Time-evolution of age-dependent mortality patterns in mathematical model of heterogeneous human population

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(collaboration avec Demetris Avraam et Bakhti Vasiev, Université de Liverpool)

Longevity 11
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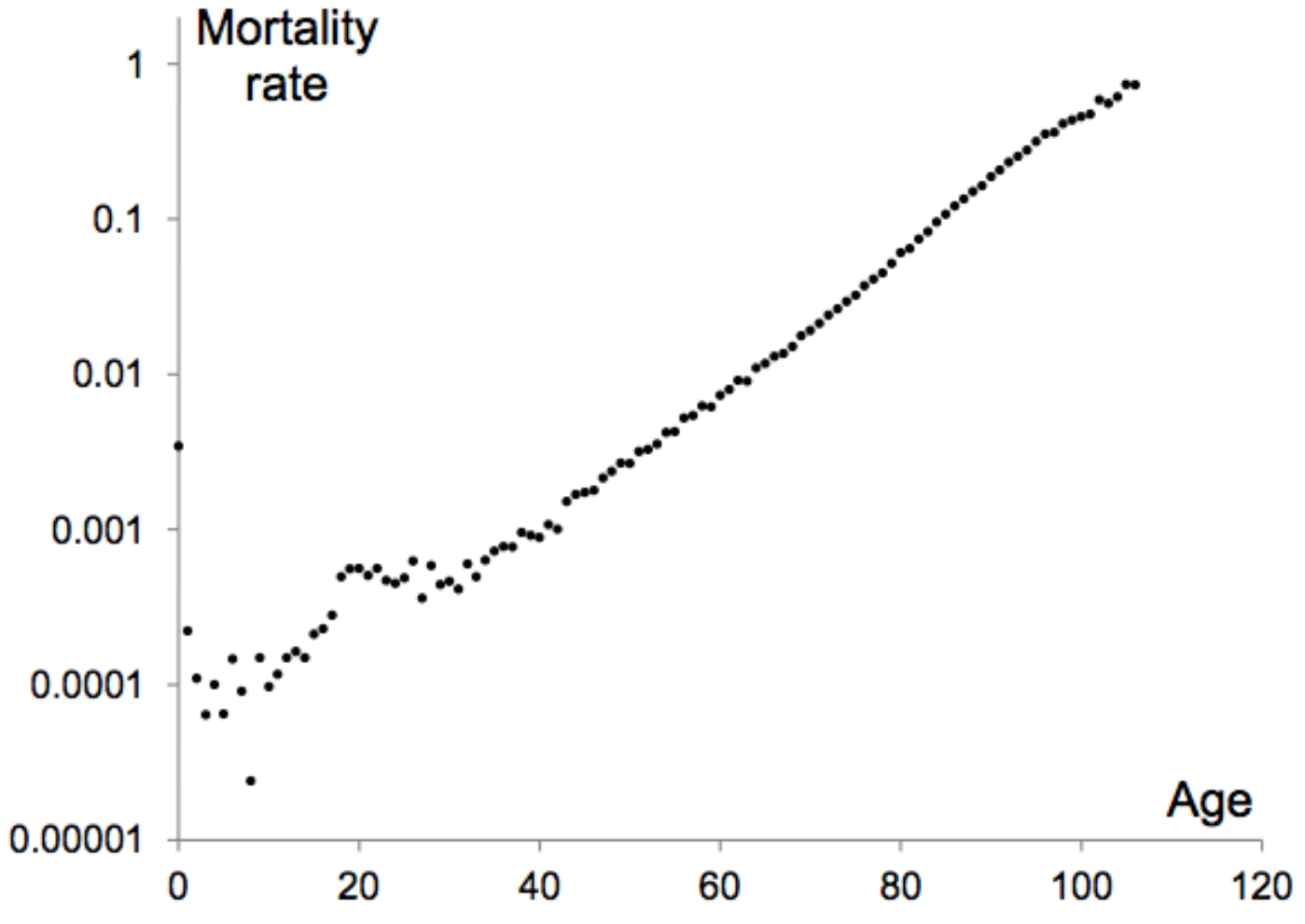

Figure: Mortality Rates, Sweden, 2000

Gompertz law of mortality

[Gompertz, 1825]:

$$m_x = m_0 e^{\beta x}$$

Gompertz law of mortality

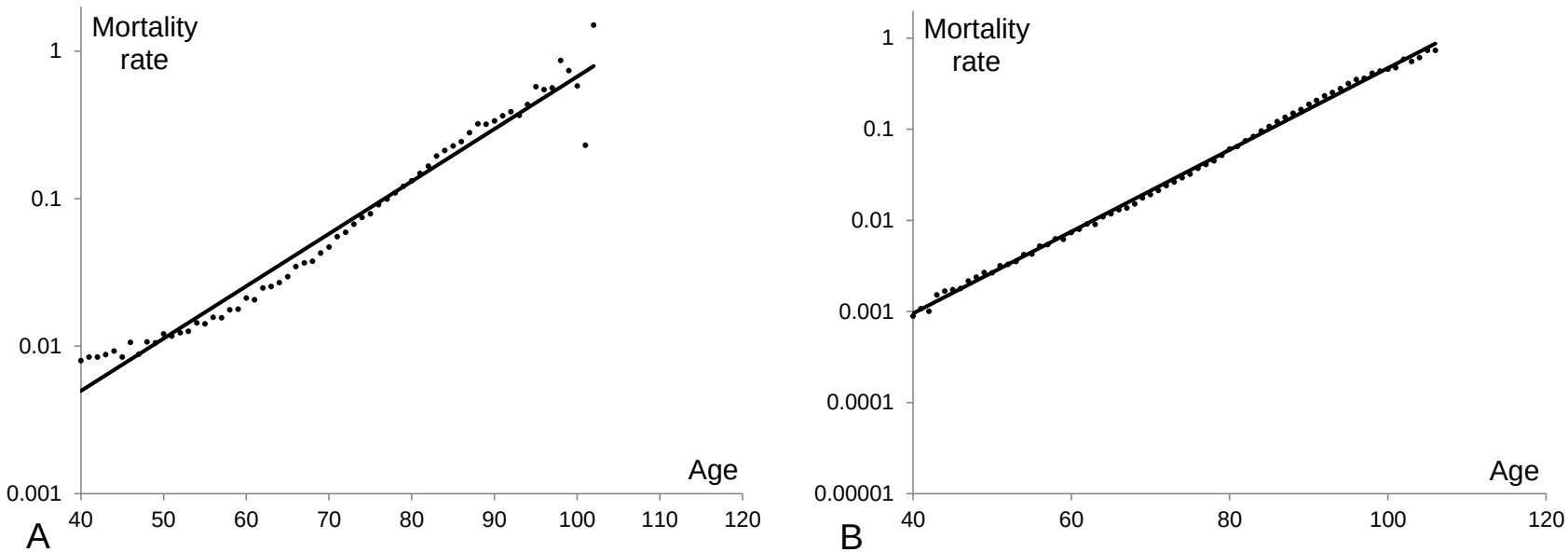

Figure: Mortality Rates, Sweden, 1900 (Panel A) and 2000 (Panel B)

Compensation effect

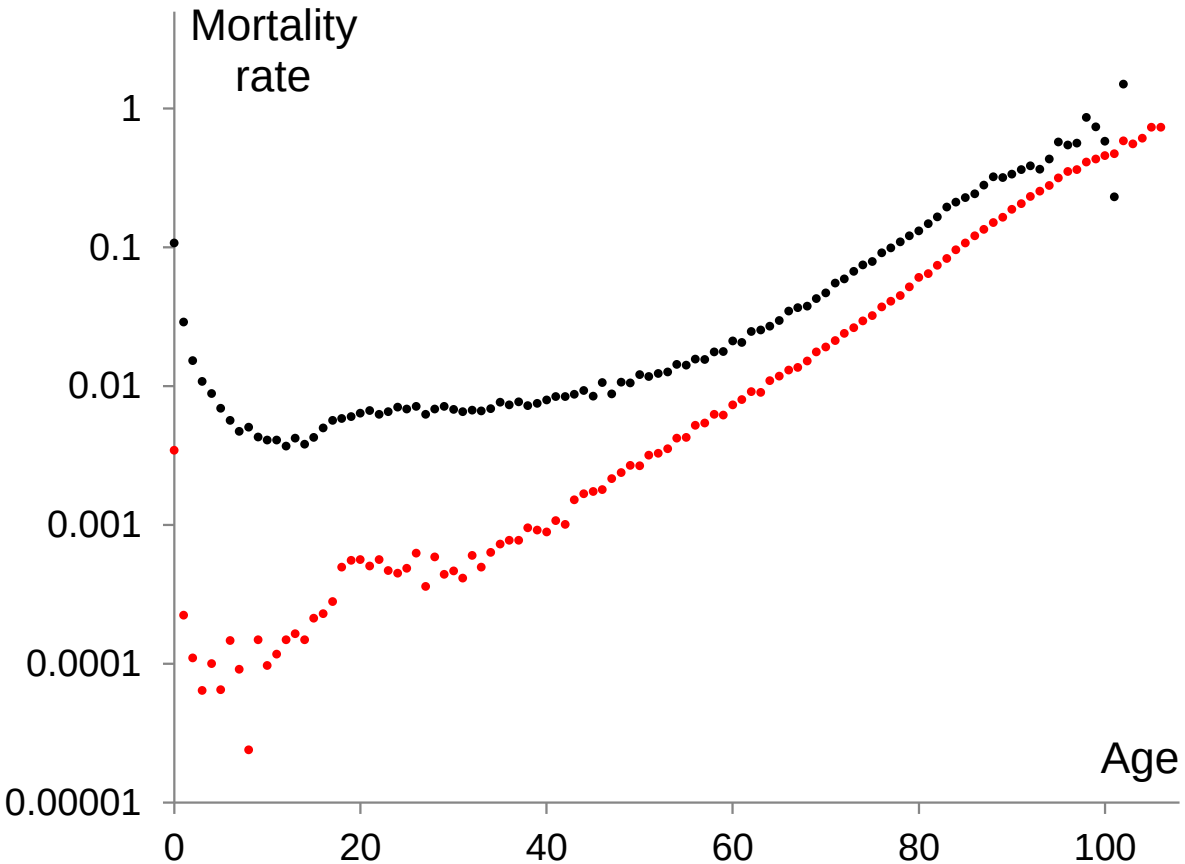

Figure: Mortality Rates, Sweden, 1900 (black) and 2000 (red)

