Wrocław University of Economics

Longevity and other types of risk – an integrated approach to measuring risk of household financial plan

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General concept

- 2 adult persons (main members of the household)
- 2 financial goals considered:
  - Retirement
  - Bequest
- 3 sub-classes of consumption:
  - Common (fixed and not assigned to any particular person),
  - Consumption of person 1
  - Consumption of person 2
- 2 investments: of person 1 and person 2 (contributions fixed in real terms)
- 2 available assets: risky and risk-free (returns fixed in real terms)

Included types of risk

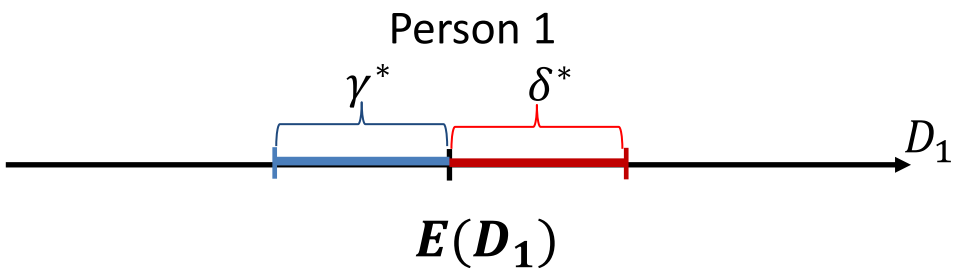
Basic version:

- Life-length risk of two persons (basic version)
  - premature death risk
  - longevity risk

Extended version:

- Time of a child's birth (for each child)
- Rate of return on investments
- Interest rate of loans

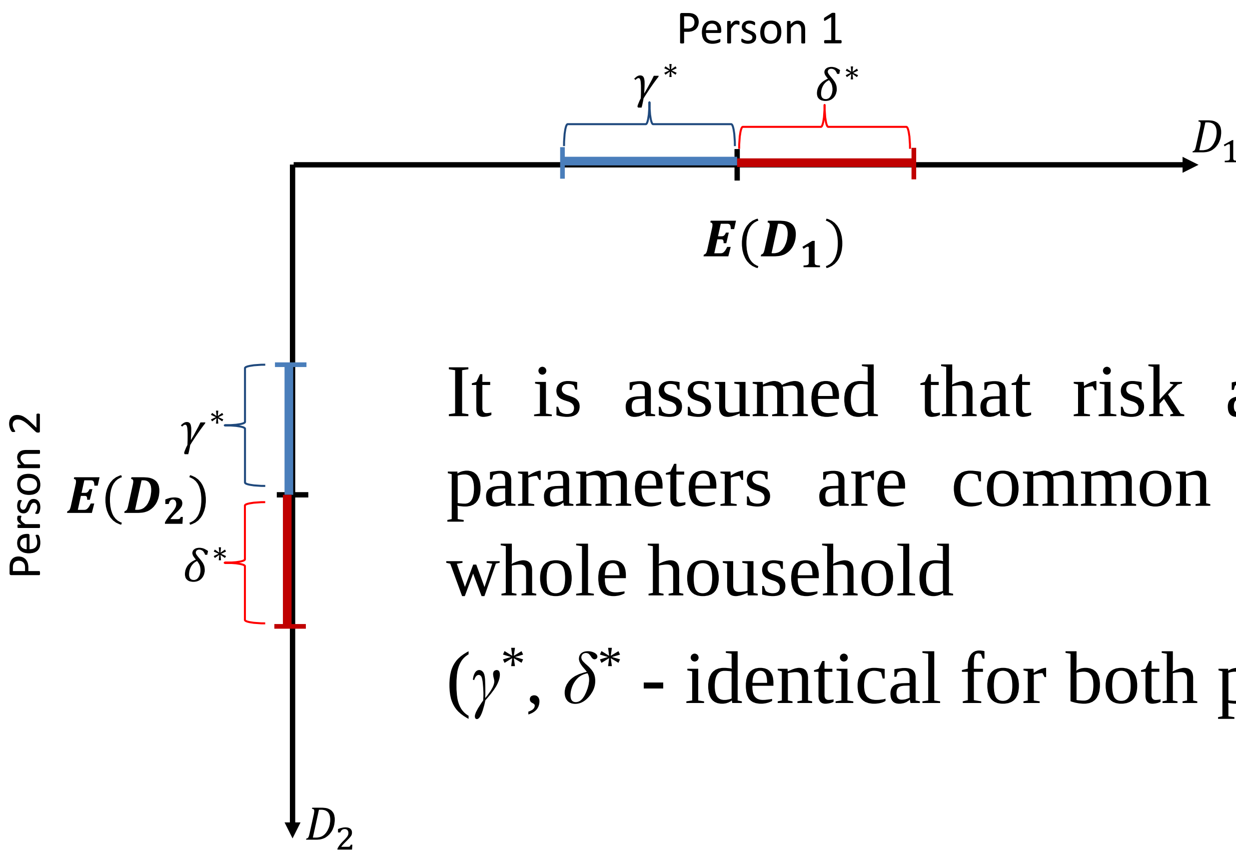
Basic version



For each person, a set of survival scenarios around the expected further life time is considered

It is associated with household's declaration of life-length risk aversion (the broader range the higher aversion)

