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On the Instability of Long-run Money Demand and the Welfare Cost of Inflation in the U.S.

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On the Instability of Long-run Money Demand and the Welfare Cost of Inflation in the U.S.

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Abstract

We evaluate the policy implications of measuring the welfare cost of inflation accounting for instabilities in the long-run money demand for the U.S. over the period 1900-2013. We extend the analysis and reassess the results reported in Lucas (2000) and Ireland (2009), also considering the recent theoretical contributions of Lucas and Nicolini (2015) and Berentsen et al. (2015). Breaks in the long-run money demand give rise to regime-dependent welfare cost estimates. We find that the welfare cost is about 0.1% of annual income over 1976-2013, as compared to 0.8% over 1945-1975. Overall, these values are substantially lower than those reported in the literature.

Keywords: Money Demand, Structural Changes, Welfare Cost of Inflation

JEL: C22, E41, E52

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1. Introduction

The main aim of this paper is to measure the welfare cost of inflation for the U.S. in the money demand framework developed by Lucas (2000) and in presence of potentially detected instabilities in the underlying money demand function. The evaluation is undertaken by mapping changes in the structural parameters of the money demand function (mainly changes in the interest-elasticity) to the measures of the welfare cost of inflation. According to Lucas (2000), the welfare cost of inflation can be defined as the social gain/utility obtained by reducing the steady-state nominal interest rate from a positive level to the near-zero level, as prescribed by the Friedman (1969)'s optimal monetary policy rule. Using U.S. data for the period 1900-1994, Lucas (2000) shows that the reduction of the annual inflation rate from 10% to 0% would imply a welfare gain of 1% of income. This result supports the view of strong intervention of monetary authorities targeting anti-inflationary policies. Lucas (2000)' contribution has generated an interesting line of theoretical research on this topic (Simonsen and Cysne, 2001; Cysne and Turchick, 2012). However, empirical contributions (see, for instance, Chadha et al., 1998, Bali, 2000, and Serletis and Yavari, 2004) have focused on the case of stable money demand to evaluate the welfare cost in the U.S., although evidence of historical instability has been reported in the literature (Ball, 2001; Ireland, 2009; Wang, 2011). In particular, the instability of money demand detected at the out-turn of the 70s and the 80s has been interpreted either as changes in the economy's transaction technology (Ball, 2001; Teles and Zhou, 2005; Ireland, 2009; Berentsen et al., 2015) or as the outcome of financial reforms and monetary policies triggered by high inflation rates (Reynard, 2004; Lucas and Nicolini, 2015). In both cases, the money demand approach advocated by Lucas (2000), which accounts for the money demand distortion brought about by positive nominal interest rates, appears a valid instrument to analyze the welfare cost of inflation (Ireland, 2009).[1] Hence, this calls for a reconsideration of the

[1]It is worth noting that the latter interpretation could imply an underestimate of the cost to the post-1980 U.S. economy. According to Dotsey and Ireland (1996), in general equilibrium, the inflation tax distorts a variety of marginal decisions, such as the holding of real cash balances and the allocation of productive resources, which are small taken individually but yield to fairly large welfare cost estimates when combined. Thus, Ireland (2009) argues that if these inefficiencies remain present in the post-1980 U.S. economy, the welfare cost could be underestimated by the measures considered in the present paper. However, according to Cysne (2003) and Cysne and Turchick (2010), the presence in the economy of monetary assets used for transaction purposes, beyond currency, paying different interest rates, may instead lead to an overestimate of the welfare cost of inflation (see Section **??**). We thank the referees for raising this point.

welfare cost of inflation when the economy moves from a regime of sustained inflation to another of moderate inflation as at the end of the 70s, or even in correspondence of a situation close to the "Friedman rule" as in most recent years, and vice-versa.

In this paper, we address these issues in our implementation of a welfare cost analysis for the U.S. Motivated by the existing literature, we estimate money demand equations using a dataset of yearly observations from 1900 to 2013.[2] Our contribution focuses on the selection of the best empirical model through a cointegration analysis accounting for the presence of regime changes, that we identify via the implementation of the testing procedure proposed by Kejriwal and Perron (2008, 2010). We estimate long-run money demand models in a single-equation framework and we provide welfare cost estimates accounting for changes in the structural parameters of the money demand function. To the best of our knowledge, this is the first contribution that measures the effect of structural instability of money demand on the welfare cost of inflation.

Our main findings are as follows. First, we find evidence of two structural breaks on the parameters of the long-run money demand relationship, located at the mid-40s and at the end of the 70s. According to our estimates, the interest-elasticity of money demand increased during the post-war from -0.1 to -0.4, but the demand curve shifted downward and became less elastic afterwards. These results are overall consistent with those reported by Ball (2001) and Ireland (2009) on U.S. data, as well as with the prediction implied by the recent theoretical contributions of Lucas and Nicolini (2015) and Berentsen et al. (2015). Second, once regimes are accounted for, welfare cost estimates are substantially lower than those reported in the literature. For instance, Lucas (2000) finds a value of 1% as opposite to a value of 0.5% in this paper, where the value drops to 0.1% in most recent decades. This means that the target of moderate inflation dictated implicitly or explicitly by the Federal Reserve (FED) would have implied very limited welfare costs to the U.S. economy in latest years.

The remainder of the paper is organized as follows. In Section 2, we briefly review the relevant seminal contributions on the issue of measuring the welfare cost of inflation. We also discuss the

[2]The dataset cannot be updated to more recent years, as in 2012 the Board of Governors of the Federal Reserve System discontinued the publication of the retail deposit sweeps data (last observation available: December 2013), which enter the monetary aggregate used in this paper (sweep-adjusted M1).

implication of the specification of the money demand function and the computation of the welfare cost. In Section 3, we describe the dataset and we report the empirical results on the selection of the specification of the cointegrating relationship where structural breaks are accounted for. Section 4 evaluates the impact of the instability of the money demand model on the welfare cost estimates. Section 5 concludes.

2. Money Demand and the Welfare Cost of Inflation: A Short Overview

There is a huge body of empirical literature since the 60s providing estimates of the structural parameters of long-run money demand and investigating its stability for monetary policy purposes (see Poole, 1988, and Hoffman and Rasche, 1991, for a review). One interesting motivation to this large literature follows the work of Bailey (1956) on the link between aggregate money demand and the welfare cost of inflationary government finance. Bailey (1956) argues that the response of the demand for real balances to changes in the nominal interest rate could be used to quantify the welfare gains from low-inflation monetary policies, such as the optimal zero-interest rate rule set years later by Friedman (1969). This money demand approach to the welfare cost of inflation has been further popularized by Lucas (2000), who extends the partial-equilibrium analysis in Bailey (1956) to a general-equilibrium framework (Sidrauski, 1967; McCallum and Goodfriend, 1987) and generates an interesting strand of theoretical and empirical contributions up to recent years (see for instance Ireland, 2009, and Cysne and Turchick, 2012).

In what follows, we provide a short overview of this literature and we show how the welfare cost of inflation can be mapped from a money demand framework.

2.1. Measures of the welfare cost of inflation

In a seminal paper on the excess burden generated by governments' inflationary finance, Bailey (1956) defines the welfare cost of inflation as the aggregate consumer surplus that could be gained by reducing the steady-state nominal interest rate from a positive level ($r > 0$) to a near-zero level. Given a money demand function $m(r)$, the welfare cost of inflation corresponds to the area under the inverse function $\psi(m)$, which measures the flow of productivity (utility) resulting from a given quantity of real balances used for transactions:

$$B(r) = \int_{m(r)}^{m(0)} \psi(\mu)\mathrm{d}\mu = \int_0^r -\rho m'(\rho)\mathrm{d}\rho, \tag{1}$$

where $B(r)$ is expressed as a fraction of income. It follows that decreases in the optimal level of money holding driven by a change in the interest rate (inflation rate) reduce this area, leading to an aggregate loss in productivity resulting from the destruction of real balances.

