ENDOGENOUS PRICE JUMPS & QUADRATIC HAWKES PROCESSES

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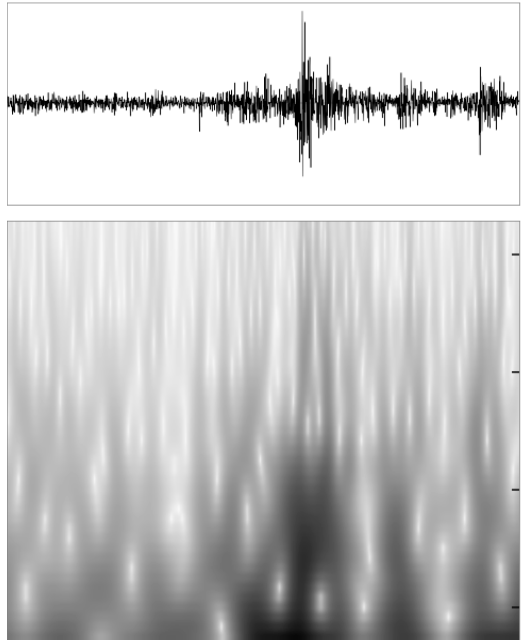
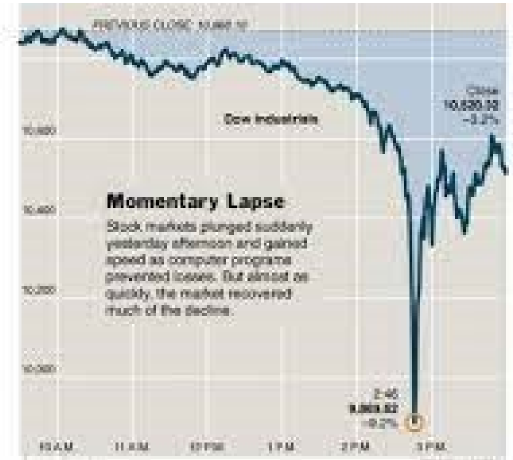


S&P500 and its wavelet representation (R. Morel)

1. Introduction

- ➢ Prices are VERY far from (geometric) Brownian motion
- Return distribution: fat tails, due to « jumps »: $P(r) \approx |r|^{-1-\mu}$ ($\mu \approx 3$)
- Volatility is a long-range memory process
- Negative returns tend to increase future volatility (Leverage effect)
- « Trends » of either sign also increase future vol. (Zumbach effect)
- ➢ We need models that encode such features mathematically and possibly shed light on the mechanisms responsible for them



1. Introduction

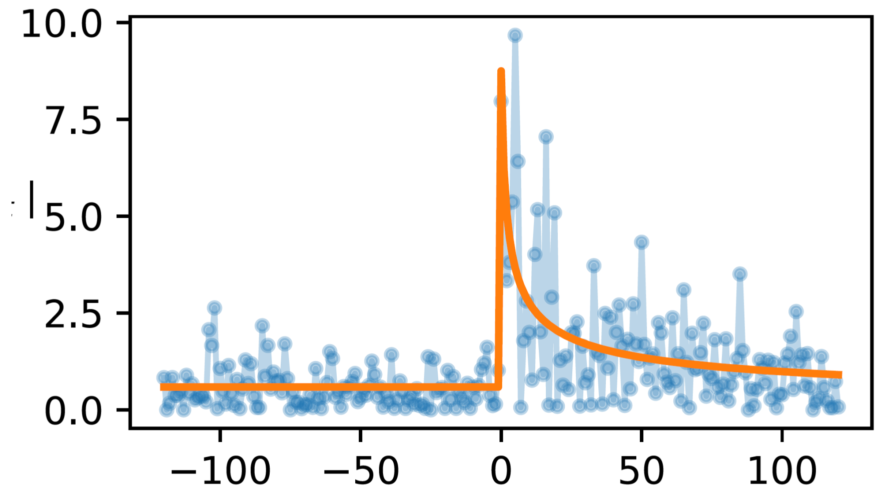
- ➢ Why do market prices jump?
- <u>Efficient Market story</u>: because some unexpected news becomes known and change the « fundamental » value – really?
- <u>Endogenous volatility story</u>: because of self-exciting feedback loops
- ➢ Of course, *some* news make prices jump, sometimes a lot
- But we know that order flow matters a lot too (cf. Reddit stocks)
- Cf. : Excess volatility puzzle in financial markets (2%/day !)

