GLWB Guarantees: Hedge Efficiency & Longevity Analysis

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- Industrial Alliance, Insurance and Financial Services is an important insurance company in Canada
- Pierre-Alexandre Veilleux is working on segregated funds
- And he is a graduate student on a "Part-time" basis working under my supervision
- The results of our paper will be in his thesis
- Inspired from a real problem in practice

Introduction

Context

- Dear Customer ...

Dear customer,

Unfortunately your flight LH494, 09.09.2015 has been cancelled.
Please use the alternatives online under My Bookings:

→ My Bookings

We apologize for any inconvenience.

Would you like to receive timely flight information per text to your mobile phone? We offer to inform you, using the mobile phone number stored in your Miles & More profile. Please amend your Miles & More profile to take advantage of this service: miles-and-more.com

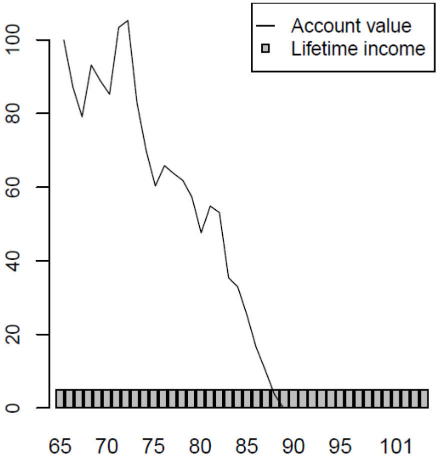
Best regards

- GLWB (Guaranteed Lifetime Withdrawal Benefit) guarantees are a special case of variable annuity
- In Canada, GLWB guarantees are called segregated fund guarantees
- They have been very popular in recent years in Canada and the United States
  - Growing need for income at retirement
  - Participation in equity markets
- Insurers now have to adequately manage the risks associated with these guarantees
- The guarantee provides the client with a lifetime income with a participation in equity markets
- It offers a combination of growth and guaranteed income
- The company is at risk when the account value is exhausted

Introduction

Context

- Illustration: Initial deposit = 100



- GLWB guarantees: complex options
- Main risks:
  - Financial markets: account value level
  - Longevity: lifetime income
  - Interest rates:
    - risk-neutral projection
    - discounting
- Consequence: Risk management of GLWB guarantees is a main concern
- Quote from Silverman & Theodore (2014, Milliman):
  - "Stochastic modeling of longevity risk can be a useful tool in the pricing and management of variable annuities with living benefits".

- Significant body of literature on segregated fund guarantees, variable annuity guarantees, and similar products
- Valuation and pricing of GLWB guarantees:
    - Shah and Bertsimas (2008)
    - Piscopo and Haberman (2011)
    - Holz, Kling, and Russ (2012)
    - Kling, Ruez, and Russ (2011)
- Risk management of GLWB guarantees:
    - Kling, Ruez, and Russ (2011) :
        - Hedge efficiency
        - Impact of modeling on hedge efficiency

- Kling, Ruez, and Russ (2011) consider 1 risk (investment) :
  - Stock markets : Heston Model (stochastic volatility)
  - Interest rate : deterministic
  - Mortality : deterministic
  - Segragate fund : stock only
- We consider 3 risks (investment, interest rate, longevity) :
  - Stock markets : Regime switching model
  - Interest rate : stochastic ($G2^{++}$)
  - Mortality : stochastic (Lee-Carter Model)
  - Segragate fund : stock & fixed income
- Analysis of the impact of stochastic mortality on hedge efficiency

- Let

$$\Omega_T = \{t_0, t_1, \ldots, t_{(\omega - x)/\Delta t}\}$$

be the times at which events can occur, where

  - $t_0 = 0$ is the contract inception date
  - x is the age at contract inception
  - ω is the maximum age
  - $t_{i+1} - t_i = \Delta t \quad \forall i$

- Financial market:
  - Stock market: $\underline{S} = \{S_{t_i}, t_i \in \Omega_T\}$
  - Bond market: $\underline{P} = \{P_{t_i}, t_i \in \Omega_T\}$

- Segregated fund:
  - $\underline{F} = \{F_{t_i}, t_i \in \Omega_T\}$
  - Diversified fund
    - Proportion ω_{t_i} in the stock market index (S_{t_i})
    - Proportion $1 - \omega_{t_i}$ in the bond market index (P_{t_i})
- Dynamic of $\underline{F}$:

$$F_{t_i} = F_{t_{i-1}} \left(\omega_{t_{i-1}} \frac{S_{t_i}}{S_{t_{i-1}}} + (1 - \omega_{t_{i-1}}) \frac{P_{t_i}}{P_{t_{i-1}}} \right) e^{-m_A \cdot (t_i - t_{i-1})}, \quad t_i \in \Omega_T,$$

where $F_{t_0} = F_0$ and m_A is the fund fee.

