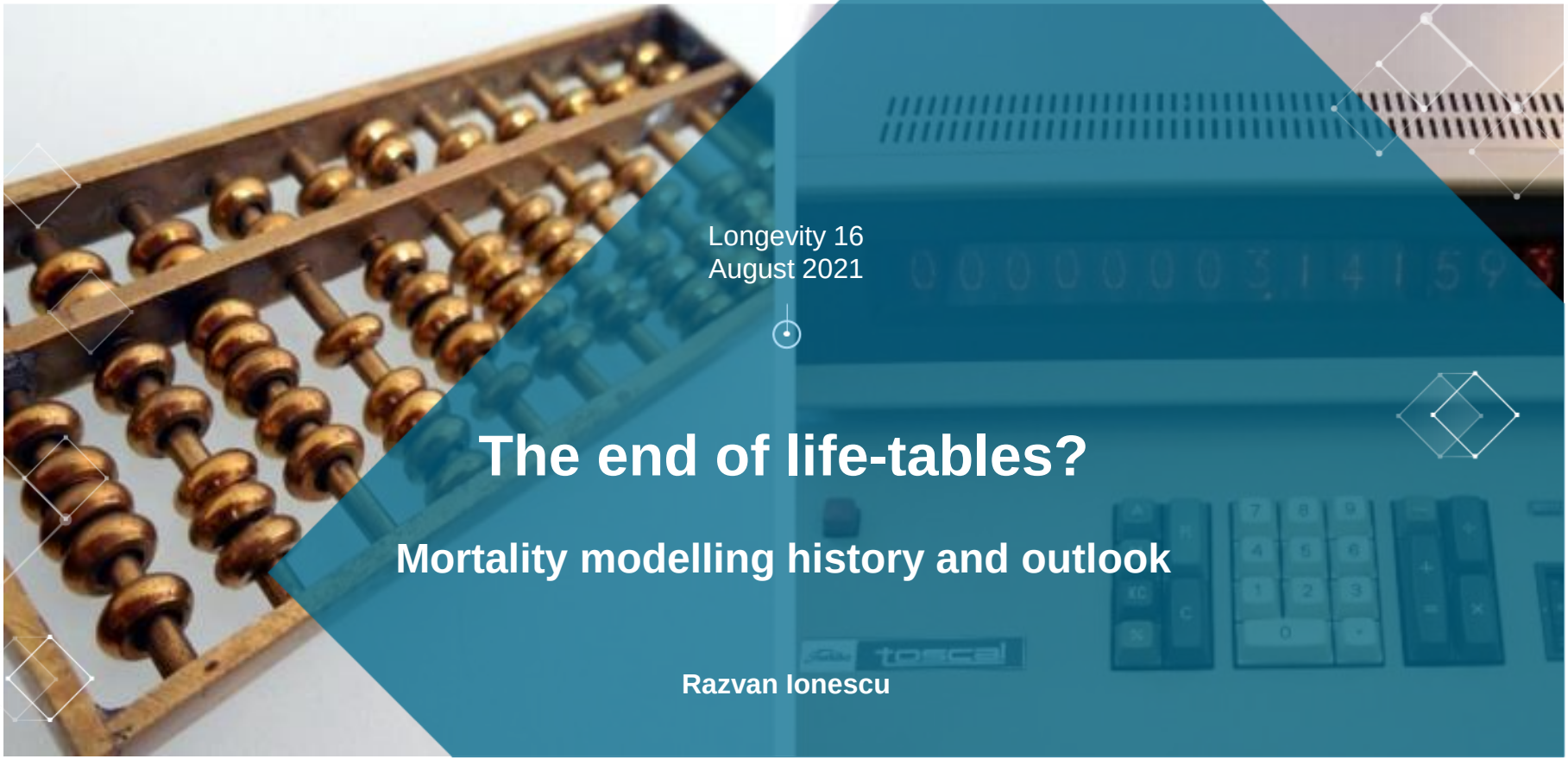

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The end of life-tables?

