

LIBOR Fallback and Quantitative Finance

Planning for the end of LIBOR
Cass Business School – 19 June 2019

Marc Henrard

LIBOR Fallback – Quantitative Finance

- 1 From basics to fallback [Pay-off]
- 2 Fallback implementation – Adjusted RFR [Measurability]
- 3 Spread Adjustment and Value transfer [Filtration]
- 4 Conclusions
- 5 Annexes

A Quant Perspective on IBOR Fallback Consultation Results

<https://ssrn.com/abstract=3308766>

Fallback transformers

<https://murisq.blogspot.com/2018/10/libor-fallback-transformers.html>

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Basics: LIBOR derivatives dates

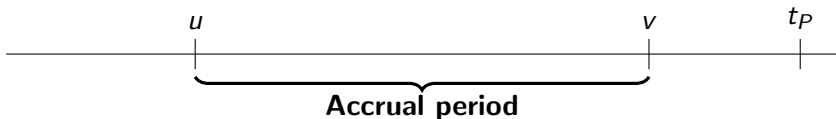
Benchmark dates

- Fixing date (θ)
- Underlying deposit effective date (σ) – T+2 for IBOR
- Underlying deposit maturity date (τ)



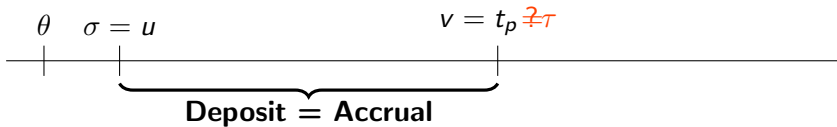
Derivative dates

- Start accrual date (u)
- End accrual date (v)
- Payment date (t_P)

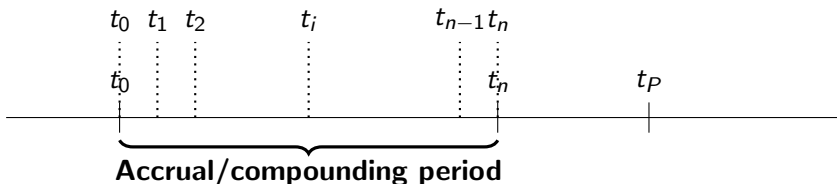


Basics: Vanilla IRS and OIS

LIBOR IRS



OIS



Typically: $t_p =$ last fixing publication + 2 business days

$$\text{Cmp}(I^O, (t_i)_{i=0, \dots, n}) = \left(\prod_{i=0}^{n-1} \left(1 + \delta_i^O I^O(t_i) \right) - 1 \right).$$

Basics: pricing derivatives

LIBOR coupon pay-off in t_P :

$$L^j(\theta)$$

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$$L^j(\theta)$$

Present value in $s < t_P$ (textbook):

$$N_s^c E^{\mathbb{X}} \left[(N_{t_P}^c)^{-1} L^j(\theta) \mid \mathcal{F}_s \right]$$

Basics: pricing derivatives

LIBOR coupon pay-off in t_P :

$$L^j(\theta)$$

Present value in $s < t_P$ (textbook):

$$N_s^c E^{\mathbb{X}} \left[(N_{t_P}^c)^{-1} L^j(\theta) \mid \mathcal{F}_s \right]$$

Present value in $s < t$ (reality):

$$N_s^c E^{\mathbb{X}} \left[(N_{t_P}^c)^{-1} (\mathbb{1}\{d > \theta\} L^j(\theta) + \mathbb{1}\{d \leq \theta\}?) \mid \mathcal{F}_s \right]$$

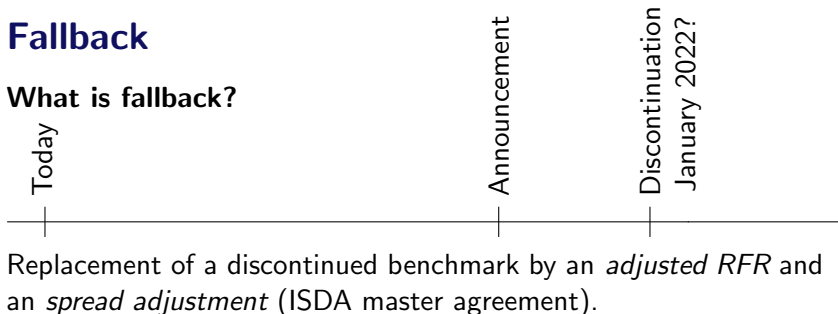
with d the discontinuation date of the LIBOR.