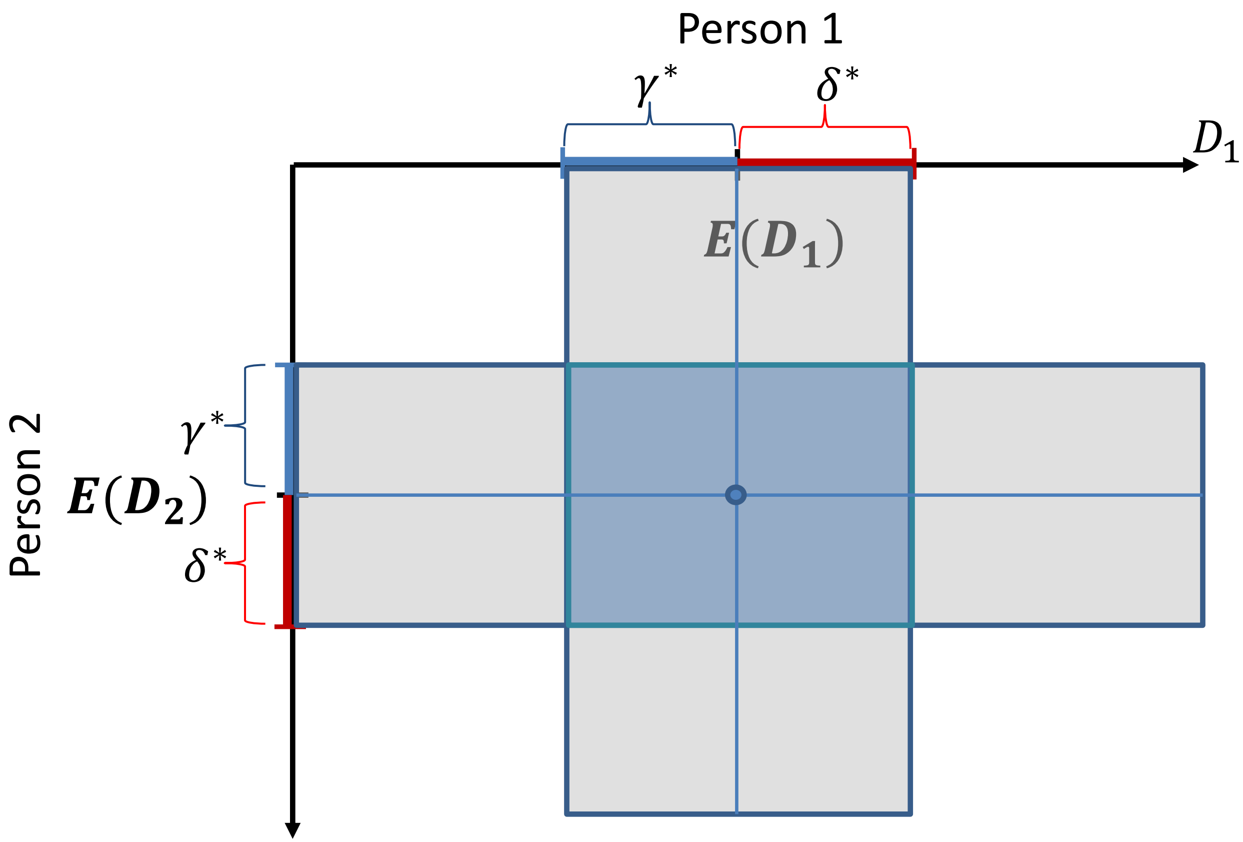
Basic version



It is assumed that risk aversion parameters are common for the whole household

(γ^*, δ^* - identical for both persons)

Basic version



The plan is optimized only for the „range of concern"

Basic version

- Value function

$$V\left(c_0, v\right) =$$

$$= \sum_{D_1^*=E(D_1)-\gamma^*}^{E(D_1)+\delta^*} \sum_{D_2^*=E(D_2)-\gamma^*}^{E(D_2)+\delta^*} p_{D_1^* D_2^*} \underbrace{\left[\begin{array}{l} \alpha \left(\sum_{t=0}^{max\left\{D_1^*, D_2^*\right\}} \frac{1}{\left(1+r_c\right)^t} u\left(C\left(t; D_1^*, D_2^*\right)\right)\left(\gamma\left(t\right)+\delta\left(t\right)\right) \right) + \\ \beta \frac{1}{\left(1+r_B\right)^{max\left\{D_1^*, D_2^*\right\}}} u\left(B\left(max\left\{D_1^*, D_2^*\right\}; D_1^*, D_2^*\right)\right) \end{array} \right]}$$

$$U(c_0, v;\ D_1^*, D_2^*) = \alpha \left(\sum_{t=0}^{max\left\{D_1^*, D_2^*\right\}} \frac{1}{\left(1+r_c\right)^t} u\left(C\left(t; D_1^*, D_2^*\right)\right)\left(\gamma\left(t\right)+\delta\left(t\right)\right) \right) +$$

$$+\beta \frac{1}{\left(1+r_B\right)^{max\left\{D_1^*, D_2^*\right\}}} u\left(B\left(max\left\{D_1^*, D_2^*\right\}; D_1^*, D_2^*\right)\right)$$

Extended version

- Value function

$$V\left(c_0, v, \boldsymbol{\kappa}; \mathbf{Z}^*\right) =$$

$$\sum_{D_1^*=E(D_1)-\gamma^*}^{E(D_1)+\delta^*} \sum_{D_2^*=E(D_2)-\gamma^*}^{E(D_2)+\delta^*} p_{D_1^* D_2^*} \left[\sum_{Ch1^*=t_0}^{W(D_1^*;D_2^*)} \sum_{Ch2^*=Ch1^*}^{W(D_1^*;D_2^*)} p_{Ch1^* Ch2^*} \left[\sum_{S^*(T_H^*)=S_L(T_H^*)}^{S_U(T_H^*)} \sum_{S^*(T_B^*)=S_L(T_B|S^*(T_H^*))}^{S_U(T_B|S^*(T_H^*))} p_{S^*(T_H^*) S^*(T_B^*)} \left[\sum_{R_M^*=R_L}^{R_U} \sum_{\eta^*=\eta_L}^{\eta_U} p_{R_M^* \eta^*} \left[U\left(c_0, v, \kappa; Z^*\right) \right] \right] \right] \right]$$

