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Cause-of-Death Mortality: What Can Be Learned From Population Dynamics?

A Study of a Heterogeneous Portfolio Dynamic

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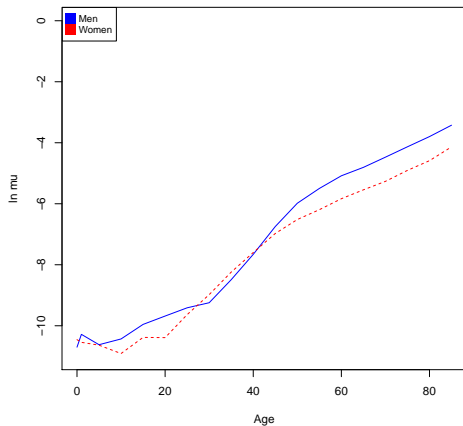
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joint work with S. Arnold, A. Boumezoued and N. El Karoui

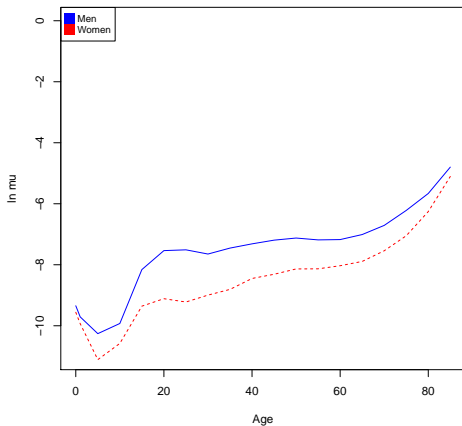
Longevity 11 Conference, Lyon, France
September 7, 2015

Introduction

French death rates for cancers in 2008



French death rates for external causes in 2008



Source : The World Health Organization (WHO)

Introduction

Death rates by socio-economic category in 2007 for males in England

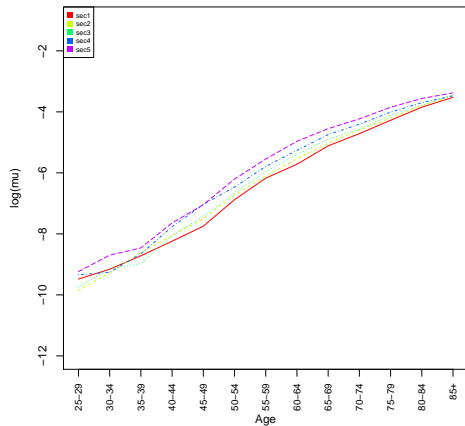


Figure : Cancers

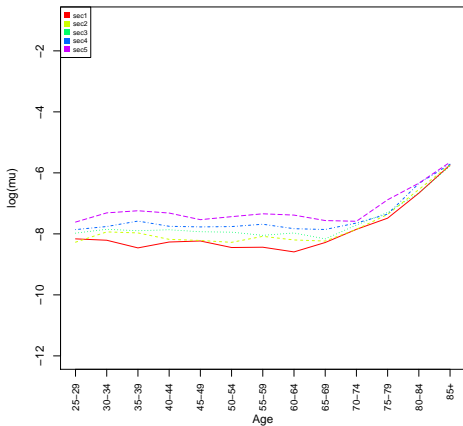


Figure : External causes

1. Population Dynamics Model

1.1 Deterministic equation without population flows

1.2 Application : French data

2. Portfolio Dynamics Model

2.1 Deterministic equation without arrivals

2.2 Application : English data

1. Population Dynamics Model

1.1 Deterministic equation without population flows

1.2 Application : French data

2. Portfolio Dynamics Model

Objective :

Study impacts of changes in cause-of-death mortality on the whole population age structure

➤ Model population dynamics

- ▷ By taking into account deaths and births :
 - with birth and death rates depending on gender and age, invariant over time
- ▷ Reference : Bensusan, Boumezoued, El Karoui and Loisel (working paper)

- The population structure described by the vector :

$$\mathbf{g}(a, t) = \begin{pmatrix} g(f, a, t) \\ g(m, a, t) \end{pmatrix}$$

- $\mathbf{g}(a, t)$: average number of individual with age a at time t
- The population dynamics without population flows is defined by :

