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On the Relationship between Cross-Sectional and Time Series Measures of Uncertainty

Abstract. In this paper, we provide a coherent theoretical investigation of the relationship between cross-section and time series measures of uncertainty, which are often employed as perfect substitutes in empirical applications. The main finding of our analysis is that there exists an ambiguous sign in the discrepancy between the two measures of uncertainty arising from the presence of cross-sectional dependence amongst individuals. Thus our study underpins the importance of accounting for cross-sectional dependence, in line with recent inferential theory of panel data models.

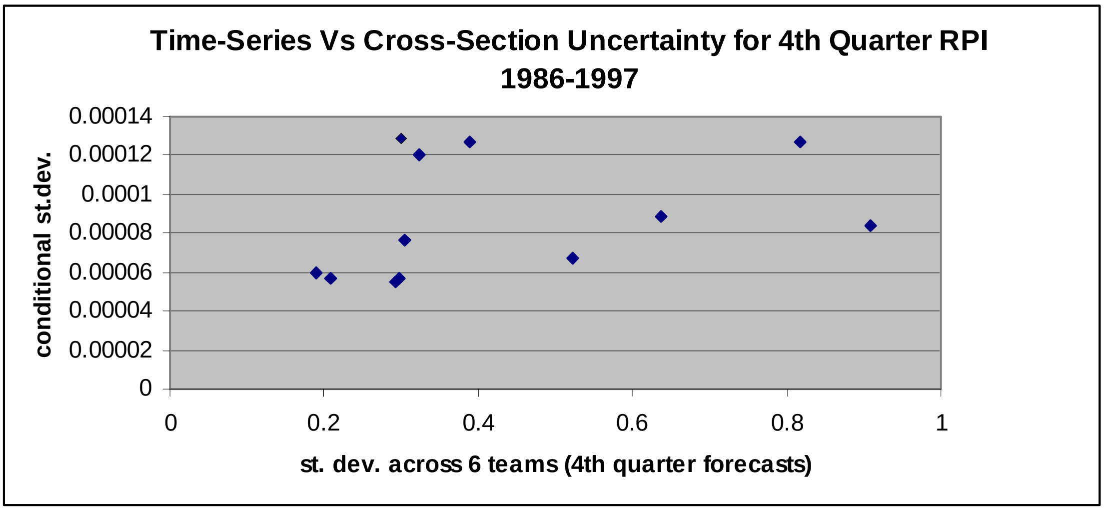
J.E.L. Classification Numbers: C21, C22.
Keywords: Cross-section and Time series, Expectations, Uncertainty, GARCH Models.

1 Introduction

There are at least two sets of literature on the economics of uncertainty. The first is concerned with the effect of uncertainty on economic activity, primarily investment. The empirical component of this enquiry now dominates over theoretical work and is now large enough to have spawned at least two meta studies (Carruth *et al* 2000, Koetse *et al* 2006). Parallel with this is a smaller literature which focuses on the definition, measurement and interpretation of uncertainty and which addresses both theoretically and empirically the relationship between distinct measures of uncertainty (Giordani and Soderlind 2003). While the second theoretical literature ought to guide the more empirically oriented applications of the first set, that is often not the case in recent published work. Indeed it is common for authors to choose measures of uncertainty without much reference to their construction or interpretation. Particularly, two main measures of uncertainty are customarily employed in the literature (see e.g. Engle, 1983). A first class of measures assesses the degree of uncertainty in a series by using the time series properties of the series itself (e.g. volatility). The second class considers the extent of disagreement among different predictions for a series as a measure of the uncertainty of the series.

The lack of correlation between different measures of uncertainty has been noted in several studies (Batchelor and Dua, 1993; Lahiri and Liu, 2006). To underscore the importance of distinguishing between time-series estimates of uncertainty and cross-section estimates we report simple comparison of the two measures using data for the UK retail price index (RPI), 4th quarter. Our cross-section measure is the standard deviation of forecast for the 4th quarter RPI 1986-97 across the six main private non-financial sector forecasting teams in the UK, where the forecasts are made in the 4th quarter itself. The time-series measure is the conditional standard deviation constructed from a GARCH analysis on the RPI[1]. Using these data one obtains the scatter

[1]We use GARCH (1,1) models for the actual RPI series from Datastream (code UKOCP074F). The data for the cross-section analysis is taken from HM Treasury Economic Prospects Team publications: "HM Treasury forecasts for the UK economy: a comparison of independent forecasts", various issues. The organisations involved are: The