The evidence that large market moves occur on days without identifiable major news casts doubts on the view that price movements are fully explicable by news...(Cutler-Poterba-Summers, 1989; R. Fair, 2002, Joulin et al. 2008)

→ A desperate attempt: the almighty market knows things that nobody knows about

Table 4: Fifty Largest Postwar Movements in S&P Index and Their "Causes"

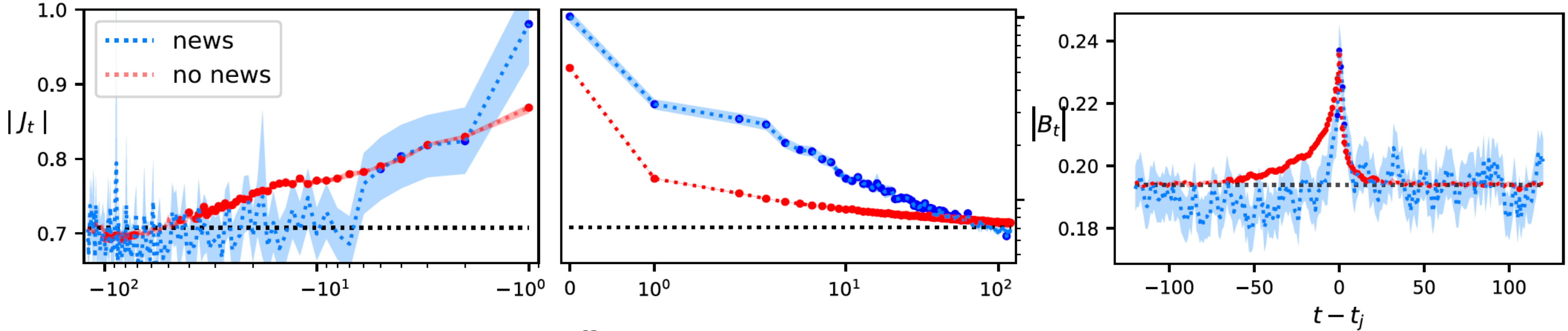
	Date	Percent Change	New York Times Explanation
1	Oct. 19, 1987	-20.47%	Worry over dollar decline and trade deficit; Fear of US not supporting dollar.
2	Oct. 21, 1987	9.10%	Interest rates continue to fall; deficit talks in Washington; bargain hunting.
3	Oct. 26, 1987	-8.28%	Fear of budget deficits; margin calls; reaction to falling foreign stocks
4	Sep. 3, 1946	-6.73%	"...no basic reason for the assault on prices."
5	May 28, 1962	-6.68%	Kennedy forces rollback of steel price hike.
6	Sep. 26, 1955	-6.62%	Eisenhower suffers heart attack.
7	Jun. 26, 1950	-5.38%	Outbreak of Korean War.
8	Oct. 20, 1987	5.33%	Investors looking for "quality stocks".
9	Sep. 9, 1946	-5.24%	Labor unrest in maritime and trucking industries.
10	Oct. 16, 1987	-5.16%	Fear of trade deficit; fear of higher interest rates; tension with Iran.



2. Intraday Price Jumps

➢ In order to improve statistics, we study 300 US stocks (2015-2020) with a one minute bin resolution (cf. Joulin, Lefèvre, Grunberg, JPB, 2008)

- A « jump » is defined as a > 4-σ event with respect to a one-day past local vol. (overnight and first/last 15 min discarded)
- Price time series are synchronised with the Bloomberg news feed containing stock name, ID or company name
- Idiosyncratic stock jumps occur mostly (≈ 95%) without news



$$J_t = \frac{r_t}{\sigma_t f_t}$$

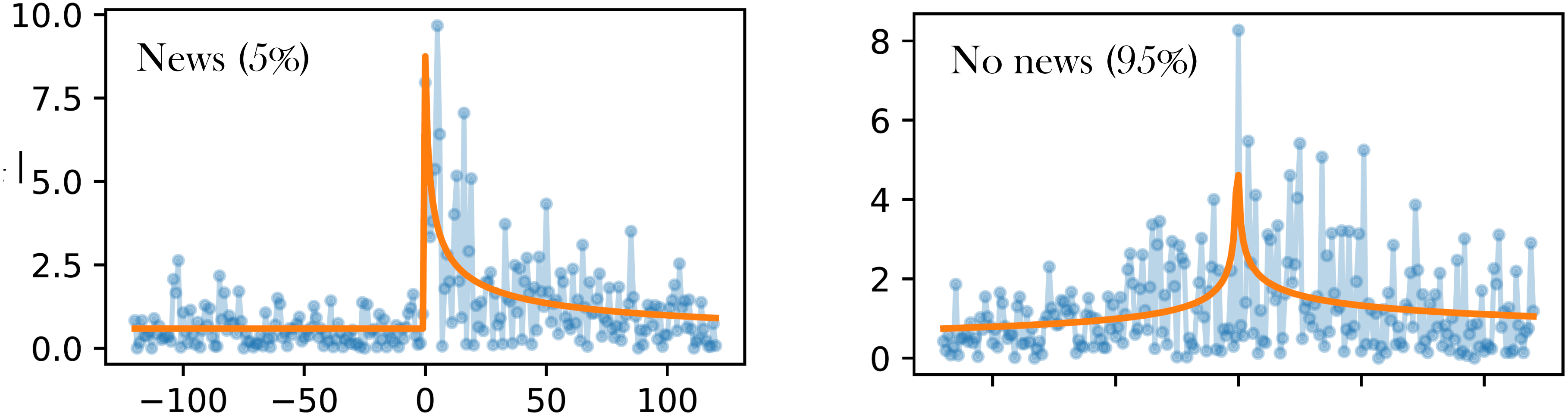
$$B_t = \kappa \frac{J_t}{|J_t|} + (1-\kappa) B_{t-1}$$

