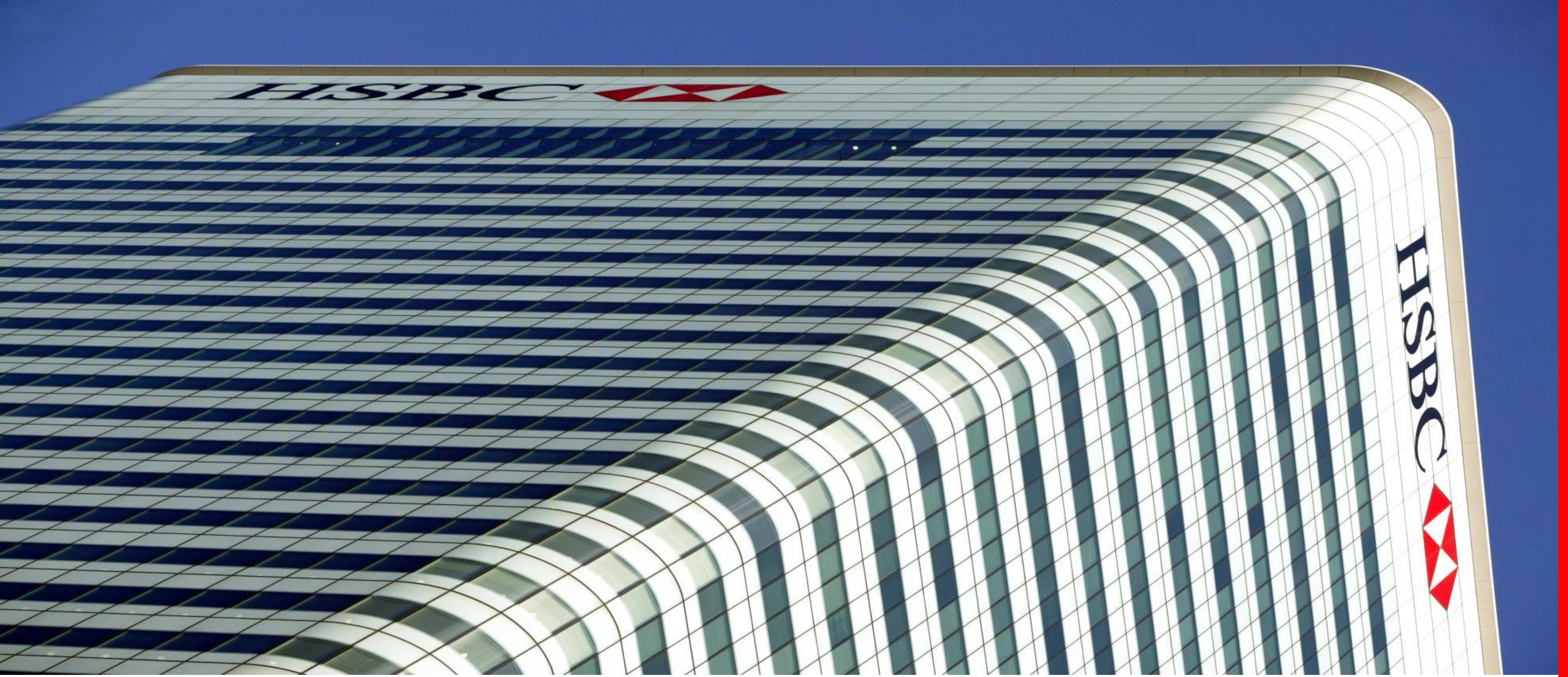


Market Making in Foreign Exchange

A. Barzykin

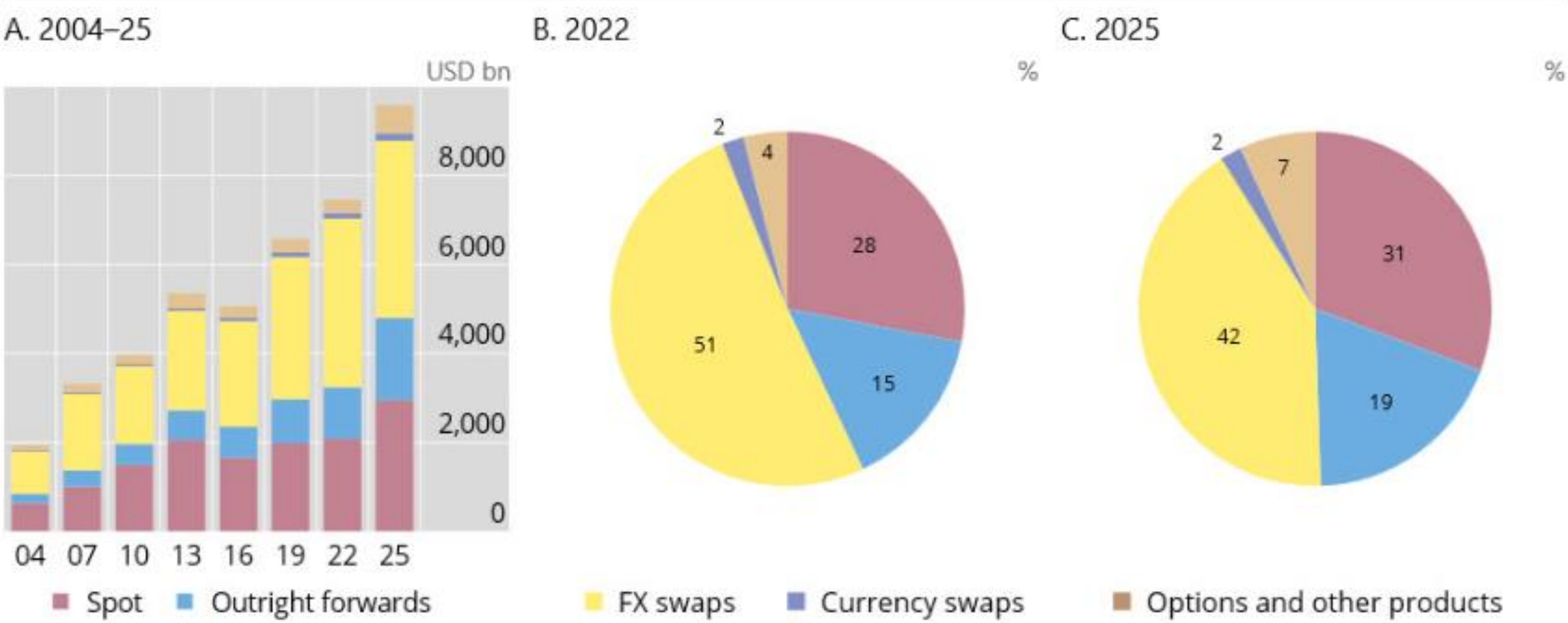
2025.10.29 Bayes Business School

HSBC

Foreign exchange market turnover by instrument[1]

