

Longevity 19, Amsterdam

Some practical benefits of continuous-time methods

Stephen J. Richards and Angus S. Macdonald

17th September 2024



1. Foreword

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2. Benefits of continuous time

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2. Benefits of continuous time
3. Conclusions

**ON
CONTEMPORARY
MORTALITY
MODELS**
FOR ACTUARIAL USE

By S. J. Richards & A.S. Macdonald

The Longevity logo, consisting of a gear icon and the word "LONGEVITAS" in a serif font, positioned at the bottom of the book cover image.

PDF available at: <https://www.longevity.co.uk/published-paper/contemporary-mortality-models-actuarial-use>

2 Benefits of continuous time

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4. Superior management information.

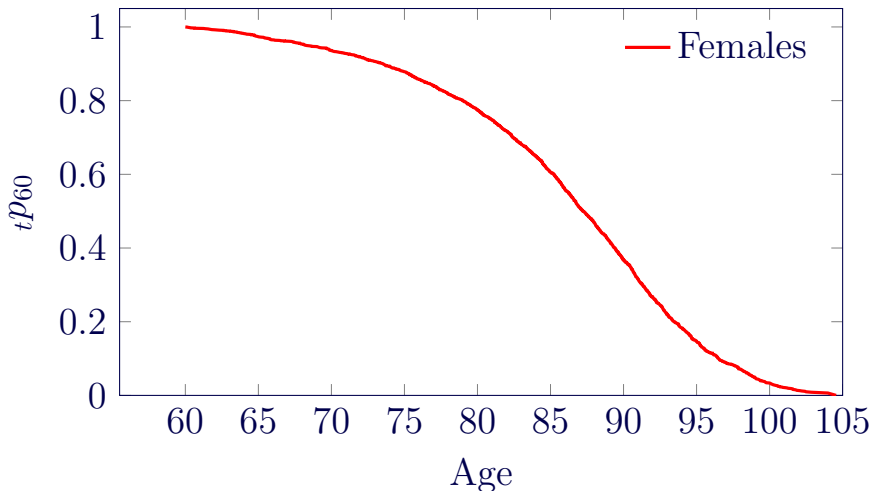
Kaplan and Meier [1958] presented a non-parametric estimate of the survival curve, ${}_t p_x$:

$${}_t \hat{p}_x = \prod_{t_i \leq t} \left(1 - \frac{d_{x+t_i}}{l_{x+t_i^-}} \right), \quad (1)$$

- x is the outset age for the survival function,
- $\{x + t_i\}$ is the set of distinct ages at death,
- $l_{x+t_i^-}$ is the number of lives alive immediately before age $x + t_i$ and
- d_{x+t_i} is the number of deaths occurring at age $x + t_i$.

Benefit 1: Data quality checks LONGEVITAS

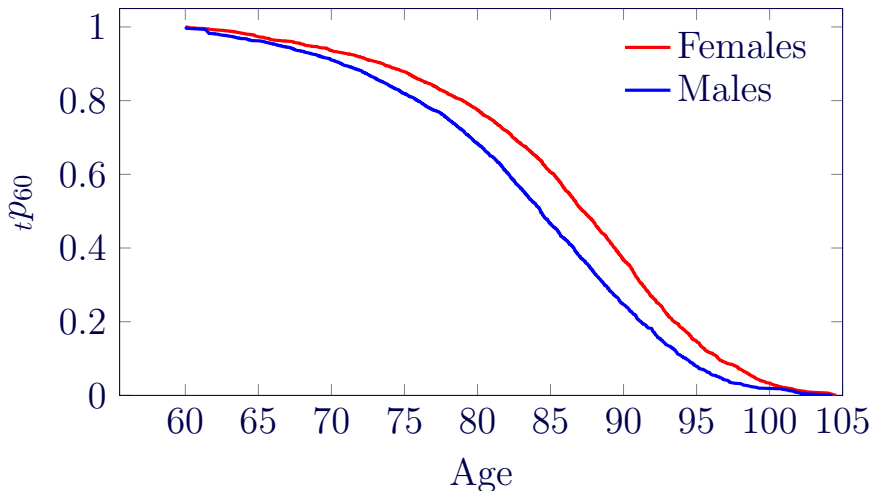
Survival curves for Dutch pension scheme:



Source: past consulting work.