The question mark in the formula is not a typo, it is the **fallback!**

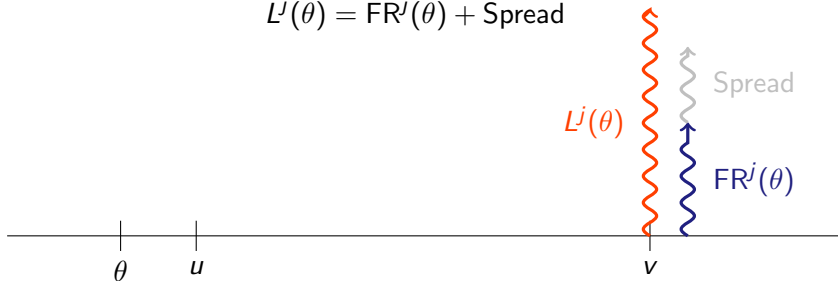
Pricing LIBOR derivatives is **not basic** anymore!

Fallback

What is fallback?



$$L^j(\theta) = FR^j(\theta) + \text{Spread}$$



Fallback and quant finance

After the fallback the pricing formula will become

$$N_s^c E^{\mathbb{X}} \left[(N_v^c)^{-1} \left(\mathbb{1}\{d > \theta\} P^j(\theta) + \mathbb{1}\{d \leq \theta\} (FR^j(\theta) + S(X, [a - l, a])) \right) \mid \mathcal{F}_s \right].$$

where d is the discontinuation date, a the announcement date and S the spread. The notations X and l will be explained later.

The dates d and a are **stopping times**.

Fallback

Criteria

- Consistency
- Absence Value Transfer
- Absence Manipulation (RFR and spread)
- Similarity with existing instruments
- Valuation simplicity
- Risk management
- Achievable!

An ISDA consultation took place from 12 July to 23 October 2018 (GBP, CHF, JPY, AUD). The results of the consultation have been published on **27 November 2018**.

USD fallback consultation **in progress**. Consultation expected related EUR. Document on the technical details and the legal wording expected Q3 2019.

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Fallback – Adjusted RFR options

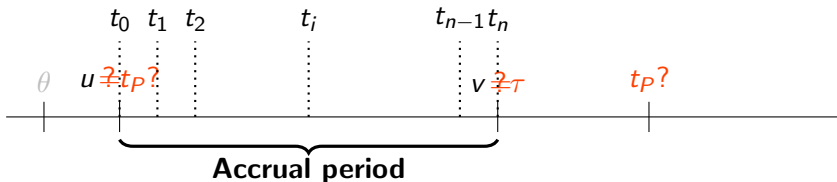
Option 3: Compounded Setting in Arrears Rate

Consultation text: “The fallback could be to the relevant RFR observed over the relevant IBOR tenor and compounded daily during that period.”

$$FR^j(t_0) = \frac{1}{\delta^j(t_0)} \left(\prod_{i=1}^n \left(1 + \delta_i^O I^O(s_{i-1}) \right) - 1 \right).$$

Pro: Interest rate, same term, similar to OIS, available?

Con: Available late, achievable?, wrong period? FRA?



Fallback – Adjusted RFR options

The adjusted rate is computed between $t_0 = \sigma$ and $t_n = \tau$;

$$FR^j(\theta) = \text{Cmp}(I^O, (t_i)_{i=0, \dots, n}).$$

is not \mathcal{F}_θ -measurable anymore, it is only \mathcal{F}_{t_n} -measurable.

In many case $t_p < \tau$. The new pay-off is **not measurable on its payment date!**

The measurability requirement may appear as a technical term of no practical importance, but is it not; it is only the precise description of the very practical requirement that **before you are able to pay an amount, you need to know what amount should be paid.**

Fallback – Adjusted RFR options

Option X1: Term rate / OIS Benchmark

Option not proposed in the ISDA consultation.

In t_0 the rate of the OIS associated to the IBOR start and maturity deposit dates is measured to create an OIS benchmark.

Pro: Term interest rate, correct period, similar to OIS

Con: Not in ISDA consultation. No term rate benchmark yet. FSB OSSGs message.

ICE swap rate. ARRC recommendations for bond, loan, securitisation include term rates. Term SOFR expected by end 2021. Consultation Euro term rate as EURIBOR-fallback. Working group on Term SONIA Rate (TSRR) + 3 term rate proposals. FCA'Bailey comment at BoE panel (June 2019): *enough liquidity in Sonia-linked instruments for a forward-looking term rate*

*[...] very large legacy [...] various ways to deal with this and our view on this is that **nothing is off the table.***

Fallback – Forward-looking v backward-looking

OIS benchmark and *compounded in arrears* are the same before the fixing date in term of valuation and risk management.

