Population dynamics for longevity risk

Alexandre Boumezoued

with Nicole El Karoui (Paris 6) and Stéphane Loisel (Lyon 1)
Paris 6 University

Probability and Random Models Laboratory, UMR-CNRS 7599

Work partially funded by Chair "Risques financiers" and ANR project "Lolita"

LONGEVITY 11, Lyon, 2015 September 7th

Aim of this talk

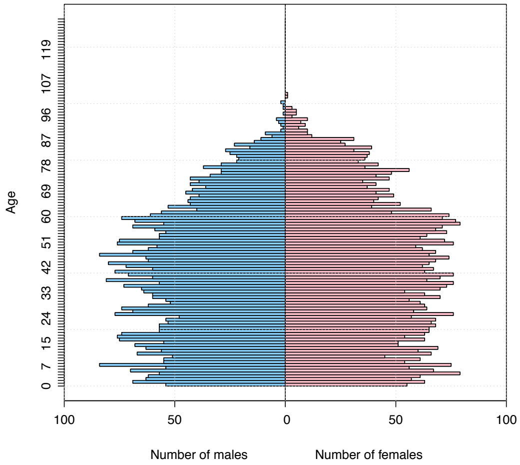
Why population dynamics ?

How do birth patterns interact with mortality ?

1. Focus on **age pyramid** dynamics
2. Focus on **heterogeneity** dynamics

Age pyramid

- **Age pyramid:** the number of individuals by age class



Age pyramid

- Evolves over time due to **several demographic events**:
  - **Deaths**
  - **Births**
  - Migration flows

Age pyramid

- Evolves over time due to **several demographic events**:
  - **Deaths**
  - **Births**
  - Migration flows
- Let $g(a,t)$: number of individuals with exact age a at exact time t ⇒ Continuous age and time setting

Age pyramid

- Evolves over time due to **several demographic events**:
  - **Deaths**
  - **Births**
  - Migration flows
- Let $g(a, t)$: number of individuals with exact age a at exact time t ⇒ Continuous age and time setting
- Example: $\int_{a_1}^{a_2} g(a, t)\mathrm{d}a$
  the number of individuals with exact age in $[a_1, a_2)$ at time t

Age pyramid

- Evolves over time due to **several demographic events**:
  - **Deaths**
  - **Births**
  - Migration flows
- Let $g(a, t)$: number of individuals with exact age a at exact time t ⇒ Continuous age and time setting
- Example: $\int_{a_1}^{a_2} g(a, t)\mathrm{d}a$
  the number of individuals with exact age in $[a_1, a_2)$ at time t
- Example: [intergenerational issues] Dependency ratio

$$r_t = \frac{\int_{65}^{\infty} g(a, t)\mathrm{d}a}{\int_{15}^{65} g(a, t)\mathrm{d}a}.$$

Mortality force & Cohort dynamics

- Let $\mu(a, t) \equiv$ mortality force at exact age a and exact time t
- Drives the time evolution of a given cohort
- Let $g(0, \nu)$ be given (number of newborns at time ν)

Mortality force & Cohort dynamics

- Let $\mu(a, t) \equiv$ mortality force at exact age a and exact time t
- Drives the time evolution of a given cohort
- Let $g(0, \nu)$ be given (number of newborns at time ν)
- The number of survivors at age a in the cohort is

$$g(a, \nu + a) = g(0, \nu) \exp\left(-\int_0^a \mu(s, \nu + s)\mathrm{d}s\right)$$

Mortality force & Cohort dynamics

- Let $\mu(a,t) \equiv$ mortality force at exact age a and exact time t
- Drives the time evolution of a given cohort
- Let $g(0,\nu)$ be given (number of newborns at time ν)
- The number of survivors at age a in the cohort is

$$g(a,\nu+a) = g(0,\nu)\exp\left(-\int_0^a \mu(s,\nu+s)\mathrm{d}s\right)$$

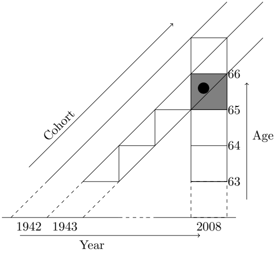
- Differentiation (age and time) leads to the...