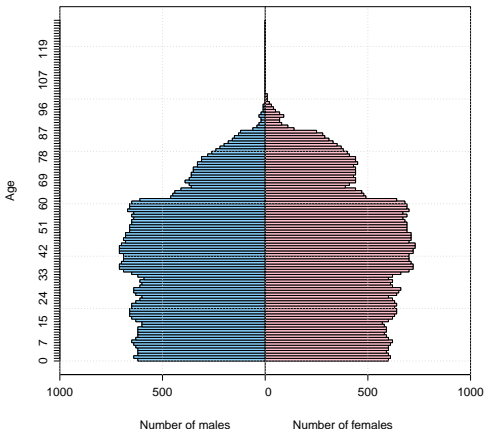
- **Deaths** : $(\partial_a + \partial_t)\mathbf{g}(a, t) = - \begin{pmatrix} \mu^f(a) & 0 \\ 0 & \mu^m(a) \end{pmatrix} \mathbf{g}(a, t)$

- **Births** : $\mathbf{g}(0, t) = \left(\int_{\mathbb{R}_+} g(f, a, t) b^f(a) da \right) \begin{pmatrix} p \\ 1 - p \end{pmatrix}$

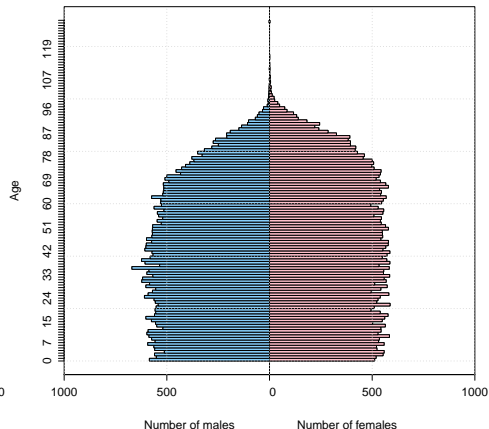
μ depends on age a ; p : probability for a newborn to be a female

Population modelling

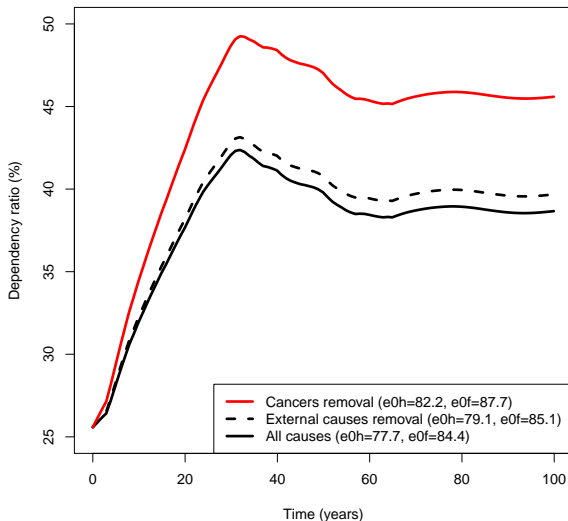
Age pyramid in 2008



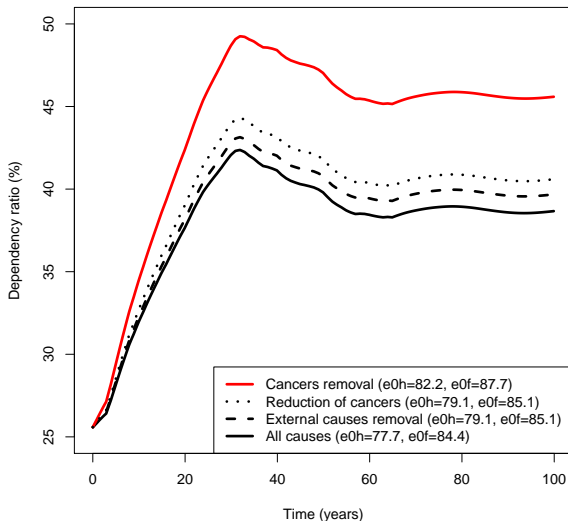
Age pyramid in 2108



Cause removal : Age Dependency Ratio from 2008 to 2108



Cause reduction : Age Dependency Ratio from 2008 to 2108



Results

- With a population dynamics model, we study impacts of cause-of-death reductions on the population age structure :
 - ⇒ **Studying the whole population dynamics gives additional informations :**
 - ▷ With the same life expectancy at birth, causes reductions can have different impacts on the age dependency ratio
 - ⇒ **Test the sensitivity to population flows and fertility :**
 - ▷ Population flows modify the age dependency ratio : cause reductions have similar impacts