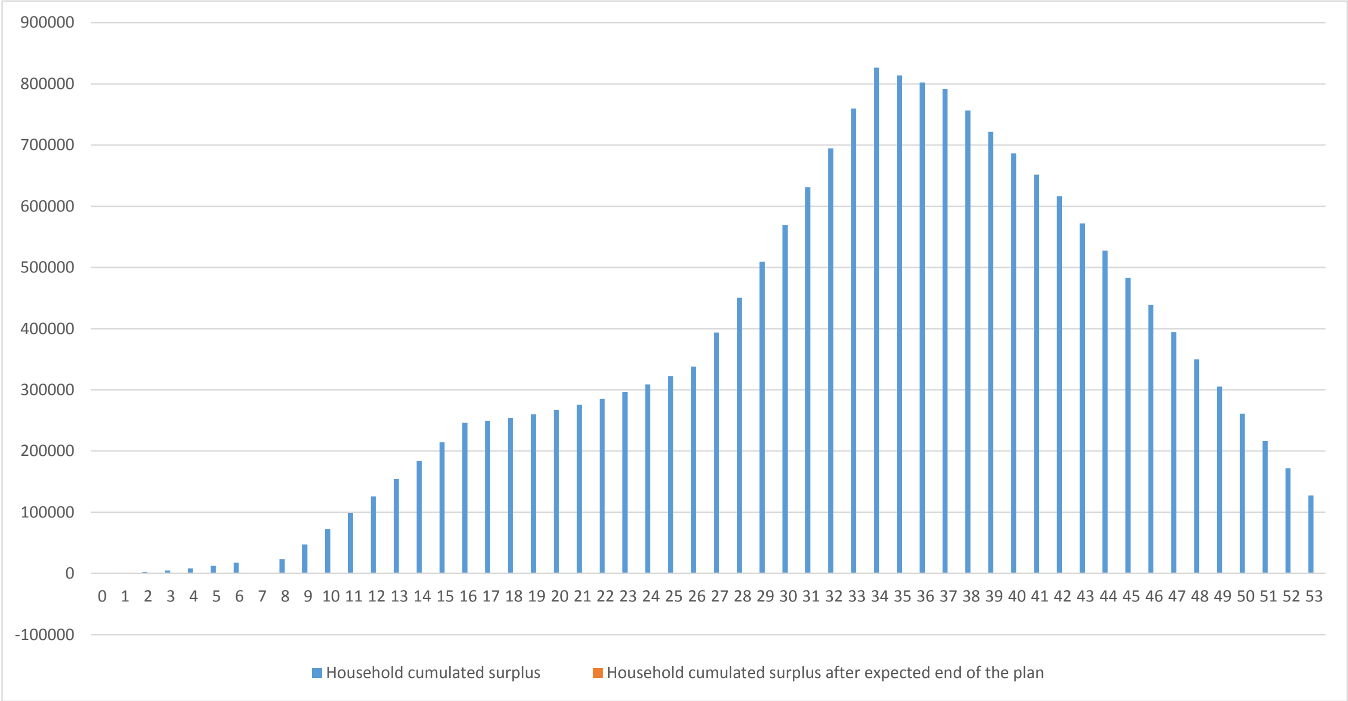
$$U\left(c_0, v;\ Z^*\right) = \alpha \left(\sum_{t=0}^{max\left\{D_1^*, D_2^*\right\}} \frac{1}{\left(1+r_c\right)^t} u\left(C\left(t; Z^*\right)\right)\left(\gamma\left(t\right) + \delta\left(t\right)\right) \right) +$$

$$+\beta \frac{1}{\left(1+r_B\right)^{max\left\{D_1^*, D_2^*\right\}}} u\left(B\left(max\left\{D_1^*, D_2^*\right\}; Z^*\right)\right)$$

$$\mathbf{Z}^* = \left[D_1^*, D_2^*, Ch_1^*, \ldots, Ch_m^*, R_M^*, \eta^*, S^*\left(T_H^*\right), S^*\left(T_B^*\right)\right]$$

Extended version

- Trajectory of a cumulated net cash flow under a given $\mathbf{z}^*$ scenario



Extended version

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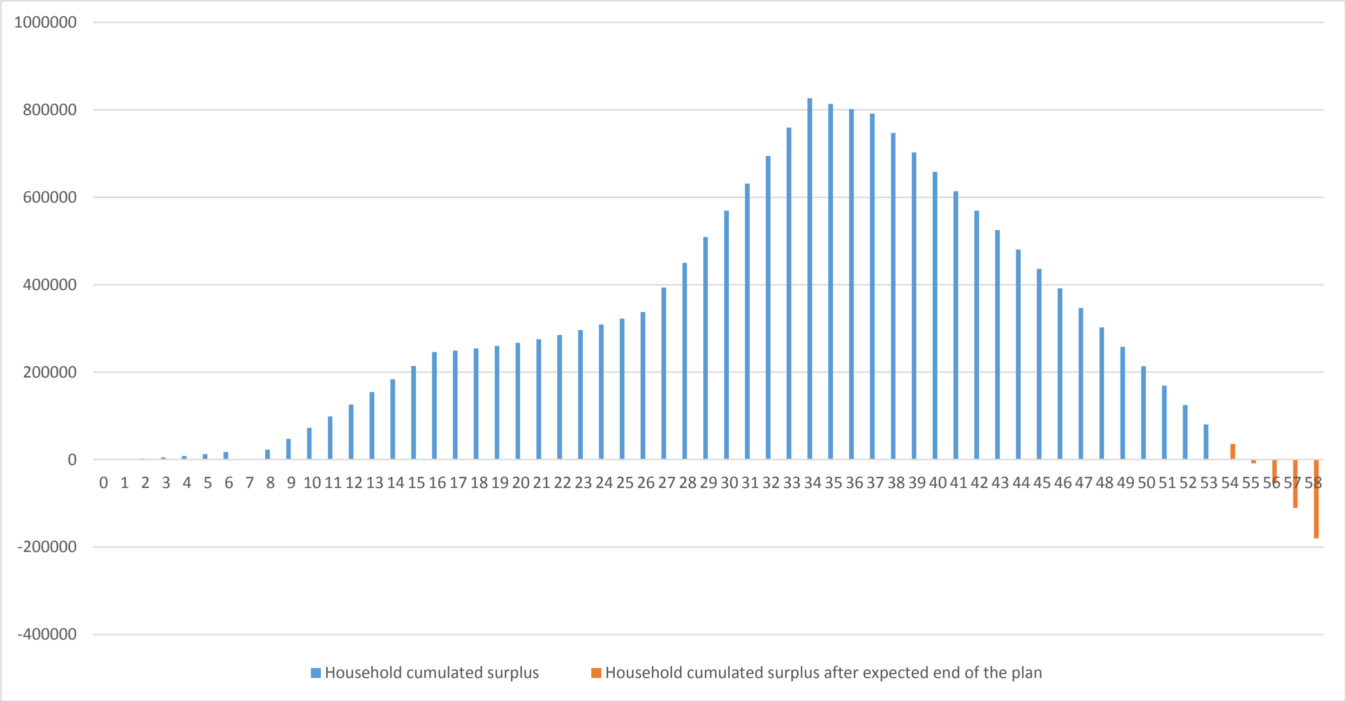