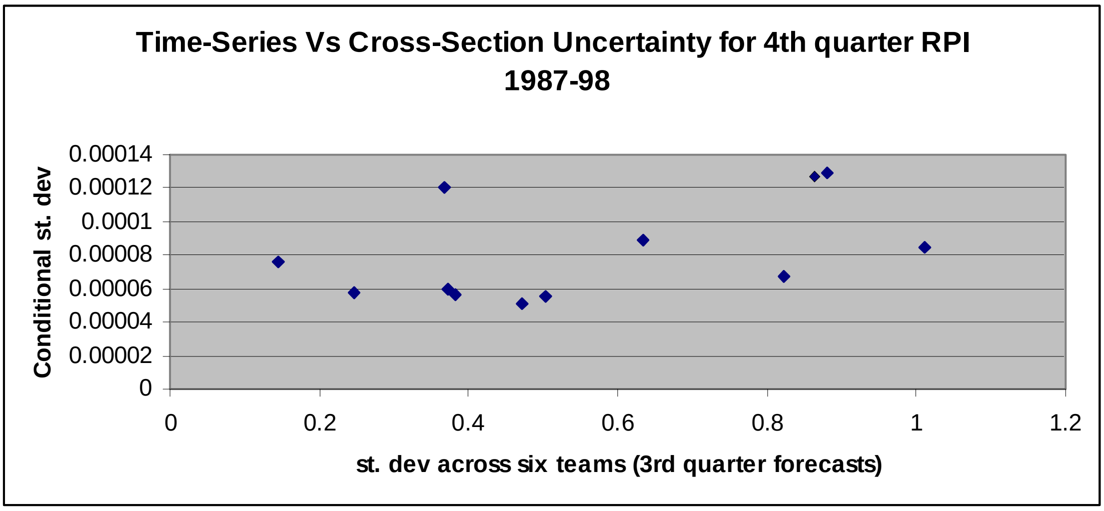
shown in Figure 1 where the simple correlation measure is 0.31 as compared to a one-tailed 5% significance critical value of 0.50 for 12 data points.

A variant of this analysis is shown in Figure 2 where the forecasts of 4th quarter RPI are made in the 3rd quarter of the year for the sample 1987-1998. Again the correlation is insignificant, though this time somewhat higher at 0.467.

This limited analysis proves that the two classes of measures of uncertainty do not have the same information content; therefore, caution should be used when employing either measure of uncertainty.

From an econometrician's perspective, the uncertainty measures that are commonly employed result from the cross-sectional aggregation of a panel of individual forecasters. The cross sectional properties of a panel data are well known to be heavily dependent on the assumption one makes on the presence of contemporaneous correlation among the individuals. The early literature (Engle, 1983) has based its analysis on the crucial assumption of absence of any form of correlation among the individual forecasters, thereby deriving

National Institute of Economic and Social Research, The London Business School, The Henley Centre, Liverpool University, The Confederation of British Industry, and Oxford University (OEF). Two of these organisations ceased to forecast at the end of the period, while the data commences in the last quarter of 1986 which is why the data series in Figure 1a and Figure 1b are confined to the periods shown.



measures of uncertainty that did not take account of possible interactions or common features/knowledge. It is well known that such assumption has a heavy influence on the results of cross-sectional aggregation, and attention should be paid as to the consequences of neglecting non negligible contemporaneous correlation. In our contribution, we address the issue of the relationship between different measures of uncertainty relaxing this assumption of no cross-sectional correlation. We prove that the impact of the presence of interactions among individuals is non trivial and potentially reverses some existing results.

This paper examines the theoretical relationship between time series and cross-section measures of uncertainty which is one of the basic distinctions in the metrics commonly employed. We use conditional variance to express time series uncertainty and we use dispersion across agents to reflect cross-section uncertainty. In Section 2 we show that is possible to express a formal relationship between our two chosen measures and we detail on how the relationship depends on different assumptions.

Our framework follows that of Engle (1983), where an outcome variable y_t is assumed to have a DGP determined by presently available information and error terms, some of which may reflect privately held information. We define time series uncertainty as the expectation of y_t conditional on the common information set. Without private information, this uncertainty may be constant for a stationary process but time variation is created when private information also has a time dimension. We define cross section uncertainty as the dispersion of (rational) forecasts of y_t about the mean for any given time period. With a large number of forecasting agents (n) the time series uncertainty is approximately equal to the sum of cross section uncertainty and a term that reflects the volatility of the outcome around the time-averaged mean (Cukierman and Wachtel, 1979)[2]

Thus, a possible representation for y_t - as in Engle (1983) - is

$$y_t = \beta y_{t-1} + \eta_t + \sum_{i=1}^{n} \alpha_{it} \varepsilon_{it}, \tag{1}$$

where the random noise η_t satisfies the usual assumptions, the random terms ε_{it} are assumed to be iid across i and t with variance α_{it}^2. Therefore the above model assumes no cross sectional dependence across individual agents.

The most important contribution of this paper (reported in Section 2) is to show that the relaxation of this assumption of independence undermines the well known result in the literature that the variance of the time series forecast error must exceed the variance of cross-section forecasts. In section 3 we discuss the consequences of cross-sectional dependence on serial correlation in the time series that represent uncertainty over time. Section 4 concludes.

2 Cross-section and time series measures of uncertainty

Private information creates dispersion across agents; hence, the conditional variance can be viewed as the sum of dispersion and volatility about the time-averaged mean (Engle, 1983).

