Life-annuities reserving in Algeria : comparison of some mortality models

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Outline

- Motivation
- Methodology
- Data
- Best model selection
- Construction of a prospective life-table
- Simulation with life annuities
- Main funding

Mortality forecasting and life-annuities in Algeria

- During the past half-century, the Algerian population has earned about 30 years in life expectancy at birth and more than 6 years in life expectancy at 50 (ONS, 2012).
- Life-insurance calculations in Algeria are still based on static life table constructed on old mortality data (CNA, 2004).
- In a previous works (Flici, 2013. Flici, 2014-a, Flici, 2015), we have tried to construct prospective life-tables for the Algerian population applying Lee carter (1992) and variants(RH and APC).
- The objective of the present paper is to propose a dynamic life table for the algerian population aged 50 years and over.
- We aim to improve the quality of the fitting / forecasting by a comparison of a set of mortality models (LC and variants VS CBD and variants)

Backgroung in mortality modeling

Lee Carter model and variants

- Lee and Carter, (1992). M1:

$$ln(\mu_{xt}) = \alpha_x + \beta_x * \kappa_t + \xi_{xt}$$

- Renshaw and Haberman (2006) M2: M1 + Cohort effect:

$$ln(\mu_{xt}) = \alpha_x + \beta_x^{(1)} * \kappa_t + \beta_x^{(2)} * \gamma_{t-x} + \xi_{xt}$$

- Currie (2006): M3 = simplified M2 with constant $\beta_t^{(1)}$ and $\beta_t^{(2)}$:

$$ln(\mu_{xt}) = \alpha_x + \frac{1}{n}\kappa_t + \frac{1}{n}\gamma_{t-x} + \xi_{xt}$$

Backgroung in mortality modeling

Cairns-Blake-Dowd model and variants

- CBD Model (linear form) :

$$ln(\frac{q_{xt}}{1-q_{xt}}) = \beta_t^{(1)} + \beta_t^{(2)} * (x - \bar{x}) + \xi_{xt}$$

- CBD model with Cohort effect : M6=M5 + Cohort effect:

$$ln(\frac{q_{xt}}{1-q_{xt}}) = \beta_t^{(1)} + \beta_t^{(2)} * (x - \bar{x}) + \gamma_{t-x} + \xi_{xt}$$

- CBD model- Quadratic form: M7 :

$$ln(\frac{q_{xt}}{1-q_{xt}}) = \beta_t^{(1)} + \beta_t^{(2)} * (x - \bar{x}) + \beta_t^{(3)} * ((x - \bar{x})^2 - \sigma_x^2) + \xi_{xt}$$

- CBD model -quadratic form + cohort effect: M7bis = M7+cohort effect

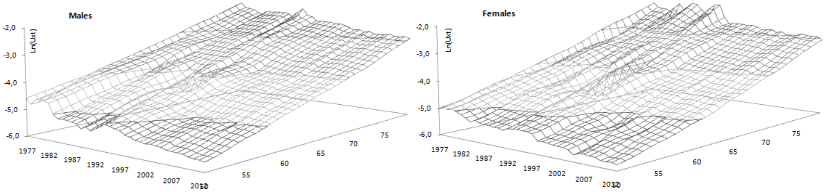
$$ln(\frac{q_{xt}}{1-q_{xt}}) = \beta_t^{(1)} + \beta_t^{(2)} * (x - \bar{x}) + \beta_t^{(3)} * ((x - \bar{x})^2 - \sigma_x^2) + \gamma_{t-x} + \xi_{xt}$$

Mortality data in Algeria

- The first Algerian life-table based on the civil registration data has been constructed in 1977 by The Algerian office for National Statistics (ONS).
- there are some calendar years missing life-tables for the period 1977 - 1997. The missing years were: 1979, 1984, 1986, 1988, 1990, 1992 and 1997.
- Some life tables were closed-out before the open age group [80 and +[. For the period 1983-1987, the closing age was [70 and +[. for the period 1993-1996, the published life-tables were closed-out at the age group [75 and +[. For the rest, it was [80 and +[or higher.
- In a previous work (Flici, 2014-b), we proposed to complete the missing data using a modified Lee Carter model with age-time segmentation.