The floating leg amount paid is the spot OIS rate as measured at the fixing date t_0 for the period $[u, v]$ and it is paid in v .

$$\begin{aligned}
 & E^{\mathbb{X}} \left[(N_v^c)^{-1} N \delta F^c(t_0; u, v) \right] \\
 &= N E^{\mathbb{X}} \left[(N_{t_0}^c)^{-1} \delta N_{t_0}^c E^{\mathbb{X}} \left[(N_v^c)^{-1} F^c(t_0; u, v) \mid \mathcal{F}_{t_0} \right] \right] \\
 &= N E^{\mathbb{X}} \left[(N_{t_0}^c)^{-1} \left(\frac{P^c(t_0, u)}{P^c(t_0, v)} - 1 \right) P^c(t_0, v) \right] \\
 &= N E^{\mathbb{X}} \left[(N_{t_0}^c)^{-1} (P^c(t_0, u) - P^c(t_0, v)) \right] \\
 &= N (P^c(0, u) - P^c(0, v))
 \end{aligned}$$

(1) tower property of conditional expectation. (2) $F^c(t_0; u, v)$ is \mathcal{F}_{t_0} -measurable, the formula of the forward and the definition of $P^c(t_0, v)$. (3) martingale property of $P^c(\cdot, x)$.

Fallback – Forward-looking v backward-looking

The cash flow in the backward-looking option is also paid in v but based on the composition of the daily rates r_i compounded over the period. This is similar to the floating payment of an OIS. The present value is given by

$$\begin{aligned} E^{\mathbb{X}} \left[(N_v^c)^{-1} N \left(\prod_{i=1}^n (1 + \delta_i r_i) - 1 \right) \right] &= NP^c(0, v) \left(\frac{P^c(0, u)}{P^c(0, v)} - 1 \right) \\ &= N(P^c(0, u) - P^c(0, v)) \end{aligned}$$

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Fallback - spread options

When?

Values computed just before the discontinuation's announcement.

Option 1: Forward Approach

The spread adjustment [is] calculated based on observed market prices for the forward spread between the relevant IBOR and the adjusted RFR in the relevant tenor at the time the fallback is triggered.

Option 2: Historical Mean/Median Approach

The spread adjustment [is] based on the mean or median spot spread between the IBOR and the adjusted RFR calculated over a significant, static lookback period (e.g., 5 years, 10 years) prior to the relevant announcement or publication triggering the fallback provisions.

Pro: No manipulation, easy to compute and verify.

Con: Value transfer. Not related to current market situation.

Adjustment Spread Computation

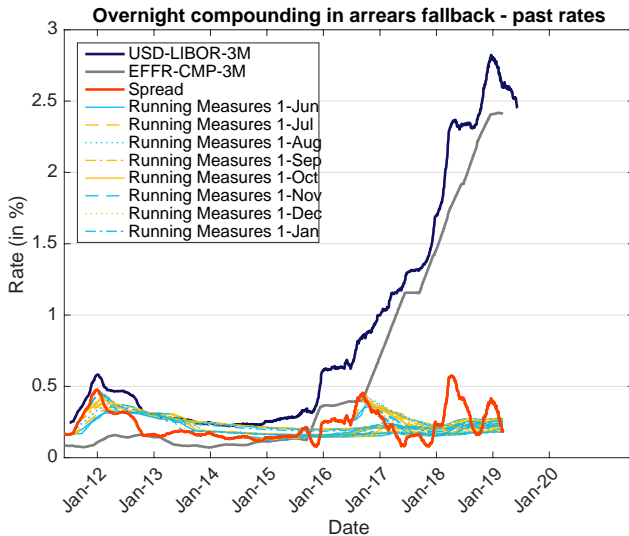
The adjustment spread is denoted $S(X, [a - l, a])$.

It depends on methodology parameters X still to be decided (mean/median, data trimming, transition period). It also depends on market data measured in the period $[a - l, a]$. The announcement date is unknown (stopping time) and the look-back period l is an element of the methodology parameters. All the LIBOR derivatives are now path dependent up to the discontinuation's announcement date.

