A Practical Approach of Natural Hedging for Insurance Companies

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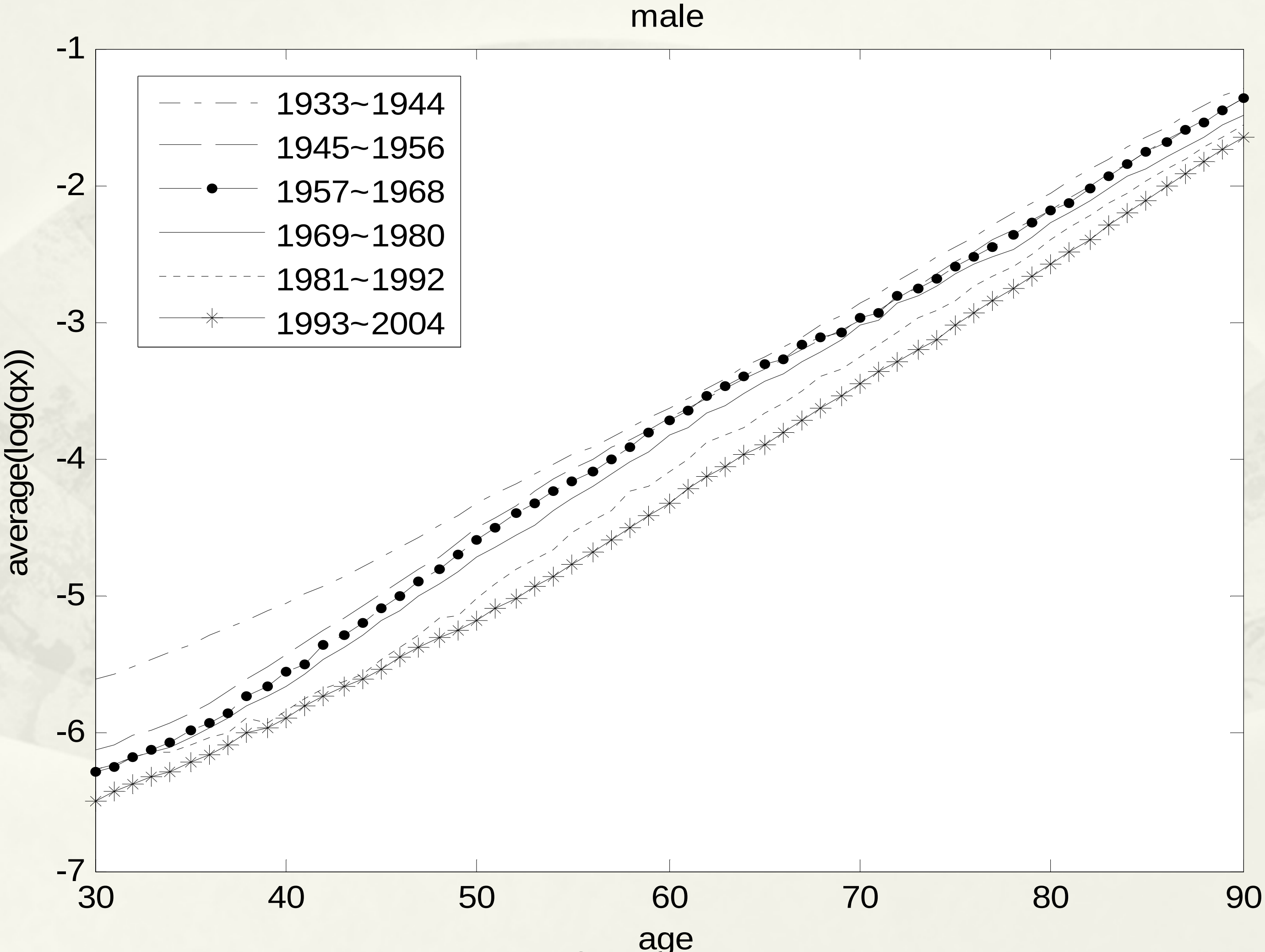
Background
–Mortality Improvement–



Background
–Mispricing Problems–

- Currently, actuaries still have mispricing problems with both life and annuity products for the reason of without considering enough mortality improvement.