Mortality data in Algeria

Figure: Algerian Mortality Surface (ln(Uxt)) for Males and Females : 1977-2013



- The single-age death rates were interpolated by the Karup-king formula
- Ages : 50 -79, Time : 1977 - 2013

M1: recall

- in (Lee and Carter, 1992), we first estimate α_x : $\alpha_x = ln(\prod_{t=T_1}^{T_n}(\mu_{xt}))^{\frac{1}{(T_n-T_1)}}$
- in second, we decompoze the residual matrix into two vectors: $ln(\mu_{xt}) - \alpha_x \approx \hat{\beta}_x * \hat{\kappa}_t$ with $\sum_{x=X_1}^{X_n} \beta_x = 1$ and $\sum_{t=T_1}^{T_p} \kappa_t = 0$
- a two stages decomposition process was proposed. In the first stage we decompose the residual matrix by SVD minimising: $minS(1) = \sum_{x=0,t=1}^{n,p}[ln(\mu_{xt}) - \alpha_x\text{-}\hat{\beta}_x * \hat{\kappa}_t]^2$
- in the second estimation stage, $\hat{\beta}_x$and $\hat{\kappa}_t$are adjusted to fit the observed number of deaths at every year (t). $minS(2) = \sum_{t=1}^{p}\sum_{x=0}^{n-1}[exp(\alpha_x + \hat{\beta}_x * \hat{\kappa}_t)L_{xt} - D_{xt}]$

D_{xt}: observed number of deaths at age x and time t,

L_{xt}: the exposure to the death risque at age x and time t (population at risk).

Best model selection: some notes

- Models to be compared (Lee-Carter, CBD) : M1, M2, M3, M5, M6, M7, M7*

Selection criteria:

- Quantitative : Weithed least squared errors, BIC, AIC.
- Qualitative : Robustness, predective capacity, sex-differencial mortality, Forecasted life expectancy, Regularity.

In the way to improve the fitting quality for all the models, we propose some modifications related to the estimation process:

- In Lee Carter(1992) , the alpha parameter $\alpha_x$is defined to be the mean over time of the ln of the central death rate. Some authors accepted that this relation can be partially respected. In the way to improve the quality of the model,

M1: recall

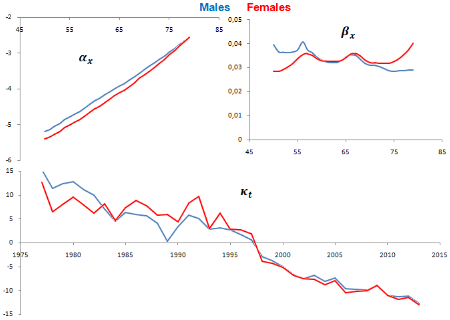
- Wilmoth (1993) proposed a one stage decomposition process based on the Weighed Least Squared Errors. (Weighted SVD)

$$minS = \sum_{x=0,t=1}^{n,p} W_{xt}[ln(\mu_{xt}) - \alpha_x \text{-} \hat{\beta}_x * \hat{\kappa}_t]^2$$

- The weight W_{xt} can be the observed number of deaths at each point x and t D_{xt}.
- Renshaw and Haberman (2006) used the original values of $\alpha_x$as a starting values which was re-estimated by the same optimization process with all the parameters in RH model. To insure the same fitting quality for all models.
- We use XL-Solver for all applications of the present work.

M1: results

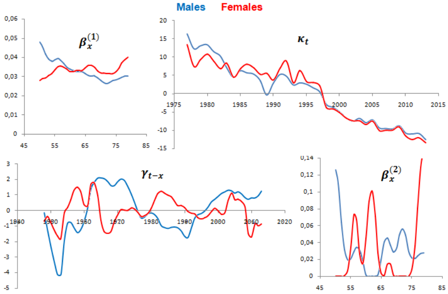
Figure: M1- Parameters estimation (1977-2013)



- mortality trend index: the two populations marks a high mortality level by the beginning of the black decade (1990th).