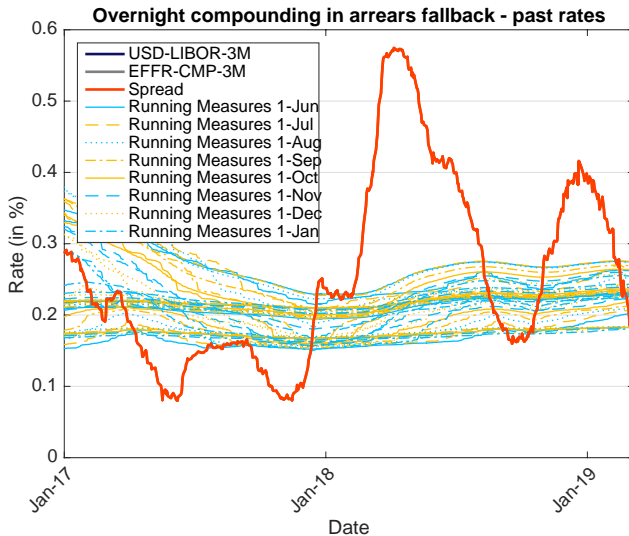
$$N_s^c E^X \left[(N_v^c)^{-1} (\mathbb{1}\{d > \theta\} P^j(\theta) + \mathbb{1}\{d \leq \theta\} (FR^j(\theta) + S(X, [a - l, a]))) \mid \mathcal{F}_s \right].$$

When is the value transfer taking place?

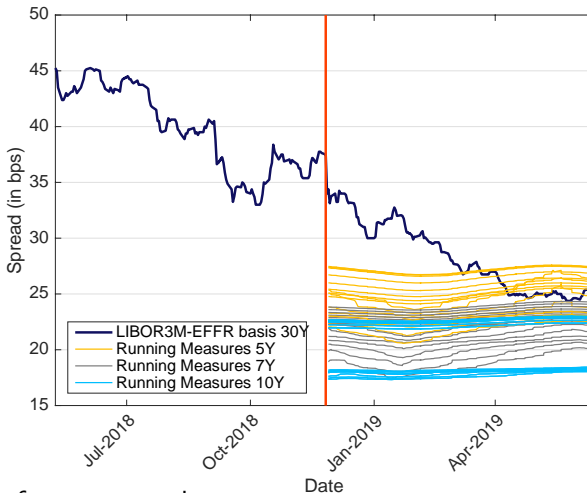
Has the value transfer started?



Has the value transfer started?



Has the value transfer started?



See blog for more graphs:

<http://multi-curve-framework.blogspot.com/2019/05/making-money-on-libor-fallback-5.html>

Fallback in the language of quants – Spread

The spread $S(X, [a - l, a])$ is \mathcal{F}_a -measurable.

Two filtrations (or maybe many): the pure market one, that we have denoted \mathcal{F}_s , and a second one, containing the fallback methodology with material non-public information \mathcal{G}_s : $\mathcal{F}_s \subset \mathcal{G}_s$.

What is the difference between

$$N_s^c E^{\mathbb{X}} \left[(N_w^c)^{-1} X(\text{FR}^j(\theta) + S(X, [a - l, a])) \mid \mathcal{F}_s \right]$$

and

$$N_s^c E^{\mathbb{X}} \left[(N_w^c)^{-1} X(\text{FR}^j(\theta) + S(X, [a - l, a])) \mid \mathcal{G}_s \right].$$

The historical spreads used to compute S cannot be manipulated.

Is it possible to manipulate the methodology and \mathcal{G} ?

Is it possible to know (part of) \mathcal{G} before the general public that knows only \mathcal{F} ?

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Benchmarks – Timetable

- Jul 2014 FSB's paper *Reforming Interest Rate Benchmarks*
- Apr 2017 GBP preferred RFR: Reformed SONIA
- Jun 2017 USD preferred RFR: SOFR
- Apr 2018 First SOFR rate published
- Sep 2018 EUR preferred RFR: ESTER
- Oct 2018 First ISDA consultation on fallback
- Nov 2018 Results of first ISDA consultation on fallback
- May 2019 Second ISDA consultation on fallback
- ??? 2019 Brexit?
- ??? 2019 New ISDA definitions including fallback provisions
- Oct 2019 First ESTER rate published
- Oct 2019 EONIA recalibration
- ??? 20?? SOFR/ESTER as collateral rate
- End 2021 End LIBOR agreement FCA/LIBOR panel banks
- Dec 2021 End transition period for EU Benchmark Regulation
- Dec 2021 End of EONIA

Conclusions

- A clear fallback procedure is required. The ISDA consultation and its results have left many details open, including the achievability of the option selected. A substantial work is still required to obtain a non-ambiguous fallback procedure. [Pay-off and Measurability]
- Spread methodology selected responsible for value transfer. Some transfer may have taken place already and some may take place later. [Filtration]
- The selected transition and fallback solutions require a substantial work from a quant finance and systems perspective.