The economic rationale behind the Bailey (1956)'s partial-equilibrium welfare cost formula is investigated by Lucas (2000) in a simplified Sidrauski (1967) general-equilibrium money-in-utility framework. In this model, the representative household maximizes utility $U[c, z]$, where c is consumption of a single non-storable good and $z = M/P$ are real balances, in an economy on a balanced growth equilibrium path, on which the money growth rate is maintained on a constant ratio of transfers to income (y). In this case, the money-income ratio $m = z/y$ is also constant, such as the inflation factor $1 + \pi$. Under standard budget constraints, first-order conditions set the equilibrium interest rate $r = U_z/U_c$.[3] Defining $m = m(r)$ as the money demand function satisfying the equilibrium, and $r = \psi(m)$ as the inverse demand function, the flow utility enjoyed by the household on the balanced path, $U[y, m(r)y]$ with $c = y$ at the equilibrium, is maximized for $r = 0$, *i.e.* under the Friedman (1969) optimal monetary policy rule. The welfare cost of inflation $w(r)$ is thus equivalent to the increase in income necessary to leave the representative household indifferent between a positive steady-state nominal interest rate and the optimal policy ($r = 0$), and it is given by the solution to the following differential equation:

$$\left\{ \begin{array}{rcl} \overline{w}'(r) & = & -\psi\left(\frac{m(r)}{1+\overline{w}(r)}\right) m'(r) \\ \overline{w}(0) & = & 0 \end{array} \right. , \tag{2}$$

with $\overline{w}'(r) > 0$ and $m'(r) < 0$. Cysne and Turchick (2012) consider an alternative way of interpreting the definition of the welfare cost of inflation in the Sidrauski framework, $\overline{w}(r)$, by taking as reference

[3] Lucas (2000) assumes homothetic preferences and a constant relative risk aversion (CRRA) form for the utility function, $U[c, z] = \frac{1}{1-\sigma}\left[c\varphi\left(\frac{z}{c}\right)\right]^{1-\sigma}$, with $\sigma > 0$ and $\sigma \neq 1$. These are consistent with a non-trending money-income ratio and with balanced growth, respectively. It follows that the equilibrium interest rate is given by $r = \varphi'(m)/(\varphi(m) - m\varphi'(m))$. It is also worth noticing that the Bailey (1956)'s formula provides an exact general-equilibrium measure of the welfare costs of inflation under the assumption of quasi-linear preferences (Cysne, 2009).

a zero steady-state nominal interest rate. Under this reverted perspective, the welfare cost of inflation $\underline{w}(r)$ is equivalent to the decrease in income necessary to leave the household indifferent between the current optimal policy and $r > 0$, and it is given by:

$$\underline{w}(r) = 1 - \mathrm{e}^{-\int_{m(r)}^{m(0)} \frac{\psi(\mu)}{1+\mu\psi(\mu)}\mathrm{d}\mu}. \tag{3}$$

Lucas (2000) also investigates alternative welfare interpretations provided by the McCallum and Goodfriend (1987) shopping-time framework, robust to the classic Clower (1967) critique. In this model, the representative household is endowed with a unit of time, which can be used either to transact or to produce the consumption good through the technology $c = (1-s)y$. Hence, s is the fraction of time devoted to carry out transactions and not produce ($s = 1 - c/y$). The household maximizes utility $U[c]$ in an economy on a balanced growth equilibrium path, subject to standard budget constraints and transaction-technology $G[m, s] \equiv m\phi(s) = (1-s)$, where $m = z/y$.[4] The first-order condition sets the equilibrium interest rate $r = G_m/G_s$, which can be rearranged to solve for $s = s(r)$ and $m = m(r)$. Since the time spent to carry out transactions has the dimension of a percentage reduction in production and consumption, $s(r)$ is a measure of the welfare cost of inflation supported by the household economizing on cash holdings (allocated to interest bearing assets) and spending time in transacting, and it is given by the solution to the following non-separable differential equation:

$$\begin{cases} s'(r) &= -\frac{rm'(r)(1-s(r))}{1-s(r)+rm(r)} \\ s(0) &= 0 \end{cases}, \tag{4}$$

with $s'(r) > 0$. Since the higher the nominal interest rate the lower the production and consumption, the flow utility enjoyed by the household is maximized under the Friedman (1969) optimal monetary policy rule ($r = 0$).

[4] The utility function used by Lucas (2000) takes again the CRRA form $U[c] = \frac{1}{1-\sigma}c^{1-\sigma}$, with $\sigma > 0$ and $\sigma \neq 1$. This is equivalent to a shopping-cost version of the McCallum-Goodfriend framework, due to the absence of leisure from the utility function. It is worth noticing that the functional form of the transaction technology $G[m, s]$ is consistent with a unit income elasticity of money demand.

2.2. *An ordering of the measures*

As shown by Simonsen and Cysne (2001) and Cysne and Turchick (2012), the welfare cost measures presented above do not have any obvious closed-form solution in the general case, but they can be conveniently arranged in an ascending order and hence approximated by a bounded interval. This ordering of the welfare cost measures leads implicitly to a confidence region of cost estimates, which may prove very useful in empirical applications. Considering the measure $s(r)$, the non-separable differential equation in (4) can be reasonably approximated by the bounded interval $\underline{A}(r) < s(r) < \overline{A}(r)$, where:

$$\overline{A}(r) = \int_0^r -\frac{\rho m'(\rho)}{1+\rho m(\rho)} \mathrm{d}\rho, \tag{5}$$

$$\underline{A}(r) = 1 - \mathrm{e}^{-\int_0^r -\frac{\rho m'(\rho)}{1+\rho m(\rho)} \mathrm{d}\rho} = 1 - \mathrm{e}^{-\overline{A}(r)}. \tag{6}$$

From expressions (5) and (1), it follows that $\overline{A}(r) < B(r)$.[5] Further, through the transformation $\mu \equiv m(\rho)$, expression (5) can be rewritten as $\overline{A}(r) = \int_{m(r)}^{m(0)} \frac{\psi(\mu)}{1+\mu\psi(\mu)} \mathrm{d}\mu$. Thus, from (3) and (6), it follows that $\underline{w}(r) = \underline{A}(r)$. The same measure can be hence obtained under two different theoretical frameworks. Now, noticing that $\int_0^r \overline{w}'(\rho)\mathrm{d}\rho > \int_0^r -\psi(m(\rho))m'(\rho)\mathrm{d}\rho \equiv \int_0^r -\rho m'(\rho)\mathrm{d}\rho$, for $\rho \in (0, r]$, it follows that $B(r) < \overline{w}(r)$. An inequality chain can then be formed to order the welfare cost measures considered:

$$\underline{w}(r) = \underline{A}(r) < s(r) < \overline{A}(r) < B(r) < \overline{w}(r), \tag{7}$$

so that for a given r the width of the region of cost estimates is given by $\mathcal{R}(r) = \overline{w}(r) - \underline{w}(r)$ and the relative percentage difference is given by $\mathcal{D}(r) = \overline{w}(r)/\underline{w}(r) - 1$.