- Life insurers may gain a profit whilst annuity insurers may suffer losses because of longevity risk,

Background
–Mispricing Problems–

- This mispricing problem commonly exists in the countries with official static life or annuity tables issued by governments or actuarial societies in which all insurance companies follow these official tables to price life or annuity products. (e.g., Taiwan)

Solutions
–Mispricing Problems–

- Constructing good mortality models for the use of pricing is one solution to hedge longevity risk for both life insurance and annuity products.

- However, because of market competition,

  -- not able to sell annuity with accurate prices since it might be too expensive to sell these annuity products.

  -- not willing to sell their life products with lower prices because most insurance companies have higher prices by using the concept of static mortality table to price life products.

Solutions
–Mispricing Problems–

- Another possible solution to hedging longevity risk is to use the mortality derivatives, such as

  --Survival Bonds (Blake and Burrows , 2001)

  ---Survival Swap (Cairn, Blake and Dowd , 2006)

- Mortality derivatives are easy and convenient to use, but Insurance companies may pay a large amount of transaction cost on mortality derivatives.

Solutions
–Mispricing Problems–

- Another alternative solution is

  "Natural Hedging Strategy".

- They can optimize the allocation of annuities and life insurances to hedge longevity risk internally with low cost.

Literature Review

- Wang, Yang and Pan (2003) investigate the influence of the changes of mortality factors and propose an immunization model to hedge mortality risks.
- Wang, Huang, Yang and Tsai (2010) analyze the immunization model mentioned above and use effective duration and convexity to find the optimal product mix for hedging longevity risk.
- Wang, Huang and Wang (2013) propose a natural hedging model that account for both the variance and mispricing effects of longevity risk at the same time. They use experience mortality rates, obtained from life insurance companies, rather than population mortality data for life insurance and annuity products.

Flaws of Previous Literatures

- Previous literatures ignore the risk of mispricing due to misjudging the correct trend of future mortality improvement and assume that the only uncertainties are volatilities of mortality error term.
- Previous literatures can only deal with a simple case which is "hedging of a group of certain age life insurance requires a group of another certain age annuity".

Contributions

- This paper can deal with the cases of complicate combinations of life and annuity policies in practice. With this optimal strategy, insurance company can hedge the whole portfolios of life and annuity by simply controlling the suitable proportions of premiums in each age of policyholders for both life and annuity products.

Mortality Model: Mitchell et al. (2013) model

$$\log m(t+1,x) - \log m(t,x) = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\kappa_t^{(3)}$$

The model is calibrated by SVD.

To consider the copula structure into the model, we reduce model as

$$\log m(t+1,x) - \log m(t,x) = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)}$$

The method of the estimation is the same as SVD.

	ml $\kappa_t^{(2)}$	ma $\kappa_t^{(2)}$	fl $\kappa_t^{(2)}$	fa $\kappa_t^{(2)}$
skewness	-0.774	-0.202	-0.295	0.165
skew std.	0.594	0.594	0.594	0.594
Excess kurtosis	0.507	2.300	-0.914	0.543
kur std.	1.188	1.188	1.188	1.188
JB	1.882	3.864	0.839	0.286
p-value	0.126	0.044	0.482	>0.50

Which model is better?
Normal, VG or NIG?

ma $\kappa_t^{(2)}$

	LLF	AIC	BIC	LLF Rank	AIC Rank	BIC Rank
Normal	-63.316	64.316	64.733	3	3	3
VG	-60.466	61.466	61.882	2	2	2
NIG	-58.547	59.547	59.964	1	1	1