Benefit 1: Data quality checks

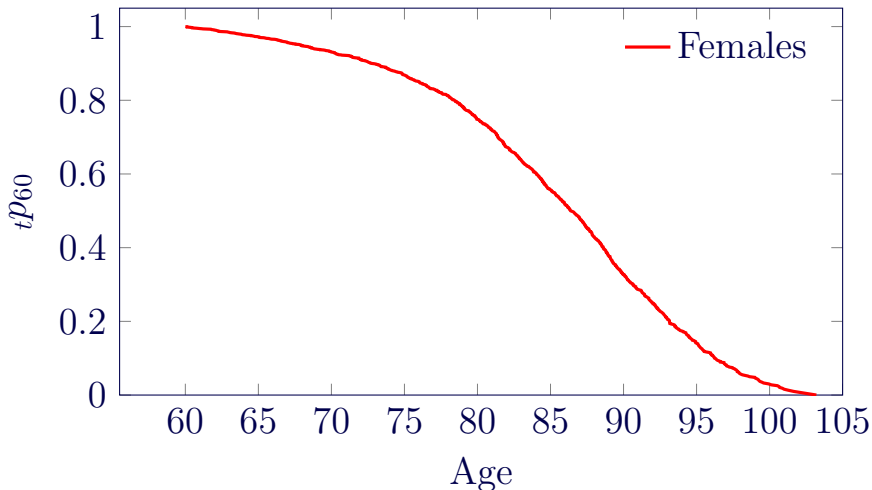
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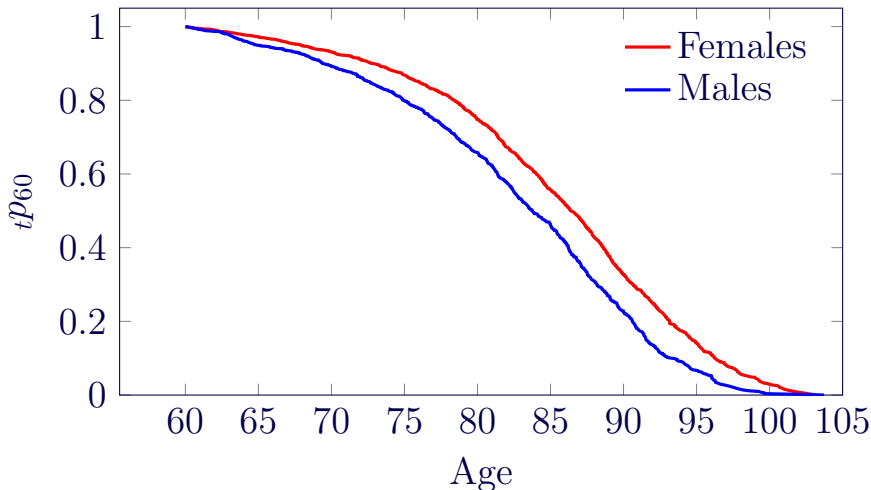
Survival curves for Scottish pension scheme:



Source: Richards and Macdonald [2024, Figure 12(a)].