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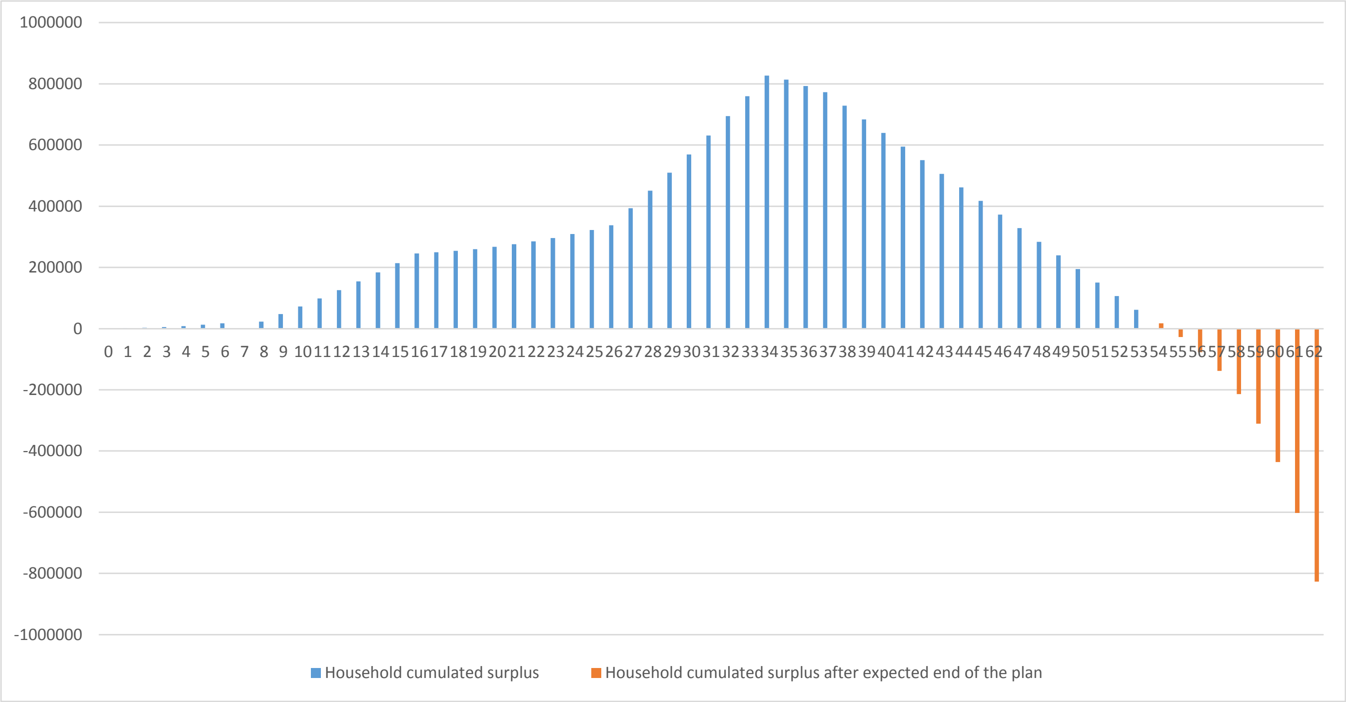

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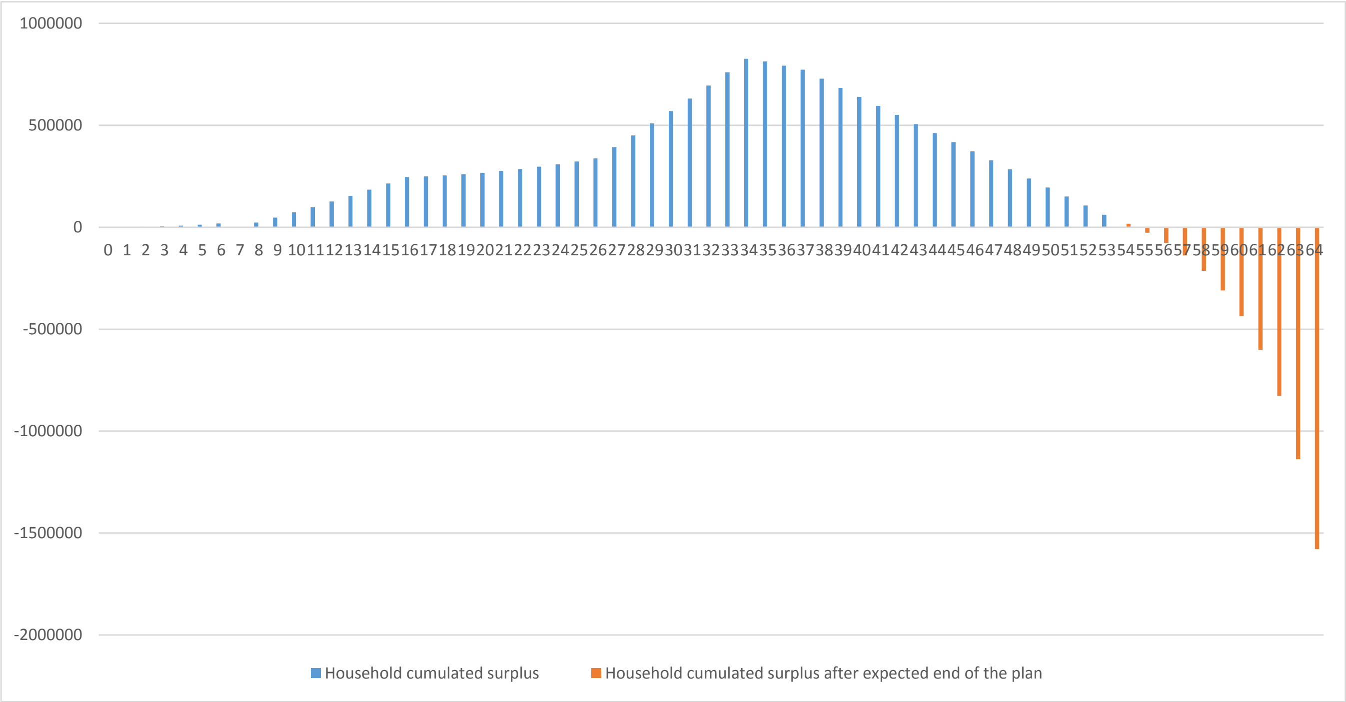