Net-net basis, daily averages in April

Graph 1

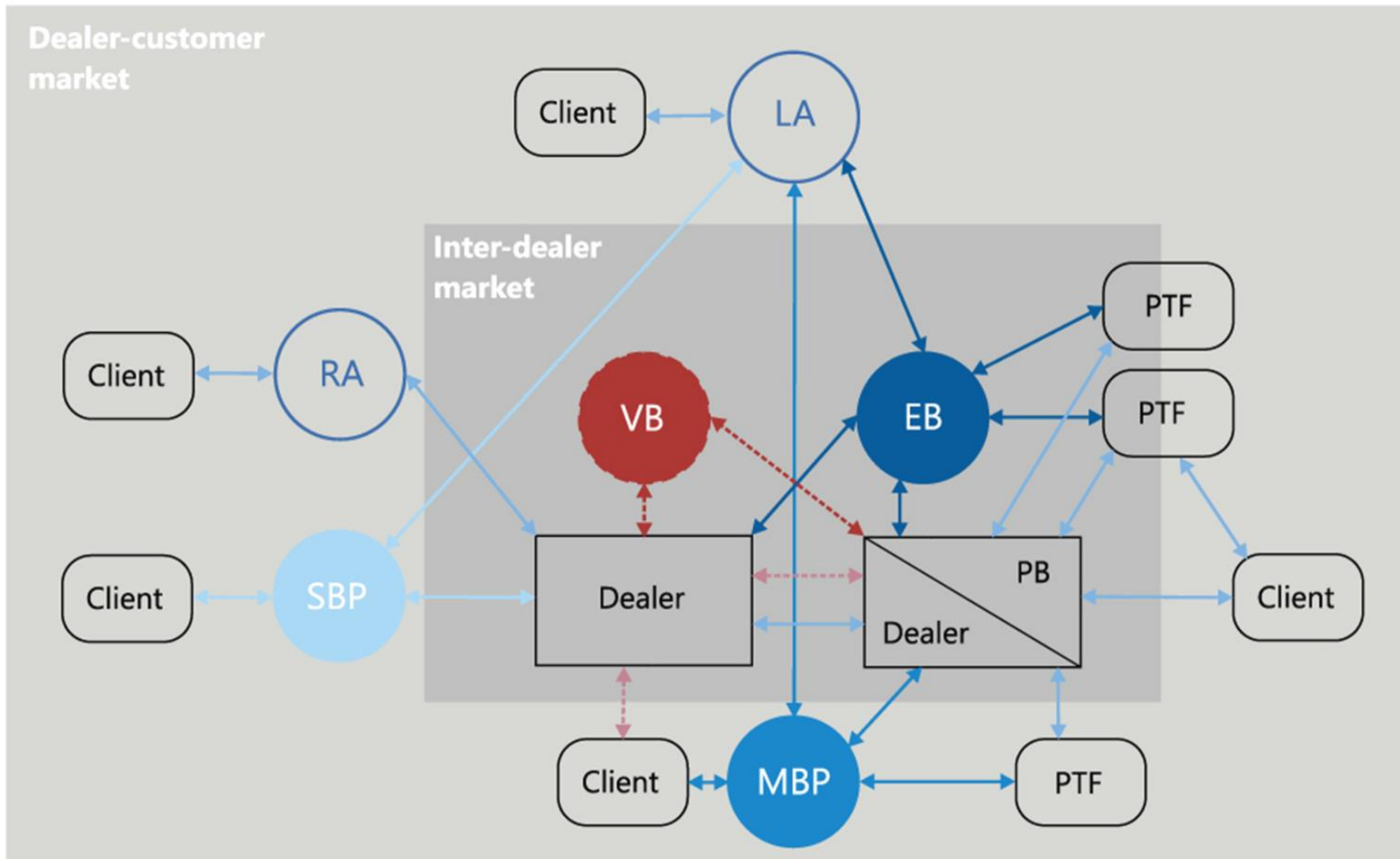


[1] Adjusted for local and cross-border inter-dealer double-counting, ie "net-net" basis.

Source: BIS Triennial Central Bank Survey. For additional data by instrument, see Table 1.

Source: Bank for International Settlements, 2025

Stylised Structure of the FX Market



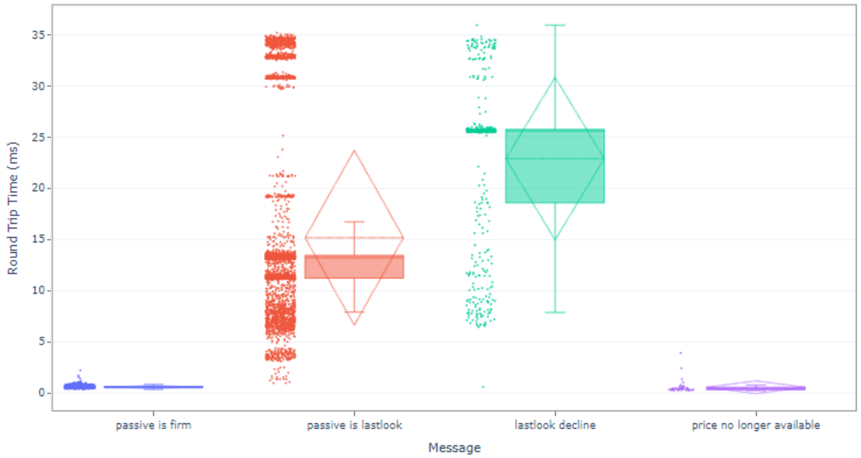
EB = electronic broker; LA = liquidity aggregator; MBP = multi-bank platform; PB = prime broker; PTF = principal trading firm; RA = retail aggregator; SBP = single-bank platform; VB = voice broker. Dashed lines indicate voice execution; solid lines indicate electronic execution.