...transport component of McKendrick-Von Foerster equation

$$(\partial_a + \partial_t)g(a,t) = -\mu(a,t)g(a,t).$$

National mortality

- How to estimate mortality force $\mu(a, t)$ on a national basis ?
- Assumption that mortality force is piecewise constant. Why ?
  1. **Classical non-parametric estimation for continuous age and time is not possible (see e.g. Keiding 1990)**
  2. Due to the lack of data
- Uses of the **Lexis diagram** to regroup individuals by **age classes** $\underline{a}$ and **years of observation** $\underline{t}$ (e.g. 1 or even 5 years)



[May vary from one country to another]

National mortality

- Therefore mortality force is estimated as

$$\hat{\mu}(\underline{a}, \underline{t}) = \frac{D(\underline{a}, \underline{t})}{E(\underline{a}, \underline{t})}$$

- $D(\underline{a}, \underline{t})$ is (e.g.) the number of deaths in year $\underline{t}$ of people age $\underline{a}$ at date of death
- $E(\underline{a}, \underline{t})$ is the *famous* **exposure to risk** $\equiv$ total time lived during year $\underline{t}$ by individuals aged $\underline{a}$

Exposure to risk depends on underlying age pyramid dynamics

$$E(\underline{a}, \underline{t}) = \int_{\underline{t}}^{\underline{t}+1} \int_{\underline{a}}^{\underline{a}+1} g(u, s) \mathrm{d}u \mathrm{d}s$$

Linking birth patterns with mortality data

- Richards, S.J. (2008).
  Detecting year-of-birth mortality patterns with limited data.
  Journal of the Royal Statistical Society, Series A, 171.
  - Highlights that "some question marks remain about the population estimates for years of birth with sharp swings in fertility"
- Cairns, A. J., D. Blake, K. Dowd, A. Kessler (2014).
  Phantoms never die: Living with unreliable mortality data.
  Tech. rep., Herriot Watt University, Edinburgh.
  - Propose a methodology to detect and manage exposure errors based on monthly/quarterly birth data

Simple population philosphy

- "People of a birth cohort share the fact that they are born from the same population"

Renewal component of the McKendrick-Von Foerster equation

$$g(0,\nu) = \int_0^{\infty} g(a,\nu) b(a,\nu) \mathrm{d}a.$$

Recall the transport component :

$$(\partial_a + \partial_t) g(a,t) = -\mu(a,t) g(a,t).$$

Simple population philosphy

- "People of a birth cohort share the fact that they are born from the same population"

Renewal component of the McKendrick-Von Foerster equation

$$g(0,\nu) = \int_0^{\infty} g(a,\nu) b(a,\nu) \mathrm{d}a.$$

$\Rightarrow$ By the way, all this shows why the Human Fertility Database* is also useful for Mortality

*HFD, Max Planck Institute for Demographic Research (Germany) and Vienna Institute of Demography (Austria). *www.humanfertility.org*

Simple population philosphy

- "People of a birth cohort share the fact that they are born from the same population"

Renewal component of the McKendrick-Von Foerster equation

$$g(0,\nu) = \int_0^\infty g(a,\nu)b(a,\nu)\mathrm{d}a.$$

$\Rightarrow$ Let us extend this to a stochastic setting

Stochastic setting and micro/macro link

- Due to the finite population size, demographic events (individual births and deaths) occur at random times
  $\Rightarrow$ Microscopic point of view
- Need of stochastic modeling to account for idiosyncratic risk

Stochastic setting and micro/macro link

- Due to the finite population size, demographic events (individual births and deaths) occur at random times
  $\Rightarrow$ Microscopic point of view
- Need of stochastic modeling to account for idiosyncratic risk
- $Z_t([a_1, a_2)) \equiv$ the stochastic number of individuals with age in $[a_1, a_2)$ at exact time t

Stochastic setting and micro/macro link

- Due to the finite population size, demographic events (individual births and deaths) occur at random times
  $\Rightarrow$ Microscopic point of view
- Need of stochastic modeling to account for idiosyncratic risk
- $Z_t([a_1, a_2)) \equiv$ the stochastic number of individuals with age in $[a_1, a_2)$ at exact time t

Micro-macro consistency*

$$\mathbb{E}\left[Z_t([a_1, a_2))\right] = \int_{a_1}^{a_2} g(a,t)\mathrm{d}a \quad \text{[Linear model]}$$

Stochastic setting and micro/macro link

- Due to the finite population size, demographic events (individual births and deaths) occur at random times
  $\Rightarrow$ Microscopic point of view
- Need of stochastic modeling to account for idiosyncratic risk
- $Z_t([a_1, a_2)) \equiv$ the stochastic number of individuals with age in $[a_1, a_2)$ at exact time t