- Account value: $\underline{A} = \{A_{t_i}, t_i \in \Omega_T\}$

$$A_{t_i} = \begin{cases} \max\left(A_{t_{i-1}} \frac{F_{t_i}}{F_{t_{i-1}}} \mathrm{e}^{-g_A \times (t_i - t_{i-1})} - \frac{1}{n} L_{t_i - 1}; 0\right) & \text{, if withdrawal} \\ A_{t_{i-1}} \frac{F_{t_i}}{F_{t_{i-1}}} \mathrm{e}^{-g_A \times (t_i - t_{i-1})} & \text{, otherwise} \end{cases},$$

where

  - $L_{t_i}, t_i \in \Omega_T$ is the annual withdrawal amount at time t_i,
  - g_A is the guarantee fee
  - n is the withdrawal frequency.

- Let $\underline{V} = \{V_{t_i}, t_i \in \Omega_T\}$ be the guarantee liability process :

  V_{t_i}= expected PV of the benefits - expected PV of the "premiums"

- It means

$$V_{t_i} = E_{\underline{\mu}}\left[E^{\mathbb{Q}}\left[\sum_{j=\max(k^*,i)+1}^{(\omega-x)/\Delta t} {}_{t_j}p_x^{(\underline{\mu})} e^{-\int_{t_i}^{t_j} r_s ds} \frac{1}{n} L_{t_{j-1}} 1_{\{nt_j \in \mathbb{N}\}} - {}_{t_k^*}p_x^{(\underline{\mu})} e^{-\int_{t_i}^{t_{k^*}} r_s ds} B_{t_{k^*}} 1_{\{t_i < t_{k^*}\}} \middle| \underline{\mu}, \mathcal{F}_{t_i} \right] \middle| \mathcal{G}_{t_i} \right]$$

$$- E_{\underline{\mu}}\left[E^{\mathbb{Q}}\left[\sum_{j=i}^{(k^*-1)} {}_{t_j}p_x^{(\underline{\mu})} A_{t_j} \left(1 - e^{-g_A(t_{j+1}-t_j)}\right) e^{-\int_{t_i}^{t_j} r_s ds} \middle| \underline{\mu}, \mathcal{F}_{t_i} \right] \middle| \mathcal{G}_{t_i} \right],$$

where

- $\mathcal{F}_{t_i}$ and $\mathcal{G}_{t_i}$ are the σ-algebras containing all financial and mortality information respectively
- $\underline{\mu} = \{\mu_{t_0}, \mu_{t_1}, \ldots, \mu_{t_{(\omega-x)/\Delta t-1}}\}$ is the force of mortality vector
- $t_{k^\star}$ is the time at which the account value is exhausted
- $B_{t_{k^*}} = A_{t_{k^*-1}} \frac{F_{t_i}}{F_{t_{i-1}}} e^{-g_A \cdot (t_{k^*} - t_{k^*-1})} - \frac{1}{n} L_{t_{k^*-1}}$

- A common strategy in the insurance industry is dynamic hedging:
  - Liquid asset portfolio
  - Frequent rebalancing
- This strategy consists in compensating our guarantee liability sensitivity to various risk factors:
  - Stock market (delta)
  - Bond market (delta)
  - Interest rates (rho)
- Sensitivities are valued using finite difference techniques for all risk factors:
  - Stock market index $\underline{S}$
  - Bond market index $\underline{P}$
  - Interest rate curve sections

- Let $V_t \equiv V_t(\theta_1, \ldots, \theta_i, \ldots, \theta_m)$ be the guarantee liability and $\theta_i, i = 1, \ldots, m$ the risk factors that affect its value.
- We have

$$\frac{\partial V_t}{\partial \theta_i} \approx \frac{V_t(\theta_1, \ldots, \theta_i, \ldots, \theta_m) - V_t(\theta_1, \ldots, \theta_i - h, \ldots, \theta_m)}{h}.$$

- The asset portfolio, H_t, is then built such that

$$\frac{\partial H_t}{\partial \theta_i} = \frac{\partial V_t}{\partial \theta_i}$$

using simple financial instruments:

  - Short positions on S_t and P_t
  - Long positions in zero-coupon bonds

- Goal: Assess how modeling of the guarantee liability impacts hedge efficiency
- We are now working under two perspectives:
  - Projection under the real-world measure $\mathbb{P}$
  - Valuation under the risk-neutral measure $\mathbb{Q}$
- Steps:
  1. Simulation of a real-world scenario ($\mathbb{P}$)
  2. For all $t_i \in \Omega_T$ in the real-world scenario,
     1. calculate the guarantee liability ($\mathbb{Q}$)
     2. calculate deltas and rhos ($\mathbb{Q}$)
     3. determine the asset portfolio ($\mathbb{Q}$)
     4. calculate the hedge gains and losses ($\mathbb{P}$)
  3. Discount the hedge gains and losses ($\mathbb{P}$)