2.3. *The specification of money demand and the welfare cost measures*

As indicated by Lucas (2000), the specification of the money demand function $m(r)$ is crucial in determining the accurate size of the welfare cost of inflation. To undertake this analysis, we hence need to specify an appropriate money demand relationship and estimate its relevant parameters. Lucas (2000) and Ireland (2009) consider two standard competing empirical specifications: log-

[5] Numerical simulations reported by Cysne and Turchick (2012) suggest that $s(r)$ can be accurately approximated by $\overline{A}(r)$, with a maximum relative error of about $\overline{A}(r)/2$.

log and semi-log specifications.[6] The former relates the natural logarithm of the money-income ratio m to the natural logarithm of the nominal interest rate r (Meltzer, 1963) and it is a straight development of the theoretical solution proposed, for instance, in Sidrauski (1967), Brock (1974) and McCallum and Goodfriend (1987), under a set of assumptions on the utility function and the budget constraint. The latter relates the natural logarithm of m to the level of r (Cagan, 1956) and can be derived from the class of inventory-theoretic models, such as in Baumol (1952), Tobin (1956), Miller and Orr (1966) and Bar-Ilan (1990), among others. Important differences arise from the two alternative specifications with respect to the implications for monetary policy. On the one hand, the semi-log specification implies an increasing interest-elasticity of real balances, meaning that as r increases real balances converge to zero.[7] However, as r approaches zero, real balances reach a finite satiation point. On the other hand, the log-log specification implies a constant interest-elasticity of money demand. Hence, the degree of substitution between real balances and alternative assets does not depend on the level of the interest rate. Hence, and not surprisingly, the shape of the welfare cost function depends on the specification of $m(r)$. For instance, the log-log specification implies that the welfare cost is an unbounded strictly concave function of the interest rate, while the semi-log specification implies a bounded concave (upwards to downwards) function.[8] This means that the money demand specification must be accurately chosen in order to fit the data properly

[6] The log-log specification $m = Kr^{-\zeta}$, where ζ is the absolute value of the interest-elasticity of money demand, can be derived from the first-order conditions with an utility function belonging to the class of preferences defined in footnote 3. The semi-log specification $m = K\exp(-\xi r)$, where ξ is the absolute value of the interest semi-elasticity of money demand, can be obtained by considering an utility function belonging to the class of monotonic transformations of the quasi-linear utility function $c + \lambda(z)$, such as $U[c, z] = g(c + \lambda(z))$, with $g' > 0$, $g'' \leq 0$, $\lambda' > 0$, $\lambda'' < 0$ (Cysne, 2009). Setting $\lambda(z) = \frac{z}{\xi}\left(1 + \ln\left(\frac{K}{z}\right)\right)$, with these preferences we have that the first order conditions lead to the equilibrium $r = \lambda'(z) = \frac{1}{\xi}\ln\left(\frac{K}{z}\right)$, which gives the semi-log specification after solving for $m(r)$.

[7] As noted by Anderson and Rasche (2001), from a microeconomic perspective, and assuming that at least some money is held by households in general equilibrium, this mechanism seems in contradiction with the diminishing marginal rate of substitution implicit in the Inada conditions.

[8] Consider the Bailey (1956) measure of the welfare cost, $B(r)$. For the log-log specification $m = Kr^{-\zeta}$, this measure has limiting property $\lim_{r\to\infty} B(r) = +\infty$ and derivatives $B'(r) > 0$ and $B''(r) < 0$. For the semi-log specification $m = K\exp(-\xi r)$, the limiting property is $\lim_{r\to\infty} B(r) = K/\xi$ and derivatives $B'(r) > 0$ and $B''(r) \gtrless 0$ with an inflection point at $r = 1/\xi$.

and avoid miscalculation of the cost of inflation.[9] Following Lucas (2000) and Ireland (2009), we choose the log-log specification which seems to fit quite well the historical annual U.S. data used in the present empirical analysis. This econometric specification takes the following (static) form:

$$\ln(m_t) = \alpha + \beta \ln(r_t) + \varepsilon_t, \tag{8}$$

where α is the constant, β is the interest-elasticity of money demand, and ε_t is the regression error term. With the log-log specification estimated in (8), the implied money demand function is $m = \exp(\alpha) r^{\beta}$, with β expected to be negative, and the welfare cost measures reported in the inequality (7) have closed-form solutions:

$$\begin{aligned}
\overline{w}(r) &= -1 + (1 - \exp(\alpha) r^{1-|\beta|})^{\frac{|\beta|}{|\beta|-1}} \\
B(r) &= \tfrac{|\beta|}{1-|\beta|} \exp(\alpha) r^{1-|\beta|} \\
\overline{A}(r) &= \tfrac{|\beta|}{1-|\beta|} \log(1 + \exp(\alpha) r^{1-|\beta|}) \\
\underline{w}(r) &= 1 - (1 + \exp(\alpha) r^{1-|\beta|})^{\frac{|\beta|}{|\beta|-1}}
\end{aligned}$$

It is worth noticing that real solutions for $\overline{w}(r)$ are only obtained for $r \in [0, \exp(\alpha)^{\frac{1}{|\beta|-1}}]$, which represents a realistic economic interval for reasonable values of α and β. It follows that the width of the region of cost estimates $\mathcal{R}(r)$ has also a bounded real solution, which is strictly increasing in r with $\mathcal{R}'(r) > 0$ and $\mathcal{R}''(r) > 0$. The relative percentage welfare cost difference $\mathcal{D}(r)$ (which, for the log-log specification, can be well approximated by $\overline{w}(r)/|\beta|$ for reasonable values of r; see Cysne and Turchick, 2012) has also a bounded real solution and is strictly increasing in r, where $\mathcal{D}'(r) > 0$ but $\mathcal{D}''(r) \gtrless 0$.

Based on the functional form selected, in the next section we estimate a long-run money demand specification for the U.S. and we investigate the presence of long-run instabilities in a cointegrating

[9] As pointed out by Ireland (2009), both specifications can be estimated econometrically, but "require one to adopt a somewhat schizophrenic view of the data". Indeed, in a linear framework, the econometric treatment of the log-log specification requires $\ln(r)$ to be an integrated process with *percentage change* stationary distribution, while the same analysis on the semi-log specification requires r to be an integrated process with *difference* stationary distribution. However, if r is I(1), then $\ln(r)$ cannot be I(1), and vice-versa. Bae and De Jong (2007) show that this issue can be solved by treating the log-log specification as a nonlinear function of the interest rate and the semi-log specification as a linear function, under the common assumption that r is an integrated process.

framework. We will then make use of the relevant estimated parameters to map a correct measure of the welfare cost of inflation.

3. Money Demand for the U.S.: Data and Empirical Estimates

3.1. The dataset

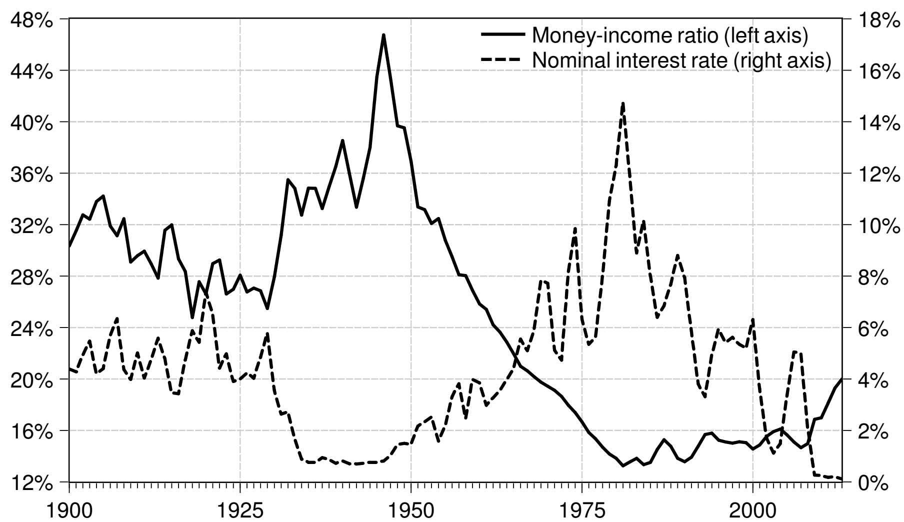
For the empirical analysis, we extended to 2013 the dataset used by Ireland (2009), which is in turn closely comparable to that of Lucas (2000). We have $T = 114$ annual observations spanning from 1900 to 2013 for money, income and interest rates. Money is measured in terms of M1, which includes mainly currency held by the public, non-interest-bearing demand deposits, and, since 1980, interest-bearing Negotiable Order of Withdrawal accounts (NOW). Further, we follow the recent literature (Ireland, 2009; Berentsen et al., 2015) and we consider a retail sweep adjusted measure of money from 1994 onward, in order to avoid a downward estimate of M1 consistent with the introduction of retail deposit sweep programs (Dutkowsky and Cynamon, 2003).[10] Income is measured in terms of nominal GDP, computed as the real GDP multiplied by the series of implicit deflators for GNP (from 1900 to 1928) and GDP (from 1929 onward). Finally, the interest rate series is constructed using data on the six-month commercial paper rate (from 1900 to 1997) and the three-month AA nonfinancial commercial paper rate (from 1998 onward), due to a discontinuity in the statistical publication of the former. The data sources are broadly the same as in Ireland (2009), and we hence refer the reader to that contribution for further details. Figure 1 reports the graphs of the money-income ratio and the nominal interest rate.