M2: results

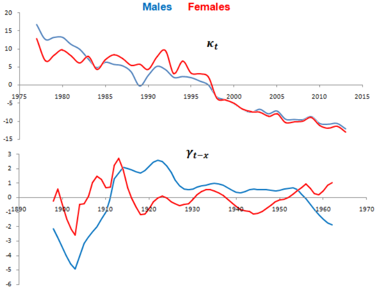
Figure: M2- Parameters estimation (1977-2013)



- alpha starting value, $\frac{1}{30}$: starting value for $\beta_x^{(1)}$and $\beta_x^{2)}$. no starting value for time and cohort components

M3: results

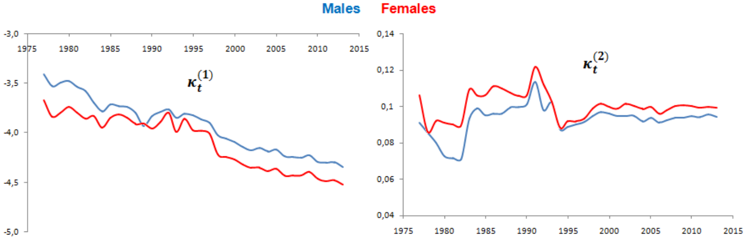
Figure: M3- Parameters estimation (1977-2013)



- the transition from M2 to M3 was by introducing $\beta_x^{(1)} = \beta_x^{(2)} = \frac{1}{n}$.
- alpha starting value, no starting value fore time and cohort component

M5: results

Figure: M5- Parameters estimation (1977-2013)

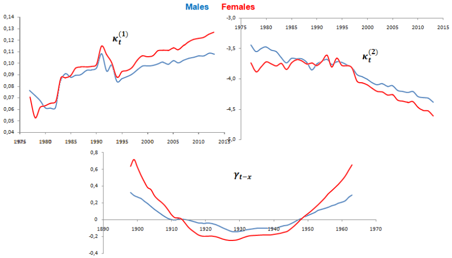


- statting values : $k_t^{(1)} = ln(\frac{q_{65,t}}{1-q_{65,t}})$. $k_t^{(2)} = \frac{Ln(\frac{q_{79,t}}{1-q_{79,t}})-Ln(\frac{q_{50,t}}{1-q_{50,t}})}{30}$.

M6: results

- Startting Values : $k_t^{(1)}$and $k_t^{(2)}$ estimated in M5. No starting value for γ_{t-x}

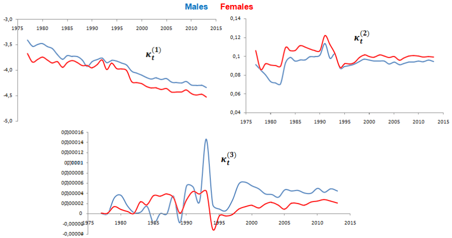
Figure: M6- Parameters estimation (1977-2013)



- Cohort effect or cosmetic effect : what about forecasting?
- Same form was observed for the belgian population (Mendes et Pochet, 2012)

M7: results

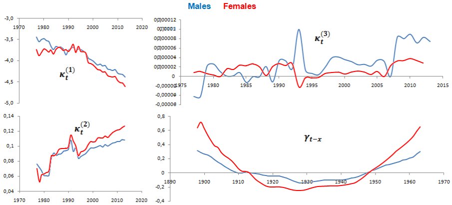
Figure: M7- Parameters estimation (1977-2013)



- Startting Values : $k_t^{(1)}$and $k_t^{(2)}$ estimated in M5. No starting value for $k_t^{(3)}$

M7*: results

Figure: M7*- Parameters estimation (1977-2013)



- Startting Values : $k_t^{(1)}$, $k_t^{(2)}$ $k_t^{(3)}$ estimated in M7 and γ_{t-x} estimated in M6.

summary

Table: comparison results

	WLS		BIC		AIC	
	Males	**Females**	**Males**	**Females**	**Males**	**Females**
M1	5353	6876	2426,5	2704,4	5089,8	5367,7
M2	*3840*	*5122*	2731,1	3050,7	4913,2	5232,9
M3	4460	6612	2476,4	2913,4	4959,3	5396,2
M5	*2926*	*7513*	*1594,9*	*2641,6*	4373,5	5420,2
M6	*1932*	*4248*	*1596,9*	*2471,4*	4044,7	4919,2
M7	2926	7511	1854,3	2900,7	4447,5	5493,8
M7*	1932	4221	*1842,3*	*2709,7*	*4114,6*	*4982,0*

- M7 and M7* are respectively an extension to M5 and M6 : No advantages
- We keep, by considering fiting quality and coherence between males and Females: M6 - M5 - M2 - M3 -M1

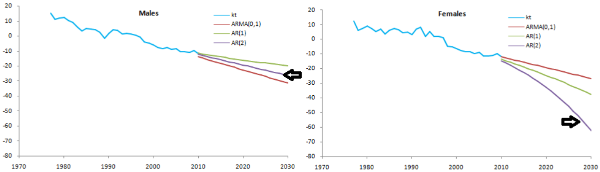
Predective capacity

- we reestimated the parameters of the three models for the period 1977 - 2010, on the basis of the obtained results, we do a forecast for the period [2011 - 2013], we compare the forcasted and the observed values.