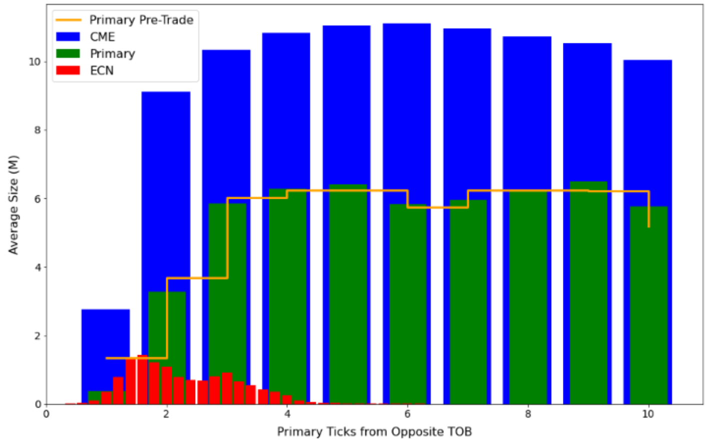
Source: King et al (2012), augmented by adding LA to depict liquidity aggregators and PTFs in their roles as both clients and intermediaries.

Source: A. Schrimpf and V. Sushko, BIS Quarterly Review, December 2019

FX liquidity landscape: global, fragmented de-localized

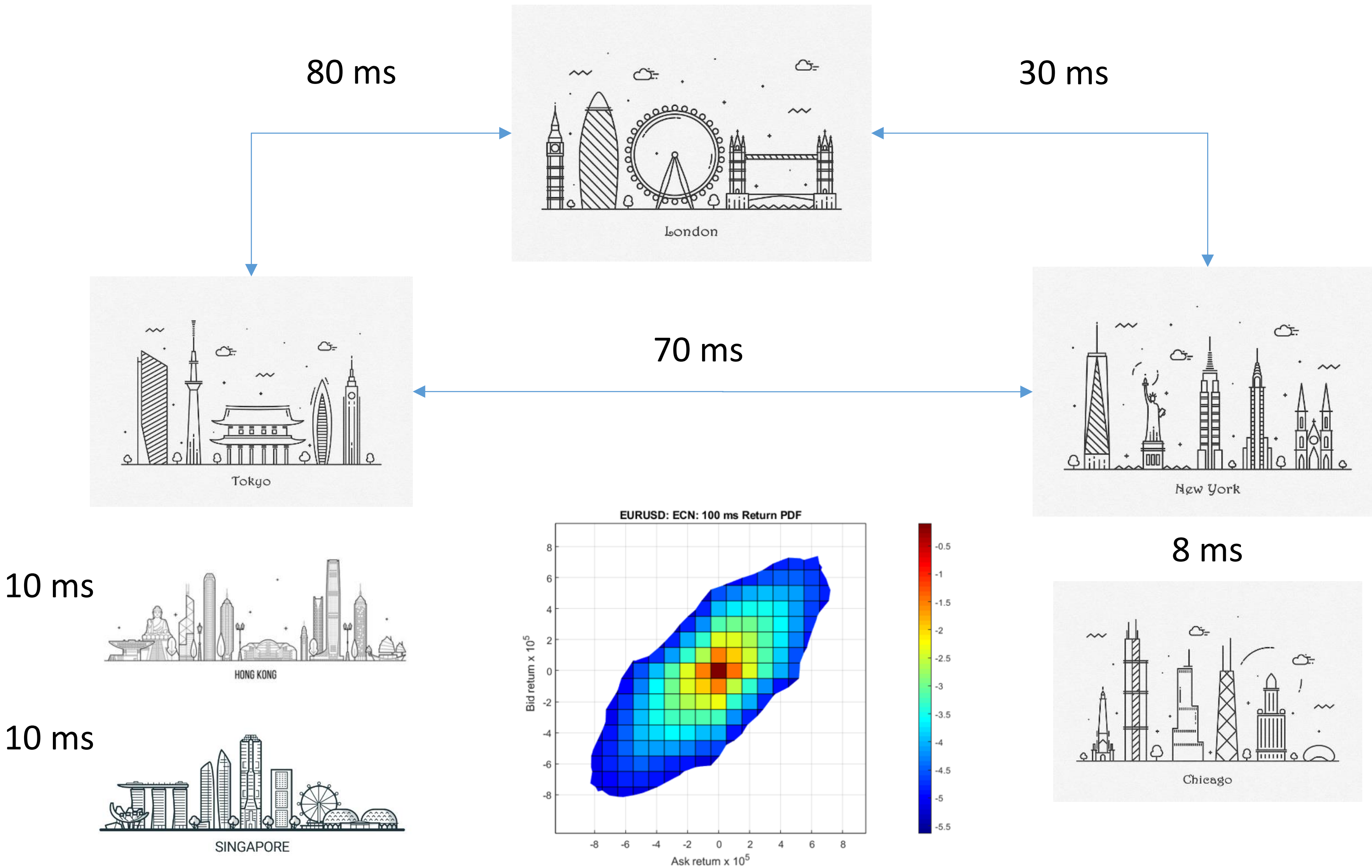
- **De-centralization:** multiple liquidity sources around the globe
  - Technological requirements for liquidity providers; co-location in several centres
  - Natural geographical **latency**
- **Firm and non-firm liquidity:**
  - **Non-firm** venues allow for **last-look** practice
- **Throttling and Randomization**:
  - Certain data feeds are **throttled**
  - **Credit screened** feeds are slow
- **Specifics of Liquidity Sources**
  - On / Off Sef: Liquidity type may change during the day
  - Mid books, post trade compensation practice
  - Indirect Liquidity: CME **futures**
- **Volume and Order Book Uncertainty**:
  - Historical and real-time volume information is incomplete
  - Icebergs with minimum visibility
  - Liquidity Recycling
  - **Different tick and lot sizes**