3.2. Searching for cointegration: long-run instability and structural changes

We first analyze the stochastic properties underlying the processes for the money-income ratio and the interest rate, both in natural logarithms. We tested for the hypothesis of unit-root

[10]NOW accounts were created in the mid-70s and represented the first formal amendment to Regulation Q of the Glass-Steagal Act, in force since 1933, which explicitly banned interest payments on checkable deposits. Up to the mid-80s, NOW accounts (as well as alternative saving accounts) had been subjected to interest rate ceilings, fixed at 5%. In 1986, interest rate restrictions were removed, but the ban on demand deposits was maintained. In 2011, Regulation Q was finally abandoned, leading to the end of the remaining restrictions on interest-bearing deposits. On the other hand, retail sweep programs were introduced in 1994 and consist in a technology allowing banks to operate back and forth reclassification of balances in checking accounts as money market deposit accounts, avoiding hence reserve requirement restrictions on these balances because reclassified in M2 (Anderson, 2003). This technology implies a severe distortion of official measures of the M1 money stock, because these shadow (overnight) deposits are accounted in M2.

Figure 1: Money-income ratio and nominal interest rate, 1900-2013



hypothesis through a battery of modified tests introduced by Ng and Perron (2001), such as the Phillips-Perron non-parametric tests (MZ_a and MZ_t) and the Elliott-Rothenberg-Stock feasible point-optimal test (MP). These tests are based upon autoregressive spectral density estimates constructed using GLS detrended data and a modified AIC criterion for the selection of the optimal lag length, where the latter is in turn constructed using OLS detrended data (Perron and Qu, 2007). The results are reported in Table 1. As expected, the null hypothesis of unit-root cannot be rejected for both series.

A cointegrating relationship between $\ln(m)$ and $\ln(r)$ is then estimated using the Dynamic OLS estimator (see Saikkonen, 1991, and Stock and Watson, 1993) to account for potential endogeneity of the interest rate, with the number of leads and lags ($\ell_T = 4$) set consistently with the upper bound condition implied by the data-dependent rule suggested by Saikkonen (1991), *i.e.* $\ell_T < T^{1/3} \approx 5$ for $T = 114$. Heteroskedasticity-autocorrelation robust standard errors are obtained through the Bartlett kernel and the Newey-West truncated automatic bandwidth selection method (Newey and West, 1994). The estimated equation is (standard errors in parenthesis):

$$\ln(m_t) = -\underset{(0.23)}{2.62} - \underset{(0.06)}{0.35}\ln(r_t) + \sum_{j=-\ell_T}^{\ell_T} \hat{\delta}_j \Delta \ln(r_{t-j}) + \hat{\epsilon}_t, \tag{9}$$

Table 1: Testing for unit roots

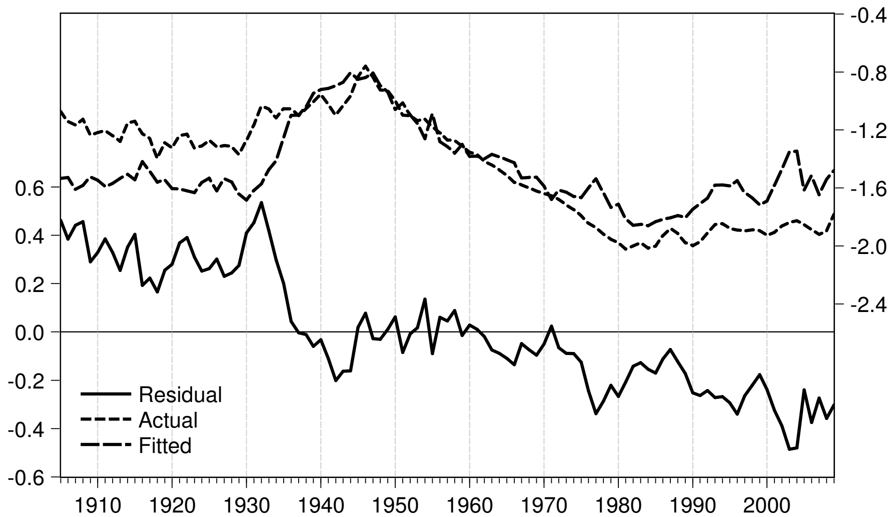
Testing for unit roots			
	MZ_a	MZ_t	MP
$\ln(m)$	-1.29	-0.72	16.50
$\ln(r)$	-3.11	-0.69	7.22

where the interest-elasticity of the money-income ratio is -0.35, which is close to, although somewhat below, the result of -0.5 consistent with a Baumol-Tobin transaction technology and reported by Meltzer (1963) and Lucas (2000). However, a visual inspection of the residuals of the regression reported in Figure 2 hints towards the presence of a substantial persistence in the regression residuals, revealing that the estimated relationship does not cointegrate. This is confirmed by the Shin (1994) test based on $\hat{\varepsilon}_t$ that strongly rejects the null hypothesis of cointegration (at 1% significance level).[11] The first important implication for the subsequent analysis is that welfare cost results, as reported by Lucas (2000) and then discussed by Ireland (2009), might be contaminated by the inconsistencies arising from the long-run relationship between $\ln(m)$ and $\ln(r)$ which does not cointegrate.

The evidence of no cointegration between money-income ratio and the interest rate reported above is also in contrast with the monetary theory and the empirical literature published in the 80s and the 90s (Lucas, 1988; Hoffman and Rasche, 1991; Stock and Watson, 1993). However, recent studies have pointed out the presence of structural instability in the money demand parameters for the U.S., especially when the estimation sample includes the last two decades of data. For instance, Ball (2001) finds an unstable long-run equation when the sample spanning up to the mid-80s used by Lucas (1988) and Stock and Watson (1993) is extended through the 90s, with changes in the structural parameters of the money demand relationship identified at the turn of the post-war period and at the beginning of the 80s. Similar conclusions about long-run instability are reported

[11] Similar conclusions are obtained by restricting the sample to the period 1900-1994, the same time span considered by Lucas (2000). The interest-elasticity is estimated to -0.4, but regression residuals confirm no cointegration. Further, we tested whether the estimated parameters reported in (9) are statistically different from those reported by Lucas (2000). From a Wald test on the joint hypothesis that $\alpha_0 = -3.02$ and $\beta_0 = -0.5$, we can reject the null at less than 1% level. When the hypothesis on the interest-elasticity is tested alone, we can reject the null at 1% level.