[2] Strictly speaking this is true only for infinite n. For finite n this relationship is modified by the fact that the private information affects both the time series and cross-section slightly differently. It remains the case however that the time-series measure exceeds the cross-section one since it contains information about common time vaiation.

Consider the variable y_t whose data generating process (DGP) is:

$$y_t = \phi\left(I_{t-1}\right) + v_t, \tag{2}$$

where $\phi\left(I_{t-1}\right)$ is a transformation of the information set available at time t and v_t an error term. Let us assume that v_t can be decomposed into two non separable components η_t and $\alpha_t'\varepsilon_t (= \sum_{i=1}^{n} \alpha_{it}\varepsilon_{it})$, with η_t representing the standard error term and $\varepsilon_t \equiv [\varepsilon_{1t}, ..., \varepsilon_{nt}]'$ an additional random component capturing private information across the n units with the corresponding vector of weights $\alpha_t \equiv [\alpha_{1t}, ..., \alpha_{nt}]'$, i.e.

$$y_t = \phi\left(I_{t-1}\right) + \eta_t + \alpha_t'\varepsilon_t \tag{3}$$

The following assumptions hold on the error term $\eta_t + \alpha_t'\varepsilon_t$:
Assumption 1: *(time dependence)* η_t and ε_t are two mutually independent, zero mean, covariance stationary processes with $Var\left(\eta_t\right) = \sigma_\eta^2$ and $Var\left(\varepsilon_{it}\right) = 1$ for all i.
Assumption 2: the α_{it}s are nonstochastic quantities that satisfy the square summability condition $\alpha_t'\alpha_t = \sum_{i=1}^{n} \alpha_{it}^2 = O\left(1\right)$ as $n \to \infty$ for all t.
Assumption 3: (*cross sectional dependence)* $E\left(\varepsilon_t\varepsilon_t'\right) = \Omega$.

Assumptions 1-3 are an extension of Engle's (1983) model. The condition $Var\left(\varepsilon_{it}\right) = 1$, as in Engle (1983), is simply a normalization rule. Assumption 2 allows the α's to be time dependent. The square summability condition prevents the variance of the error term in regression (3) from exploding as the number of individuals grows; a similar assumption is contained in Pesaran and Weale (2005). Assumption 3 considers the presence of contemporaneous correlation, and therefore it takes account of the possibility of interactions among agents, unlike Engle (1983) where cross sectional dependence among individuals was ruled out. The special case when $E\left(\varepsilon_t\varepsilon_t'\right) = I_n$ is restrictive and is explored separately.

In what follows, we refer to the element in position (i, j) in matrix Ω as ω_{ij}; moreover, we employ the following decomposition of Ω, $\Omega = I_n + \Omega^*$, where Ω^* contains the off-diagonal terms of matrix Ω.

2.1 Time series and cross-section measures of uncertainty

The terms in equation (3) contain two alternative measures of uncertainty. The first class is a time series measure, where uncertainty is defined as the squared difference/GARCH variance between the actual value of the aggregate y_t and its expected value conditional on past information. The second is a cross-sectional measure, where uncertainty is based on the dispersion of individual expectation or forecasts in surveys.

Consider the time series measure. From (3), the variance of y_t conditional on past information I_{t-1} is $Var\left[y_t|I_{t-1}\right] = \sigma_\eta^2 + \alpha_t' \Omega \alpha_t$. Each individual forecaster with inside information has conditional expectation y_t^i and forecast error ε_t^i given respectively by

$$E\left[y_t|y_{t-1}, \varepsilon_{it}\right] \equiv y_t^i = \phi\left(I_{t-1}\right) + \alpha_{it}\varepsilon_{it}$$

$$\varepsilon_t^i = y_t - y_t^i, \tag{4}$$

and the mean square error (MSE) for any forecaster i is given by

$$E\left(\varepsilon_t^i\right)^2 = \sigma_\eta^2 + \sum_{k\neq i}\sum_{j\neq i} \alpha_{kt}\alpha_{jt}\omega_{ij}.$$

Following Engle (1983), the *average* MSE is equal to:

$$\frac{1}{n}\sum_{i=1}^{n} E\left(\varepsilon_t^i\right)^2 = \sigma_\eta^2 + \frac{n-1}{n}\alpha_t'\alpha_t + \frac{n-2}{n}\alpha_t'\Omega^*\alpha_t. \tag{5}$$

Equation (5) is an assessment of the average accuracy of the individuals' forecast, and not a cross-sectional measure of dispersion/disagreement among individuals (see also Pesaran and Weale, 2005). Equation (5) approaches $Var\left(y_t|I_{t-1}\right)$ either when $n \to \infty$, or when the amount of private information α_i is small for all individuals.