Compensation effect

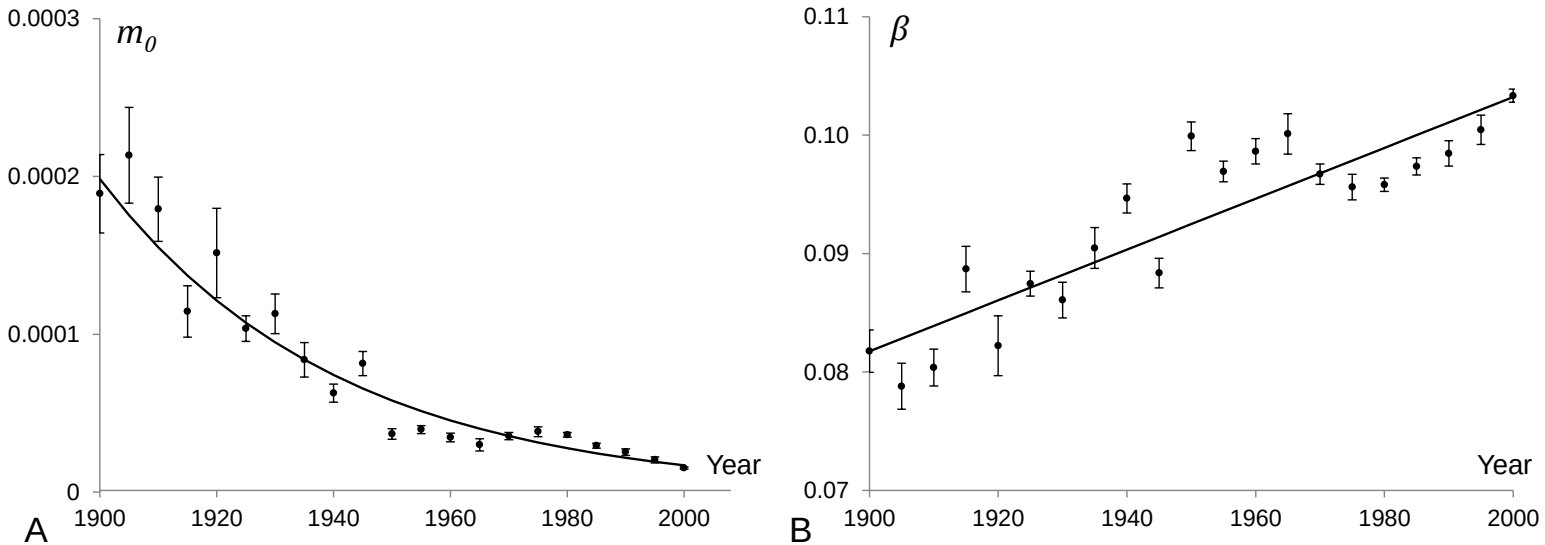

Figure: Evolution of the exponential parameters, Sweden, age 40+

$\rightarrow$ Strehler and Mildvan correlation [Strehler and Mildvan, 1960]:

$$\begin{aligned} \ln(m_0) &= \ln(M) - \beta X \\ m_0 &= M e^{-\beta X} \end{aligned}$$

Compensation effect

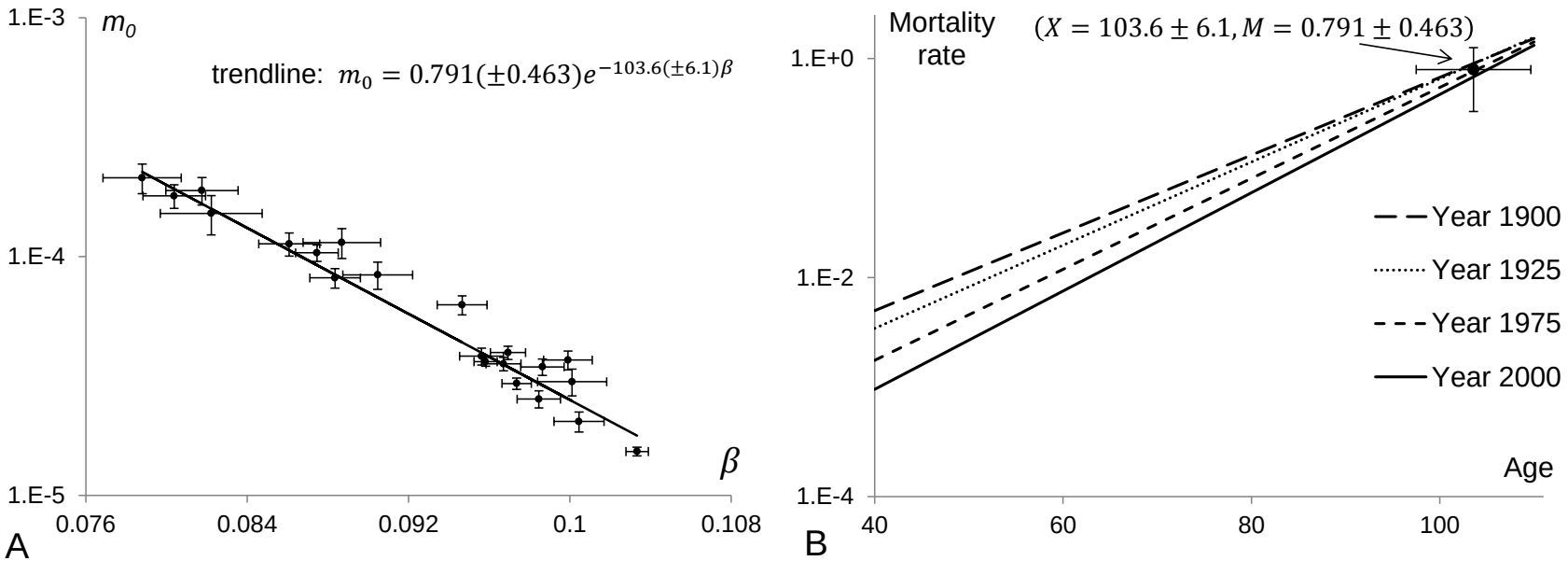

Figure: Compensation effect in 40+ mortality dynamics

Aim of this work

Populations are heterogeneous.