Figure 2: Actual, fitted and residuals from regression (9)

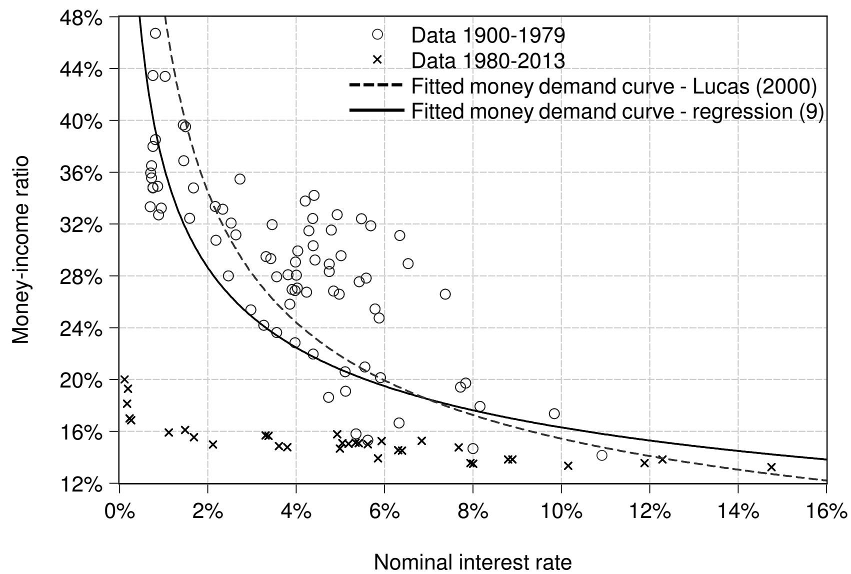


by Wang (2011), who implements a test for segmented cointegration (Qu, 2007) over roughly the same dataset as in Ball (2001) and reports evidence of multiple regime changes.[12]

Thus, these results, jointly with our findings reported above, suggest that the lack of cointegration may be explained by a misspecification in the money demand equation, likely due to unaccounted structural changes affecting the long-run equilibrium relationship between $\ln(m)$ and

[12]The economic interpretation of the presence of unmodelled instabilities is still subject to an open debate, in particular on the way in which these are accounted for in theoretical models and economic policy analyses. Ball (2001) suggests that the identified changes in the structural parameters of money demand may be viewed as changes in the economy's transaction technology. In particular, the post-war rise in velocity (for given interest rates) may reflect a downward trend in money demand arising from technological changes, such as the creation of near-monies. Teles and Zhou (2005) show that the pure transaction service feature of money (liquidity and zero-interest bearing) was clearly embedded in the M1 aggregate up to the 80s, but the distinction between M1 and M2, illiquid and interest-bearing, has been blurred by a sequence of crucial financial innovations and banking deregulation thereafter. Reynard (2004) suggests that the instability of money demand in the late 70s and in the 80s can be explained by the evolution of financial market participation, *i.e.* the share of U.S. households holding non-monetary assets (such as deposits, stocks and bonds) in their portfolio. The instability of money demand observed in the first half of the XXth century and at the beginning of the 80s has been recently explored by Lucas and Nicolini (2015), who refer to the regulatory changes on the banking sector implied by Regulation Q. Using a model of banking competition with regulation on interest rates paid on deposits (ceiling), the authors find that the interest-elasticity of real money balances in the regulated economy is higher than in the free-market economy. Berentsen et al. (2015) suggest instead that the instability of money demand is related to the introduction of retail deposit sweep programs in the first half of the 90s. The authors find that this financial innovation, modeled as an increase in the money market participation of agents, affects the aggregate money demand by reducing the interest-elasticity (agents earn higher rates on their idle balances) and shifting downward the demand curve (the money stock is allocated more efficiently).

Figure 3: U.S. money demand, 1900-2013: Lucas (2000) and regression (9)



$\ln(r)$. This intuition is confirmed by inspecting Figure 3, which plots long-run money demand curves obtained fitting the data with parameters both used by Lucas (2000) and estimated from (9): most of the observations from the 80s onward (see Ireland, 2009), as well as a large number of points located at the center of the plot, seem consistent with different theoretical curves.

Thus, a natural follow up of the above results is to test for cointegration in presence of structural changes. To this purpose, We implement the Kejriwal and Perron (2008, 2010) testing procedure based on the following steps. First, from (9), we allow both the constant and the interest-elasticity to change over time/alternative regimes:

$$\ln(m_t) = \sum_{i=1}^{n+1} \alpha_i + \sum_{i=1}^{n+1} \beta_i \ln(r_{i,t}) + \sum_{j=-\ell_T}^{\ell_T} \delta_j \Delta \ln(r_{t-j}) + \varepsilon_t, \tag{10}$$

where n is the number of breaks and $n+1$ the number of regimes. Break dates are then sequentially estimated via a dynamic programming algorithm (Bai and Perron, 2003). Finally, the null hypothesis of no breaks is tested against the alternative of fixed $k = n$ breaks via sup-Wald test

Table 2: Structural breaks tests

	$\epsilon = 20\%,\ N = 3$		$\epsilon = 15\%,\ N = 5$	
	Test statistic	Break date	Test statistic	Breaks dates
$\sup F(1)$	10.19*	1936	10.19	1936
$\sup F(2)$	9.13*	1945, 1976	9.13*	1945, 1976
$\sup F(3)$	5.43	1930, 1951, 1976	6.32	1945, 1961, 1976
$\sup F(4)$			5.11	1923, 1945, 1961, 1976
$\sup F(5)$			4.01	1923, 1945, 1961, 1976, 1991
$UD \max F$	10.19*		10.19	
BIC	-4.93	2	-4.99	3
LWZ	-4.29	2	-4.29	2

Notes: ***, ** and * denote statistical significance at the 1, 5 and 10% levels, respectively.

statistics of the form:

$$\sup_{\lambda \in \Lambda_\epsilon^k} F(\lambda, k) = \frac{SSR_0 - SSR_k}{k(T-(k+1)q)^{-1} SSR_k} \xrightarrow{p} \frac{SSR_0 - SSR_k}{k\hat{\omega}^2}, \tag{11}$$

where λ is the vector of break dates, ϵ a trimming parameter determining the minimum length of each segment explored by the dynamic algorithm, SSR_0 and SSR_k the sum of squared residuals under the null and the alternative hypothesis, respectively, q the number of parameters which are allowed to change (excluding the constant), and $\hat{\omega}^2$ a consistent estimate of the long-run variance constructed using a hybrid method proposed by Kejriwal (2009). In addition, we also consider a double-maximum test against the alternative hypothesis of an unknown number of breaks based on the maximum of the individual tests for the null of no breaks versus $n = 1, \dots, N$ breaks, where N is an upper bound:

$$UD \max F(N) = \max_n \sup_{\boldsymbol{\lambda} \in \Lambda_\epsilon^n} F(\boldsymbol{\lambda}, n). \tag{12}$$

As a robustness check, we follow Kejriwal (2008) and we compare the test results to the output of two selection criteria, namely the BIC (Yao, 1988) and its modified version proposed by Liu et al. (1997, LWZ henceforth). Results are reported in Table 2. The $\sup F$ test rejects at 10% level probability the null hypothesis against the alternative of at most 2 breaks, whether the maximum number of breaks is set to 3 or 5. Similarly, the $UD \max F$ test rejects the null against the alternative of unknown breaks, although significantly at 10% level only for $N = 3$. Finally, both information criteria tend to select 2 breaks (3 breaks in the case of BIC with $N = 5$). All in all, we have evidence of $k = 2$ structural breaks, at dates 1945 and 1976. Interestingly, these findings are broadly consistent with those reported by Ball (2001), who identifies a post-war and a post-82 regimes.[13]

Equation (13) reports the estimates of (9) with the selected breaks included (standard errors in parenthesis):

$$\begin{aligned}\ln(m_t) = &- \underset{(0.07)}{1.64}\,\iota_{t_1} - \underset{(0.02)}{0.13}\ln(r_{t_1}) - \underset{(0.06)}{2.83}\,\iota_{t_2} - \underset{(0.02)}{0.43}\ln(r_{t_2}) - \underset{(0.06)}{2.24}\,\iota_{t_3} - \underset{(0.02)}{0.11}\ln(r_{t_3}) \\ &+ \sum_{j=-\ell_T}^{\ell_T} \hat{\delta}_j \Delta \ln(r_{t-j}) + \hat{\epsilon}_t, \end{aligned} \tag{13}$$

where ι_{t_i} is a regime-dependent intercept, $t_i = T_{i-1} < t \leq T_i$, with $i = 1, \ldots, n+1$, and by convention $T_0 = 0$ and $T_{n+1} = T$. The results suggest that the interest-elasticity of the money-income ratio increased from -0.13 in the pre-war period to -0.43 up to the 80s, and then decreased back again to a low -0.11 in the last part of the sample.[14] Further, Figure 4, that plots long-run money demand curves obtained fitting the data with parameters estimated from our structural breaks regression, provides strong evidence of the downward shift of the money demand curve in the last regime. As expected, our findings on the interest-elasticity for the pre-war period are in line with those reported, for instance, by Stock and Watson (1993) and Ball (2001). The estimated elasticity for the third regime is also close to that reported by Ireland (2009), although his specification involves quarterly data spanning from 1980 to 2006. The higher elasticity observed in the post-war period up to the mid-70s is a novel result, reflecting the transition from low to high velocity of money driven by changes in the transaction technology, such as the creation of near-monies instruments, which is in turn consistent with a rise in the degree of substitution between real balances and alternative assets.[15]

[13] Carlson et al. (2000) find a stable long-run money demand, based upon monthly data of the MZM aggregate over 1964-1998, only when the estimation sample is restricted to start in 1976.