A useful decomposition for the MSE follows:

Proposition 1 *Letting* $\bar{y}_t = \frac{1}{n}\sum_{i=1}^{n} y_t^i$*, the average MSE can also be de-*

composed as:

$$\frac{1}{n}\sum_{i=1}^{n} E\left(\varepsilon_t^i\right)^2 = \frac{1}{n}\sum_{i=1}^{n} E\left[\left(y_t^i - \bar{y}_t\right)^2\right] + E\left[\left(y_t - \bar{y}_t\right)^2\right]. \tag{6}$$

Proof. See Appendix. ■

Equation (6) shows that the *average* MSE can be decomposed into two (positive) quantities: $E\left[\left(y_t - \bar{y}_t\right)^2\right]$, which takes account of the expected, average forecast error, and the term $\sum_{i=1}^{n} E\left[\left(y_t^i - \bar{y}_t\right)^2\right]$, which measures the extent of disagreement among individuals. Therefore, the average MSE and cross sectional dispersion are ordered, with the former always greater than the latter.

In the Appendix, we also derive the two ancillary results, that will be used below:

$$\sum_{i=1}^{n} E\left[\left(y_t^i - \bar{y}_t\right)^2\right] = \frac{n-1}{n}\alpha_t'\alpha_t - \frac{1}{n}\alpha_t'\Omega^*\alpha_t, \tag{7}$$

$$E\left[\left(y_t - \bar{y}_t\right)^2\right] = \sigma_\eta^2 + \left(\frac{n-1}{n}\right)^2 \alpha_t'\Omega\alpha_t. \tag{8}$$

Equation (7) indicates that when the amount of private information α_t grows large, the degree of dispersion $\sum_{i=1}^{n} E\left(y_t^i - \bar{y}_t\right)^2$ increases as well. Notice that σ_η^2 does not have an impact on the dispersion across individuals, due to common uncertainty about the future.

$Var\left(y_t|I_{t-1}\right)$ represents the time series measure of uncertainty (TS_t henceforth). $\sum_{i=1}^{n} E\left(y_t^i - \bar{y}_t\right)^2$ is a measure of the dispersion of forecasts across individuals, and therefore it can be viewed as a cross-sectional measure of uncertainty (CS_t henceforth)[3].

2.2 Cross-sectional dependence

In this section we study the relationship between these two measures when we assume the presence of cross sectional dependence. The following theorem characterizes the relationship between the two quantities:

[3]Note that this cross sectional measure of uncertainty is the same as proposed in Pesaran and Weale (2005), apart from the term $1/n$.

Theorem 1 *The relationship between TS_t and CS_t is given, for each t, by*

$$TS_t = \frac{n}{n-1} CS_t + \sigma_\eta^2 + \frac{n}{n-1} \alpha_t' \Omega^* \alpha_t. \tag{9}$$

Proof. See Appendix. ■

Equation (9) states that there is a static, deterministic relationship that links the level of time series uncertainty and the degree of dispersion among individuals uniformly across time.

The discrepancy between TS_t and CS_t has ambiguous magnitude and sign. In particular, the difference between the two measures can be written as

$$\begin{aligned} \Delta m_t &\equiv TS_t - CS_t = \frac{1}{n-1} CS_t + \sigma_\eta^2 + \frac{n}{n-1} \alpha_t' \Omega^* \alpha_t = \\ &= \sigma_\eta^2 + \alpha_t' \left[\frac{1}{n} \Omega + \Omega^* \right] \alpha_t. \end{aligned}$$

This is a very useful result in understanding the discrepancy between the measures of uncertainty. Δm_t depends on four terms: the variance of the unobservable component η_t (σ_η^2), the number of individuals n, the amount of private information α_t (Ω) and the degree of interaction among agents (Ω^*). The presence of σ_η^2 leaves no ambiguity but that Δm_t increases with σ_η^2 since TS_t explicitly takes account of the "common" uncertainty σ_η^2, whereas CS_t is only sensitive to individual specific components and therefore fails to take account of σ_η^2. On the other hand, the impact of the other three components is less clear, mainly because the matrix $n^{-1}\Omega + \Omega^*$ (and therefore of the quadratic form $\alpha_t' [n^{-1}\Omega + \Omega^*] \alpha_t$) is non (positive/negative) definite. To see this, let us assume that matrix Ω has at least two distinct eigenvalues (λ^i)[4], it can be showed that the i-th eigenvalue of $n^{-1}\Omega + \Omega^*$ is

$$\varphi^i = \frac{n+1}{n} \lambda^i - 1.$$

[4]This condition is a very general one, in that the only case when a correlation matrix has non distinct eigenvalues (all equal to 1) is when the off diagonal terms are all equal to zero, which corresponds to the case of independent agents.