Integrated measures of risk

- Household members should be informed how the financial plan may perform under **different scenarios** (that is, not only the expected one)
- They are also concerned about how the performance of their financial plan is influenced by **multiple risk factors of different nature**
- Their **decision to accept or reject a plan** depends, amongst others, on the above
- In the model, performance of **cumulated net cash flow under different scenarios reflects influence of all risk factors** that are taken into consideration in a given version of the model
- It is, thus, postulated that **integrated measures of risk** for household financial plans constructed under the model are **based on cumulated net cash flow** as the main risk variable

Integrated measures of risk

- measures based on residual wealth
    - **Residual Wealth at Risk**
    - Residual Wealth Volatility
    - Residual Wealth Aspiration Level
- measures indicating problems during whole life-cycle
  - measures indicating threats to financial plan realization
    - **Lifetime Cumulated Net Cash Flow at Risk**
    - Incremental Shortfall
    - Shortfall Scenario Probability
  - measures of household default risk
    - **Household Default Probability**

Residual Wealth at Risk

- in each scenario (i.e., for each of the considered values $\boldsymbol{Z}^*$ of the random vector $\boldsymbol{Z}$ – comp. eq.) there is calculated a deviation from the applied benchmark of cumulated net cash flow at household end,
- a left-tail quantile of deviations, corresponding to a pre-set tolerance level (small significance level), is determined on the basis of the scenarios,

Residual Wealth at Risk

$$P(CSp_{T_B} \leq XCSp_{T_B} - RWaR) = q$$

May by 0

where:

$XCSp_{T_B}$ – benchmark of cumulated net cash flow at household end,

CSp_{T_B} – cumulated net cash flow at household end,

$RWaR$ – Residual Wealth at Risk,

q – $RWaR$ tolerance level (a small significance level).

Residual Wealth at Risk

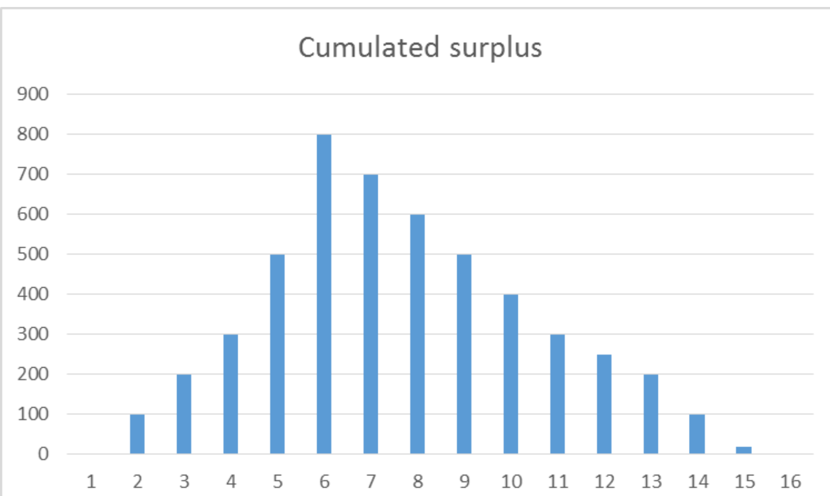
Advantages:

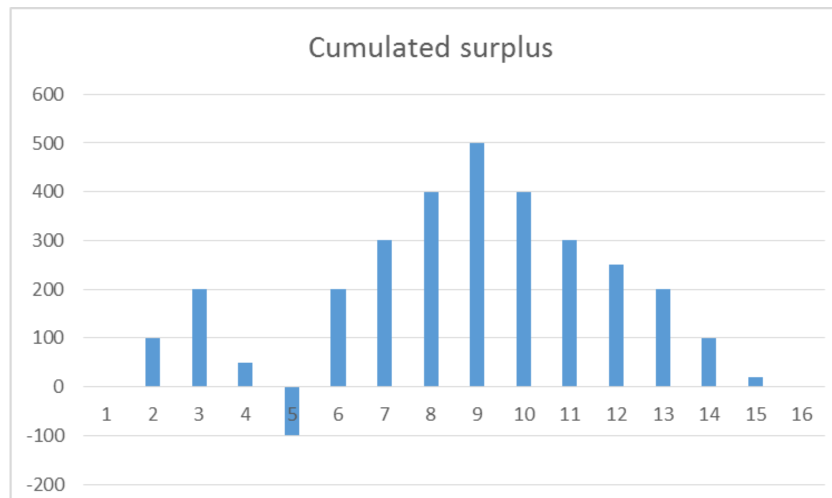
- grasps in one measure the most important information about riskiness of a given plan, namely potential shortfall at the household end (all shortfalls that the household faces along the line but then recovers from are less important than the residual shortfall of the plan)
- straightforward interpretation: with probability 1-q the residual wealth of the household will not fall below the benchmark level of $XCSp_{T_B}$