M1: Time index projection

- comparison between three models : AR(1), AR(2) and Arima (0, 1, 0)
- The best model is AR(2)

Figure: M1 : time mortality index forecasting

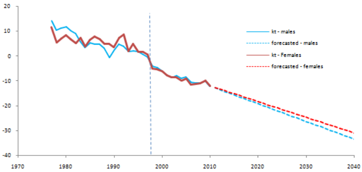


- it leads to incoherent forecasting if we compare males and females;
- The idea is to use only the period [1998 - 2010]

M1: Time index projection (2)

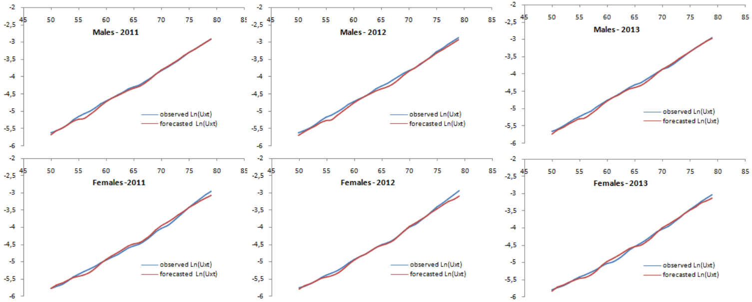
- model Arima (0, 1, 0)
- period : 1998 - 2010

Figure: M1 : time mortality index forecasting



M1: Observed VS Forecasted

Figure: M1 : Observed VS Forecasted

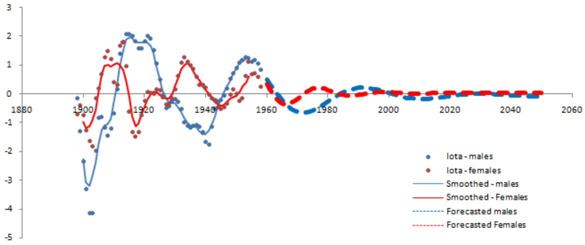


M2: Time / Cohort effect projection

Time index projection : same as in M1

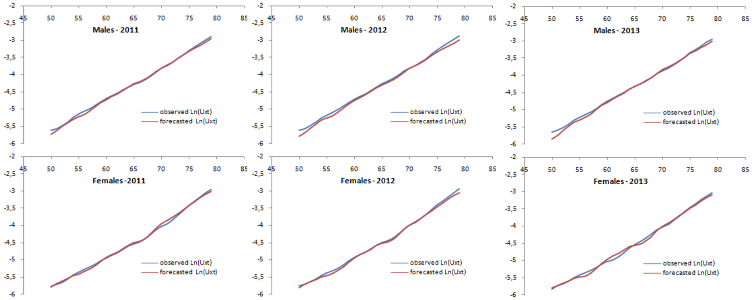
Cohort effect projection: The best model is AR(2)

Figure: M2 : Cohort effect projection



M2: Observed VS Forecasted

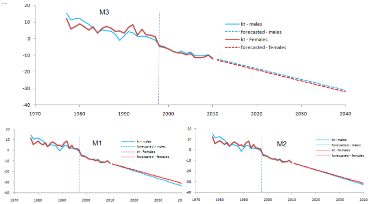
Figure: M2 : Observed VS Forecasted



M3; Time index projection

- The best model is ARIMA(0,1,0) with drift (1998 - 2010)
- adverse to results obtained with M1 and M2, $k_t^{Females} \leq k_t^{Males}$:

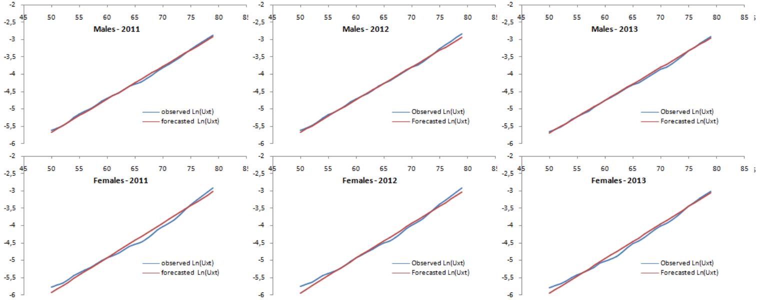
Figure: M3 : time mortality index forecasting