Since the largest eigenvalue of Ω, say $\lambda^{\max}$, is $\lambda^{\max} > 1$, we have that $\varphi^{\max} > 0$. As far as other eigenvalues are concerned, it is possible that $\varphi^{i} < 0$ for some i. This always happens if $n \to \infty$. In this case the trace of matrix $n^{-1}\Omega + \Omega^{*}$ is equal to 1 and on average the eigenvalues are equal to zero. Therefore, given that $\varphi^{\max} > 0$, there must be at least one negative eigenvalue In the light of these considerations, $\alpha_t' \left[n^{-1}\Omega + \Omega^{*}\right] \alpha_t$ can have positive or negative sign, and in the latter case it can offset the term σ_η^2. Thus Δm_t can be positive or negative sign.

Note that our findings in respect to the discrepancy between TS_t and CS_t does not hold if we assume that there is no interaction among agents, as in Engle (1983). In such case, equation (9) reduces to

$$TS_t = \frac{n}{n-1} CS_t + \sigma_\eta^2,$$

and therefore we would have $TS_t > CS_t$.[5] [6]

It could be argued that forecasts tend to be more similar when the objective uncertainty (say, measured by the common σ_η^2) is high. This would entail a herding behaviour since agents would tend to rely less and less upon their private information when the amount of uncertainty is very high. A

[5] In this case, the discrepancy between the two measures is

$$\Delta m_t \equiv TS_t - CS_t = \frac{1}{n-1} CS_t + \sigma_\eta^2 = \frac{1}{n} \sum_{i=1}^{n} \alpha_{it}^2 + \sigma_\eta^2.$$

Δm_t depends now on three terms: σ_η^2, n and the α_{it}s. Δm_t increases as σ_η^2 and the amount of private information increase (ceteris paribus), whilst the number of individuals differs substantially depending on whether n is finite or infinite. As $n \to \infty$, from the square summability condition for the α_{it}, it follows that $\Delta m_t = \sigma_\eta^2$.

[6] Ambiguity arises for finite n. Suppose n increases to $n+1$, with the new contribution of an agent whose amount of private information is $\alpha_{n+1,t}$. Then the difference between the level of discrepancy with $n+1$ individuals, referred to as $\Delta m_t^{(n+1)}$, and that with n agents, $\Delta m_t^{(n)}$, is equal to

$$\begin{aligned} \Delta m_t^{(n+1)} - \Delta m_t^{(n)} &= \frac{1}{n+1} \left(\sum_{i=1}^{n} \alpha_{it}^2 + \alpha_{n+1,t}^2 \right) + \sigma_\eta^2 - \frac{1}{n} \sum_{i=1}^{n} \alpha_{it}^2 + \sigma_\eta^2 = \\ &= \frac{1}{n+1} \left[\alpha_{n+1,t}^2 - \frac{1}{n} \sum_{i=1}^{n} \alpha_{it}^2 \right], \end{aligned}$$

possible (simple) way of modelling herding is making the alphas dependent on σ_η^2 as $\alpha_i = 1/\sigma_\eta^2$. This would result in

$$CS_t = \frac{n-1}{\sigma_\eta^2} - \frac{1}{n}\frac{i'\Omega^* i}{\sigma_\eta^2}.$$

Thus, as $\sigma_\eta^2 \to \infty$ we have $CS_t = 0$ - which is the main consequence of herding, namely there not being any discrepancy among individuals. However, as $\sigma_\eta^2 \to \infty$

$$\Delta m_t \approx \sigma_\eta^2,$$

which is not zero, but infinity. Thus, high uncertainty that results in herding does not reduce the discrepancy between TS_t and CS_t, but contrary to intuition it enhances it

The discrepancy between cross sectional and time series measures of uncertainty is even more relevant when we consider the presence of another common term (x_t). In this case the specification for the DGP of y_t is

$$y_t = \phi(I_{t-1}) + x_t + \sum_{i=1}^{n} \alpha_{it}\varepsilon_{it} + \eta_t, \tag{10}$$

where with respect to (2) we add x_t as a random variable with variance σ_x^2, representing common information available to all agents (but not to the econometrician). The conditional variance of y_t is now given by $Var[y_t|I_{t-1}] = \sigma_x^2 + \sigma_\eta^2 + \alpha_t'\Omega\alpha_t$. Also, each individual i forecasts the level of y_t employing both common information y_{t-1} and x_t and private information ε_{it}, obtaining

$$E[y_t|y_{t-1}, x_t, \varepsilon_{it}] \equiv y_t^i = \phi(I_{t-1}) + x_t + \alpha_{it}\varepsilon_{it}$$

with forecast error ε_t^i given as in (4). Similar calculations as before show that

$$E\left(\varepsilon_t^i\right)^2 = \sigma_\eta^2 + \sigma_x^2 + \sum_{k\neq i}\sum_{j\neq i} \alpha_{kt}\alpha_{jt}\omega_{ij},$$

$$\frac{1}{n}\sum_{i=1}^{n}E\left(\varepsilon_t^i\right)^2=\sigma_\eta^2+\sigma_x^2+\frac{n-1}{n}\alpha_t'\alpha_t+\frac{n-2}{n}\alpha_t'\Omega^*\alpha_t.$$