Residual Wealth at Risk

Disadvantages:

- insensitive to shortfalls along the line



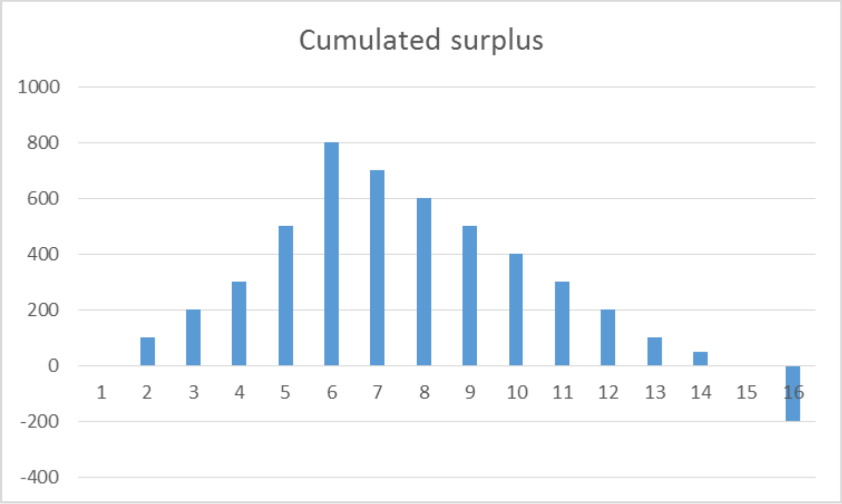


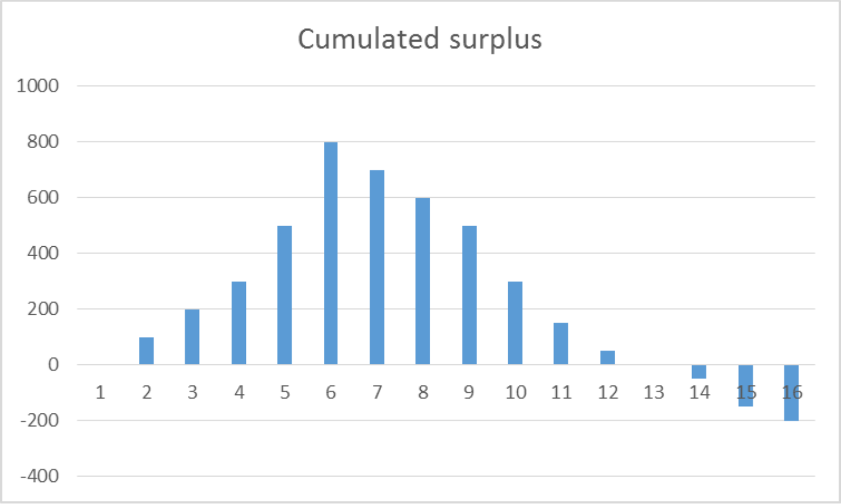
Does not distinguish between scenarios like these

Residual Wealth at Risk

Disadvantages:

- insensitive to the time during which the household incurs a shortfall





Does not distinguish between scenarios like these

Lifetime Cumulated Net Cash Flow at Risk

- in each of the considered scenarios, all cumulated shortfalls are summed up (cumulating of cumulated shortfalls),
- from all scenarios there is taken a quantile of the sum of cumulated shortfalls, corresponding to a predefined small tolerance level.

Lifetime Cumulated Net Cash Flow at Risk

$$P(CCSh \leq -LCaR) = q$$

where q is $LCaR$ tolerance level (a small significance level)

$$CSh_t = \begin{cases} CSp_t & \text{if} \quad CSp_t < 0 \\ 0 & \text{if} \quad CSp_t \geq 0 \end{cases}$$

$$CCSh^{(\mathbf{Z}^*)} = \sum_{t=1}^{T_B^*} CSh_t^{(\mathbf{Z}^*)}$$

where:

$CSh_t^{(Z^*)}$ - cumulated shortfall at a moment t, under a scenario $\mathbf{Z}^*$,

$CCSh^{(Z^*)}$ - sum of cumulated shortfalls from the start of the plan until the end of the particular scenario $\mathbf{Z}^*$

Lifetime Cumulated Net Cash Flow at Risk

Advantages:

- takes into account cumulated shortfalls occurring during the whole life of the household
- takes into account the length of time during which the household remains under cumulated shortfalls
- may be interpreted as the sum of all loans taken for contingency financing

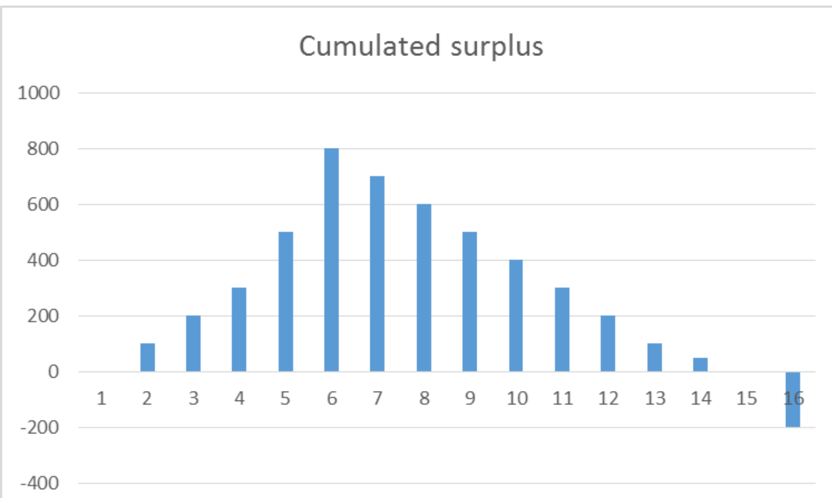
Lifetime Cumulated Net Cash Flow at Risk