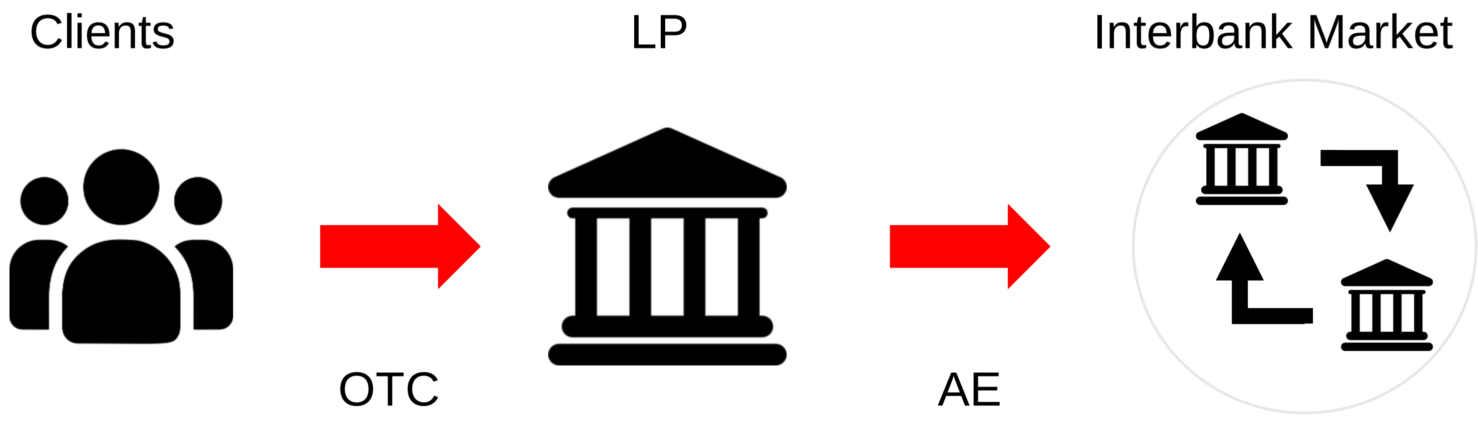
Source: HSBC

FX liquidity landscape: latency



Source: HSBC data

Motivation



Internalization

- \+ Making spread
- \- Flow uncertainty, risk

Externalization

- \+ Certainty of execution
- \- Transaction cost, market impact

- Understanding the role of liquidity conditions, trading styles and controls within a single framework
- Understanding client tiers
- Academic literature only addressed either pure OTC market making or algorithmic execution
- Multi-currency valuation challenge and the curse of dimensionality

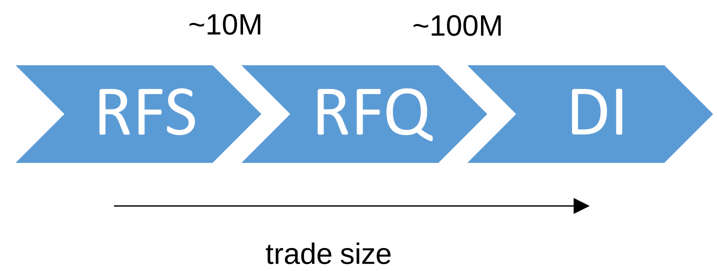
Understanding Client Flow

HSBC

S_t reference market price

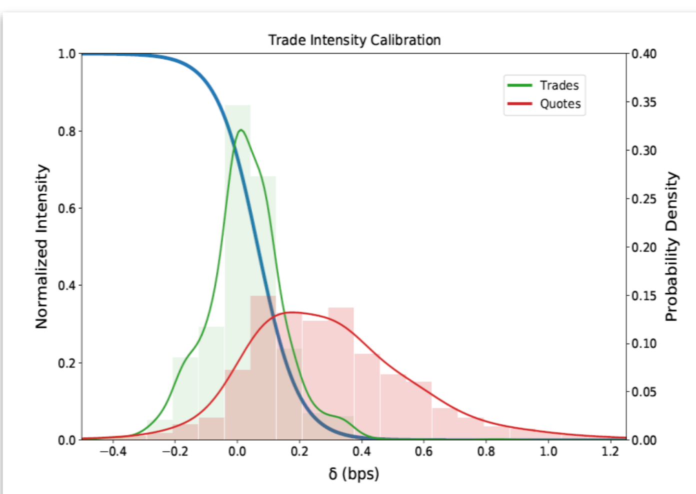
$S^b(t,z) = S_t(1 - \delta^b(t,z))$ quoted bid price

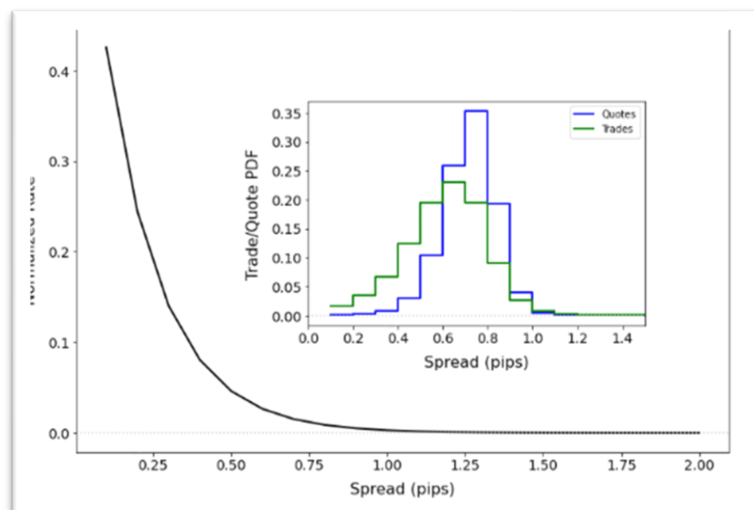
$S^a(t,z) = S_t(1 + \delta^a(t,z)))$ quoted ask price



$$
\begin{aligned}
\mathrm{LL}_k^{b/a} &= \sum_{i \in \mathcal{I}_k^{b/a}} \log(\Lambda_k^{b/a}(\delta^i)) - \sum_{j \in \mathcal{J}_k^{b/a}} \Lambda_k^{b/a}(\delta^j)\tau^j \\
&= I_k^{b/a} \int_\delta \log(\Lambda_k^{b/a}(\delta)) f_k^{b/a,\mathcal{T}}(d\delta) - \bar{\tau} \int_\delta \Lambda_k^{b/a}(\delta) f_k^{b/a,\mathcal{Q}}(d\delta)
\end{aligned}
$$

$$\Lambda_k^{b/a}(\delta) = \frac{\lambda_k^{b/a}}{1+e^{\alpha_k^{b/a}+\beta_k^{b/a}\delta}}$$ trade intensity