Mortality modelling history and outlook

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SCOR The Art & Science of Risk | Life

Summary

- For many centuries, selling life annuities was a popular and quick way for European governments to fill the state reserves. This triggered the need to assess annuity buyer's life expectancy. Life Tables proved to be a useful tool. To better reflect annuitants' mortality, during the recent past, Life Tables have been complexified by introducing, in addition to age, additional dimensions such as gender and time.

- However, with the advent of information technologies and big data, the computation power and the available information have been significantly increased. Today, Machine Learning techniques allow to exploit the whole set of information available, to estimate mortality risk.

- Will these new methods make Life Tables obsolete? After a description and comparison of advantages of both approaches, we will try to answer to this question.

Life annuities: a 2000 years old story

While life annuity exists since antiquity, the estimation method considered were crude for several centuries.

Statue of Ulpian

Ulpian, a famous Roman jurist, proposed around 200 AD an update of life annuities value estimation by refining the price variation by age.

Ulpian's tables is considered as the oldest known life table. However, it is most certainly more an *educated guess* than an accurate risk estimation.

In the following centuries, selling life annuities was a popular and quick way for European kingdoms and governments to fill the state reserves.

This triggered the need to assess annuity buyer's life expectancy. However, methods of estimation haven't evolved up to the 17th century.

The moneychanger and his wife
Marinus van Reymerswaele

The use of probabilities and mortality statistics in life annuities valuation starts in the 17th century.

Johan de Witt

Jan de Witt, *Grand Pensionary* of Holland, proposed in **1671** to use mathematics to valuate life annuities relying on mortality probabilities. It is considered as the first application of probabilities in finance.

His calculations showed that the annuities were mispriced and proposed a transformation of the system into age-dependent scheme. However, after his violent death in 1972 the *Staten* decided to go back to the old pricing system.

In **1693**, Edmond Halley, famous English astronomer, using the birth and death records of the city of Breslau (Poland) constructed a life table and used it to price life annuities.

His work greatly influenced the development of actuarial science.

Edmond Halley

Actuarial practices were not immediately full adopted. The importance of a correct mortality assessment appears at the start of the 19th century.

Bastille 14th of July 1789

In France, after the seven years' war, the flat-rate pricing of life annuities has been reinstated. This led to a severe crisis as life annuities become the largest kingdom debt component by 1788. Some historians claim that it is one of the root causes of the French revolution in 1789.

Following the revolution, in **1794,** Pierre-Joseph Cambon, president of the *Comité des Finances,* restructured the life annuity debt relying on actuarial techniques.

In **1808,** to face Napoleonian invasion threat, British government decided to sell life annuities to raise money. However, the use of unappropriated data lead to underestimation of the price.

Indeed, the mortality data originating from 1780 church records in Northampton was not reflecting the lower mortality of annuitants that belongs to the highest social classes.

Caricature of Napoleon

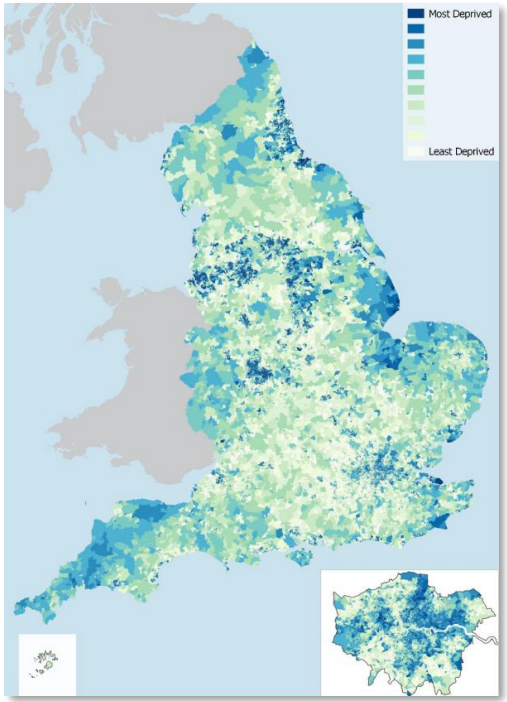
Fast forward to present days

Underwritten annuities, a product that requires more than a *simple* segmentation modelling approach.