- We have the tools required for steps 2(a) - 2(c)
- But we must make an appropriate link between
  - the scenario under the $\mathbb{P}$ measure
  - the valuation of guarantee liability under the $\mathbb{Q}$ measure
- Then, we complete steps 2(d) and 3

- Let $\underline{GP} = \{GP_{t_i}, t_i \in \Omega_T\}$ be the process of hedge gains with

$$GP_{t_i} = H^-_{t_i} - H_{t_{i-1}} + {}_{t_i}p_x^{(\mu)}(R_{t_i} - C_{t_i}) - (V_{t_i} - V_{t_{i-1}}),$$

  where
  - R_{t_i}: revenue from the guarantee fee at time t_i
  - C_{t_i}: claim payment made by the company at time t_i
  - $H^-_{t_i}$: asset portfolio value before rebalancing at time t_i
  - $H_{t_{i-1}}$: asset portfolio value after rebalancing at time t_{i-1}
- Let *PVGP* be the present value of gains and losses under the $\mathbb{P}$ measure :

$$PVGP = \sum_{i=1}^{\frac{\omega-x}{\Delta t}} GP_{t_i} \prod_{j=0}^{i-1} ZC(t_j, t_{j+1}),$$

  where $ZC(t_j, t_{j+1})$ is the zero-coupon bond of maturity $t_{j+1} - t_j$ at time t_j in our real-world scenario.

- The discount function implies an investment in the money market account.
- Assessing hedging efficiency is a stochastic-on-stochastic calculation:
  - Outer loop: scenarios under the $\mathbb{P}$ measure
  - Inner loop: liability valuation and greeks under the $\mathbb{Q}$ measure
- The computation time involved is substantial.

- Stock market:
  - Lognormal model (LN): $dS_t = \mu^S S_t dt + \sigma^S S_t dW_t^S$
  - Regime-switching lognormal model (RSLN):

$$dS_t = \mu^S_{\rho_t} S_t dt + \sigma^S_{\rho_t} S_t dW_t^S,$$

  where ρ_t is a two-state continuous-time Markov process
- Stochastic mortality:
  - Let $\mu_{x,t_i} = e^{\alpha_x + \beta_x \kappa_{t_i}}$, where μ_{x,t_i} is the force of mortality for age x between t_i and t_{i+1}.
  - Constant mortality improvement (Cst): $\kappa_{t_i} = \kappa_{t_{i-1}} + \theta(t_i - t_{i-1})$
  - Lee-Carter model in discrete time (LC): $\kappa_{t_i} = \kappa_{t_{i-1}} + \theta(t_i - t_{i-1}) + \sigma^\mu \sqrt{\Delta t}\epsilon^\mu_{t_i}$, $\epsilon^\mu_{t_i} \sim N(0,1)$, $t_i \in \Omega_T$

- Interest rates:
  - Let $s(t, t+T)$ be the continuously compounded T-year spot rate at time t
  - Constant curve (Cst):

$$s(t, t+T) = \frac{1}{T}\left[(T+1)s(t, t+T+1) - s(t, t+1)\right]$$

  - G2++ model (G2):

$$s(t, t+T) = \frac{-1}{T}\left[\ln\left(\frac{P^M(0, t+T)}{P^M(0, t)}\right) - \frac{1}{2}(V(t, T) + V(0, t) - V(0, T))\right.$$
$$\left. + x(t)B(a, T-t) + y(t)B(b, T-t)\right]$$
$$dx(t) = a(\lambda_1 - x(t))dt + \sigma dW_1^r(t) \qquad x(0) = 0$$
$$dy(t) = b(\lambda_2 - y(t))dt + \eta dW_2^r(t) \qquad y(0) = 0$$
$$dW_1^r(t)dW_2^r(t) = \rho dt$$

Hedge efficiency empirical study

Parameters

- Contract holder:
  - 65-year-old male
  - $\$100,000$ single premium
  - Withdrawals deferred for 5 years (at age 70)
- Contractual parameters:
  - $n = 4$ (withdrawal frequency)
  - $g_A = 1.5\%$ (guarantee fee)
  - $m_A = 3.0\%$ (fund fee)
  - $l_{70} = 5.5\%$ (withdrawal rate)

- Projection: Monthly $\left(\Delta t = \frac{1}{12}\right)$
  - Financial variables
  - Mortality
  - Hedge portfolio rebalancing
- Stock market: Canadian stock market (TSX TR)
- Interest rates:
  - Canadian swap curve as of December 31, 2014
  - G2++: Babbs and Nowman (1999)
- Longevity: Canadian males