Marginal distribution

ml $\kappa_t^{(2)}$: Normal

ma $\kappa_t^{(2)}$: NIG

fl $\kappa_t^{(2)}$: Normal

fa $\kappa_t^{(2)}$: Normal

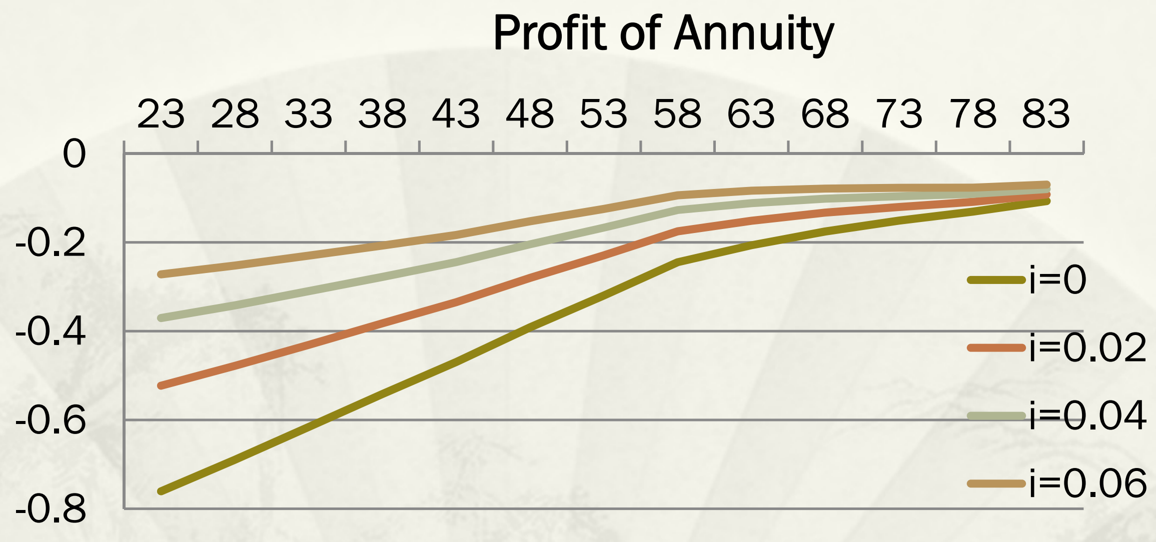
Correlation

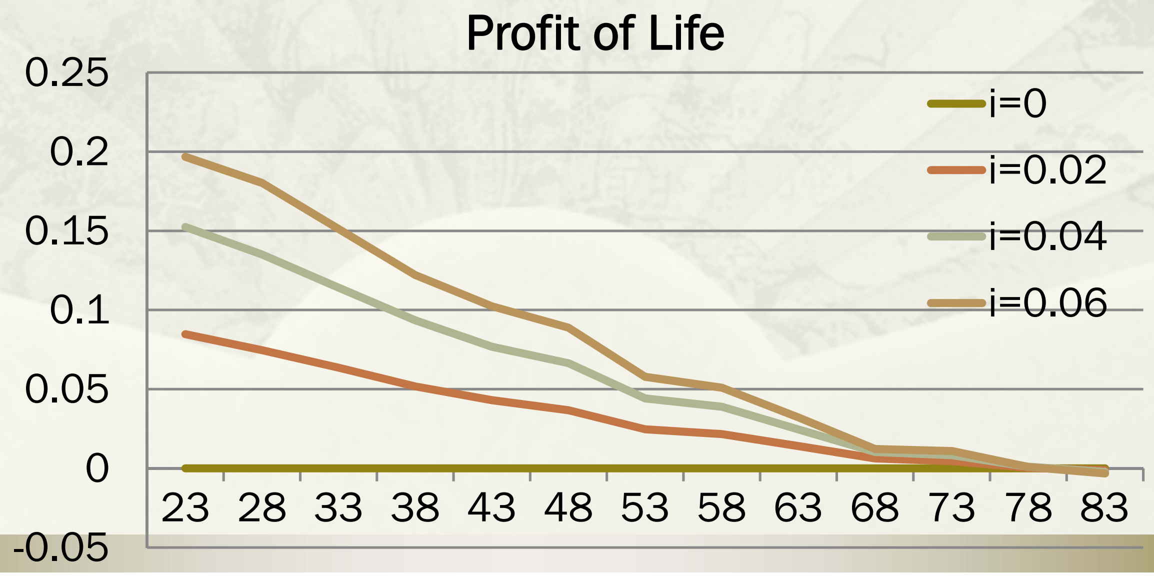
$\kappa_t^{(2)}$	ml	ma	fl	fa
	Pearson's Linear correlation			
ml	1	0.6473	0.0324	0.5163
ma	0.6473	1	-0.2609	0.7345
fl	0.0324	-0.2609	1	-0.1960
fa	0.5163	0.7345	-0.1960	1
	Spearman's rho			
ml	1	0.4853	0.1029	0.3995
ma	0.4853	1	-0.4118	0.5539
fl	0.1029	-0.4118	1	-0.2475
fa	0.3995	0.5539	-0.24755	1
	Kendall's tau			
ml	1	0.3676	0.1176	0.2794
ma	0.3676	1	-0.2794	0.4118
fl	0.1176	-0.2794	1	-0.1912
fa	0.2794	0.4118	-0.1912	1