[14] For the second regime, the estimated coefficients are close to those reported by Lucas (2000). However, we can reject again at less than 1% level both the joint null hypothesis $\alpha_0 = -3.02$ and $\beta_0 = -0.5$ and the individual hypothesis on the interest-elasticity. Thus, we can conclude that in our long-run analysis (with or without breaks) there is no statistical evidence in favor of the Baumol-Tobin transaction technology advocated by Meltzer (1963) and Lucas (2000).

[15] A downward trend in the velocity of money is often explained by a decrease in the income-elasticity of real balances from unity to 0.5. Lucas (2000) suggests that a technical change in the provision of transactions services would produce a downward trend in the money-income ratio.

Figure 4: U.S. money demand, 1900-2013: regression (13) with breaks in 1945 and 1976

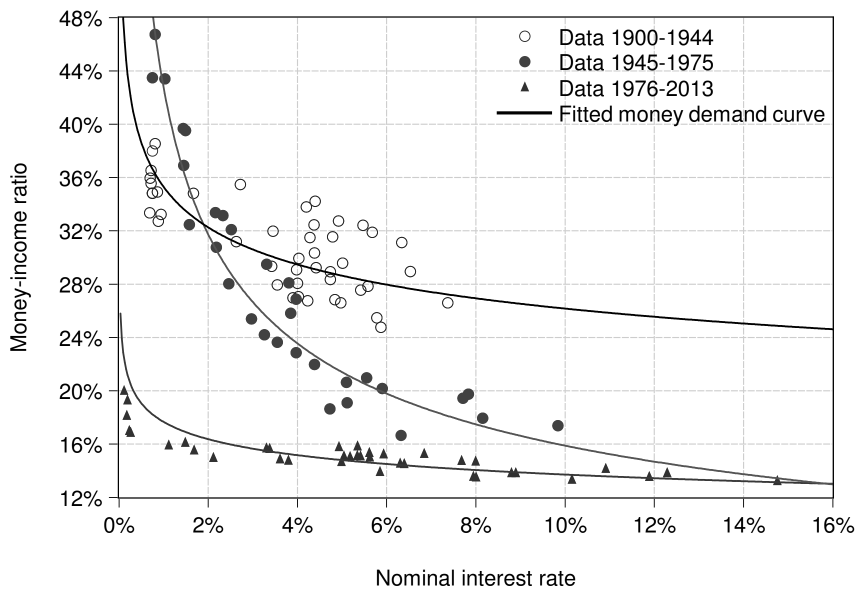
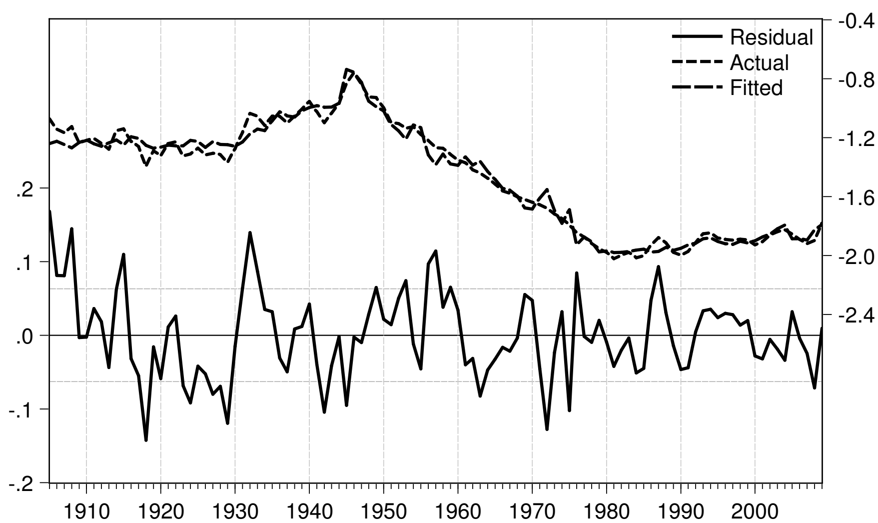


Figure 5: Actual, fitted and residuals from regression (13) with breaks in 1945 and 1976



In the empirical literature, this progressive shift in the money demand curve has been approximated by ratchet variables and linear trends (see, for instance, Carlson et al., 2000). As noted by Lucas (2000), the money-income ratio is essentially trendless over the entire century, despite the stepwise decline of money holdings observed in the post-war period (see Figure 1). Hence, a trend in the cointegrating relationship covering the post-war period up to the 80s would be necessary to obtain a stable long-run relationship. However, a visual inspection of the residuals estimated from our structural changes model (see Figure 5) reveals that deterministic trends are here unnecessary to obtain non-trending errors. A formal test of cointegration with breaks (Carrion-i-Silvestre and Sansò, 2006; Arai and Kurozumi, 2007) does not reject the null of stationary residuals, after controlling for initial conditions in the residuals vector. It is worth noticing that the test cannot reject the null at any standard significance level when exact critical values are used, and at 10% level with asymptotic critical values.[16] We have evidence of stable long-run relationship with 2 structural breaks. In what follows, we explore its impact in measuring the welfare cost of inflation.

4. Instability in Money Demand and Welfare Cost Estimates for the U.S.

We provided robust econometric evidence of an unstable money demand specification attributed to changes in the structural parameters of the long-run relationship between real balances and interest rates. An interesting but completely unexplored field is represented by the policy implications of measuring the welfare cost of inflation in presence of instabilities in the money demand function. Thus, we now turn to the evaluation of the welfare cost of inflation for the U.S. using the theoretical specifications presented in Section 2.1 and the estimated calibration parameters ($\hat{\alpha}$ and $\hat{\beta}$) reported in Section 3.2.

We exploit the inequality chain (7) and in the analysis we only consider the theoretical bounds represented by $\overline{w}(r)$ and $\underline{w}(r)$, which can be computed from closed-form solutions reported in Section 2.2 and from the estimated calibration parameters ($\hat{\alpha}$ and $\hat{\beta}$). Moreover, it is reasonable to account for the uncertainty affecting the estimated parameters when computing welfare cost estimates. To this purpose, we consider the confidence region of $\overline{w}(r)$ and $\underline{w}(r)$ using the 90% level confidence values for $\hat{\alpha}$ and $\hat{\beta}$. This requires the construction of confidence intervals using the Bonferroni inequality, *i.e.* $\tilde{\alpha} = \hat{\alpha} \pm \mathcal{B}\sigma_\alpha$ and $\tilde{\beta} = \hat{\beta} \pm \mathcal{B}\sigma_\beta$ with $\mathcal{B}$ the Bonferroni multiple and σ the standard

[16] Critical values are affected by nuisance parameters such as the number and the location of breaks, as well as the number of regressors. Since there is no tabulation for our specific case of two 2 breaks located around 40% and 70% of the sample, we computed critical values by simulation using 10,000 steps and 20,000 replications.

error. Hence, upper and lower bound analogs of $\overline{w}(r)$ and $\underline{w}(r)$, that is $\overline{w}(r)^+$ and $\underline{w}(r)^-$, are computed using these joint confidence bounds.