The following Proposition characterizes the discrepancy between TS_t and CS_t in this case:

Proposition 2 *Let y_t be generated by model (10). Then the relationship between TS_t and CS_t is given, for each t, by*

$$TS_t=\frac{n}{n-1}CS_t+\sigma_\eta^2+\sigma_x^2+\frac{n}{n-1}\alpha_t'\Omega^*\alpha_t. \tag{11}$$

Proof. See Appendix. ■

Equation (11) leads to the same conclusions as Theorem 1. The discrepancy between TS_t and CS_t is affected by the same terms as before, plus the extra term σ_x^2 which only increases further the difference between the two measures of uncertainty. The term σ_x^2 is common to all individuals and thus enters the conditional variance $Var\left[y_t|I_{t-1}\right]$ but it does not enter into the definition for CS_t. It is interesting to note that in this case the discrepancy between the two measures TS_t and CS_t increases because of the increase of the terms that are common among individuals, irrespective of whether they refer to common ignorance (η_t) or common knowledge (x_t).

3 Serial and contemporaneous dependence in TS_t and CS_t

In Section 2 we derived a static relationship between time series uncertainty and cross-section uncertainty and we showed that the magnitude of the discrepancy cannot be ordered unless we assume cross-sectional independence. In this section, we expand our framework to the case of the presence of serial dependence in TS_t and, as a consequence, in CS_t. We prove that the presence of serial correlation is a consequence of the presence of cross-sectional dependence in the data.

The existence of autocorrelation in the time evolution of TS_t and CS_t is likely to play a pivotal role whenever designing any empirical application.

This is because normally one would have time series data for TS_t and CS_t, and any analysis to find a relationship between the two measures would involve finding a correctly specified model.

Here, we propose a unified framework that provides a theoretical justification for both dependence across individuals and serial correlation, based on the consideration that individuals may have an adaptive rule to update their predictions on the variable y_t, whether this be in a weighted average with rational forecasts or not. This is on a different note with respect to Section 2, where forecasts where assumed to be rational, but it allows for a simple and concise discussion. Consider a set of n individuals with adaptive forecasts for the process $\{y_t\}_{t=0}^{\infty}$. For the purpose of a concise discussion, assume that y_t is an iid process with $E(y_t) = 0$ and $E(y_t^2) = \sigma^2$. With respect to equation (1), this entails that $\beta = 0$. In an adaptive forecast framework, the i-th individual's prediction $\hat{y}_t^i$ is given by

$$\hat{y}_t^i = y_{t-1} + \gamma_i \left(\hat{y}_{t-1}^i - y_{t-1} \right),$$

where γ_i is the single individual's weighting of the previous prediction error. Note that $\gamma_i \left(\hat{y}_{t-1}^i - y_{t-1} \right)$ represents the ith individual's private information, since it depends on $\hat{y}_{t-1}^i$ (the individual's own prediction) weighted according to the individual's perception. The forecast error here is

$$\hat{\varepsilon}_t^i = y_t - \hat{y}_t^i,$$

and it holds that

$$\hat{\varepsilon}_t^i = y_t - y_{t-1} - \gamma_i \left(\hat{y}_{t-1}^i - y_{t-1} \right). \tag{12}$$

Thus, the prediction errors $\hat{\varepsilon}_t^i$ are correlated with each other due to the presence of the "common factors" y_t and y_{t-1}. From equation (12), it can be shown that the presence of an adaptive forecasting rule can induce simultaneously:

- cross-correlation, i.e. $E\left[\hat{\varepsilon}_t^i \hat{\varepsilon}_t^j\right] \neq 0$ for two different individuals i and j;
- autocorrelation, i.e. $E\left[\hat{\varepsilon}_t^i \hat{\varepsilon}_{t-h}^i\right] \neq 0$ for any lag h.

To show this, note first that after some algebra

$$\hat{\varepsilon}_t^i = (1+\gamma_i) \sum_{k=1}^{\infty} (-1)^k \gamma_i^{k-1} y_{t-k} - y_t,$$

where, since the y_ts are iid zero mean it also holds that

$$\begin{aligned} Var\left[\hat{\varepsilon}_t^i\right] &= E\left[\left(\hat{\varepsilon}_t^i\right)^2\right] \\ &= (1+\gamma_i)^2 \sum_{k=1}^{\infty} \gamma_i^{2(k-1)} \sigma^2 + \sigma^2 \\ &= \sigma^2 \frac{(1+\gamma_i)^2}{1-\gamma_i^2} + \sigma^2 \\ &= \frac{2\sigma^2}{1-\gamma_i}. \end{aligned}$$