* Marcaccioli, JPB, Benzaquen, 2021

2. Intraday Price Jumps

- ➢ <u>More interesting:</u> volatility and trend profiles before news induced jumps and no-news jumps are <u>markedly different</u>, both on average and event-wise (see later)
- No-news jump profiles are more symmetric and decay slower
- Trends are clearly building up before no-news jumps
- Order book volume starts going down earlier for no-news jumps
- ➢ Many of these results confirm and sharpen those of Joulin et al. 08

Individual Price jumps



* Marcaccioli, JPB, Benzaquen, 2021

3. A Hawkes Inspired Fit

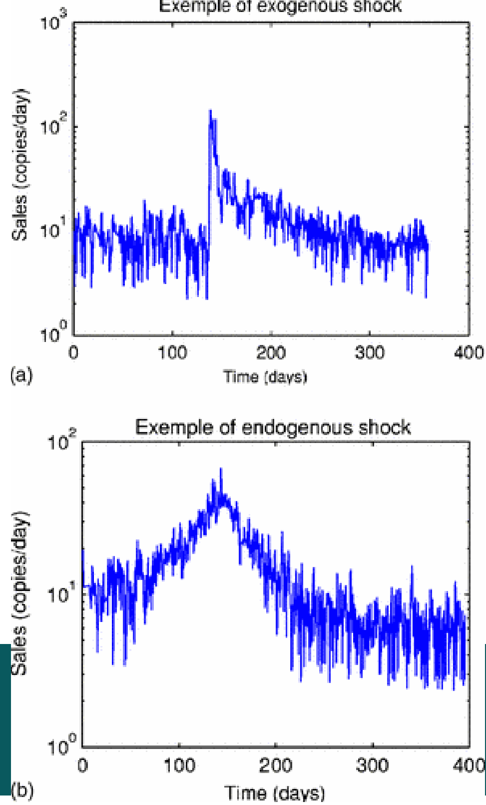
- Assuming an underlying near-critical Hawkes process (see below)

$$|J_t| \propto \begin{cases} (t-t_j)^{\theta-1}, & \text{EMC}, \quad t > t_j, t-t_j \ll (1-n)^{-\frac{1}{\theta}}; \\ (t-t_j)^{-\theta-1}, & \text{EMC}, \quad t > t_j, t-t_j \gg (1-n)^{-\frac{1}{\theta}}; \\ |t-t_j|^{2\theta-1}, & \text{SEC}, \quad t \lesssim t_j \end{cases}$$

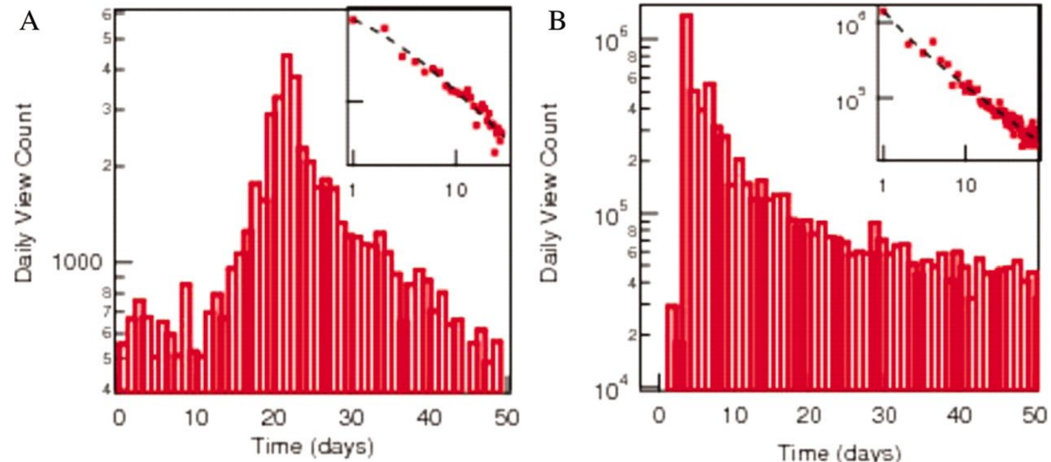
EMC: Efficient Market Class, SEC: Self-Excited Class