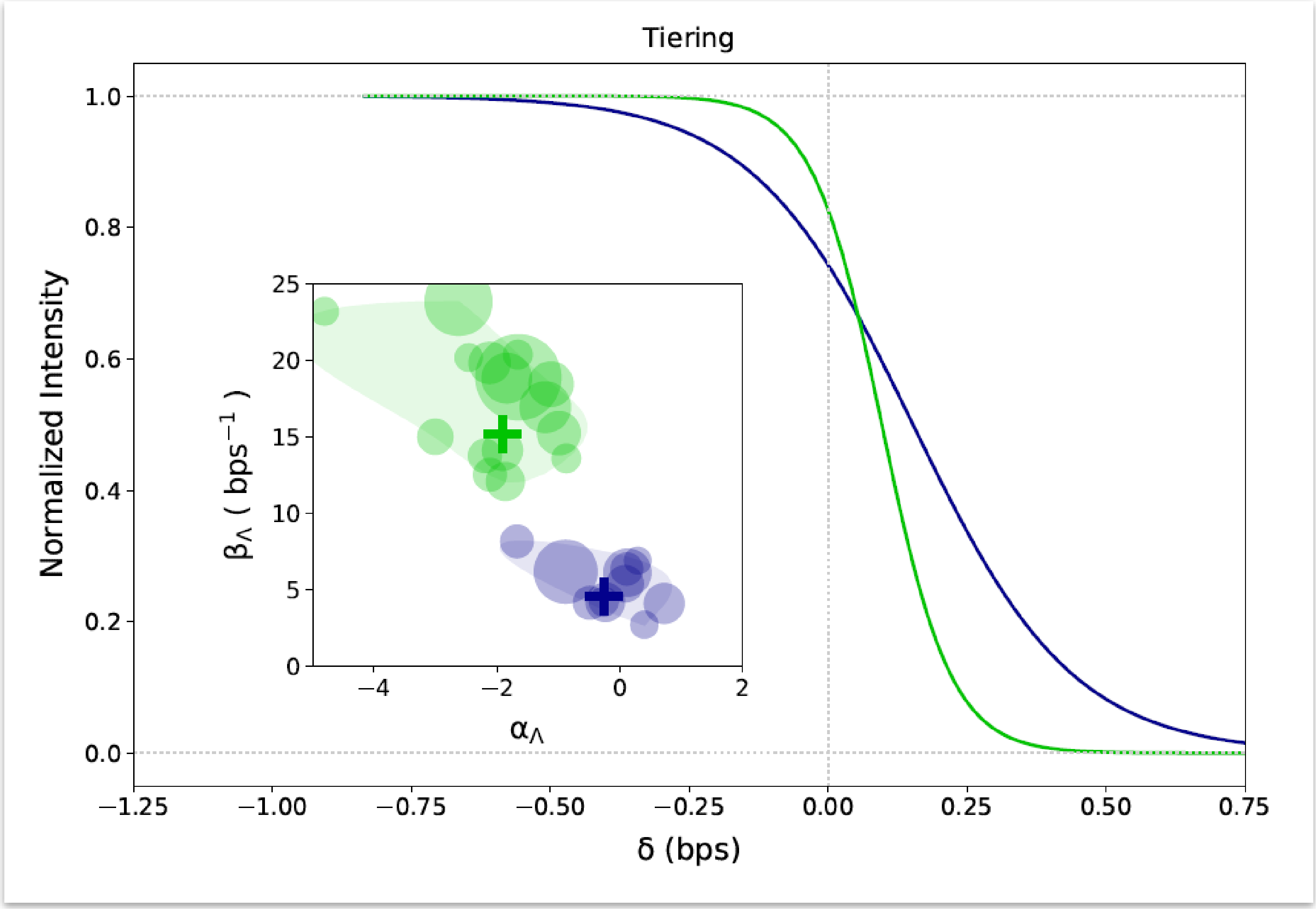




Source: BBG, Risk, 2022

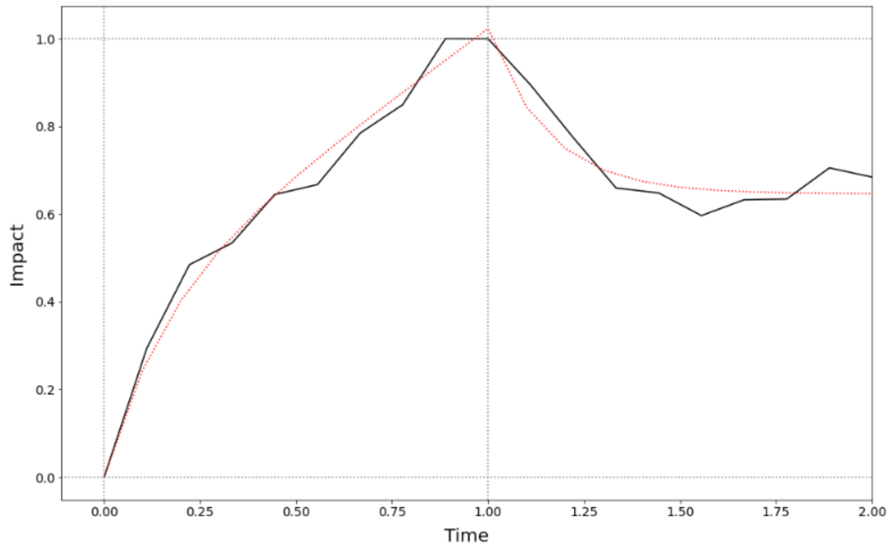
Source: HSBC Data

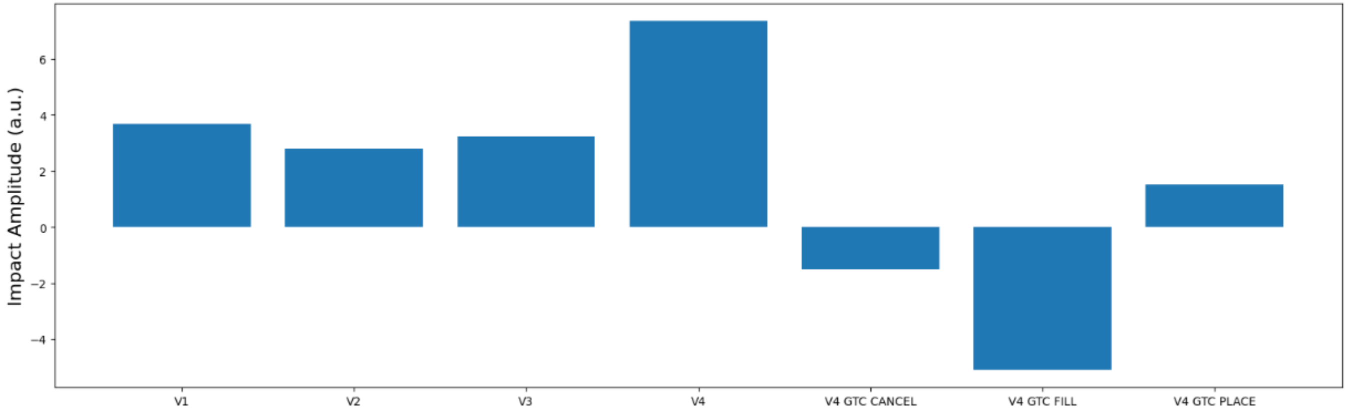
Client Tiers by Clustering



Source: BBG, Risk, 2022

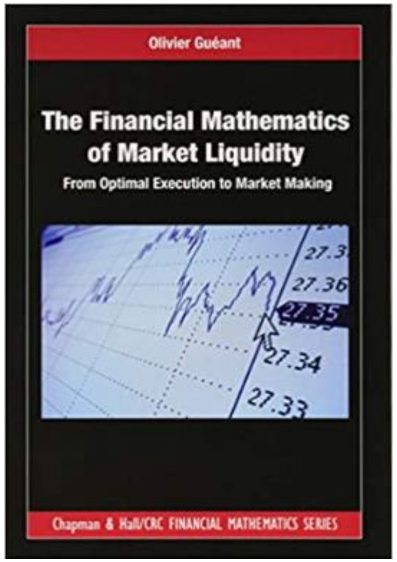
Algorithmic Execution Calibration

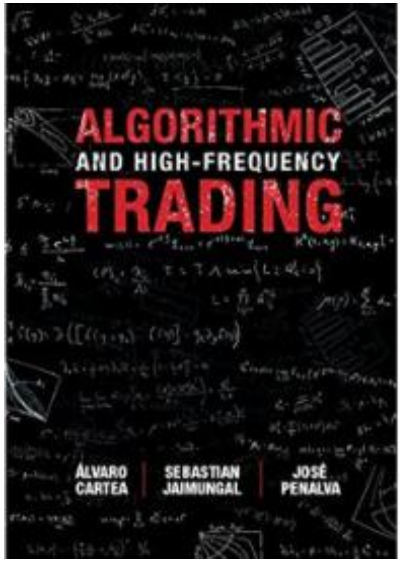




Source: HSBC Data

- M. Avellaneda and S. Stoikov, High-frequency trading in a limit order book, 2008.
- R. Almgren and Chriss, Optimal execution of portfolio transactions, 2001.





HSBC FX Research Initiative

P. Bergault

O. Guéant

Stochastic Optimal Control

HSBC

Inputs

- Random market price with known volatility
- Client flow intensity and size distribution (jump process)
- Cost and impact of interbank trading

Controls

- Bid/ask pricing $\delta^{b,n}(t,z), \delta^{a,n}(t,z)$
- Interbank hedging rate v_t

Price: $$dS_t = \underbrace{\sigma S_t dW_t}_{\textit{volatility}} + \underbrace{k v_t S_t dt}_{\textit{market impact}}$$

Inventory: $$dq_t = \underbrace{\sum_{n=1}^{N}\int_{z=0}^{\infty} z J^{b,n}(dt,dz) - \sum_{n=1}^{N}\int_{z=0}^{\infty} z J^{a,n}(dt,dz)}_{\textit{client flow}} + \underbrace{v_t dt}_{\textit{hedging}}.$$