Note that, as far as cross-dependence is concerned, we have:

$$\begin{aligned} Cov\left[\hat{\varepsilon}_t^i, \hat{\varepsilon}_t^j\right] &= E\left[\hat{\varepsilon}_t^i \hat{\varepsilon}_t^j\right] \\ &= \sigma^2 + \sigma^2 (1+\gamma_i)\left(1+\gamma_j\right) \sum_{k=1}^{\infty} \gamma_i^{k-1} \gamma_j^{k-1} \\ &= \sigma^2 \frac{\left(1+\gamma_i+\gamma_j+\gamma_j\gamma_i\right)\gamma_j\gamma_i + 1 - \gamma_i\gamma_j}{1-\gamma_i\gamma_j} \neq 0. \end{aligned}$$

Also, as far as autocorrelation is concerned:

$$\begin{aligned} Cov\left[\hat{\varepsilon}_t^i, \hat{\varepsilon}_{t-h}^i\right] &= E\left[\hat{\varepsilon}_t^i \hat{\varepsilon}_{t-h}^i\right] \\ &= \sigma^2 (1+\gamma_i)^2 \sum_{k=1}^{\infty} (-1)^{2k+h} \gamma_i^{k-1+k+h-1} - \sigma^2 (-1)^h (1+\gamma_i) \gamma_i^{h-1} \\ &= \sigma^2 \gamma_i^{h-1} (1+\gamma_i) (-1)^h \frac{2\gamma_i - 1}{1-\gamma_i}, \end{aligned}$$

which is different from zero as long as $\gamma_i \neq 0$.

Note that this explanation as to the possible source of autocorrelation is not exhaustive. Another well documented source of persistent time dependence in time series is the presence of structural change.

4 Concluding remarks

The main aim of the paper was to provide a coherent theoretical investigation of the relationship between cross-section and time series measures of uncertainty, which are often employed as perfect substitutes in empirical applications. Most of the results in the earlier literature assume independent individuals, thereby neglecting the possible presence of common information among forecasters. We derive a deterministic relationship that links the time series and the cross sectional measures of uncertainty. Our analysis assumes the presence of an unobservable component as well as private information in the process that generates the economic variable to be forecast. We show the existence of a gap between the two measures. We prove that the discrepancy between the two measures depends on the unconditional dispersion, the dispersion of the component of information available to all agents (but unavailable to or ignored by econometricians) and a measure of the extent of private information, available only to individual agents. The interesting and important result in the paper is the ambiguous sign in the discrepancy between time series and cross section measures of uncertainty, that we show arises from the presence of cross-sectional dependence. Thus, our paper provides further theoretical support for recent inferential theory of panel data models.

Our findings play a crucial role in empirical investigations (e.g. using CBI data or HM Treasury Economic Prospects Team data). This goes beyond the scope of this paper but of course we leave this for future work.

Aknowlegments

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Appendix

Proof of Proposition 1. In light of the definition of ε_t^i, it can be written:

$$\begin{aligned}\frac{1}{n}\sum_{i=1}^{n} E\left(\varepsilon_t^i\right)^2 &= E\left[(y_t-\bar{y}_t)^2\right]+\frac{1}{n}\sum_{i=1}^{n} E\left[\left(y_t^i-\bar{y}_t\right)^2\right] \\ &\quad -\frac{2}{n}\sum_{i=1}^{n} E\left[(y_t-\bar{y}_t)\left(y_t^i-\bar{y}_t\right)\right]=I+II+III.\end{aligned} \tag{A.1}$$

Proposition 1 states that term III is equal to zero. Consider the following passages

$$\begin{aligned}&E\left[(y_t-\bar{y}_t)\left(y_t^i-\bar{y}_t\right)\right] \\ =\ & E\left[\left(\eta_t+\frac{n-1}{n}\alpha_t'\varepsilon_t\right)\left(\frac{1}{n}\alpha_t'\varepsilon_t-\alpha_{it}\varepsilon_{it}\right)\right] \\ =\ & E\left[\left(\frac{n-1}{n}\alpha_t'\varepsilon_t\right)\left(\frac{1}{n}\alpha_t'\varepsilon_t-\alpha_{it}\varepsilon_{it}\right)\right],\end{aligned}$$

since η_t is independent of ε_t; therefore

$$\begin{aligned}&E\left[\left(\frac{n-1}{n}\alpha_t'\varepsilon_t\right)\left(\frac{1}{n}\alpha_t'\varepsilon_t-\alpha_{it}\varepsilon_{it}\right)\right] \\ =\ & \frac{n-1}{n^2}E\left[\left(\alpha_t'\varepsilon_t\right)^2\right]-\frac{n-1}{n}E\left(\alpha_t'\varepsilon_t\alpha_{it}\varepsilon_{it}\right),\end{aligned}$$

so that

$$\begin{aligned}III &= -\frac{2}{n}\sum_{i=1}^{n}\left\{\frac{n-1}{n^2}E\left[\left(\alpha_t'\varepsilon_t\right)^2\right]-\frac{n-1}{n}E\left(\alpha_t'\varepsilon_t\alpha_{it}\varepsilon_{it}\right)\right\} \\ &= -\frac{2(n-1)}{n^2}E\left[\left(\alpha_t'\varepsilon_t\right)^2\right]+\frac{2(n-1)}{n^2}E\left(\alpha_t'\varepsilon_t\sum_{i=1}^{n}\alpha_{it}\varepsilon_{it}\right) \\ &= -\frac{2(n-1)}{n^2}E\left[\left(\alpha_t'\varepsilon_t\right)^2\right]+\frac{2(n-1)}{n^2}E\left[\left(\alpha_t'\varepsilon_t\right)^2\right]=0.\end{aligned}$$