Benefit 1: Data quality checks

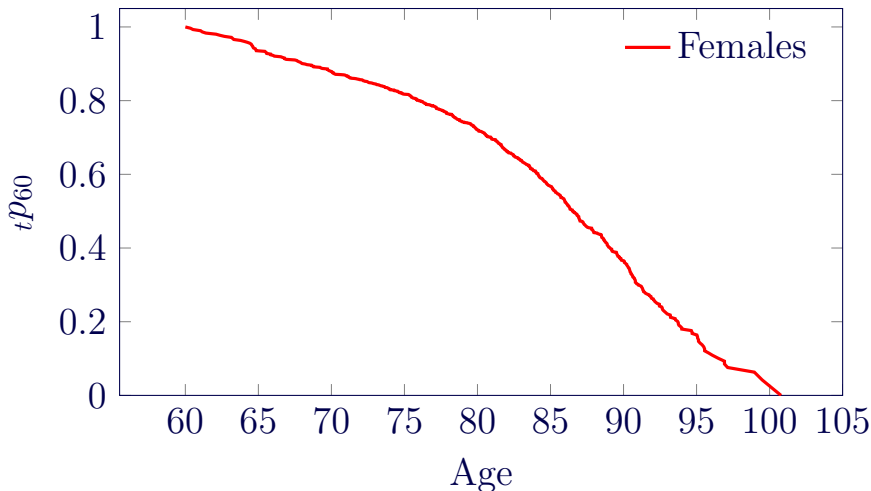
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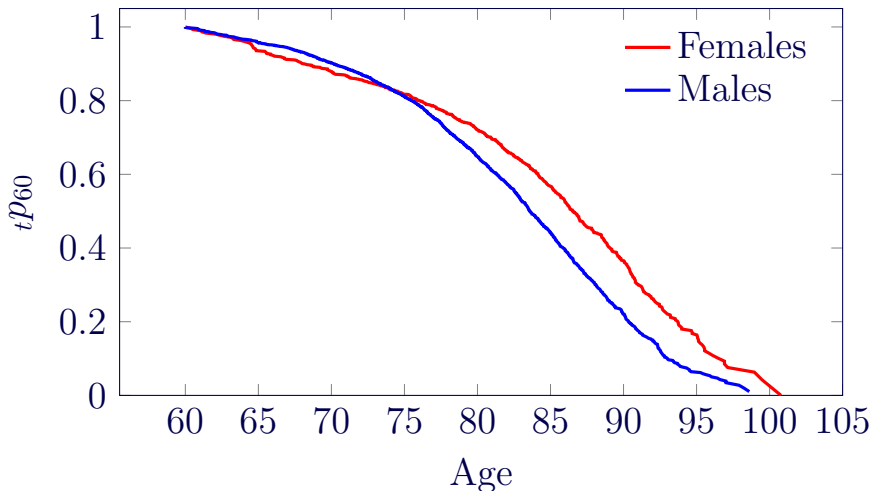
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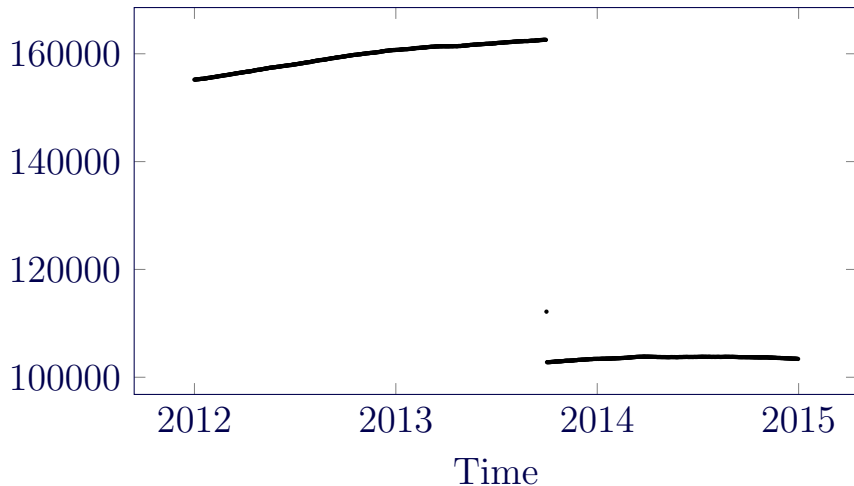
- As checks for data quality.
- For communicating to non-specialists.

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- However, observation can be interrupted in real world...

Number of in-force annuities at each date for a UK insurer:



Source: Richards and Macdonald [2024, Figure 3(a)].

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- Commutation of small pensions.

- Survival models handle interrupted observations as *right-censored* records.

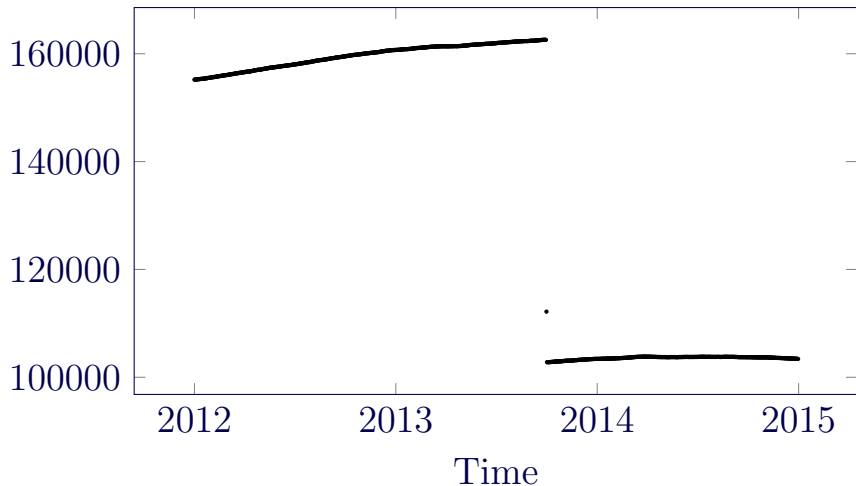
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- Early exits are treated like survivors, just with an earlier censoring date.