- Good fits with a unique θ ≈ **0.3** – same value as in other self-exciting social phenomena: Amazon books/YouTube views

3. Universal Endo/Exogenous Profiles?

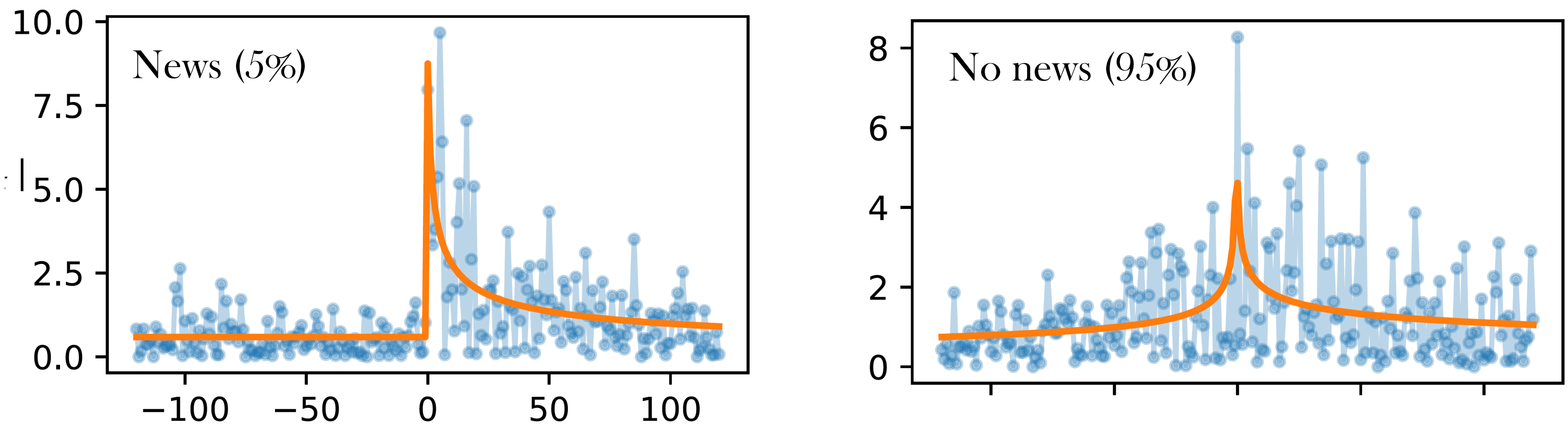


Amazon book sales (Sornette et al.)