This proves the proposition. As far as I and II in equation (A.1) are concerned, for the sake of completeness we have

$$\begin{aligned} y_t - \bar{y}_t &= y_t - \frac{1}{n}\sum_{i=1}^{n} y_t^i = y_t - \phi(I_{t-1}) - \frac{1}{n}\alpha_t'\varepsilon_t = \\ &= \eta_t + \alpha_t'\varepsilon_t - \frac{1}{n}\alpha_t'\varepsilon_t = \eta_t + \frac{n-1}{n}\alpha_t'\varepsilon_t, \end{aligned}$$

such that

$$I = E\left[(y_t - \bar{y}_t)^2\right] = \sigma_\eta^2 + \left(\frac{n-1}{n}\right)^2 \alpha_t'\Omega\alpha_t.$$

Also, since

$$\begin{aligned} \bar{y}_t - y_t^i &= y_t^i - \frac{1}{n}\sum_{i=1}^{n} y_t^i = \phi(I_{t-1}) + \alpha_t'\varepsilon_t - \phi(I_{t-1}) - \alpha_{it}\varepsilon_{it} = \\ &= \frac{1}{n}\alpha_t'\varepsilon_t - \alpha_{it}\varepsilon_{it}, \end{aligned}$$

it holds that

$$\begin{aligned} \sum_{i=1}^{n} E\left[\left(y_t^i - \bar{y}_t\right)^2\right] &= \frac{1}{n}E\left[\left(\alpha_t'\varepsilon_t\right)^2\right] + \sum_{i=1}^{n} E\left(\alpha_{it}^2\varepsilon_{it}^2\right) - \frac{2}{n}E\left(\alpha_t'\varepsilon_t\sum_{i=1}^{n}\alpha_{it}\varepsilon_{it}\right) = \\ &= \frac{1}{n}\alpha_t'\Omega\alpha_t + \alpha_t'\alpha_t - \frac{2}{n}\alpha_t'\Omega\alpha_t \\ &= \frac{n-1}{n}\alpha_t'\alpha_t - \frac{1}{n}\alpha_t'\Omega^*\alpha_t, \end{aligned}$$

which proves equation (7). ■

Proof of Theorem 1. From equation (7), it is known that the cross sectional measure of dispersion, CS_t, can be expressed as

$$CS_t = \sum_{i=1}^{n} E\left[\left(y_t^i - \bar{y}_t\right)^2\right] = \alpha_t'\alpha_t - \frac{1}{n}\alpha_t'\Omega\alpha_t.$$

From the definition of TS_t we have $TS_t = \sigma_\eta^2 + \alpha_t'\Omega\alpha_t$. Given that $\alpha_t'\Omega\alpha_t =$

$n\alpha_t'\alpha_t - nCS_t$ and

$$\alpha_t'\alpha_t = \frac{n}{n-1}CS_t + \frac{1}{n-1}\alpha_t'\Omega^*\alpha_t,$$

we have

$$\begin{aligned} TS_t &= \sigma_\eta^2 + \alpha_t'\Omega\alpha_t \\ &= \sigma_\eta^2 + n\alpha_t'\alpha_t - nCS_t \\ &= \sigma_\eta^2 + \frac{n^2}{n-1}CS_t + \frac{n}{n-1}\alpha_t'\Omega^*\alpha_t - nCS_t \\ &= \frac{n}{n-1}CS_t + \sigma_\eta^2 + \frac{n}{n-1}\alpha_t'\Omega^*\alpha_t. \end{aligned}$$

■

Proof of Proposition 2. Since

$$\frac{1}{n}\sum_{i=1}^{n} E\left(\varepsilon_t^i\right)^2 = \sigma_\eta^2 + \sigma_x^2 + \frac{n-1}{n}\alpha_t'\alpha_t + \frac{n-2}{n}\alpha_t'\Omega^*\alpha_t,$$

all the considerations and derivations previously made still hold. Therefore even in this case we have

$$CS_t = \alpha_t'\alpha_t - \frac{1}{n}\alpha_t'\Omega\alpha_t.$$

Given that

$$TS_t \equiv Var\left(y_t|I_{t-1}\right) = \sigma_\eta^2 + \sigma_x^2 + \alpha_t'\alpha_t,$$

we also have

$$TS_t = \sigma_\eta^2 + \sigma_x^2 + \frac{n}{n-1}CS_t + \frac{n}{n-1}\alpha_t'\Omega^*\alpha_t.$$

■