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- Binomial models are not well suited to this...
...but survival models are.

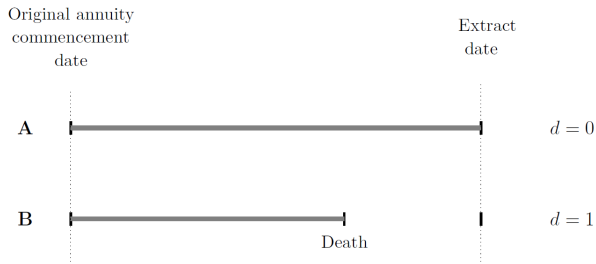
Original annuity
commencement
date

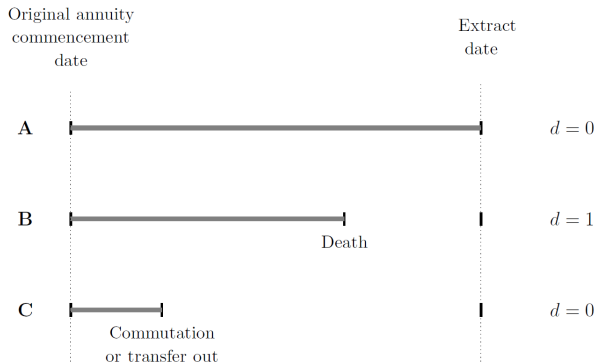
Extract
date

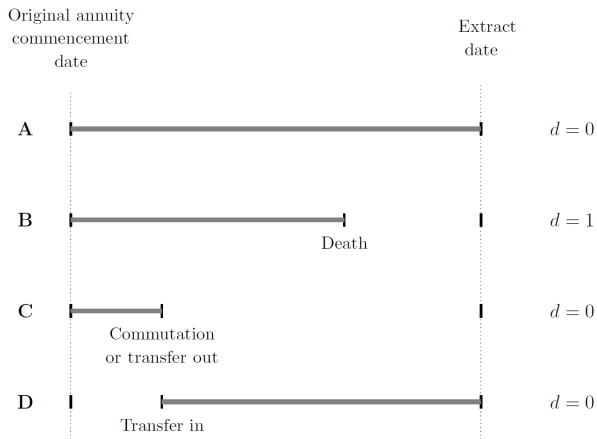
A

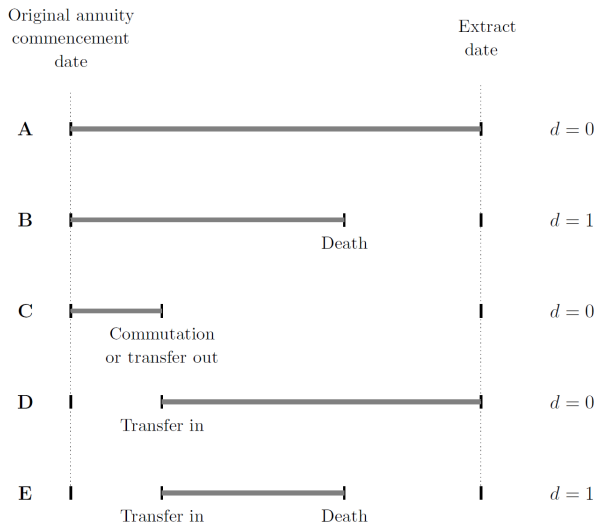


$d = 0$



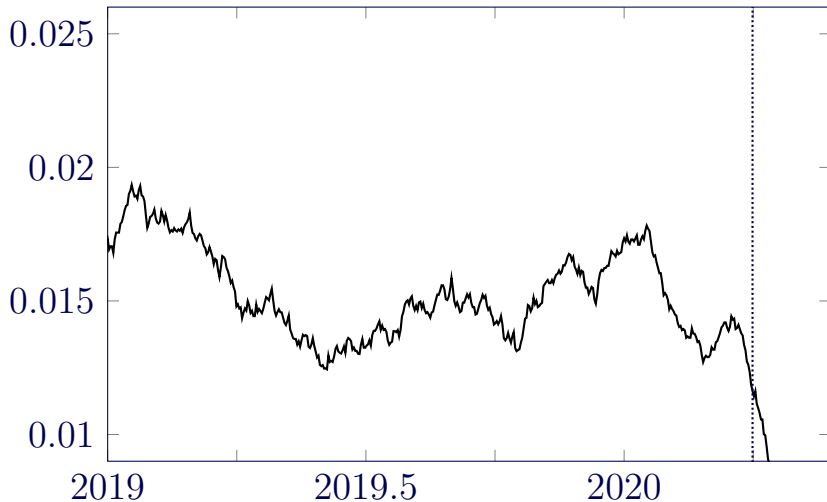






Benefit 4: Management information

Mortality hazard using June 2020 extract:

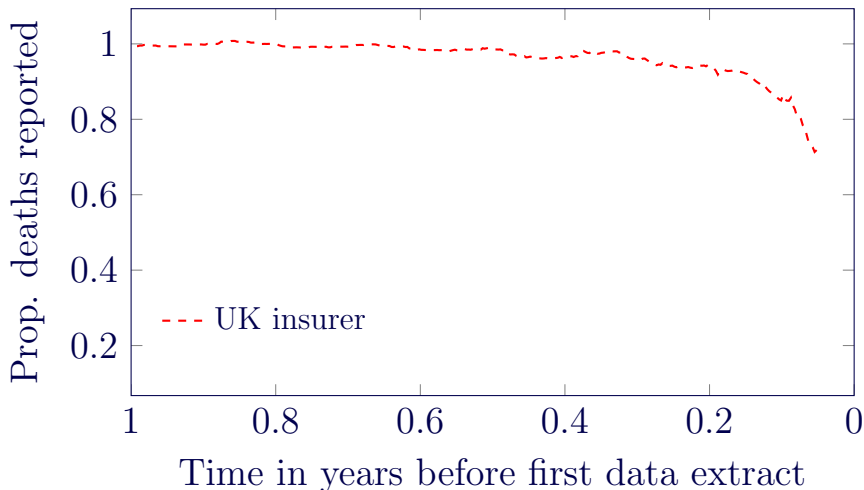


Source: Richards and Macdonald [2024, Figure 15(a)].

1. No sign of pandemic mortality by June 2020.

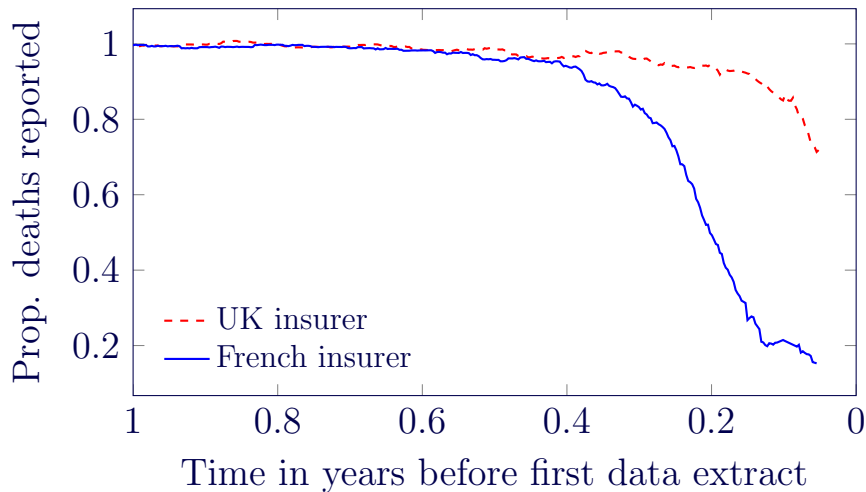
1. No sign of pandemic mortality by June 2020.
2. Problem of delays in reporting deaths (IBNR/OBNR)...

Estimated proportion of deaths reported for two annuity portfolios:



Source: Richards [2022a, Section 4].