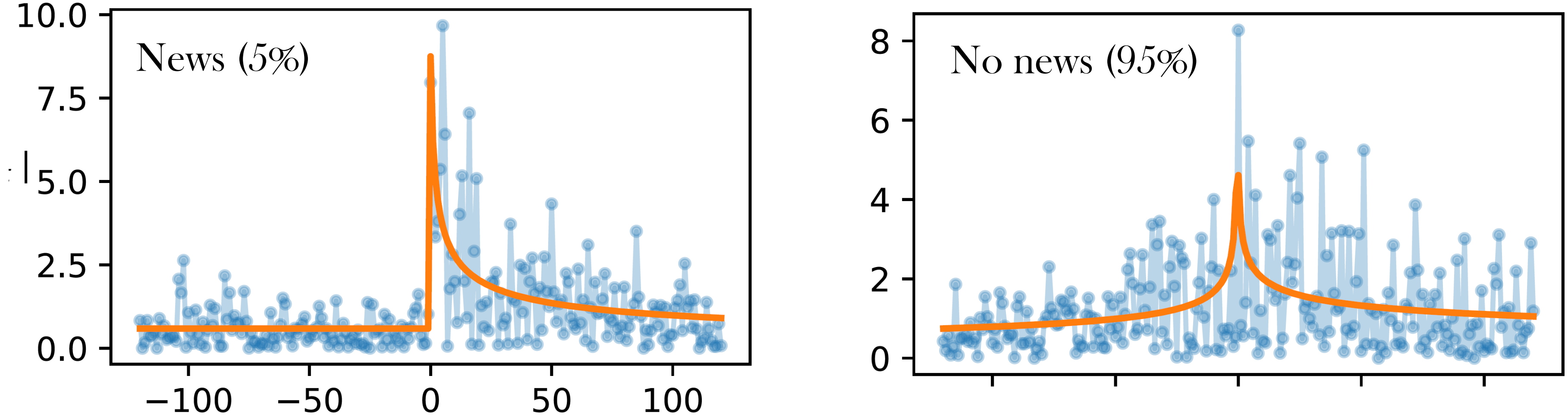


YouTube views (Sornette et al.)

Price jumps (US stocks)



Individual Price jumps



* Marcaccioli, JPB, Benzaquen, 2021

3. Unsupervised Classification

- Based on the left exponent p_ℓ, right exponent p_r and asymmetry $\mathscr{A}$ one can classify jumps into news related/no-news much better than luck (Note: Out-of-Sample AUC : 0.72)
- More current work using wavelets and PCA

(with C. Aubrun & R. Morel)

➢ **But where do endogenous jumps come from?**

→ <u>Hawkes processes</u>

	Logit
p_ℓ	-0.432***
	(0.080)
p_r	0.469***
	(0.131)
$\mathcal{A}$	-1.897***
	(0.211)
const.	-3.623***
	(0.110)
AUC	0.73

$$\lambda_t = \lambda_\infty + \int_{-\infty}^{t} \phi(t-s)\, \mathrm{d}N_s$$

$$n \equiv \int_0^\infty \phi(\tau) d\tau$$

4. Hawkes Processes

- Hawkes processes describe many « self-exciting » systems (earthquakes, crime, riots, bank defaults, financial activity)
- Consider a time-dependent Poisson process of rate $\boldsymbol{\lambda_t}$ ($\approx$ vol^2)
- This rate depends on past events **dN** through a certain kernel ϕ
- Financial markets: near-critical (**n** $\approx$ 1!) with power-law kernels $\phi(\tau) \approx \tau^{-1-\theta}$, encoding excess volatility & « long memory »

(see e.g. E. Bacry, I. Mastromatteo, J.F. Muzy, Hawkes Processes in Finance)

$$\lambda_t = \lambda_\infty + \int_{-\infty}^{t} \phi(t-s)\, \mathrm{d}N_s$$

4. Hawkes Processes

- Hawkes processes however fail to account for:

→ Power-law tails for the distribution of returns

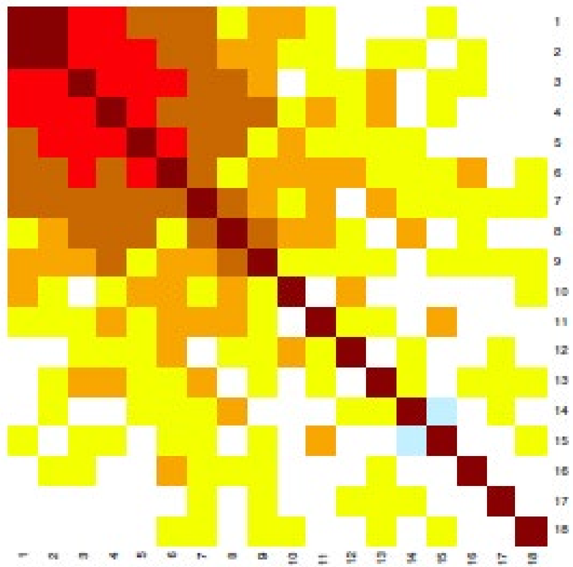
→ Absence of Time Reversal Invariance (Zumbach: past low frequency vol (« trends ») increases high frequency volatility, beyond « leverage »: not accounted by most stoch. vol. models)

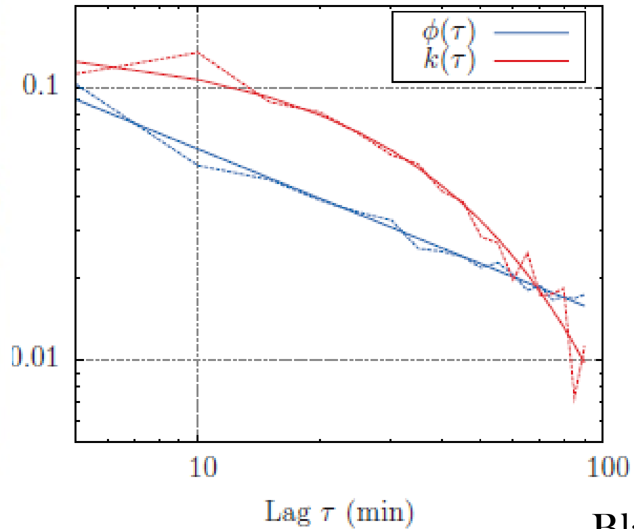
- Need to generalise Hawkes processes to include returns (d**P**) feedback on top of activity (d**N**) feedback → « Q-Hawkes »