→ Each subpopulation obeys the exponential law.

→ Can we model the mortality of the entire population as a mixture of weighted exponential terms?

→ If yes, do we observe the compensation effect in each subpopulation?

[Avraam et al., 2014]

Gompertz and an extension

Gompertz law of mortality [Gompertz, 1825]:

$$m_x = m_0 e^{\beta x}$$

An extension of [Avraam et al., 2013]

$$m_x = \sum_{j=1}^{n} \rho_{jx} m_{jx} = \sum_{j=1}^{n} \rho_{jx} m_{j0} e^{\beta_j x}$$

[Booth and Tickle, 2008]

All ages excluding the extrinsic causes of death

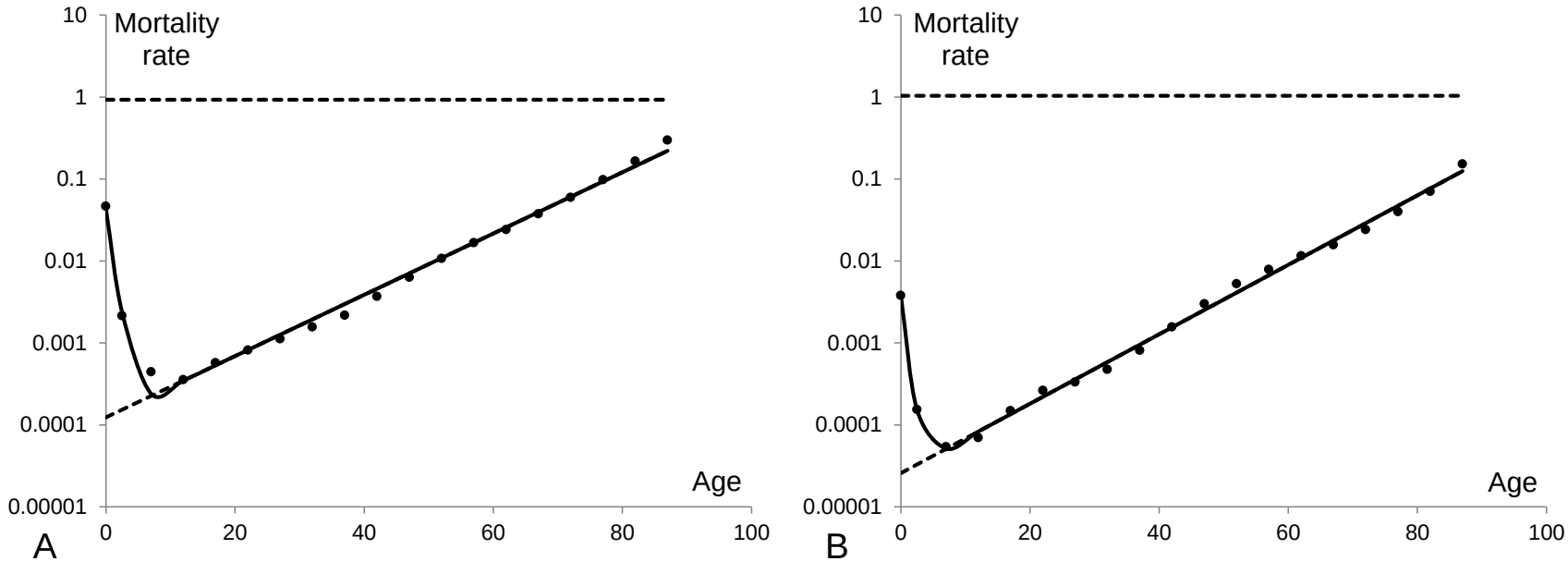

Figure: Heterogeneous model, Sweden, 1951 (Panel A) and 2010 (Panel B)