- Cohort effect : same as in M2

M3:Observed VS Forecasted

Figure: M3 : Observed VS Forecasted



M5:k(1) projection

- The best model is AR(1)

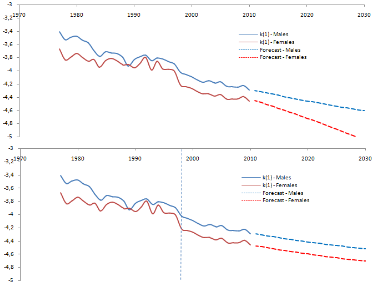


Figure: M5 k(1) projection

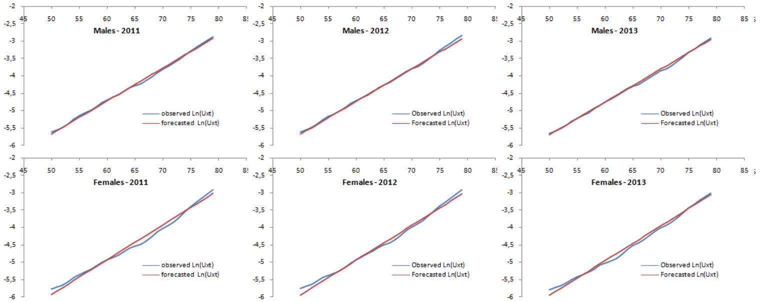
M5: k(2) projection

- The best model is AR(1)

Figure: M5 k(1) projection

M5: Observed VS Forecasted

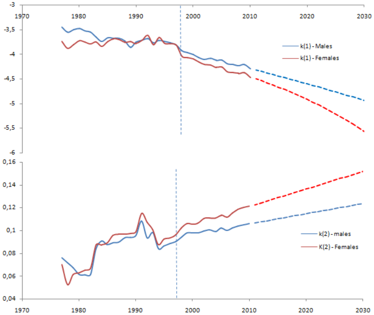
Figure: M5 : Observed VS Forecasted



M6 : k(1) and k(2) projection

- The best model is Random walk with drift for K(1) and AR(1) for K(2)

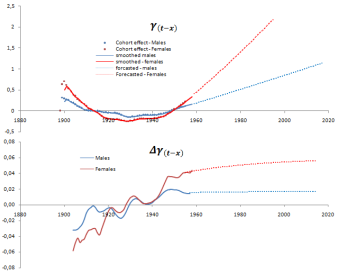
Figure: M6 k(1) and k(2) forecast



M6: Cohort effect projection

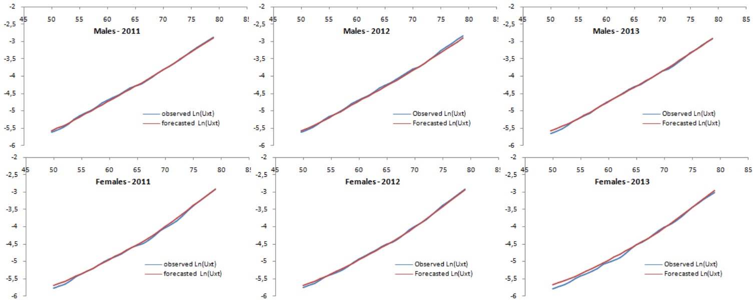
- Cohort effect stationarized by differentiaion (1st difference) : ARIMA(1,2,0)

Figure: M6 cohort effect forecast



M6: Observed VS Forecasted

Figure: M6 : Observed VS Forecasted



Comparison

Figure: comparison

	Males					Females				
	2011	2012	2013	Sum	rank	2011	2012	2013	Sum	rank
M1	0,048	0,094	0,054	0,196	4	0,101	0,069	0,077	0,247	4
M2	0,042	0,101	0,085	0,227	5	0,054	0,044	0,074	0,172	2
M3	0,014	0,063	0,014	0,092	2	0,012	0,050	0,033	0,095	1
M5	0,042	0,034	0,068	0,144	3	0,248	0,175	0,202	0,625	5
M6	0,022	0,041	0,022	0,085	1	0,050	0,030	0,109	0,189	3