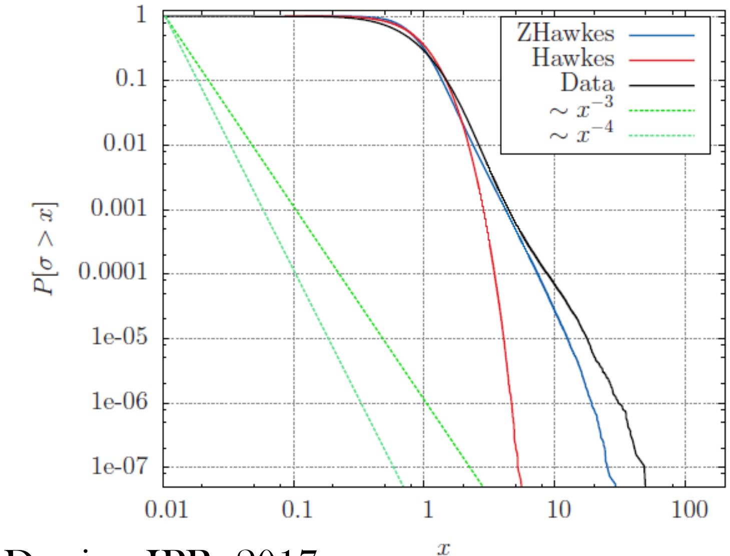
$$\boldsymbol{\lambda}_t = \boldsymbol{\alpha}_0 + \int_0^t \boldsymbol{\phi}(t-s)\, d\boldsymbol{N}_s + \int_0^t \boldsymbol{L}(t-s)\, dP_s + \int_0^t \int_0^t \boldsymbol{K}(t-s, t-u)\, dP_s dP_u$$

5. Quadratic Hawkes Processes

- ➢ Need to generalise Hawkes processes to include returns (**dP**) feedback on top of activity (**dN**) feedback → « Q-Hawkes »
- Φ: describes the Hawkes feedback (activity on itself)
- L: describes the leverage effect (with constraints to ensure positivity)
- K: describes the Zumbach effect
- ➢ Micro-foundation for « vol roughness » and « path dependent vol » (Gatheral, Rosenbaum et al., Guyon)







Blanc, Donier, JPB, 2017

5. Quadratic Hawkes Processes

- ➢ Q-Hawkes calibrated using correlations → kernels (YW eqs.)

→ Hawkes Φ is power-law with high values of **n** ($\approx$ 0.8)

→ K is well approximated by diagonal + rank 1 (Zumbach)

$$K(\tau, \tau') \approx \phi(\tau)\delta_{\tau-\tau'} + k(\tau)k(\tau')$$

→ Model reproduces well the power-law distribution of returns, generated by a <u>small</u> Zumbach k ($\approx$ 0.06) + TRI violations