- To capture annuitants' mortality, actuaries are considering life tables by segments, or groups. For instance, for life annuities, life tables can vary by:
  - Gender (Male, Female)
  - Marital status (Married, Widow, Never-married)
  - Socioeconomic group (postcodes-based groups)
  - Annuity amount group
- This may lead practitioners to deal with a significant number of life tables.

- Underwritten annuities products: applicants with poor health are offered cheaper annuities to compensate for their reduced life expectancies. To price these kind of products, in addition of the variables listed above, underlying mortality assumptions must capture the impact of health status and behavior (BMI, tobacco, Blood Pressure, Cholesterol...) on applicants' mortality.

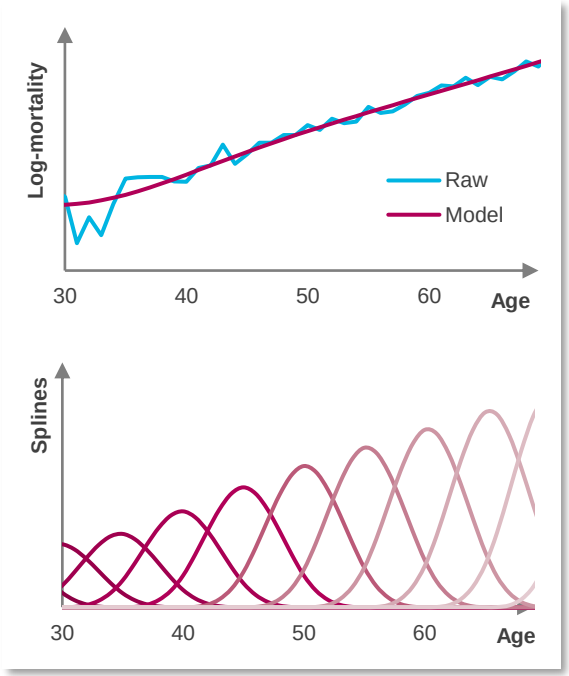
IMDs in England



Life tables are favored by actuaries and models are mainly used to construct them.

- In 1825 Gompertz proposed the first mortality model that captures the exponential increase of mortality with age. However, the mortality pattern with age is more complex to represent.
- Recently (late 90's) actuaries started to use non-parametric approaches (such as Local Likelihood or P-Splines) allowing to model non-linear effects and to properly capture mortality patterns.
- However, practitioners are using models mainly to extrapolate mortality and smooth raw mortality rates, i.e. to construct nice life tables.

Poisson P-Splines illustration



Big Data and Machine Learning

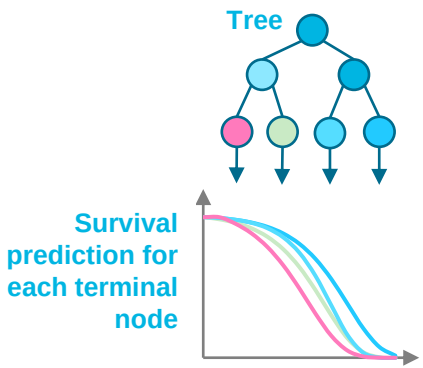
Machine Learning approaches allows to take advantage of large dataset.

- Machine Learning (ML) refers to powerful models that can deal with large dataset, i.e., take advantage of all information available.
- However, this comes with a cost as ML models are complex and often compared to a "*black box*". Nonetheless, solutions exist to interpret ML model such as *Partial Dependance or SHAP*.
- In Finance, the ML techniques are applied, and improvement is observed when compared to previous methods. But this ability to deal with large dataset, mainly triggered a race to get new *exotic* data.
- Over time more and more information will be available for mortality risk prediction. One can expect that the amount of information collected today would generate very rich mortality experience dataset in the next decades.

Survival Tree and Random Survival Forests models as an illustration.

