and globally.
- Hedging strategy can be categorized as an internal or external method.
- Most of existing literature deal with either internal method or external method.

Introduction

- Internal method (Natural Hedging)
  - Insurer can hedge longevity risks with their own business products between life insurance and annuity because these two types of products are sensitive in opposing ways to the changes in mortality rates.
  - Natural Hedging Strategy & Immunization model (Wang et al 2010; Wang et al. 2013)

Introduction

- External method
  - Using capital market solutions
  - Securitization of longevity risk (Lin and Cox 2005; Cox et al. 2006; Dowd et al. 2006; Blake et al. 2006; Denuit et al. 2007; Biffis and Blake, 2009; Blake et al. 2010; Dawson et al., 2010)
  - E.g., EIB/BNP longevity bond & Q-forward contract & Kortis notes

Introduction

- Restriction of internal hedging
  - Need to adjust sales volume of life and annuity product
  - Cox and Lin (2007) suggest that natural hedging is good but may be too expensive to be effective in the context of internal life insurance and annuity products.
- Purpose of this research
  - We propose a hedging strategy that combines both natural hedging and external hedging.
  - The annuity providers usually has more annuity policies than life insurance policies in their liability, i.e. the longevity risk cannot be fully natural hedged and the insurer can consider the external hedging by using the longevity-linked securities to deal with the remaining longevity risk.

Introduction

- Purpose of this research
  - Longevity risk exposure may be different in different lines of business, mortality index underlying the hedging instrument.
    - Coughlan et al. (2011) point out the importance of basis risk in longevity hedging.

  - Deal with basis risk in the hedging strategy.

Introduction

- Deal with basis risk in the following three aspects.
  - First, *Instead of population mortality data*, we employ a unique experience mortality data from life insurance industry that includes the mortality experience for both life insurance and endowment insurance for men and women separately.
    - These incidence data include more than 50,000,000 policies, collected from all Taiwanese life insurance companies.

Introduction

- Deal with basis risk in the following three aspects.
  - Second, we measure the basis risk for the longevity exposed business based on Yang and Wang (2013)'s multi-population mortality framework.
    - This model is calibrated to capture the basis risk between different lines of insurance policies.

Introduction

- Third, we take into the multi-population mortality model in the longevity hedging framework based on the profit function for different lines of life insurance business and the longevity-linked securities.

- The profit function is calculated based on both the cash flow of the insurer's liability and the cash flow of the longevity-linked securities.

- We obtain the analytic solution for the hedging strategy.

Modeling Mortality Dynamics

- Consider N product lines for life insurance, $m_{x,t}^{j}$ then , $j=1,\dots,N,$ is the mortality force at age x for in the j^{th} insurance product during calendar year t.

$$\ln m_{x,t}^{j} = a_x^j + b_x^j k_t^j + e_{x,t}^j, \qquad j=1,\dots,N$$

  - where the parameters b_x^j and k_t^j are subject to $\sum_x b_x^j = 1$ and , $\sum_t k_t^j = 0$ to ensure the model identification.

Modeling Mortality dynamics

- The future mortality rates for a person of age x at time t in matrix form

$$\begin{bmatrix} \ln m_{x,t}^{1} \\ \ln m_{x,t}^{2} \\ \vdots \\ \ln m_{x,t}^{N} \end{bmatrix} = \begin{bmatrix} a_x^{1} \\ a_x^{2} \\ \vdots \\ a_x^{N} \end{bmatrix} + \begin{bmatrix} b_x^{1} & 0 & \cdots & 0 \\ 0 & b_x^{2} & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & b_x^{N} \end{bmatrix} \begin{bmatrix} k_t^{1} \\ k_t^{2} \\ \vdots \\ k_t^{N} \end{bmatrix} + \begin{bmatrix} e_{x,t}^{1} \\ e_{x,t}^{2} \\ \vdots \\ e_{x,t}^{N} \end{bmatrix}$$

Calibration of Mortality Model

- The data covers more than 50,000,000 policies issued by the life insurance companies in Taiwan from the period of 1972 to 2008.

- Divide all policies into two groups
  - With endowments
  - Without endowments
    - The insurance company in Taiwan has a special product strategy that selling endowment insurance instead of annuity business.
    - Endowments as the proxy for annuity policies