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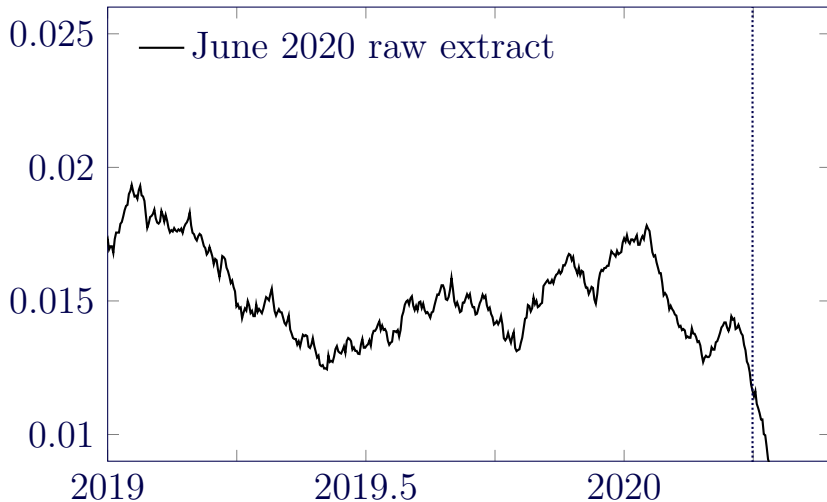
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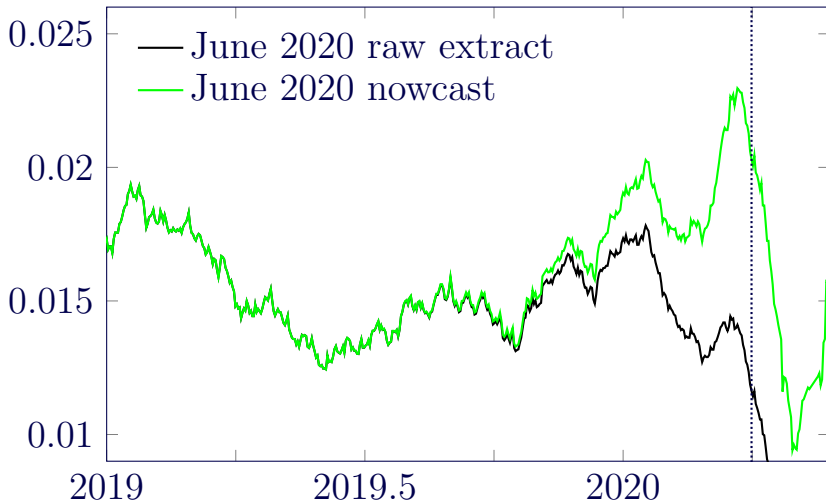
1. Estimate the delay function.
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3. Bańbura et al. [2013] call this a “nowcast”...

Mortality hazard:



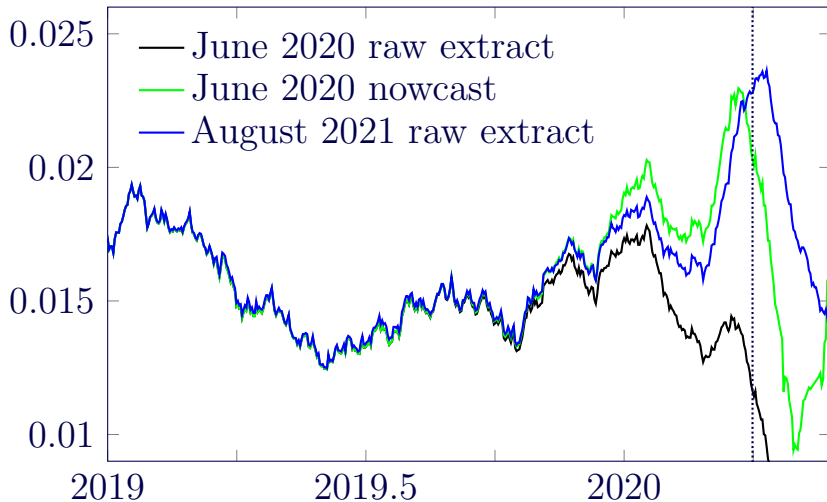
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3 Conclusions

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With continuous-time methods actuaries can:

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4. Get timelier management information.

- M. Bańbura, D. Giannone, M. Modugno, and L. Reichlin. Chapter 4 — Now-Casting and the Real-Time Data Flow. In Graham Elliott and Allan Timmermann, editors, *Handbook of Economic Forecasting*, volume 2, pages 195–237. Elsevier, 2013. doi: 10.1016/B978-0-444-53683-9.00004-9.
- E. L. Kaplan and P. Meier. Nonparametric estimation from incomplete observations. *Journal of the American Statistical Association*, 53:457–481, 1958.

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- S. J. Richards. Allowing for shocks in portfolio mortality models. *British Actuarial Journal*, 27:1–22 (with discussion), 2022b. doi: 10.1017/S1357321721000180.
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