Wealth: $$dX_t = \underbrace{\sum_{n=1}^{N}\int_{z=0}^{\infty} S^{a,n}(t,z) z J^{a,n}(dt,dz) - \sum_{n=1}^{N}\int_{z=0}^{\infty} S^{b,n}(t,z) z J^{b,n}(dt,dz)}_{\textit{spread capture}} - \underbrace{v_t S_t dt - L(v_t) S_t dt}_{\textit{trading cost}}.$$

Objective: $$\mathbb{E}\left[\underbrace{X_T + q_T S_T}_{\textit{P\&L}} - \underbrace{\frac{C}{2}\int_0^T q_t^2 d[S]_t}_{\textit{Risk}}\right]$$

- Maximizing wealth while minimizing inventory risk

HSBC

Notations:

$$\gamma = CS_0$$ *risk aversion*

$$f^n(\delta) = \frac{1}{1 + e^{\alpha^n + \beta^n \delta}}$$ *client pricing signature*

HJB PIDE:

$$\begin{aligned} 0 \;=\; & \partial_t \theta(t,q) - \frac{\gamma}{2}\sigma^2 q^2 + \sum_{n=1}^{N} \int_0^\infty z H^n\left(\frac{\theta(t,q) - \theta(t,q+z)}{z}\right)\lambda^n(z)dz \\ & + \sum_{n=1}^{N} \int_0^\infty z H^n\left(\frac{\theta(t,q) - \theta(t,q-z)}{z}\right)\lambda^n(z)dz + \mathcal{H}\left(\partial_q \theta(t,q) + kq\right), \qquad \forall (t,q) \in [0,T) \times \mathbb{R} \end{aligned}$$

with terminal condition $\theta(T,\cdot) = 0$, where

$$H^n : p \in \mathbb{R} \mapsto \sup_\delta f^n(\delta)(\delta - p) \qquad \text{and} \qquad \mathcal{H} : p \in \mathbb{R} \mapsto \sup_v \left(vp - L(v)\right).$$

Optimal Controls:

$$\delta^{b,n*}(t,z) = \bar{\delta}^n\left(\frac{\theta(t,q_{t-}) - \theta(t,q_{t-} + z)}{z}\right),$$