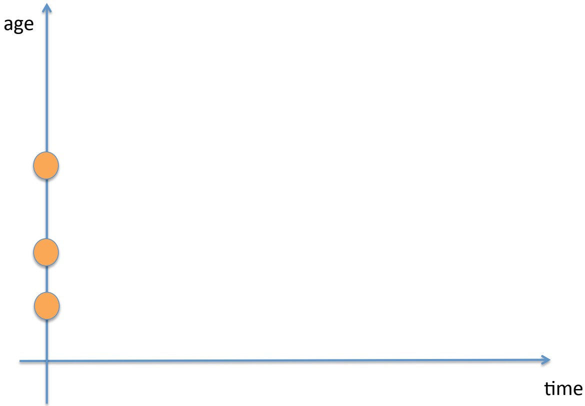
Micro-macro consistency*

$$\mathbb{E}\left[Z_t([a_1, a_2))\right] = \int_{a_1}^{a_2} g(a, t)\mathrm{d}a \quad \text{[Linear model]}$$

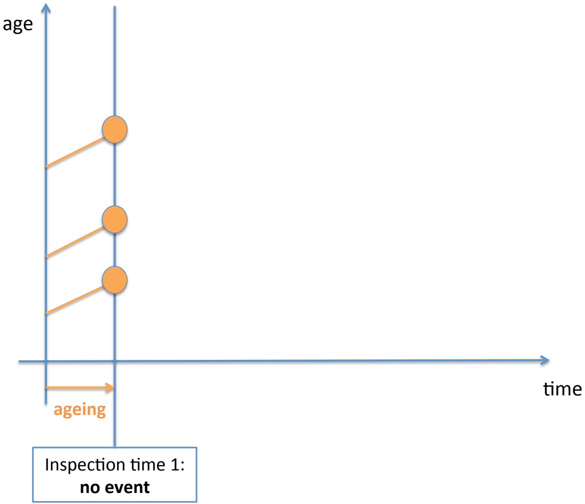
- Simulation by means of the *Thinning algorithm*

* Convergence of sequence of renormalized population processes (large number effect) also holds

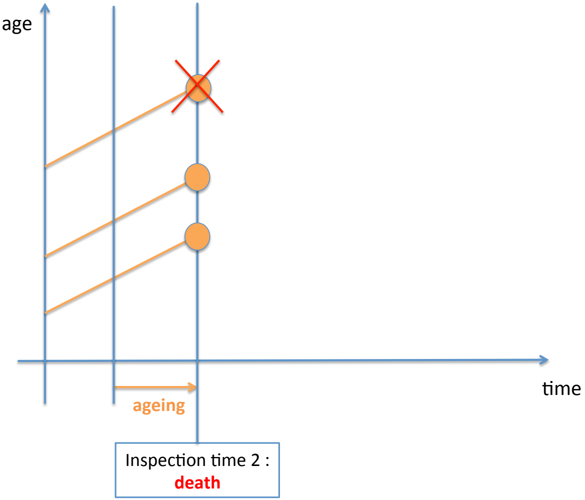
Simulation algorithm: stochastic setting



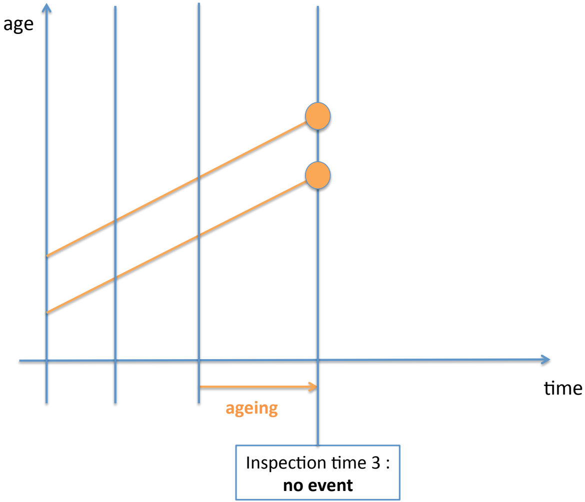
Simulation algorithm: stochastic setting