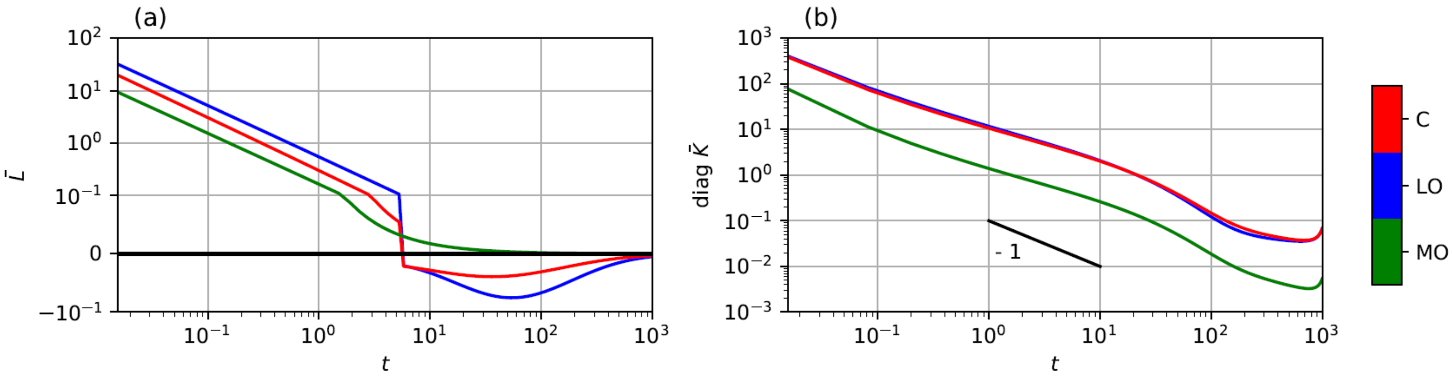
6-vectors 6 x 6 matrix 6-vectors

$$\boldsymbol{\lambda}_t = \boldsymbol{\alpha}_0 + \int_0^t \boldsymbol{\phi}(t-s)\,d\boldsymbol{N}_s + \int_0^t \boldsymbol{L}(t-s)\,dP_s + \int_0^t \int_0^t \boldsymbol{K}(t-s,t-u)\,dP_s dP_u$$

* Fosset, JPB, Benzaquen, 2021

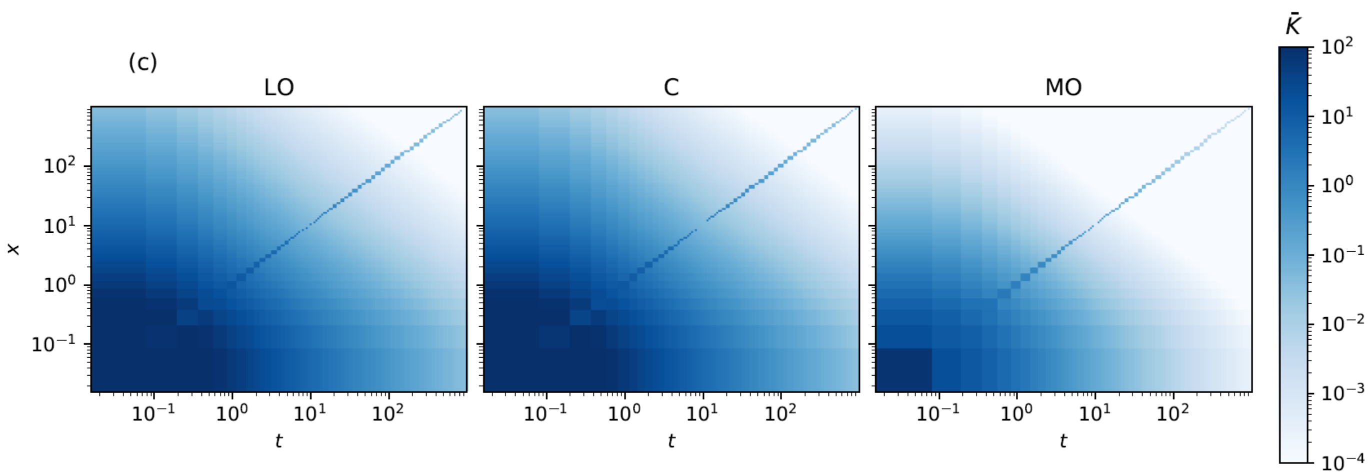
6. A Quadratic Hawkes for Order Book Activity

- Consider the two best limits and 6 event-types: MO, LO, CA, described by a 6-dimensional rate vector $\boldsymbol{\lambda}_t$ (→ 3 by symmetry)
- These rates depend on past events **dN** and past price changes **dP**
- The second term is a Hawkes feedback (bid/ask symmetric)
- The third term is a « leverage » feedback (bid/ask antisymmetric)
- The last term couples past volatility **K(u,u)** and past trends **K(u,v)** to present rates (bid/ask symmetric) – cf. the Zumbach effect