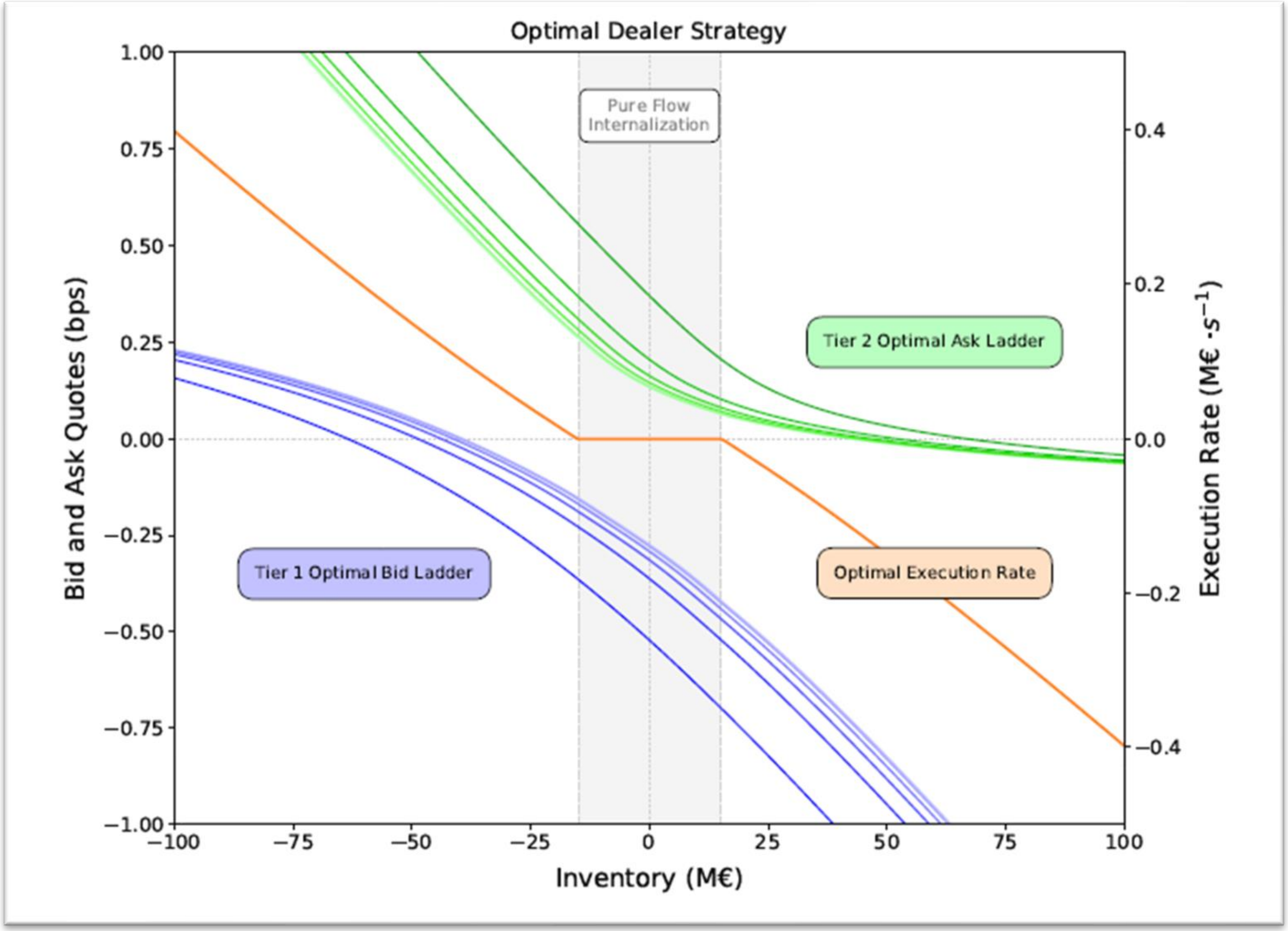
$$\delta^{a,n*}(t,z) = \bar{\delta}^n\left(\frac{\theta(t,q_{t-}) - \theta(t,q_{t-} - z)}{z}\right),$$

and

$$v_t^* = \mathcal{H}'\left(\partial_q \theta(t,q_{t-}) + kq_{t-}\right),$$

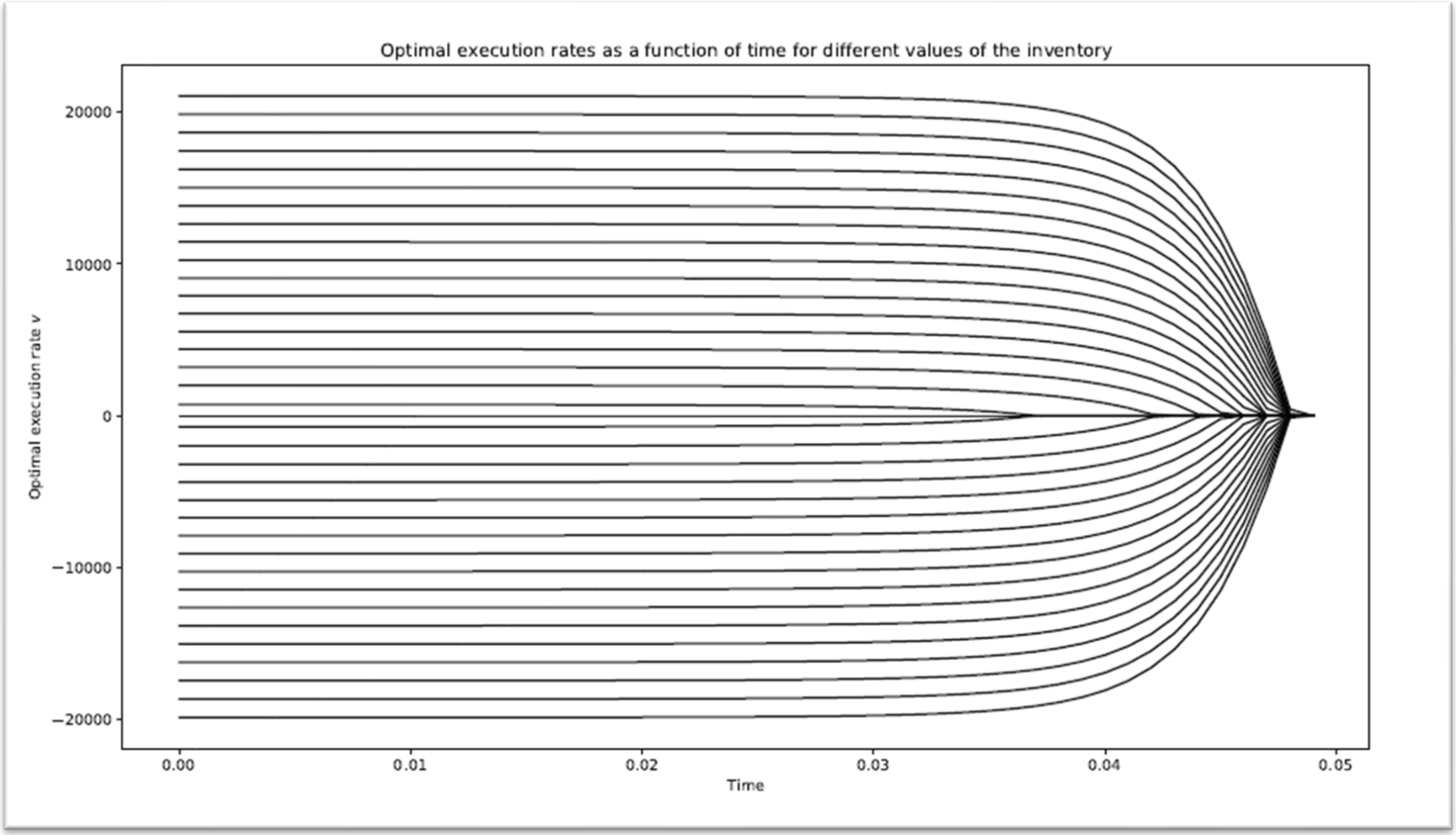
where $\bar{\delta}^n(p) = (f^n)^{-1}\left(-H^{n\prime}(p)\right)$.