Contacts

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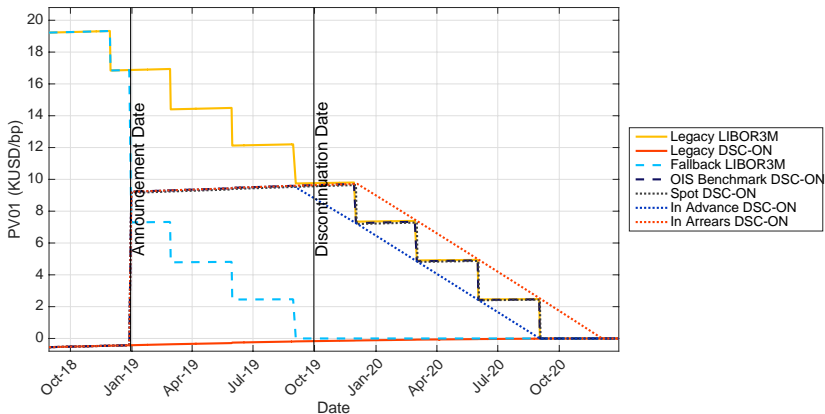
Email: info@murisq.com

Quantitative finance
Derivatives market
Model validation
Market infrastructure

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Risk Transition through the Fallback



Risk – mixed delta

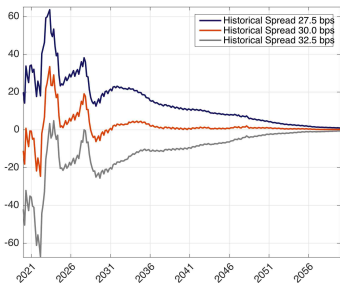
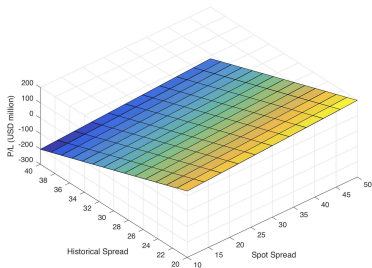
USD-DSCON-OIS		USD-LIBOR3M-IRS	
	USD		USD
USD-OIS-1M	-12	USD-FRA3M-1Mx4	-2,512
USD-OIS-3M	15	USD-FRA3M-3Mx6	-2,472
USD-OIS-6M	-41	USD-FRA3M-6Mx9	0
USD-OIS-9M	1	USD-IRS3M-1Y	-4
USD-OIS-1Y	-144	USD-IRS3M-2Y	-12
USD-OIS-2Y	-340	USD-IRS3M-3Y	16,881
USD-OIS-3Y	-17,412	USD-IRS3M-4Y	16,393
USD-OIS-4Y	-17,009	USD-IRS3M-5Y	0
USD-OIS-5Y	37,439	USD-IRS3M-7Y	0
USD-OIS-7Y	12,639	USD-IRS3M-10Y	0
USD-OIS-10Y	0	USD-IRS3M-15Y	0
USD-OIS-15Y	0	USD-IRS3M-20Y	0
USD-OIS-20Y	0	USD-IRS3M-30Y	0
USD-OIS-30Y	0		
Total	15,136	Total	28,274

Delta risk/bucketed PV01. Risk as of December 2018 under the hypothesis that discontinuation will take place on 1 Jan 2022.

Produced by scalable tools based on production-grade libraries.

Estimating the value transfer

Exact methodologies (spreads) are not known yet. Nevertheless it is possible to estimate the value impacts for different scenarios.



Spot/historical spread impact

Discontinuation date impact

Example: USD 1000 swap portfolio, change adjustment spread and discontinuation date (extract from POC with test portfolio).

Vanilla becoming exotics? Cap/floor

Vanilla IBOR caplet with strike \bar{K} , expiry t_0 and paid-off in v :

$$\delta^j(t_0)(I^j(t_0) - \bar{K})^+.$$

Amount in the case of *compounding setting in arrears*? Amount still paid in v and is given, for a spread S , by

$$\delta^j(t_0) \left(\frac{1}{\delta^j(t_0)} \left(\prod_{i=1}^n (1 + \delta_i^0 I^O(s_{i-1})) \right) - 1 \right) + S - \bar{K} \Big)^+$$

The important difference is the the *expiry date*, which is now delayed to $s_{n-1} = v - 1d$. The pay-off can be written as

$$\max \left(\prod_{i=0}^{n-1} (1 + \delta_i F^O(s_i, s_i, s_{i+1})) , 1 + \delta K \right) - 1$$

The vanilla IBOR cap/floor are becoming Asian options using compounding as averaging method on rates.

Is the GBP spread curve flat?

