Modeling Multi-population Longevity Risk with Mortality Dependence:

A Lévy Subordinated Hierarchical Archimedean Copula Approach

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Extended Abstract

In recent years a number of multi-population mortality models, particularly the two-population mortality models, have been proposed. For example, Li and Lee (2005) developed an augmented common factor model for a group of populations, imposing a common mortality change by age but allowing each its own age pattern and level of mortality. Li and Hardy (2011) recommended four plausible ways of generalizing a single-population mortality model to one that fits two or more populations. Cairns et al. (2011) introduced a general framework for producing consistent mortality forecasts for a pair of related populations. Dowd et al. (2011) (see also Jarner and Kryger, 2011) designed a gravity mortality model for two related but different sized populations. Zhou et al. (2013) introduced a two-population mortality model with transitory jump effects and used it for pricing catastrophic mortality securitizations. By using co-integration analysis, Yang and Wang (2013) and Zhou et al. (2014) investigated the long-run equilibrium in multi-country mortality data and applied a vector error correction model for mortality forecasts. Copula-based approach for capturing the mortality dependence between populations has been proposed in Lin et al. (2013) and Chen et al. (2013). Former paper used the Gaussian copula while and latter paper used the factor based copula to capture the mortality dependence.

This paper contributes to the literature on mortality modelling by proposing a new multi-population mortality method which provides a greater flexibility in modeling mortality dependence across various populations. More specifically, as demonstrated in Wang et al. (2013) that the mortality model Renshaw and Haberman (RH, 2006) with non-Gaussian residuals provides better out-of-sample performance for England and Wales, France, and Italy. Hence, our proposed work begins by first employing the extended RH model of Wang et al. (2013) to capture the evolution of marginal long-term mortality data. Then the Lévy subordinated hierarchical Archimedean copula (LSHAC) is used to construct mortality dependence structure of multi-population time and cohort indices simultaneously. There are some advantages of using LSHAC for capturing mortality dependence. Among them the foremost advantage is that LSHAC is an enriched class of method for modeling dependence. The classical Archimedean copula, although has the advantage of simplicity in that it is completely specified by a "generator" involving only a very small number of parameters (irrespective of the dimensions), it suffers from the serious drawback that all the random variables are assumed to be exchangeable. While the hierarchical Archimedean copula (HAC) has been proposed to partially overcome the exchangeability by "nesting" two or more Archimedean copulas with appropriate grouping, its generator, on the other hand, must fulfill the completely monotonic conditions to ensure that the resulting HAC yields a valid multivariate distribution. These conditions, however, can be difficult to verify and hence also restrict its practical application. This led to the development of LSHAC by Hering et al. (2010) (see also Mai and Scherer, 2012). They illustrated a stochastic representation of two-level LSHAC model and demonstrated that the stochastic representation can be constructed in higher levels in an iterative way. Motivated by these findings, the goal of this paper is to employ the multi-layer LSHAC involving Archimedean copula generator functions with Lévy subordinators (such as Gamma and Inverse Gaussian subordinators) to model the hierarchical structure of multi-population mortality dependence of time and cohort indices drawing from RH model with non-Gaussian residuals for the long-term multi-population mortality data. The goodness of fits of multi-layer LSHAC models and the hierarchical structure of multi-population mortality dependence are assessed using some empirical mortality data.

Finally, we take the hierarchical mortality dependence structure across population into account and apply a multivariate Wang transform to price multi-population longevity securities. We illustrate the applicability of our proposed multi-population mortality model and the pricing framework by pricing some multi-population linked securities including a bond with structure similar to Kortis longevity bond.

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