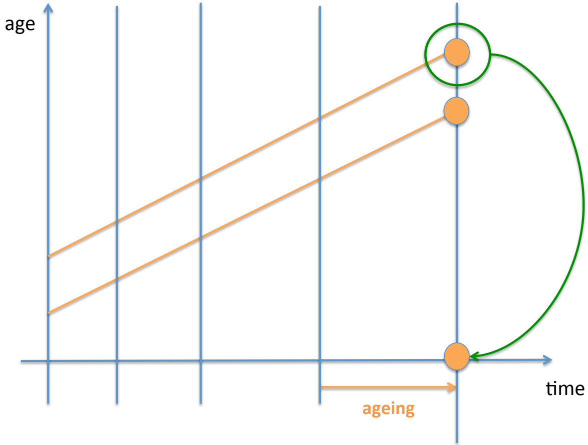
Simulation algorithm: stochastic setting



Simulation algorithm: stochastic setting



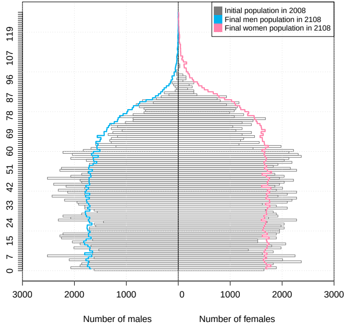
Simulation algorithm: stochastic setting

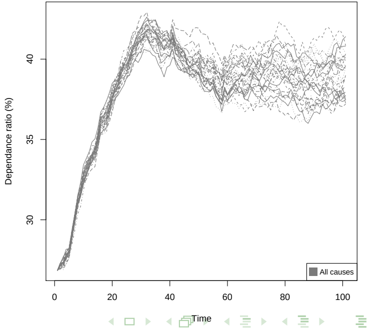


Inspection time 4 :
birth

Simulation example

- From Arnold, Boumezoued, Labit Hardy, El Karoui (2015). *Cause-of-death mortality : What can be learned from population dynamics ?* [hal-01157900]
- Age pyramid and dependency ratios $R_t = \frac{Z_t([65,\infty))}{Z_t([15,65))}$.





What is heterogeneity at the individual level ?

- The way an individual behaves in the population may differ depending on particular factors, or **characteristics** $x \in \mathcal{X}$
- The population is made of several sub-groups:

$$g(a,t) = \sum_{x \in \mathcal{X}} g(x,a,t).$$

- An individual with **characteristics** $x \in \mathcal{X}$ and age a at time t,
  - Dies at rate $\mu(x,a,t)$
  - Gives birth at rate $b(x,a,t)$

What is heterogeneity at the individual level ?

- The way an individual behaves in the population may differ depending on particular factors, or **characteristics** $x \in \mathcal{X}$
- The population is made of several sub-groups:

$$g(a,t) = \sum_{x \in \mathcal{X}} g(x,a,t).$$

- An individual with **characteristics** $x \in \mathcal{X}$ and age a at time t,
  - Dies at rate $\mu(x,a,t)$
  - Gives birth at rate $b(x,a,t)$
    and the newborn has characteristics $x' \sim K^b(x,a,\mathrm{d}x')$

What is heterogeneity at the individual level ?

- The way an individual behaves in the population may differ depending on particular factors, or **characteristics** $x \in \mathcal{X}$
- The population is made of several sub-groups:

$$g(a, t) = \sum_{x \in \mathcal{X}} g(x, a, t).$$

- An individual with **characteristics** $x \in \mathcal{X}$ and age a at time t,
  - Dies at rate $\mu(x, a, t)$
  - Gives birth at rate $b(x, a, t)$
    and the newborn has characteristics $x' \sim K^b(x, a, \mathrm{d}x')$

Characteritics can change during life

Changes its characteristics during life at rate $e(x, a, t)$

What is heterogeneity at the individual level ?

- The way an individual behaves in the population may differ depending on particular factors, or **characteristics** $x \in \mathcal{X}$
- The population is made of several sub-groups:

$$g(a,t) = \sum_{x \in \mathcal{X}} g(x,a,t).$$

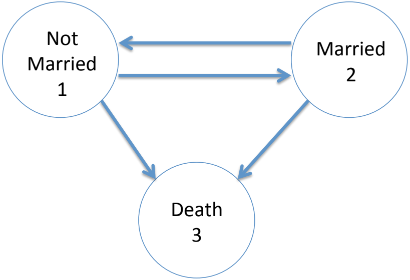
- An individual with **characteristics** $x \in \mathcal{X}$ and age a at time t,
  - Dies at rate $\mu(x,a,t)$
  - Gives birth at rate $b(x,a,t)$
    and the newborn has characteristics $x' \sim K^b(x,a,\mathrm{d}x')$