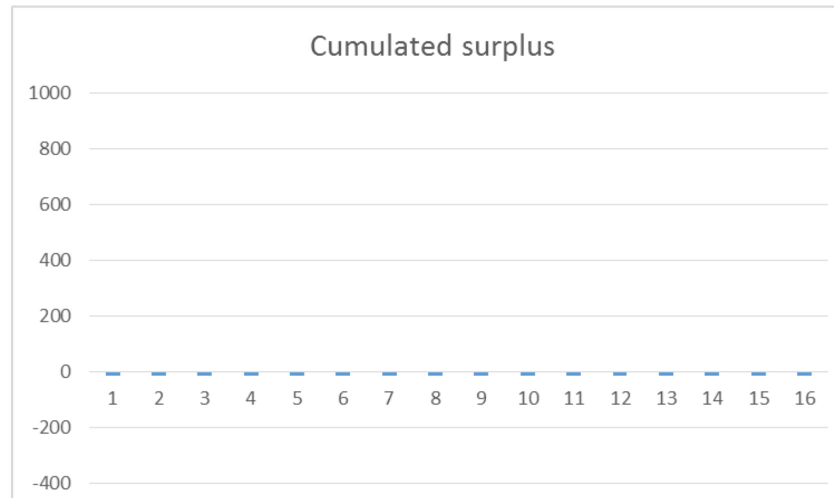
Disadvantages:

- using the approach in which cumulated shortfall is once more cumulated may seem awkward
- does not distinguish between a series of small cumulated shortfalls year by year and one big cumulated shortfall if the sum of the first is identical as the second (time value of money taken into account)

Lifetime Cumulated Net Cash Flow at Risk

Disadvantages:



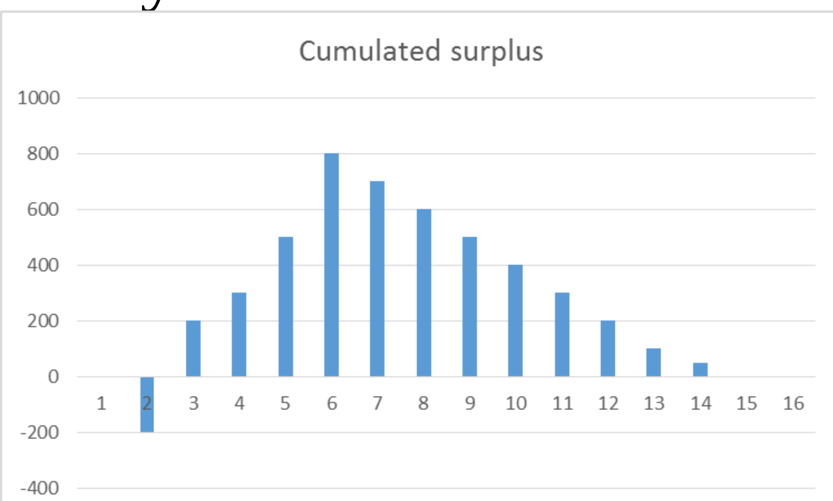


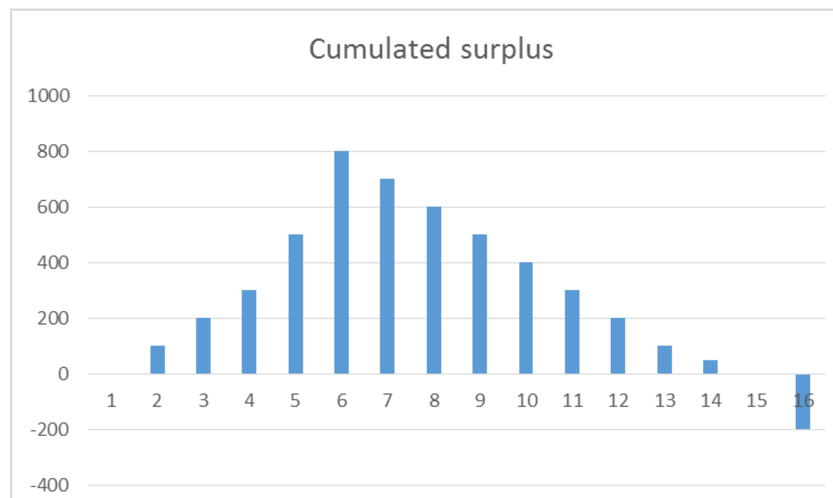
Does not distinguish between scenarios like these

Lifetime Cumulated Net Cash Flow at Risk

Disadvantages:

- nor does it distinguish between a recoverable shortfall at the beginning and unrecoverable one at the end if they are of an identical size





Does not distinguish between scenarios like these

Household Default Probability

- a default threshold is determined at the moment t_0 (e.g., creditworthiness of the household at t_0); then, the default threshold changes in time with financial situation of the household.
- scenarios in which cumulated shortfall exceeds the default threshold are identified as default scenarios; formally there are defined in the formula:

$$\boldsymbol{T}^* = \boldsymbol{Z}^* : \underset{t=1,..,T_B^*}{\exists} CSp_t^{\left(\boldsymbol{z}^*\right)} < DTh_t^{\left(\boldsymbol{z}^*\right)}$$

Household Default Probability

- Probabilities of default scenarios are probabilities of such scenarios that fulfill definition given by the equation . A default scenario probability is defined by the formula:

$$p_{\boldsymbol{T}_i^*} = \begin{cases} p_{\boldsymbol{z}_i^*} & \text{if} \quad \underset{t=1,..,T_B^*}{\exists} CSp_t^{(\boldsymbol{z}_i^*)} < DTh_t^{(\boldsymbol{z}_i^*)} \\ 0 & \text{if} \quad \underset{t=1,..,T_B^*}{\forall} CSp_t^{(\boldsymbol{z}_i^*)} \geq DTh_t^{(\boldsymbol{z}_i^*)} \end{cases}$$