The results are compared to the benchmark values provided by the welfare cost estimates reported by Lucas (2000) and Ireland (2009). However, in this paper we extend their analysis by computing intervals for welfare cost estimates, based on both historical values for nominal and real interest rates and alternative counterfactual scenarios. We compute sample-specific averages of the nominal interest rate ($\bar{r}$), the inflation rate ($\bar{\pi}$) and the implicit real interest rate ($\bar{\rho}$). Results are reported in Panel A of Table 3. According to the values of the calibration parameters reported by Lucas (2000), average interest and inflation rates computed over the sample 1900-1994 (4.6% and 3.1%, respectively, leading to $\bar{\rho} = 1.5\%$) imply an average cost of inflation of about $1.0 - 1.1\%$ of the income. This value is very close to what we would obtain if we assume that the steady-state real interest rate (ρ_{ss}) ranges between 3% and 5% under the policy of zero inflation or price stability. In this case, the prevailing nominal interest rate would also range between 3% and 5%, with a cost to the economy of about $0.85 - 1.1\%$ of income. Under a policy of positive inflation matching the average inflation rate, the cost of positive nominal rates (6.1% and 8.1%, respectively) would range instead between 1.2% and 1.4% of income. Finally, assuming $\rho_{ss} = 3\%$ would imply a cost of 1.1% of income for a policy of 2% inflation, and $1.7 - 1.8\%$ of income for a policy of 10% inflation. All in all, the results based on the calibration reported by Lucas (2000) and a combination of reasonable values for inflation policy and steady-state rates suggest that the welfare cost of inflation for the U.S. should range between 1% and 2% of GDP.

We now turn to welfare cost estimates based on the cointegrating regression performed over the sample 1900-2013 and the calibration parameters reported in Equation (9). The results are reported in Panel B of Table 3. Average interest and inflation rates (4.3% and 2.9%, respectively, leading to $\bar{\rho} = 1.4\%$) are very close to the values reported in the previous paragraph. However, here they imply an average cost of inflation of about 0.5% according to point estimates, or ranging between 0.3% and 0.8% according to confidence intervals. Assuming a steady-state real interest rate ranging between 3% and 5%, the policy of zero inflation or price stability would cost the economy about $0.4 - 0.6\%$ of income, with confidence intervals ranging between 0.2% and 0.8%. Under a policy of

Table 3: Welfare cost estimates: no structural changes models

Panel A. Lucas (2000) money demand parameters						
ρ_{ss}	r	π	$\underline{w}(r)$	$\overline{w}(r)$	$\mathcal{R}(r)$	$\mathcal{D}(r)$
3.0	3.0	0.0	0.84	0.85	0.01	1.70
5.0	5.0	0.0	1.08	1.10	0.02	2.21
1.5	*4.6*	*3.1*	1.03	1.06	0.02	2.11
3.0	6.1	*3.1*	1.19	1.22	0.03	2.43
5.0	8.1	*3.1*	1.37	1.41	0.01	2.81
3.0	5.0	2.0	1.08	1.10	0.02	2.21
3.0	13.0	10.0	1.73	1.79	0.06	3.58

Panel B. Regression (9) estimated parameters						
ρ_{ss}	r	π	$\underline{w}(r)^{-}$	$\overline{w}(r)^{+}$	$\mathcal{R}(r)$	$\mathcal{D}(r)$
3.0	3.0	0.0	0.25	0.62	0.37	152.33
5.0	5.0	0.0	0.36	0.82	0.45	125.02
1.4	*4.3*	*2.9*	0.32	0.75	0.43	132.39
3.0	5.9	*2.9*	0.41	0.89	0.48	117.08
5.0	7.9	*2.9*	0.51	1.04	0.53	103.52
3.0	5.0	2.0	0.36	0.82	0.45	125.02
3.0	13.0	10.0	0.75	1.36	0.62	82.63

Notes: values reported are expressed in percentage points. Values in italic denote the empirical average over the estimation period.

positive inflation matching $\bar{\pi}$, the cost of positive nominal rates (5.9% and 7.9%, respectively) would be about $0.6 - 0.8\%$ of income, with confidence intervals ranging between 0.4% and 1.0%. Finally, assuming $\rho_{ss} = 3\%$ would imply a cost of 0.6% of income (with intervals $0.4 - 0.8\%$) for a policy of 2% inflation, and $1.0 - 1.1\%$ of income (with intervals $0.7 - 1.4\%$) for a policy of 10% inflation. It is worth noticing that both $\mathcal{R}(r)$ and $\mathcal{D}(r)$ are fairly large compared to the values reported in Panel A. This outcome can be mainly attributed to the inclusion of parameter uncertainty in the calculation of welfare cost estimates (otherwise the values would be very similar in both panels) that leads to a substantial widening of the estimated welfare cost interval. All in all, the cost of inflation based on regression (9) is halved compared to the estimates reported above. These are very important findings that suggest a different quantitative interpretation with respect to the estimates reported by Lucas (2000), of the welfare gain implied by the Friedman rule for the U.S. We thus recommend at a first stance a downward revision of these benchmark estimates.

4.1. Welfare cost of inflation and regime changes

Let us now consider welfare cost estimates based on the cointegrating regression with regime changes performed over the sample 1900-2013 and the calibration parameters reported in Equation (13). Results are reported in Table 4.

Table 4: Welfare cost estimates: structural changes model

ρ_{ss}	r	π	$\underline{w}(r)^-$	$\overline{w}(r)^+$	$\mathcal{R}(r)$	$\mathcal{D}(r)$
Panel A. 1900-1944 regime						
3.0	3.0	0.0	0.10	0.18	0.08	76.57
5.0	5.0	0.0	0.16	0.28	0.12	71.61
1.8	*3.7*	*1.9*	0.12	0.22	0.09	74.59
3.0	4.9	*1.9*	0.16	0.27	0.11	71.84
5.0	6.9	*1.9*	0.22	0.37	0.15	68.85
3.0	5.0	2.0	0.16	0.28	0.12	71.61
3.0	13.0	10.0	0.38	0.63	0.25	64.38
Panel B. 1945-1975 regime						
3.0	3.0	0.0	0.51	0.69	0.18	34.61
5.0	5.0	0.0	0.70	0.91	0.21	30.54
0.1	*3.9*	*3.8*	0.60	0.79	0.19	32.57
3.0	6.8	*3.8*	0.84	1.08	0.24	28.26
5.0	8.8	*3.8*	0.98	1.24	0.26	26.43
3.0	5.0	2.0	0.70	0.91	0.21	30.54
3.0	13.0	10.0	1.24	1.53	0.30	23.85
Panel C. 1976-2013 regime						
3.0	3.0	0.0	0.04	0.09	0.05	146.41
5.0	5.0	0.0	0.06	0.14	0.08	137.15
2.2	*5.5*	*3.3*	0.06	0.15	0.09	135.45
3.0	6.3	*3.3*	0.07	0.17	0.10	133.21
5.0	8.3	*3.3*	0.09	0.22	0.12	128.71
3.0	5.0	2.0	0.06	0.14	0.08	137.15
3.0	13.0	10.0	0.14	0.32	0.17	121.98

Notes: See Table 3.