Characteritics can change during life

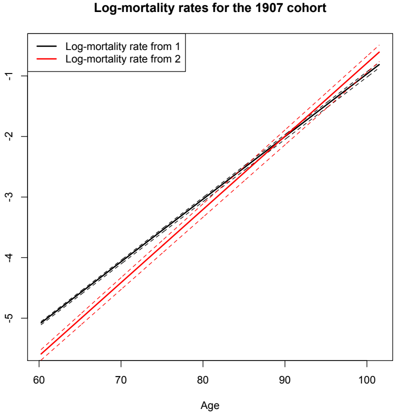
Changes its characteristics during life at rate $e(x,a,t)$ from characteristics x to $x' \sim K^e(x,a,\mathrm{d}x')$ [$e \equiv$ *evolution*]

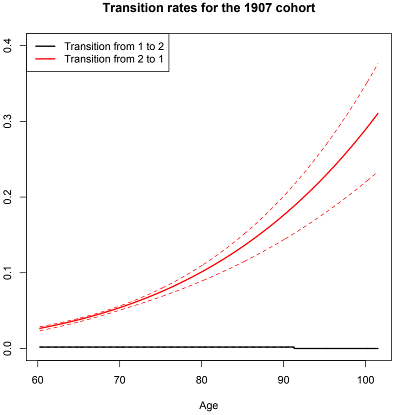
Characteristics changes during life

- Statistical example based on a French representative sample from INSEE
- Focus on **cohort dynamics**: its composition evolves according to deaths and characteristics changes (multistate model)



- Heterogenous mortality compensation law + transition rates

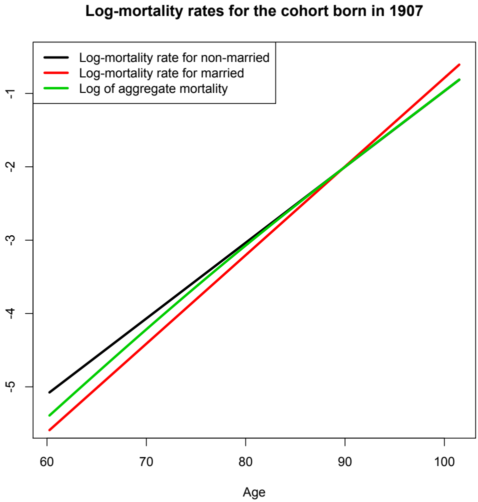




From Boumezoued, El Karoui, Loisel (2015), Measuring mortality heterogeneity dynamics with interval-censored data. *Working paper.*

- Cohort heterogenous dynamics

  $\Rightarrow$ **age pattern of aggregate mortality**



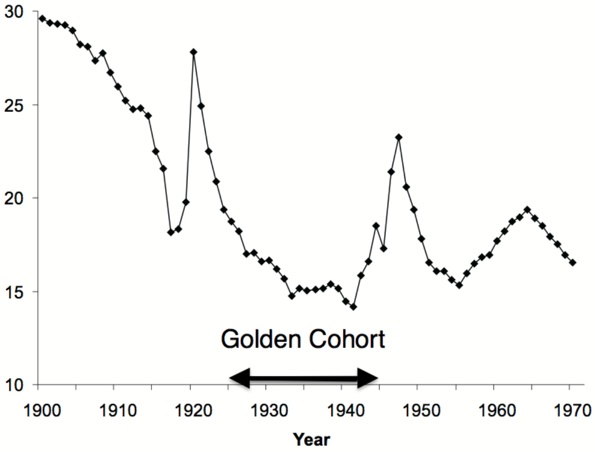
Population heterogenous dynamics

The Cohort Effect: Insights And Explanations, 2004, R. C. Willets

The Golden cohort has experienced higher mortality improvements than earlier and later generations. Some possible explanations:

- Impact of World War II on previous generations,
- Changes on smoking prevalence: tobacco consumption in next generations,
- Impact of diet in early life,
- Post World War II welfare state,
- Patterns of birth rates

Cohort effect (UK)



Data source: www.mortality.org

Figure 6. Crude birth rate per 1,000 population, England and Wales, 1900 to 1970

Impact of heterogenous birth patterns

"One possible consequence of rapidly changing birth rates is that the 'average' child is likely to be different in periods where birth rates are very different. For instance, if trends in fertility vary by socio-economic class, the class mix of a population will change."