Copula models

Copula models	LLF	AIC	BIC	LLF rank	AIC rank	BIC rank
Clayton copula	18.2989	-17.2989	-16.882	2	2	2
Gumbel copula	2.3255	-1.3255	-0.9089	3	3	3
Gaussian copula	29.4657	-23.465	-20.966	1	1	1

Mispricing
in male-life and male-annuity





Objective function

- Weight:

$$W = [w_1, w_2, \cdots, w_{52}]^T$$, w_i is the proportion invests i the ith policy

$$\text{if } i = \begin{cases} 1 \sim 13, w_i \text{ represent the weight invest in male} - \text{annuity aged}(18 + 5i) \\ 14 \sim 26, w_i \text{ represent the weight invest in male} - \text{insurance aged}(18 + 5(i - 13)) \\ 27 \sim 39, w_i \text{represent the weight invest in male} - \text{annuity aged}(18 + 5(i - 26)) \\ 40 \sim 52, w_i \text{represent the weight invest in male} - \text{insurance aged}(18 + 5(i - 39)) \end{cases}$$

- Profit: $\Pi = [\pi_1, \pi_2, \cdots, \pi_{52}]^T$

$$\pi_i = \frac{Px|_{period} - Px|_{cohort}}{Px|_{period}}$$

- Covariance

$$\Omega = \begin{bmatrix} \sigma_{1,1} & \cdots & \sigma_{1,52} \\ \vdots & \ddots & \vdots \\ \sigma_{52,1} & \cdots & \sigma_{52,52} \end{bmatrix}, \sigma_{i,j} = \mathrm{cov}(\pi_i, \pi_j)$$

Objective function

$$\mathcal{OF}: \min \; W^T \Omega W + \beta \cdot W^T \Pi$$

$$\text{subject to } \; W^T 1 = 1 \, , 0 \leq W \leq 1$$

$$\rightarrow \frac{\partial \, \mathcal{OF}}{\partial \, W} = 2\Omega \, W + \beta \Pi = 0$$