All ages excluding the extrinsic causes of death

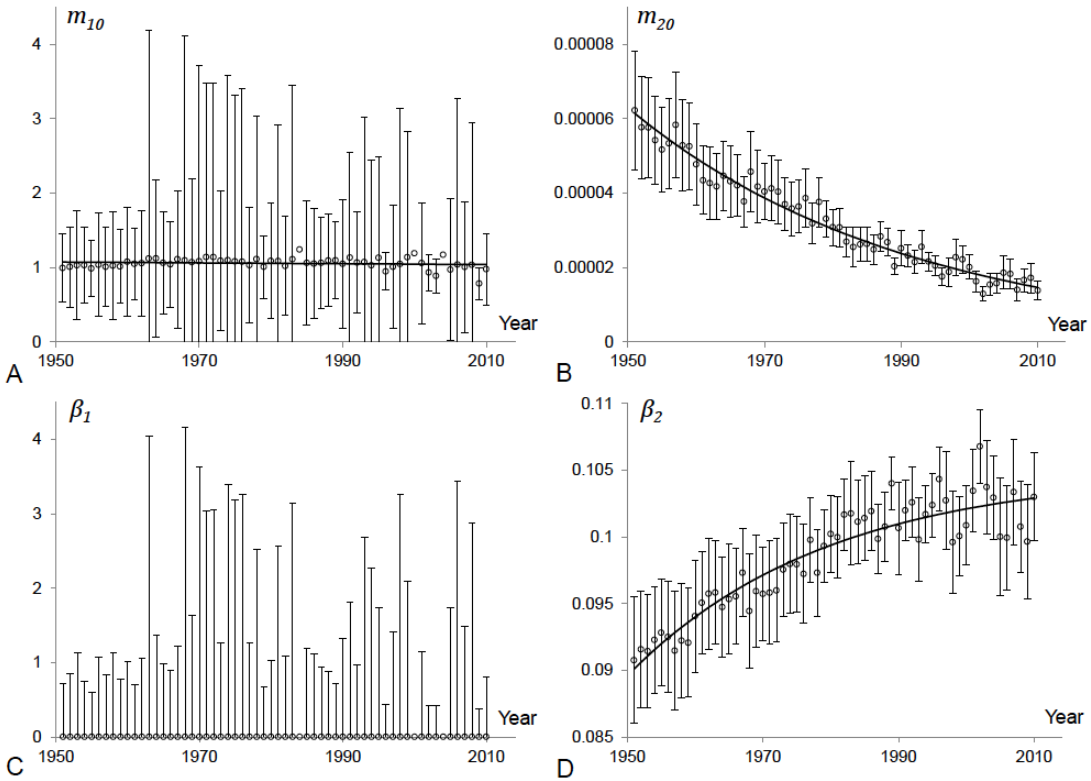

Figure: Evolution of the exponential parameters of a two-subpopulation model, Sweden

