

# Time-evolution of age-dependent mortality patterns in mathematical model of heterogeneous human population

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## Abstract

The widely-known Gompertz law of mortality states the exponential increase of mortality with age in human populations. Such an exponential increase is observed at the adulthood span roughly after the reproductive period while mortality data at young and extreme old ages deviate from it. The heterogeneity of human populations, i.e. the existence of subpopulations with different mortality dynamics, is a useful consideration that can explain the configuration of age-dependent mortality patterns across the whole life-course. Combining the heterogeneity of populations and the Gompertz law, the mortality rate of the entire population can be described by a combination of exponential terms, where each term represents the Gompertzian dynamics of one of the subpopulations. This model is capable of reproducing the entire mortality pattern including the observed deviations from the Gompertz law. Besides, the model is advantageous as compared to many other parametric mortality models since its parameters have demographic interpretations. In this work, the model of heterogeneous populations has been fitted to Swedish mortality data for consecutive periods and this permitted us to describe the evolution of mortality dynamics in terms of the evolution of the model parameters over time. We have found that the evolution of the model parameters validates the applicability of the compensation law of mortality to each subpopulation separately. Furthermore, our study has indicated that the population structure changes so that the population tends to become more homogeneous over time. Finally, our analysis of the decrease of the overall mortality in population over time shows that this decrease is mainly due to a change in the population structure and to a lesser extent to a reduction of mortality of each of the subpopulations, the latter being represented by an alteration of Gompertzian dynamics. Therefore, once the homogenization process will be over, potential future mortality improvements will be relatively small compared to what we observed over the last century.

**Keywords** Gompertz law of mortality, Population heterogeneity, Mathematical model, Model fitting, Compensation law of mortality, Homogenization.