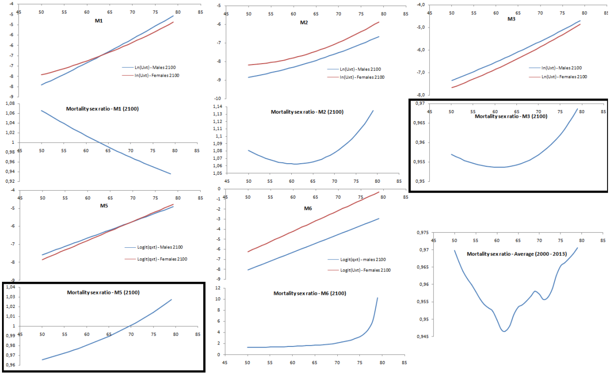
- M3 is the best model regarding to the (short term) predective capacity, followed by M6.

Long term forecasting coherence

- We forecast mortality until 2100;
- we observe the sex-differential mortality by the horizon of the forecast
- Coherance : Males mortality is almost sup than Females Mortality
- We compare to the observed sex ratio (1977-2013): $SR_{xt} = \frac{ln(u_{xt}^{Females})}{ln(u_{xt}^{Males})}$
- finally, we compare to the observed trend of the sex mortality ratio observed during the period [2000-2013].

Long term forecasting coherence

Figure: expected Mortality sex ratio (2100)



Construction of a dynamic life-table

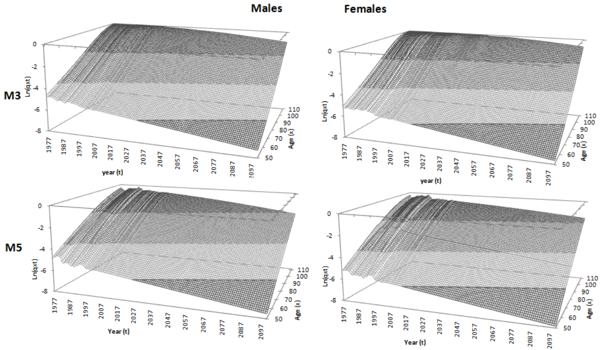
- We use M3 and M5
- The projected life tables are closed-out with Denuit and Goudemiaux Model (2005) without age limit constraint:

$$ln(q_x) = a + bx + cx^2 + \xi_x$$

parameters a, b and c are estimated on the age group [50 - 79]

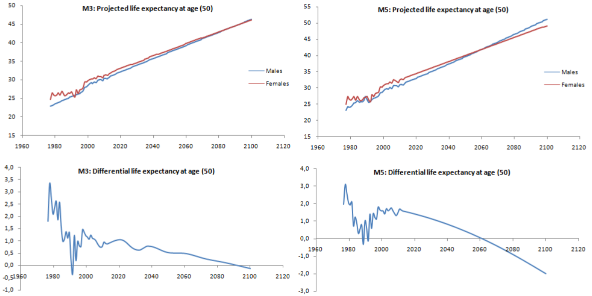
Projected mortality surface: M3 VS M5

Figure: projected mortality surfaces : M3 VS M5



Residual life expectancy: M3 VS M5

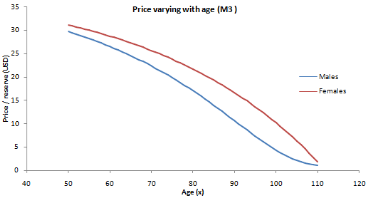
Figure: projected life expectancy at age (50) : M3 VS M5



Simulation with life annuities pricing

- Annual rente - 1 USD - payable by January, 1 st until death.
- The price of this life-annuities and its evolution over time for individual aged 50 :

Figure: Price of Life annuity in 2015 (males -Females)



Funding

- Model selection : combine quantitative and qualitative criteria
- the irregularity of the data during the 70's and 80's and the events of the terrorism decade = forecast the time mortality trends (all model) only on the basis of a short observation period (1998 - 2013): is it sufficient ?
- The cohort effect in CBD models doesnt have a regular trend (VS LC models) = need to better separate the Cohort effect from the residual term,
- the bad quality of M6 compared with M5 : related to the forecasting of the cohort effect (look for more adapted method)
- The results and conclusions are only confirmed with the considered age group (50 - 80), more regular and coherence results can be obtained when we change the age group (Flici, 2013 : age 60-80 and Flici, 2015 : age 0-80)

Thank you !!!!