All ages excluding the extrinsic causes of death

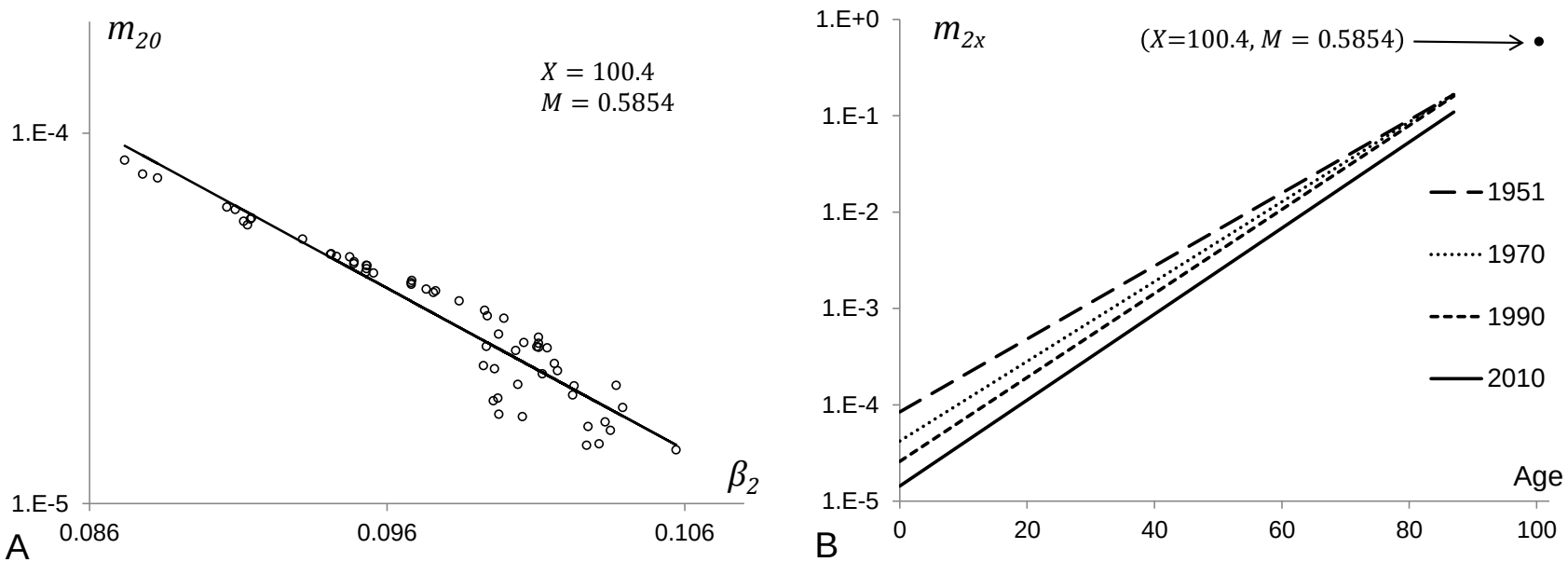

Figure: Compensation effect for the second subpopulation, Sweden

All ages excluding the extrinsic causes of death

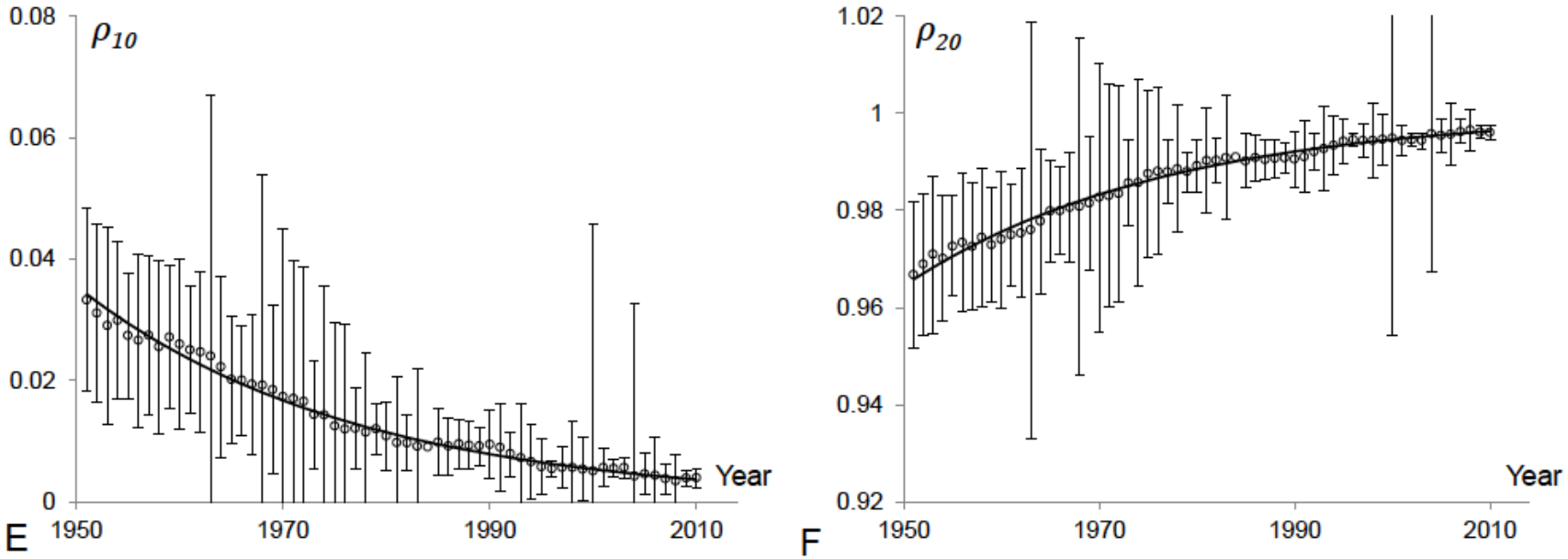

Figure: Evolution of the fractions of a two-subpopulation model, Sweden

$\rightarrow$ homogenization of the population

All ages, all causes of death

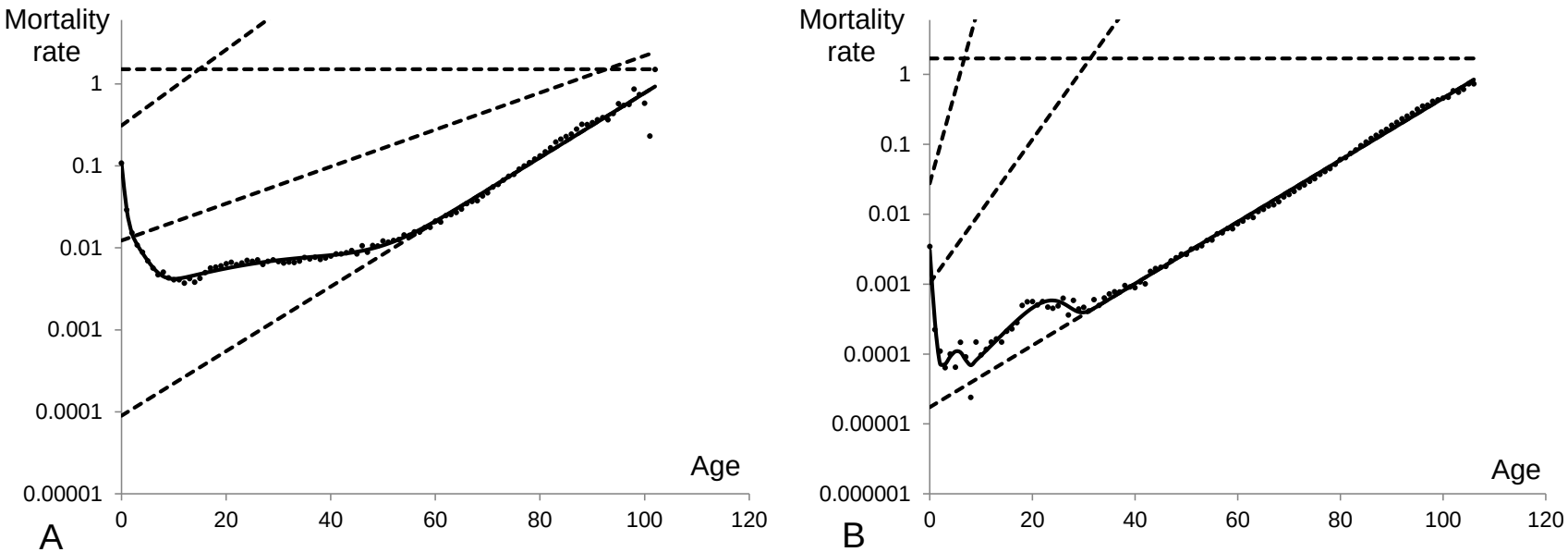

Figure: Heterogeneous model, Sweden, 1900 (Panel A) and 2000 (Panel B)