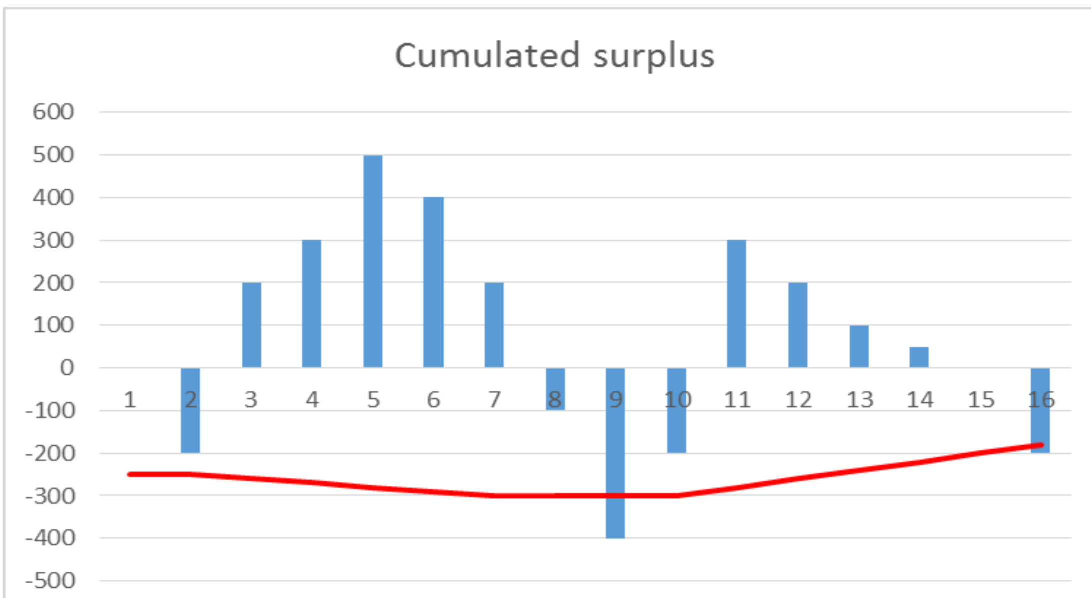
Household Default Probability

- Probabilities of default scenarios are summed up
- Probability that any of the default scenario realizes is the measure of risk is obtained:

$$HDP = \sum_{i=1}^{n} p_{\boldsymbol{T}_i^*}$$

Household Default Probability

- Default threshold
  - the minimum cumulated net cash flow (maximum cumulated shortfall) below which the household is not able to finance its shortfalls by debt any more



Household Default Probability

- Default threshold (*DTh*) – rule of determining
  - *DTh* changes over time
  - Calculated on the basis of household potential free cash flow (*PFCF*)
  - Household *PFCF* includes net cash flow from planned debt, but it does not include contingency loans
  - Loans for contingency financing are the second element determining default or non-default status
    - Minimum possible payment (*MPP*) is subtracted from *PFCF*
  - Default in a period #*t* if: $\boldsymbol{PFCF_t < MPP_t}$

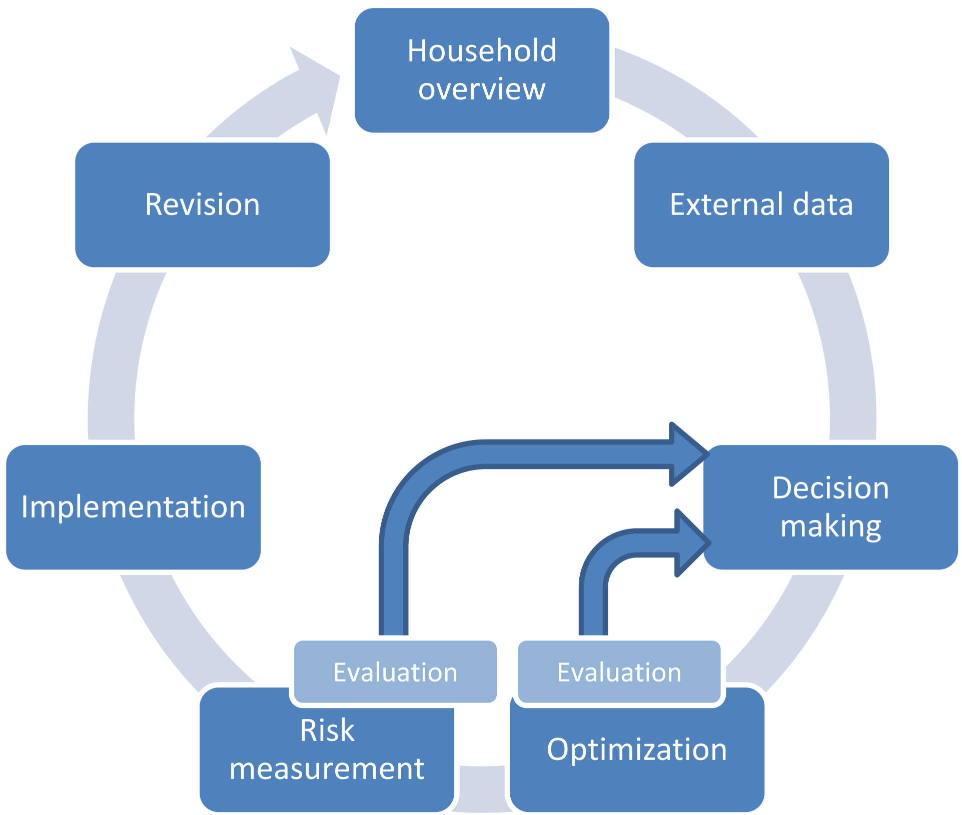
Household Default Probability

Advantages:

- takes into account a default threshold
- its interpretation is rather intuitive (probability of default)

IMHR – How to use them?

- Expected discounted utility includes information about risk after all; why IMHR are useful too?



Conclusions

- No integrated risk measures for household life-long financial plan have been proposed so far
- Integrated measures of risk supply households with a new decision criterion
- Offer a new tool for comparing financial plans
- May be used in other models of household financial planning

Thank You for Your Attention