The Cohort Effect: Insights And Explanations, 2004, R. C. Willets

Simulation example*

- Reference death rate $\bar{d}(a) = A\exp(Ba)$
- Parameters $A \sim 0.0004$ and $B \sim 0.073$ estimated on French national data for year 1925 to capture proper magnitude
- Group 1 : time independent death rate $d^1(a) = \bar{d}(a)$ and birth rate $b^1(a) = c\mathbf{1}_{[20,40]}(a)$ (c=0.1)
- Group 2 : time independent death rate $d^2(a) = 2\bar{d}(a)$ but $b_2(a,t) = 4c\mathbf{1}_{[20,40]}(a)\mathbf{1}_{[0,t_1)\cup(t_2,\infty)}(t) + 2c\mathbf{1}_{[20,40]}(a)\mathbf{1}_{[t_1,t_2]}(t)$.

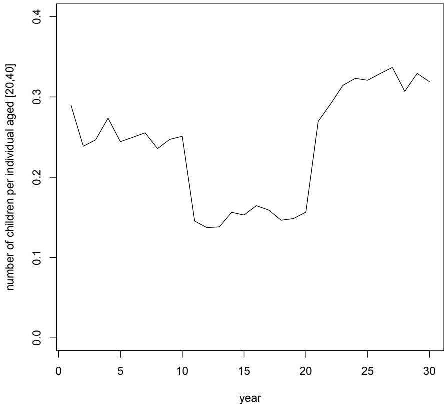
- Constant death rates but reduction in overall fertility between times t_1 (=10) and t_2 (=20)

- Aim: compute standard demographic indicators

*From Bensusan, H., A. Boumezoued, N. El Karoui, S. Loisel. 2010–2015. Impact of heterogeneity in Human population dynamics. *Working paper*

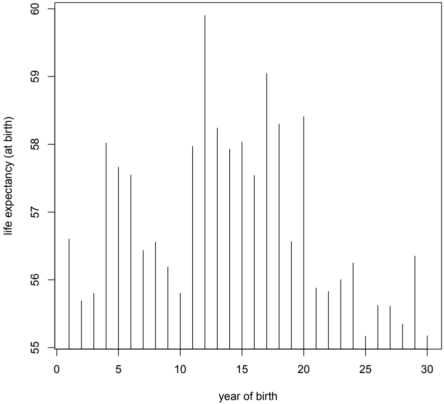
Aggregate fertility

- One trajectory with 10000 individuals (randomly) splitted between groups. Estimation of aggregate fertility



Life expectancy by year of birth

- "Cohort effect" for aggregate life expectancy



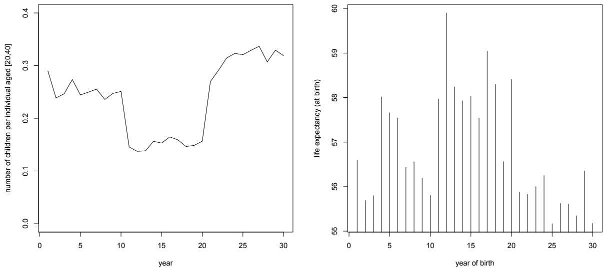


Figure : Observed fertility (left) and estimated life expectancy by year of birth (right)

- Death rates by specific group remain the same
- But reduction in fertility for "lower class" during 10-20 modifies the generations composition
  $\Rightarrow$ "upper class" is more represented among those born between 10 and 20

So, why population dynamics ?

1. A general mathematical framework to understand the joint impact on aggregate mortality of
   - Birth patterns
   - Heterogeneity
   - Demographic stochasticity (idiosyncratic risk)
   - Environment noise (systematic risk) [is included in the model]
2. A simulation toolbox for demographic scenarios generation...
3. ...gathering statistical inputs of various types (heterogenous mortality, birth rates, characteristics changes)

Bibliography

Bensusan, H., A. Boumezoued, N. El Karoui, S. Loisel. 2010–2015.
Impact of heterogeneity in human population dynamics.
Working Paper

Arnold, S., A. Boumezoued, H. Labit Hardy, N. El Karoui. 2015.
Cause-of-death mortality: What can be learned from population dynamics?
HAL preprint Id: hal-01157900

Boumezoued, A., N. El Karoui, S. Loisel. 2015.
Measuring mortality heterogeneity dynamics with interval-censored data.
Working Paper

Cairns, A. J., D. Blake, K. Dowd, A. Kessler. 2014.
Phantoms never die: Living with unreliable mortality data.
Tech. rep., Herriot Watt University, Edinburgh

Willets, RC. 2004.
The cohort effect: insights and explanations.
Cambridge Univ Press