Calibration of Mortality Model

Summary of policy numbers derived from Taiwan insurance data

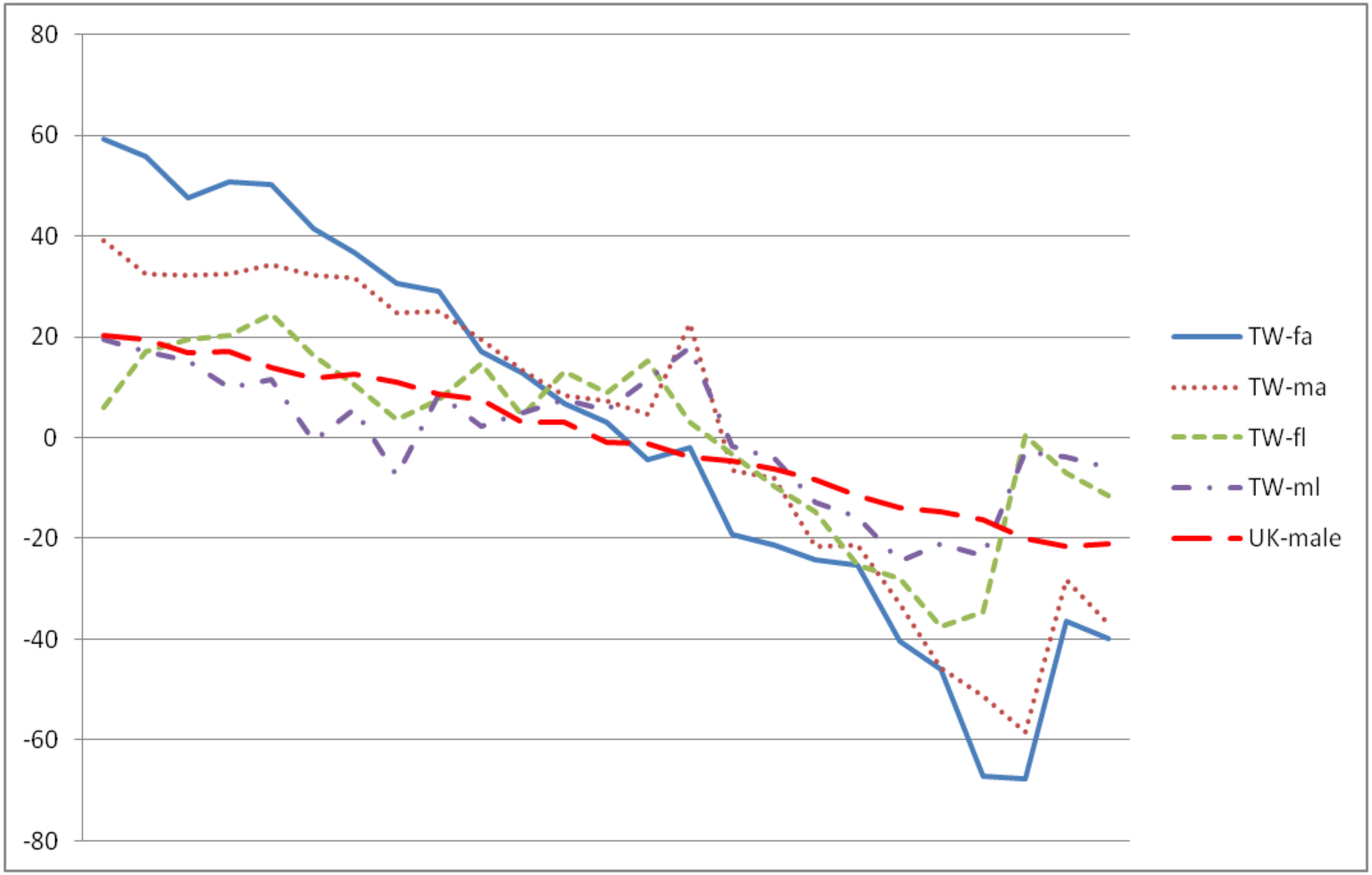
Insurance Type	Female	Male
With Endowments	7,175,200	7,509,730
No Endowments	18,254,681	18,072,776

Calibration of Mortality Model

- We demonstrate that the insurer uses q-forward contracts as the external hedging instrument.
  - Assume the underlying mortality index for the q-forward contract is based on the UK male population mortality.

- In order to model the basis risk among different populations, we apply Co-integration analysis to capture the interrelated mortality improvement among populations according to the mortality experience.

Calibration of Mortality Model



Mortality Time Trend k_t^j for Different Population Groups

Calibration of Mortality Model

- We can observe that the mortality improvement for annuity populations seems more faster than for life insurance populations and UK male population.

- All k_t's move in a similar pattern for different groups of populations
  - A long-term equilibrium relationships among all k_t's and the k_t's have cointegrated effect

Calibration of Mortality Model

- We can write down the estimated parameters for the error correction model as follows

$$\begin{pmatrix} \Delta k_{1,t} \\ \Delta k_{2,t} \\ \Delta k_{3,t} \\ \Delta k_{4,t} \\ \Delta k_{5,t} \end{pmatrix} = \begin{pmatrix} -0.9736 \\ 0.6833 \\ -0.7639 \\ 0.0224 \\ -1.9252 \end{pmatrix} + \begin{pmatrix} 7.5470 \\ 9.2306 \\ -0.0714 \\ 2.5930 \\ -0.4821 \end{pmatrix} \left[-0.2570 + \sum_{i=1}^{5} \beta_i k_{i,t-1} \right] + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \end{pmatrix}$$

Calibration of Mortality Model

- where

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} = \begin{pmatrix} -0.0250 \\ -0.0845 \\ 0.0744 \\ 0.0053 \\ 0.1729 \end{pmatrix}$$

$$\Sigma = \mathrm{Cov}\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \\ \varepsilon_{5,t} \end{pmatrix} = \begin{pmatrix} 30.11 & 7.3671 & -8.0381 & 3.3607 & 2.5366 \\ 7.3671 & 27.3574 & -13.4597 & 8.0873 & 1.775 \\ -8.0381 & -13.4597 & 97.9874 & 43.1154 & -0.1343 \\ 3.3607 & 8.0873 & 43.1154 & 67.068 & -0.8885 \\ 2.5366 & 1.775 & -0.1343 & -0.8885 & 1.7289 \end{pmatrix}$$