- $PVGP$ is the present value of hedge gains and losses
  - $PVGP > 0 \Rightarrow$ gain
  - $PVGP < 0 \Rightarrow$ loss
- To quantify risk in the left tail, we use

$$E_{\alpha}^{PVGP} = E\left[PVGP | PVGP < VaR_{1-\alpha}(PVGP)\right],$$

  the TVaR of hedge losses.
- Modeling under the $\mathbb{P}$ measure:
  - Stock market: RSLN2
  - Interest rates: Two-factor Gaussian model
  - Longevity: Lee-Carter

- Our results :

			E_α^{PVGP} as % of A_0			
Stock	Interest	Longevity	0.6	0.8	0.9	0.95
LN	Cst	Cst	-1.65	-2.38	-3.04	-3.65
RSLN	Cst	Cst	-1.80	-2.54	-3.25	-3.91
RSLN	G2	Cst	-1.26	-1.94	-2.56	-3.10
RSLN	G2	LC	-1.26	-1.94	-2.56	-3.10

- Results from Kling, Ruez, and Russ (2011):

	E_α^{PVGP} as % of A_0
Stock	0.9
LN	-4.3
Heston	-4.2

- Computation involves simulations with simulations
  - Outer loop : 1000 scenarios (under $\mathbb{P}$)
  - Inner loop : 1000 scenarios at each month to compute the value of the

Hedge efficiency empirical study

Results

- Observations and conclusions:
  - Substantial computation time
    - Optimized programming in C++ and R
    - Total computation time: 5-7 days on 8 cores in parallel
  - Adding stochastic volatility alone does not improve the hedge efficiency
    - Conclusion similar to the one of Kling, Ruez, and Russ (2011)
    - Impact is more pronounced for the RLSN model
  - Including the G2++ model materially improves the hedge efficiency
  - Stochastic longevity has a negligible effect on results:
    - The guarantee valuation looks at the average of scenarios
    - The mean and median of the Lee-Carter model are fairly close
  - Calculations at age 50 lead to similar conclusions:
    - Substantial reduction of risk when interest rate volatility is introduced
    - Relatively small longevity impact

- There are two levels in the hedge efficiency analysis:
  - Projection under the real-world measure $\mathbb{P}$
  - Valuation under the risk-neutral measure $\mathbb{Q}$
- Stochastic mortality in the guarantee liability valuation has little impact on hedge efficiency.
- What about in the real-world projection?

- We wish to allocate the risk between financial and longevity risks.
- Euler's capital allocation method:
  - Let $S = X_1 + X_2$
  - Contribution of risk X_i in $TVaR_\kappa(S)$:

$$C_\kappa^{TVaR}(X_i) = \frac{1}{1-\kappa}\int_{VaR_\kappa(X_1+X_2)}^{\infty} E[X_i \times 1_{\{S=y\}}]dy.$$

- Anecdote: Did you know ? Euler (1707-1783) also made contribution on the computation of premiums for life annuities !

SUR
LES RENTES VIAGERES,
PAR M. EULER.

- Let $S = \varphi(X_1, ..., X_n)$
- They are two possible cases for φ
- $\varphi =$ linear function of the components of $(X_1, ..., X_n)$
  - We can directly apply Euler's capital allocation method
  - $X_1, ..., X_n$: insurance contracts, annuity contracts, lines of business, assets, loans, etc.
- $\varphi =$ nonlinear function of the components of $(X_1, ..., X_n)$
  - We cannot directly apply Euler's capital allocation method
  - $X_1, ..., X_n$: risk factors such as interest rate, mortality index, inflation, etc.
  - We need to decompose S in linear components in order to apply Euler's allocation method
- Some possible decomposition methods :
  - Hoeffding decompostion (Rosen & Saunders (2010))
  - Taylor expansion (Karadey et al. (2014))
- We conclude with an illustration of the method in the next part

Longevity analysis

Risk allocation

- We use Hoeffding's decomposition:

$$\begin{aligned} PVGP &= g(Z_1, Z_2) \\ &= E[PVGP|Z_1] + (PVGP - E[PVGP|Z_1]) \end{aligned}$$

  where

  - Z_1 represents the mortality path
  - Z_2 represents financial variables
  - $PVGP$ be the present value of hedged gains with both financial and longevity risks

- Interpretation:
  - $E[PVGP|Z_1]$: Average $PVGP$ over all mortality paths
  - $PVGP - E[PVGP|Z_1]$: Additional risk caused by stochastic mortality

Allocation	E_α^{PVGP} as % of A_0			
κ	0.6	0.8	0.9	0.95
Financial Risks	-0.99	-1.55	-1.97	-2.30
Longevity Risks	-0.27	-0.39	-0.59	-0.80

Future work

- Hedge efficiency analysis
  - Sensitivity to parameters
  - Impact of a one-factor interest rate model on hedge efficiency
- Longevity analysis
  - Comparison with actuarial margins for adverse deviations

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