Average interest rates do not differ substantially in the first two regimes (3.7% in 1900-1944 and 3.9% in 1945-1975), but the high rates policy observed by the end of the 70s and during the first-half of the 80s leads to a somewhat higher average in the last regime (5.5%). It is worth noticing that latest years have been characterized by historically low interest rates, consistently with the easing policy set by the Federal Reserve (FED) in the aftermath of the Great Recession episode. Inflation is about 1.9% in the first regime, but almost doubled in the second and third regimes (3.8% and 3.3%, respectively). The low average inflation observed in the first regime is mainly due to a few deflationary episodes in the 20s and the 30s. On the other hand, recent years have been characterized by a moderate inflation (around 2%) consistent with the implicit (and explicit since 2012) target of the FED (Goodfriend, 2004). It follows that the average real interest rate is around 2% in both the first and last regime (1.8% and 2.2%), but it is very low in the intermediate regime (0.1%). The point estimates suggest that the implied cost of inflation is about 0.2% in the first regime, 0.7% in the second regime and 0.1% in the last regime. Confidence intervals are quite narrow, leading to a similar lecture of cost estimates. Assuming a steady-state real interest rate ranging between 3% and 5%, the policy of zero inflation or price stability would cost the economy about $0.1 - 0.2\%$ of income in the first and third regimes, and about $0.6 - 0.8\%$ (with intervals $0.5 - 0.9\%$) of income in the second regime. Under a policy of positive inflation matching $\bar{\pi}$, the cost of positive nominal rates would be about $0.2 - 0.3\%$ of income in the first regime (with intervals $0.2 - 0.4\%$), $1.0 - 1.1\%$ of income in the second regime (with intervals $0.8 - 1.2\%$), and $0.1 - 0.2\%$ in the third regime. Finally, assuming $\rho_{ss} = 3\%$ and a policy of 2% inflation would imply a cost of 0.2% of income (with intervals $0.2 - 0.3\%$) in the first regime, 0.8% in the second regime and 0.1% in the third regime. A policy of 10% inflation would cost the economy 0.5% of income (with intervals $0.4 - 0.6\%$) in the first regime, 1.4% (with intervals $1.2 - 1.5\%$) in the second regime and 0.2% (with intervals $0.1 - 0.3\%$) in the third regime. These findings lead to several interesting conclusions.

First, the size of the cost of inflation for the first and third regimes are broadly comparable across scenarios, which means that the two regimes share some long-term equilibrium features. This is likely related to the fact that the money demand function displays an interest-elasticity which is virtually the same across regimes. Of course, striking differences arise from the level of money-income ratio, which is around 30% on average during the pre-war period and only 15% from the late 70s on. This means that for high interest rates (not considered in Table 4) welfare cost estimates are nevertheless expected to diverge. Second, for moderate interest rates our welfare cost estimates are overall substantially lower than those reported by Lucas (2000). Infrequently exceeding 1%, they rather float mostly around $0.4 - 0.5\%$, dropping to 0.1% in most recent decades, suggesting that the priors on the welfare gain implied by the Friedman rule for the U.S. might be substantially revised downward. Third, from an approximate decomposition, we can calculate the contribution of changes in the interest-elasticity of money demand to variations of the welfare cost of inflation across regimes. This amounts to about 60% of the (positive) variation from the first to the second regime, and about 90% of the (negative) variation from the second to the third regime. Fourth, the policy of 2% inflation, dictated implicitly or explicitly in the last two decades by the

FED, seems to imply limited welfare costs to the economy: between 0.05% and 0.1%, depending on the assumed steady-state real interest rate. However, after almost three decades of sustained real rates, the monetary policy response to the Great Recession drove nominal interest rates to very low territory, while inflation kept around 1.5%. Thus, the economy has been facing negative real rates since 2009. This policy being not sustainable in the long-run, it is reasonable to expect nominal rates to rise again in the next years. Finally, compared to the literature (the only exception being Lucas, 2000), our findings are interestingly close to the quantitative results reported by Ireland (2009) and Calza and Zaghini (2011) on the post-80s period, as well to estimates obtained from calibration of theoretical models reported by Cooley and Hansen (1991), Faig and Jerez (2007), and Berentsen et al. (2015), among others. However, they are overall below the estimates reported in Fischer (1981), Lucas (1981), Craig and Rocheteau (2008), and Gupta and Majumdar (2014).

4.2. Welfare cost of inflation in presence of interest-bearing assets

The analysis reported above considers the computation of the welfare cost of inflation in a simple unidimensional framework, where only non interest-bearing monetary assets are implicitly considered. However, as discussed in Section 3, technological innovations and new regulations have increased the liquidity of interest-bearing deposits in the last decades. Thus, in an economy characterized by the presence of these financial technologies, the welfare cost specifications presented in Section 2 may be misleading, because they do not account for the existence of a possible trade-off between more liquid non interest-bearing and less liquid interest-bearing monies (Bali, 2000; Simonsen and Cysne, 2001; Cysne, 2003; Cysne and Turchick, 2010, 2012). Neglecting the existence of interest-bearing monetary assets in the portfolio hold by the representative household may result in a bias in the evaluation of the welfare costs of inflation.

In this section, we provide an evaluation of this bias by implementing the approach described by Cysne and Turchick (2010). For ease of exposition, we limit our analysis to the simple case of two groups of monetary assets, non-interest and interest bearing. We consider a Cobb-Douglas monetary-aggregator technology, with unit constant elasticity of substitution between the assets. Further, we assume that the interest-elasticity of the demand for non-interest bearing assets (ε, in absolute value) and the elasticity of substitution between the monetary assets and the consumption good (ν) in the utility function of the household are both less than 1. These assumptions are quite realistic and not unusual in theoretical monetary models, as they jointly imply a positive interest-elasticity of the demand for interest bearing assets. It is worth noticing that these assumptions also imply that the unidimensional measures of the welfare costs of inflation are expected to be biased upward, *i.e.* they always overestimate the "true" welfare costs of inflation in a framework with non-interest and interest bearing monetary assets. Accordingly, considering the Bailey (1956)'s measure of the welfare cost of inflation, the bias takes the following form:

$$\Omega(r) = \frac{(1-\nu)(1-\theta)}{\nu} > 0 \tag{14}$$

where θ is the relative share of the non-interest bearing asset in the Cobb-Douglas monetary-aggregator. We evaluate the bias $\Omega(r)$ by building on the empirical unidimensional results obtained for the last regime estimated in our sample, which is consistent with the presence in the economy of monetary assets used of transaction purposes, beyond currency, paying different interest rates. Further, we consider the benchmark case of a real interest rate at 3% and a policy of 2% inflation, implying a nominal interest rate at 5%, which is fairly close to actual average observations for the last regime, as reported in Table 4.

According to the unidimensional Bailey (1956)'s measure, we evaluate the welfare cost of inflation to 0.10% of output for the last regime, for simplicity neglecting parameters uncertainty. When we consider the bidimensional framework described above, the overestimation bias $\Omega(r)$ is in the range of 140% and 13%, for combinations of reasonable values for ν and θ in the range of 0.3 and 0.7. Accordingly, the "unbiased" welfare cost of inflation would range between 0.04% and 0.09%. These results are very close to those reported in Panel C of Table, representing an additional evidence of the low welfare cost of inflation identified for the last regime, as commented in Section 4.1.

5. Conclusions

In this paper, we evaluated the policy implications of measuring the welfare cost of inflation accounting for instabilities in the long-run money demand for the U.S. over the period 1900-2013. We extended the analysis and reassessed the results reported in Lucas (2000) and Ireland (2009), also in the light of the recent contributions by Lucas and Nicolini (2015) and Berentsen et al. (2015). We estimated a long-run money demand specification that cointegrates only when breaks are accounted for. We then evaluated the costs to the economy of inflationary policies under the assumption of regime changes and we found out that the existing empirical evaluations, based on likely misspecified money demand models, tend to overestimate the welfare cost of inflation. In particular, we found evidence of two statistically significant structural breaks (in 1945 and 1976) affecting the long-run money demand relationship. According to our estimates, the interest-elasticity of money demand increased during the post-war from -0.1 to -0.4, but the demand curve shifted downward and became less elastic afterwards. These results are consistent with those reported by Ball (2001) and Ireland (2009) on U.S. data, as well as with the prediction implied by the recent theoretical contributions of Lucas and Nicolini (2015) and Berentsen et al. (2015). Once regimes are accounted for, welfare cost estimates appear substantially lower than those reported, for instance, in Lucas (2000): usually around 0.5%, but only up to 0.1% in most recent decades. This means that the target of moderate inflation dictated implicitly or explicitly by the FED would have implied very limited welfare costs to the U.S. economy in latest years.

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