Calibration of Mortality Model

- We can treat the first difference of k_t as a time-varying multivariate normal distribution.

$$\begin{pmatrix} \Delta k_{1,t} \\ \Delta k_{2,t} \\ \Delta k_{3,t} \\ \Delta k_{4,t} \\ \Delta k_{5,t} \end{pmatrix} \sim N(\mu(k_{t-1}), \Sigma)$$

- where

$$\mu(k_{t-1}) = \begin{pmatrix} -0.9736 \\ 0.6833 \\ -0.7639 \\ 0.0224 \\ -1.9252 \end{pmatrix} + \begin{pmatrix} 7.5470 \\ 9.2306 \\ -0.0714 \\ 2.5930 \\ -0.4821 \end{pmatrix} \left[-0.2570 + \sum_{i=1}^{5} \beta_i k_{i,t-1} \right]$$

Optimal Hedging Strategy for longevity risk

- Assumed to have more annuity policies than life insurance policies in their liability
  - The longevity risk cannot be fully natural hedged. Thus, the insurer also considers the external hedge instrument and attempts to find the optimal level of longevity securities to effectively reduce longevity risk.

- The profit function is calculated according to the cash flow of the insurer's liability and the cash flow of the hedging instrument.

Optimal Hedging Strategy for longevity risk

- Let U(t) denote the insurer's profit at time t. The profit function of the insurance company including all life and annuity policies at time t is defined as follows:

$$U(t) = M \times V^{S}(t) - \sum_{j} c_{L,j} V^{L}\left(x_j, t\right) - \sum_{j} c_{A,j} V^{A}\left(x_j, t\right)$$

- $V^S(t)$ is the value of the external hedge instrument at time t and M is the amount of the hedge instrument, $V^L(x_j, t)$ is the value of *j*th life insurance policy for the insured aged x_j with face amount $c_{L,j}$; $V^A(x_j, t)$ is the value of *j*th annuity policy for the insured aged x_j with face amount $c_{A,j}$

Optimal Hedging Strategy for longevity risk

- We consider q-forward as the hedging instrument against longevity risk. The present value of payoff function of q-forward can be represent as

$$V^{S}(0)=1000(1+r)^{10}(q^{f}_{UK-male65}-q^{real}_{UK-male65})$$

  - $q^{f}_{UK-male65}$ is the q forward rate determined at the beginning of the contract and the maturity is 10 years.
  - The interest rate is assumed to be 0.02.

Optimal Hedging Strategy for longevity risk

- Our objective is to determine the optimal value of M
  - how many units of longevity-linked securities the insurer should take to reduce the variation for the insurer's profit

- To estimate the effect of the mortality change on the profit, we apply Taylor expansion of U(t) with respect to mortality trend (k_t)

Optimal Hedging Strategy for longevity risk

- We investigate three different hedging strategies based on the mean-variance framework for the insurer's profit process.
  - First, Maximizing mean-variance approach:

$$\max_{\mathrm{M}} E\left[\Delta \mathrm{U}(t)\right] - \theta Var\left(\Delta U(t)\right)$$

Optimal Hedging Strategy for longevity risk

- We investigate three different hedging strategies based on the mean-variance framework for the insurer's profit process.
  - Second, Maximizing mean- value at risk approach :

$$\max_{M} E\left[\Delta U(t)\right] - z_p \sqrt{Var\left(\Delta U(t)\right)}$$

where z_p is the inverse cumulated distribution function of standard normal distribution.

Optimal Hedging Strategy for longevity risk

- We investigate three different hedging strategies based on the mean-variance framework for the insurer's profit process.

  - Third, Maximizing mean- conditional tail expectation approach:

$$\max_{M} E\left[\Delta U(t)\right] - \frac{\sqrt{Var\left(\Delta U\left(t\right)\right)}}{p} \frac{e^{-\frac{\left[z_p\right]^2}{2}}}{\sqrt{2\pi}}$$

Number examples

- Assumption for the life insurance and annuity policies
  - male insured aged 55
  - female insured aged 55

Number examples

Optimal Hedging Strategies

	Optimal M
Strategy 1: Delta mv	4.008961 ($\theta = 0.8$) 3.267354 ($\theta = 1$) 2.772949 ($\theta = 1.2$)
Strategy 2: Delta VaR	0.810618
Strategy 3: Delta CTE	1.086048

- We can see under mean variance assumption, as θ increases, the optimal hedging units decreases
- The optimal hedging strategies of VaR and CTE need fewer units of q-forward to hedge the longevity risk.

Conclusions

- This paper analyzes the optimal hedging strategy to hedge longevity risk.
- Different to existing literature, we utilize both internal and external hedging to deal with longevity risk.
- We can find that the optimal hedging strategies analytically.