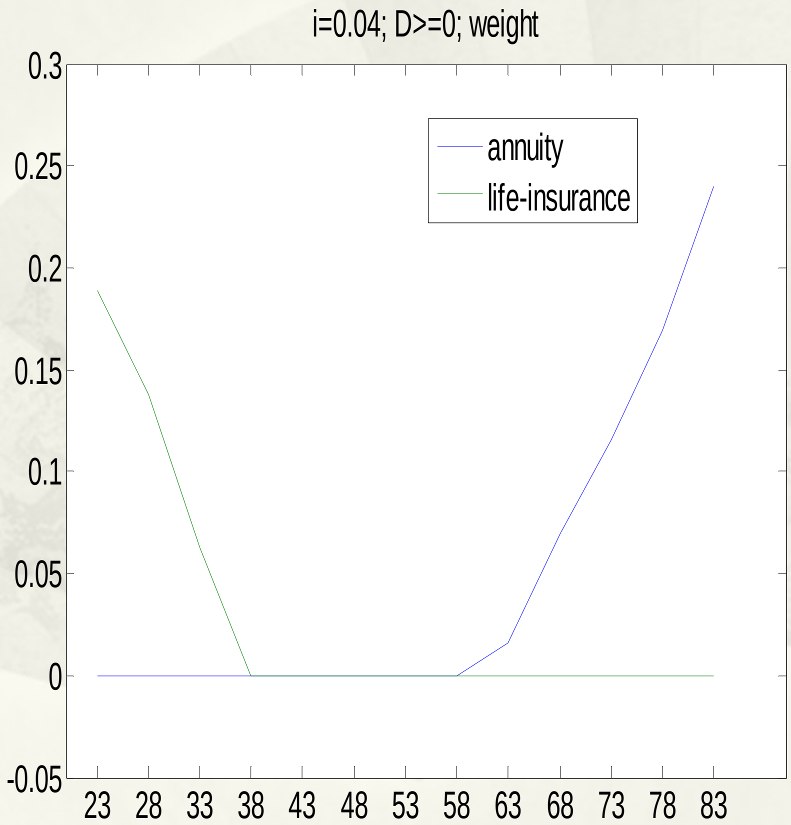
$$\rightarrow W = \; -\frac{\beta}{2} \bar{\Omega}^{-1} \Pi$$