1. Population Dynamics Model

2. Portfolio Dynamics Model

2.1 Deterministic equation without arrivals

2.2 Application : English data

Objective :

Study impacts of changes in cause-of-death mortality on insurance portfolio composed of cohorts with different socio-economic categories

- Model portfolio dynamics
 - ▷ By taking into account deaths and arrivals :
 - with cause-of-death rates depending on age, time, gender and socio-economic categories

- The portfolio is compounded by cohorts with k different socio-economic categories. The cohort structure is described by the vector :

$$\mathcal{G}(a, t) = \mathcal{G}(a) = \begin{pmatrix} g_1(a) \\ \dots \\ g_k(a) \end{pmatrix},$$

- ▷ $\mathcal{G}(a, t)$: average number of individual with age a at time t
- The cohort dynamics without arrivals is defined by deaths :

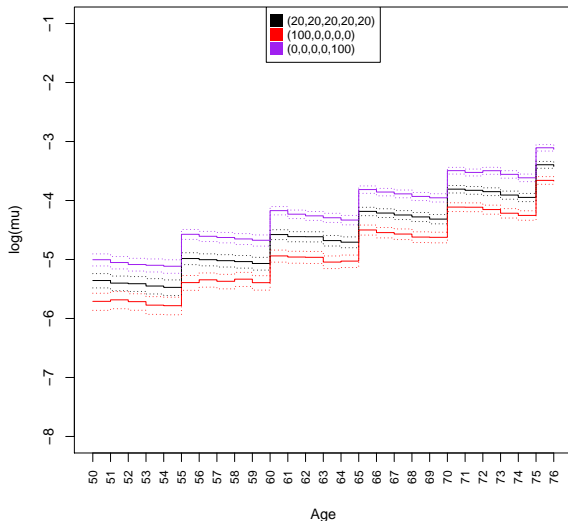
$$\frac{dg(a)}{da} = g'(a) = -d(a)g(a)$$

- Cohort death rate :

$$d(a) = -\frac{g'(a)}{g(a)} = -\frac{\sum_{1 \leq k \leq 5} g'_k(a)}{\sum_{1 \leq k \leq 5} g_k(a)}; \quad g_k(a) = g_k(a_0)S_k(a_0, a)$$

Aggregate death rate for different compositions :

English females with age 50 in 1981



Cause removal : relative difference of aggregate death rate

English females with age 50 in 1981

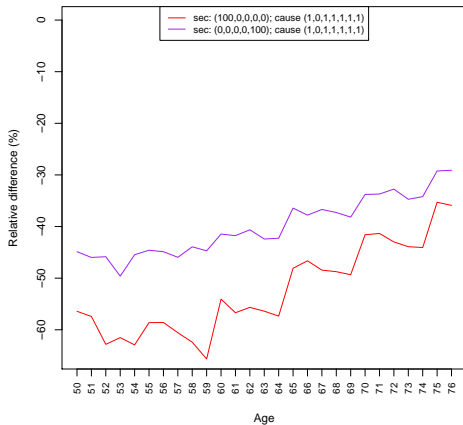


Figure : Cancers removal

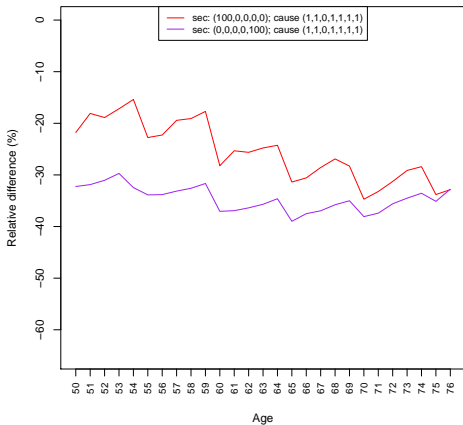


Figure : Circulatory diseases removal

First results

- With a population dynamics model, we study impacts of cause-of-death reductions on a portfolio mortality composed by different socio-economic category :
 - ⇒ **Following the portfolio structure, causes reductions can have different impacts on the aggregate mortality**
- Following
 - ▷ Portfolio dynamics with arrivals
 - ▷ Portfolio dynamics with arrivals and seniority
 - ▷ Study aggregate mortality of a population composed with different socio-economic categories

Bibliographie

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thank you!