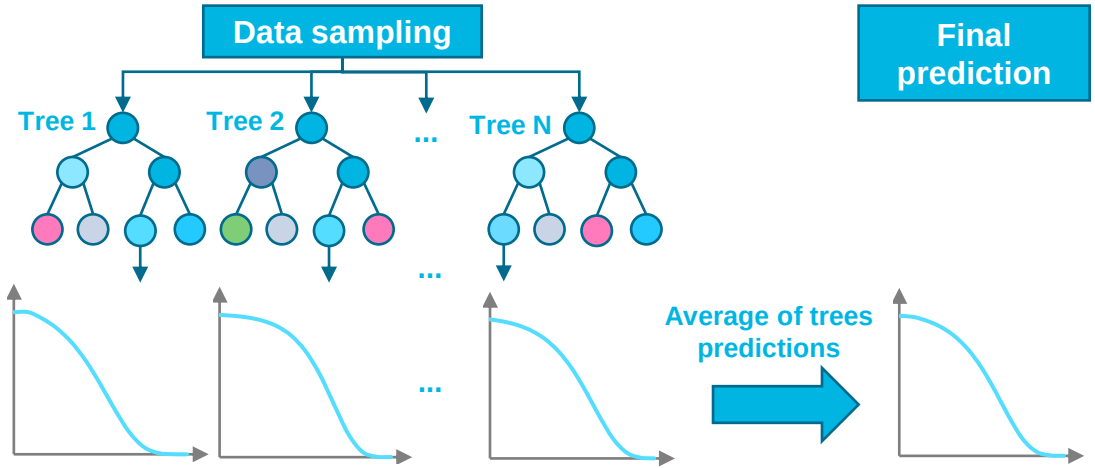
Survival Tree

- Survival Tree algorithm *segments* the population based on mortality. For each final *segment*, mortality is estimated.
- This is equivalent to dividing a population in several groups and constructing a life table for each group. It is close to current actuarial practices.



Random Survival Forests

- Survival Forest algorithm perform data sampling, then for each sample, a survival tree is constructed. Final prediction is obtained by averaging the prediction of each constructed tree.
- It is equivalent to constructing thousands of life tables and averaging them.



Outlook

Hybrid approaches: life tables and ML models can coexist.

- Relying on relational models, one can consider the following mortality hazard rates relationship:

$$\log(\mu(X)) \sim \log(\mu^{ref}(X)) + \theta(X)$$

Life table reference mortality.

Life table considered uses only subset of X to predict the mortality, e.g., only *Age* and *Gender*.

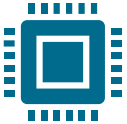
Adjustment to be applied to the reference mortality.
The model θ can rely on Machine Learning techniques. It captures all information of individual X, e.g., *Age, Gender, Socioeconomic group, Marital status, Tobacco, BMI…*

- The approach starts with a life table and uses machine learning to fine tune the model θ:
  - Imposing no intercept ensures the average predicted mortality corresponds to the reference mortality
  - Adding a penalty on *Age* allows to keep similar mortality shape as the reference mortality

- This approach allows to capture individual specificities. It also provides comfort as Machine Learning approaches outputs are adjustments applied on top of a life table. However, it doesn't solve all the problems.

Decades of actuarial practices using life tables make it difficult to quickly move to full model-based approach.

Practitioners consider life tables to be more convenient than a complex models. The *synthetic power* of life tables allow to clearly share the information.



Most of IT systems have been constructed around life tables. Systems are *picking up* the right values in the life table to perform the needed calculations.

As a life table can be easily printed on paper, it is easier to add to a (re)insurance contract or treaty appendix.

Regulation is built around life tables, as statutory life tables must be used to compute statutory reserves. From a peer review perspective: life tables appear to be more *transparent* than a complex model.


CONCLUSION

Conclusion

- While the *synthetic power* of Life Tables allows to easily share the information, its complexity, added to better reflect insured mortality, makes them less *transparent*.
- Machine Learning techniques allow to fully take advantage of all the information available in a large dataset to predict mortality. The *black box* aspect of this algorithm is slowing their adoption.
- Probably Life Tables will remain around for the next decades. But the thirst for more accurate predictions will eventually push actuaries to adopt more complex models.
- It is hard to believe that life tables will be still used for annuities valuation over another century. My prediction: "*Life tables will no longer be used (in actuarial science) before I retire*".

Thank you

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