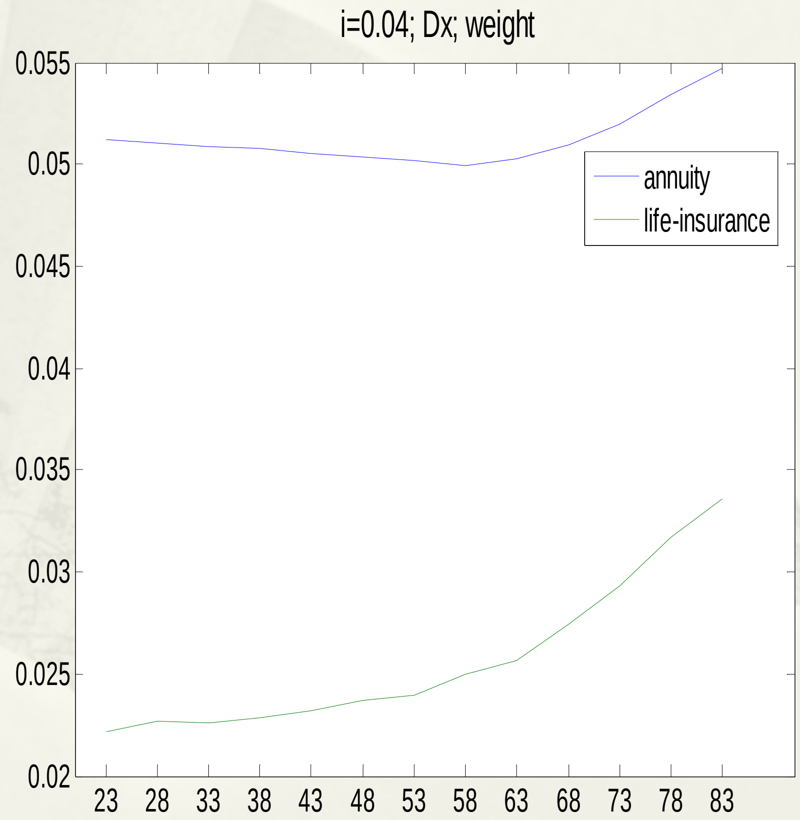
Optimal Allocation of Premiums

Min $W^T \Omega W$ where $W^T \Pi \geq 0$

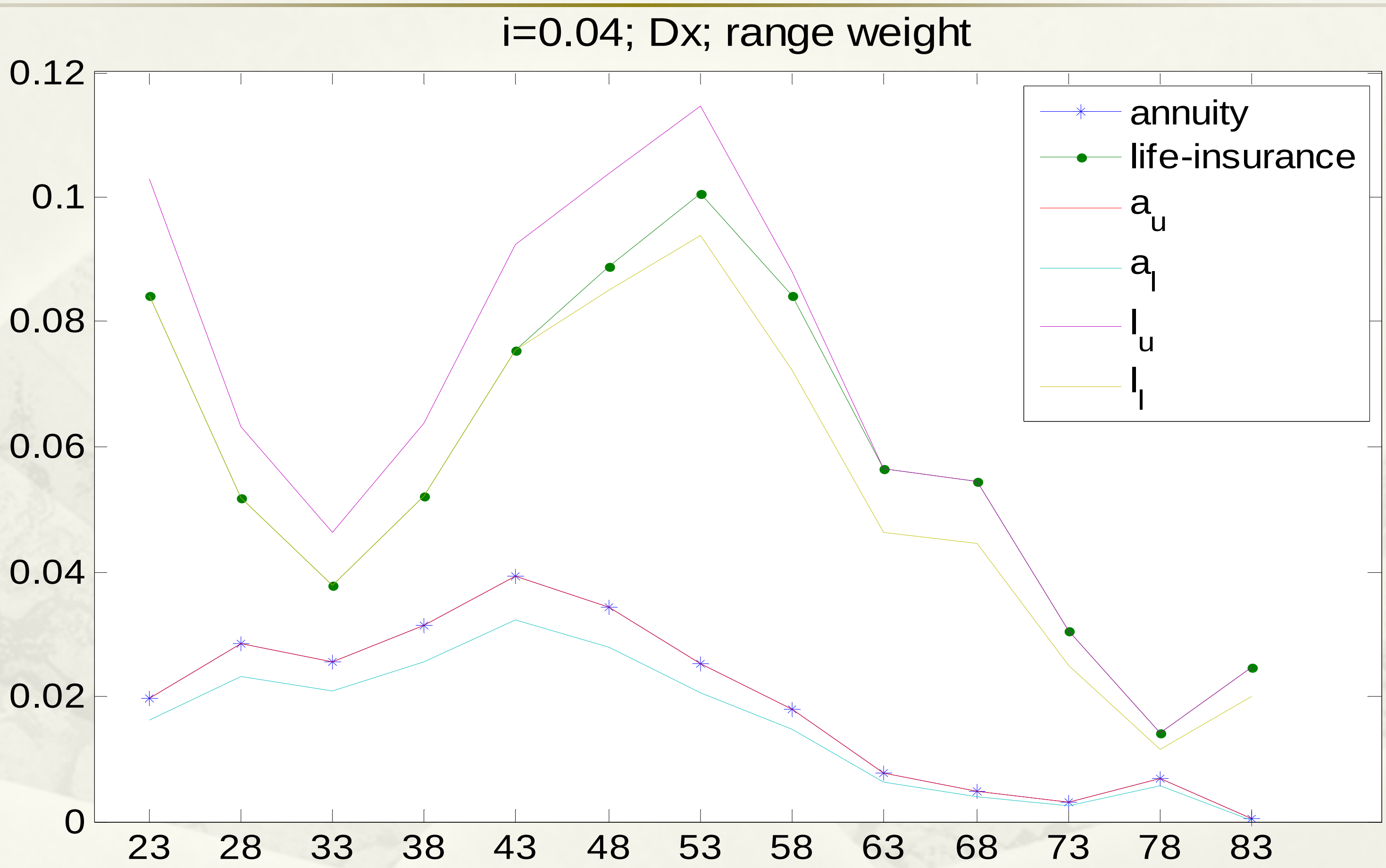
Min $W^T \Omega W$

* D=



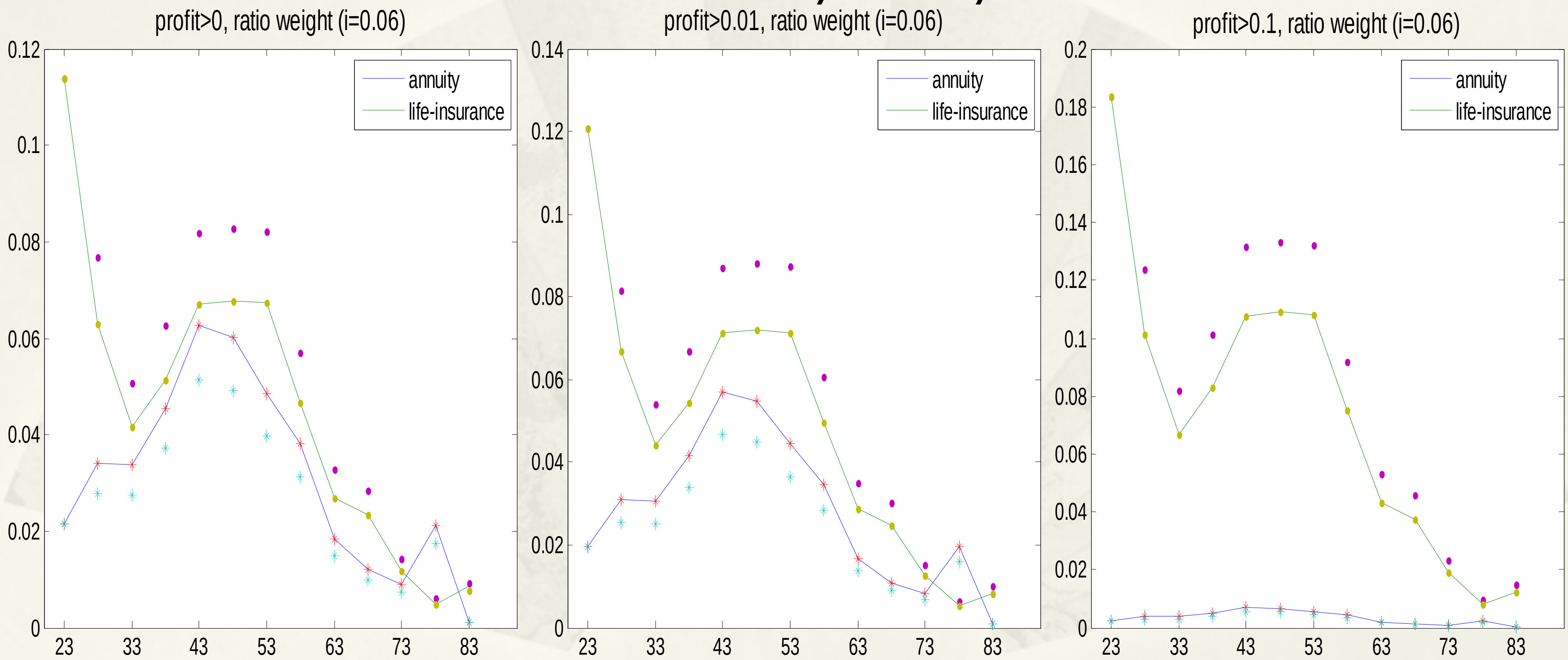


With Insurer's Current Constraints



With Insurer's Current Constraints

- Profit constrain: 0.1% / 1% / 10%



variance is 0.0048

variance is 0.0056 (profit=0.01),

variance is 0.0169 (profit=0.1)

Conclusion and Suggestion

- We find that insurance companies should hold more annuity premiums than life premiums in order to reduce the mispricing volatility.
- However, in that case, insurance companies reduce the mispricing volatility with negative profit of mispricing.

Conclusion and Suggestion

- However, ignoring the constraint of non-negative profit and only consider volatility risk, insurance companies could suffer deficit from mispricing.
- With the constraint of non-negative profit, insurance companies should hold higher proportions of life premiums for the younger policyholder and hold higher proportions of annuity premiums for the elder policyholder in order not to have a deficit due to mispricing.