All ages, all causes of death

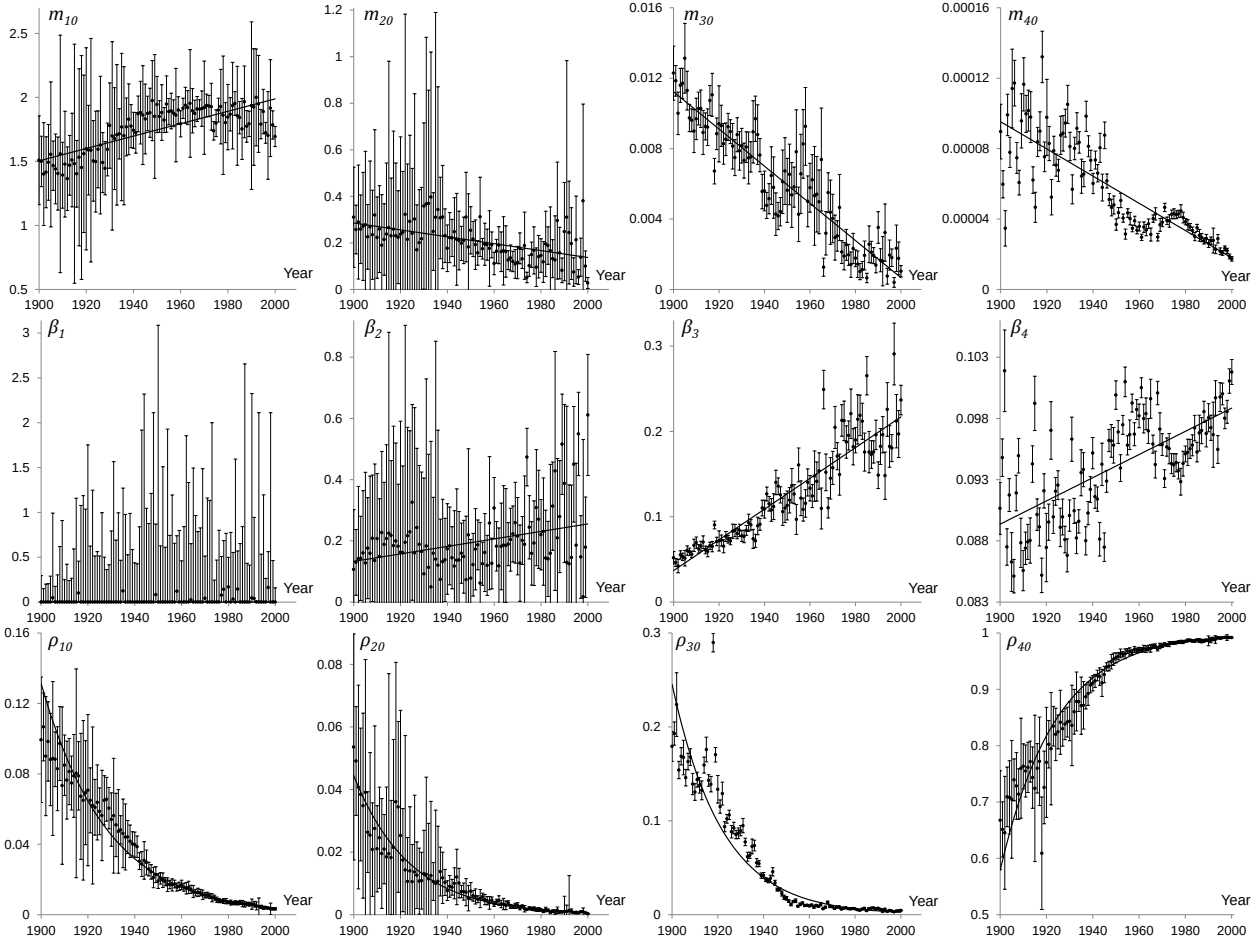

Figure: Evolution of the parameters of a four-subpopulation model, Sweden

All ages, all causes of death

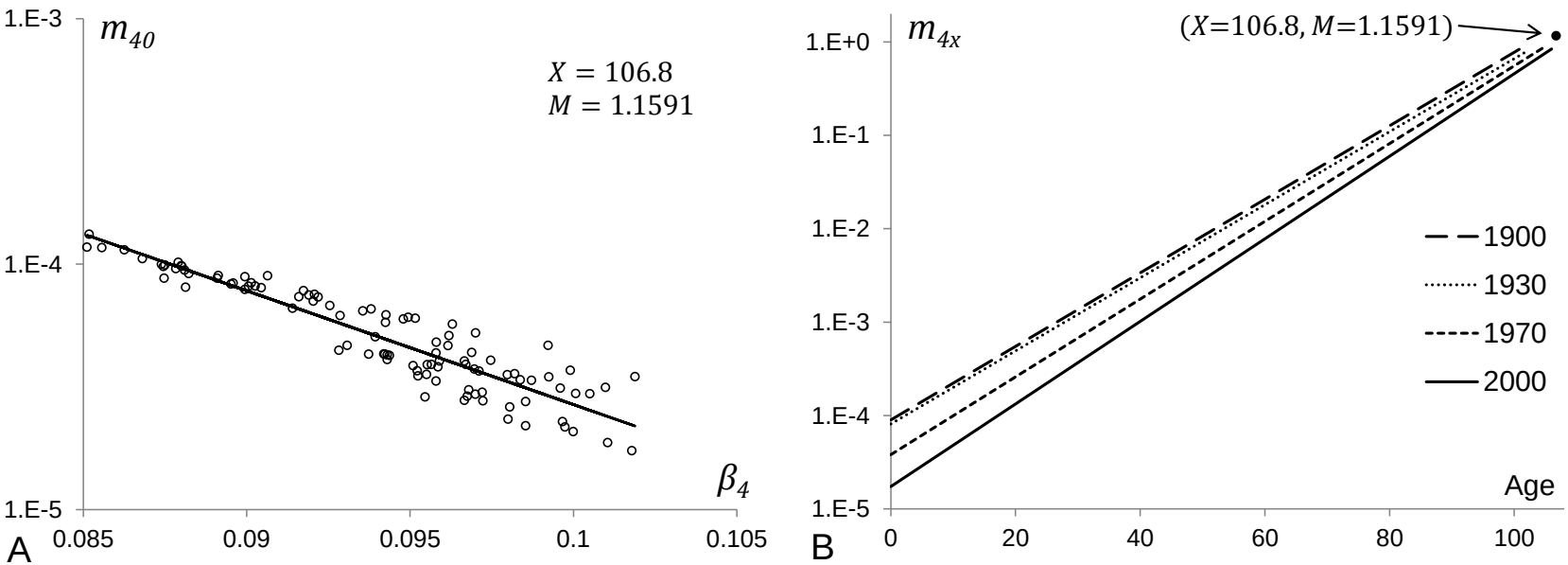

Figure: Compensation effect for the fourth subpopulation, Sweden

All ages, all causes of death

What part of past mortality decrease is due to the homogenization of the population and what part is due to a real mortality decrease?

All ages, all causes of death

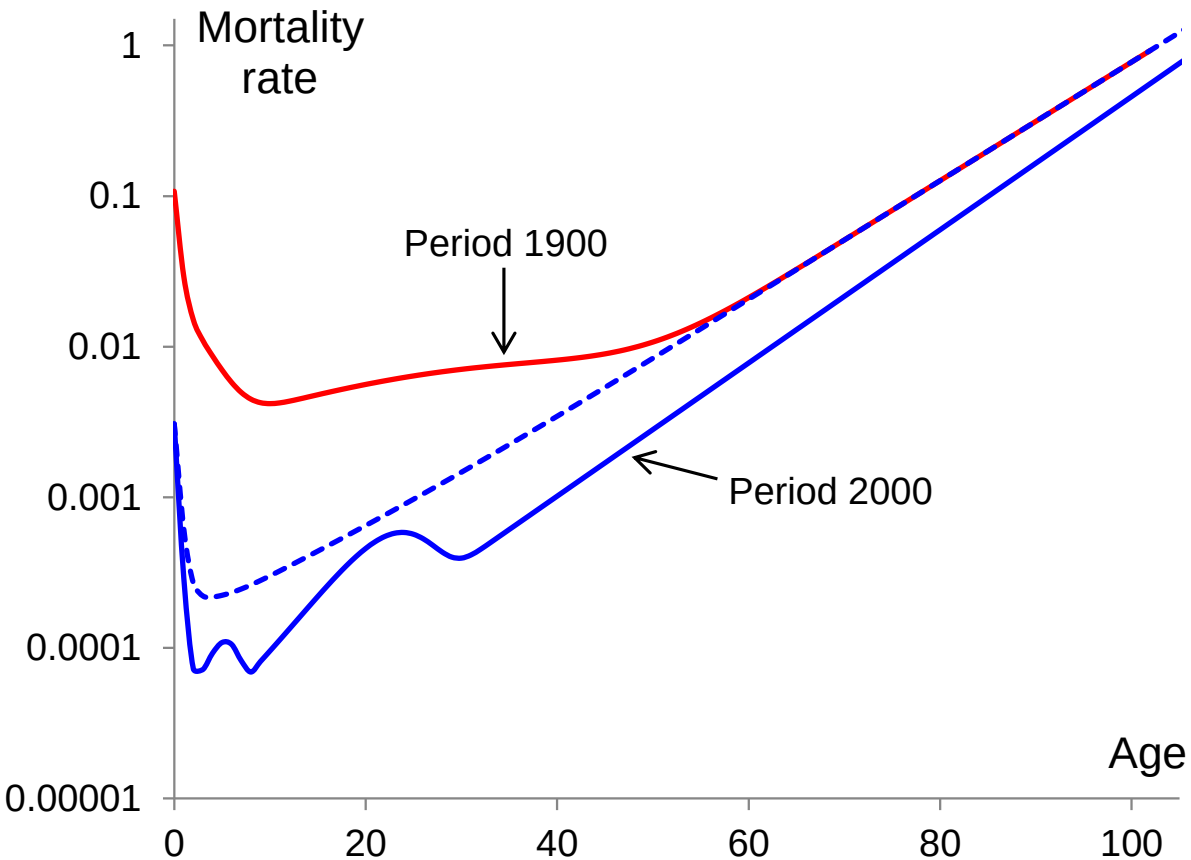

Figure: Reduction in Swedish mortality rates within one century

Concluding remarks

We consider a model of heterogeneous population composed of several subpopulations having different mortality dynamics.

→ Each subpopulation follows the Gompertz law of mortality

Two main findings:

→ The compensation law of mortality is confirmed at the subpopulation level;

→ Homogenization of the population over time.

Further steps:

→ Model giving potential insights for mortality at extreme old ages;

→ Any links with genes and natural selection?

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Thank you very much for your attention!

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