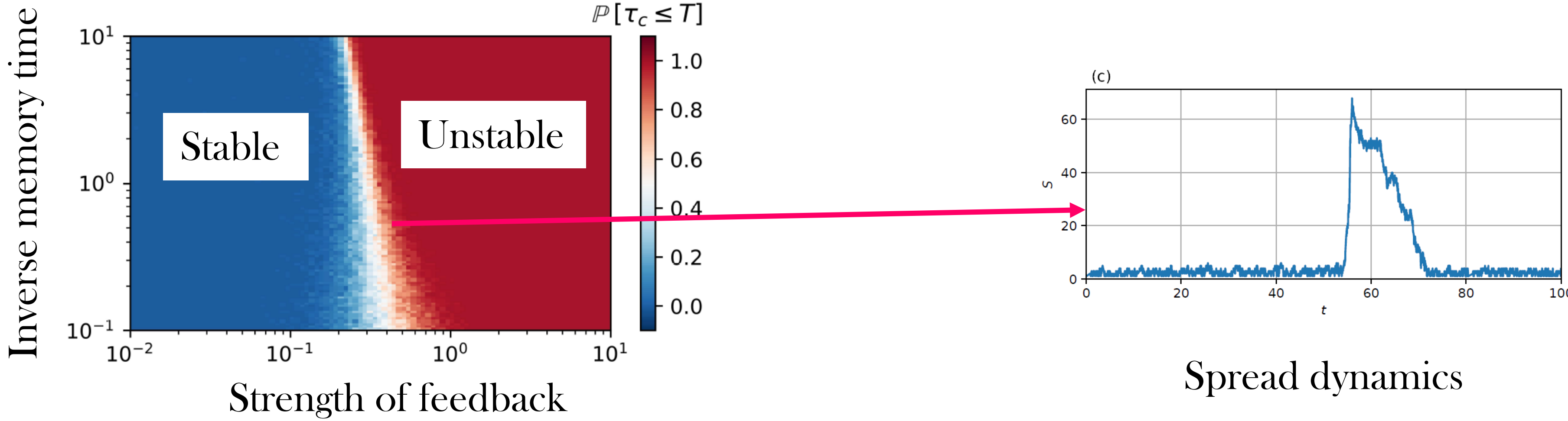


* Fosset, JPB, Benzaquen, 2021

6. Calibration (EUROSTOXX)



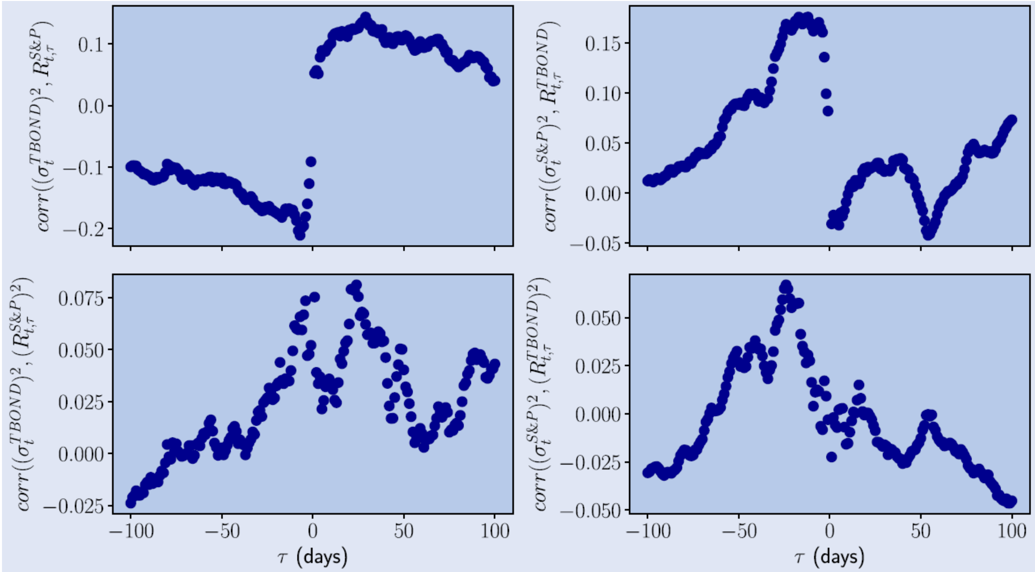
- ➢ There is a clear influence of past trends and past volatility on event rates, which decrease the volume in the order book → a possible feedback loop:



* Fosset, JPB, Benzaquen, 2021

7. Liquidity Crises

- volatility ↑→ liquidity ↓ → volatility↑ → if strong enough, this feedback loop that can lead to liquidity crises
- A genuine second-order phase transition between a <u>stable</u> and a <u>crisis-prone</u> market (difficult mathematical analysis)
- Note: such an instability also exists in the Glosten-Milgrom model, for the same reason – the fear of future price jumps is enough to induce liquidity crises & price jumps



Cross-leverage TBOND-S&P

Cross-Zumbach TBOND-S&P

8. Conclusions

- ➢ The « excess volatility puzzle » suggest that markets (& economies!) undergo turbulent endogenous dynamics, far from « rational equilibrium »
- Flows are dominant in determining price moves, cf. IMH (Gabaix-Koijen)
- ➢ Quadratic Hawkes processes provide a convenient unifying framework: « agent based » microfoundation of rough vol/path dependent vol models
- Useful in eliciting destabilization mechanisms and, possibly, detecting incipient liquidity crises/market seizures
- ➢ Multivariate Q-Hawkes and « cross-Zumbach » effects (with C. Aubrun)