Optimal Strategy



Source: BBG, Risk, 2022

Convergence



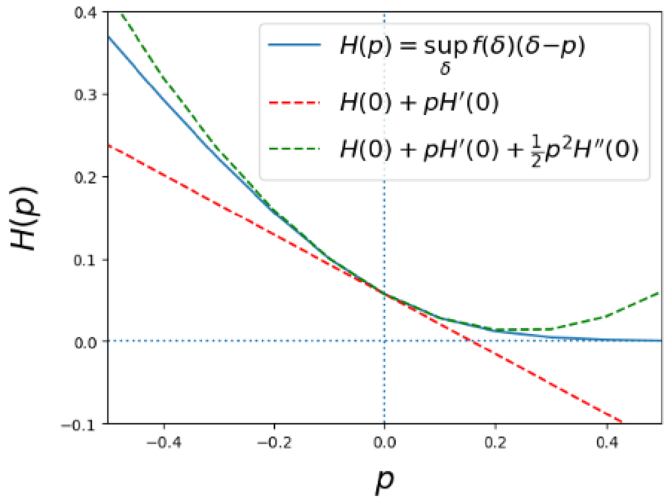
Source: BBG, Mathematical Finance, 2023

Quadratic approximation

$$H_n(p) = H_n(0) + H_n'(0)p + \frac{1}{2}H_n''(0)p^2$$

$$H_n(0) = \hat{\delta}_n f(\hat{\delta}), \qquad H_n'(0) = -f(\hat{\delta}), \qquad H_n''(0) = f(\hat{\delta})/(\beta\hat{\delta}^2)$$

$$\hat{\delta}_n = \arg\max_{\delta} \delta f(\delta)$$



Riccati

$$\theta(t,q) = -A(t)q^2 - B(t)q - C(t)$$

$$\dot{A} = -\frac{1}{2}\gamma\sigma^2 + 4hA^2, \qquad \dot{B} = \mu + 4hAB, \qquad A(T) = \kappa, \quad B(T) = 0$$

$$h = \sum_{n=1}^{N} \int_0^{\infty} H_n''(0)\lambda_n(z)z dz$$

Hedging

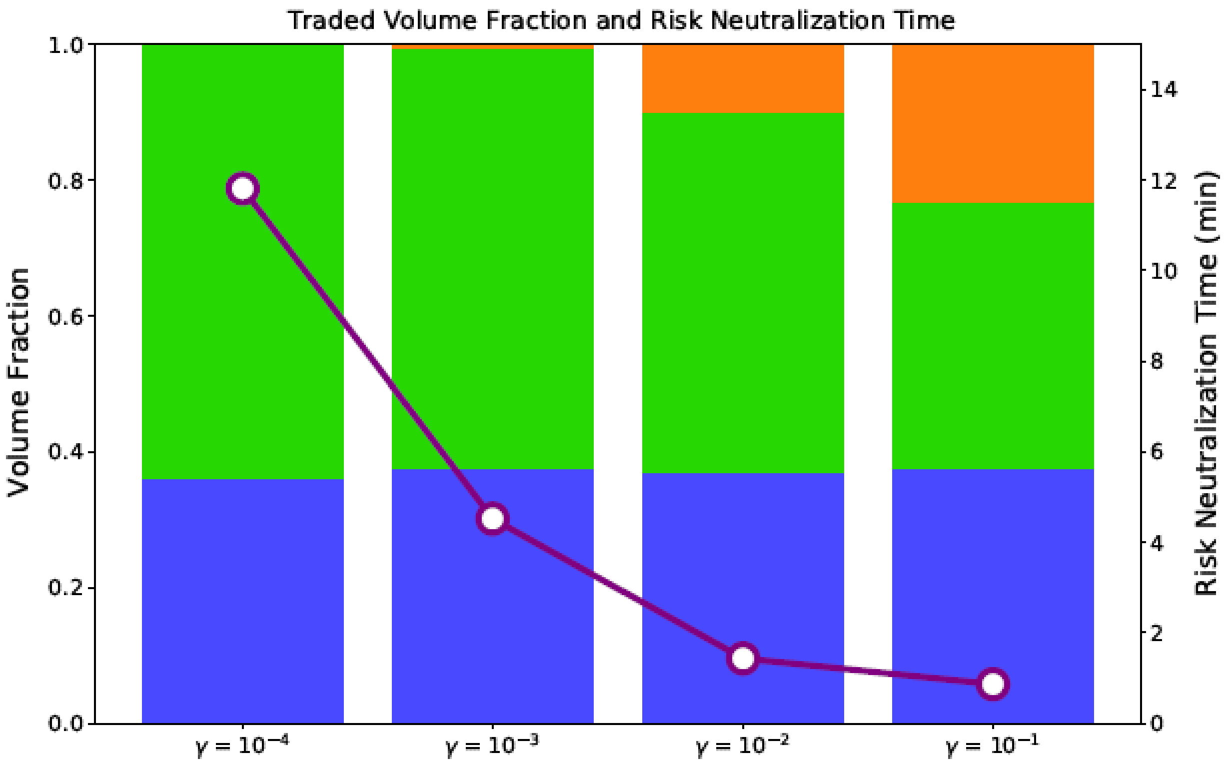
$$v^* = \frac{p - \psi}{2\eta} \quad \text{if } p > \psi, \qquad v^* = \frac{p + \psi}{2\eta} \quad \text{if } p < -\psi$$

$$p = 2Aq + B - kq.$$

Quoting

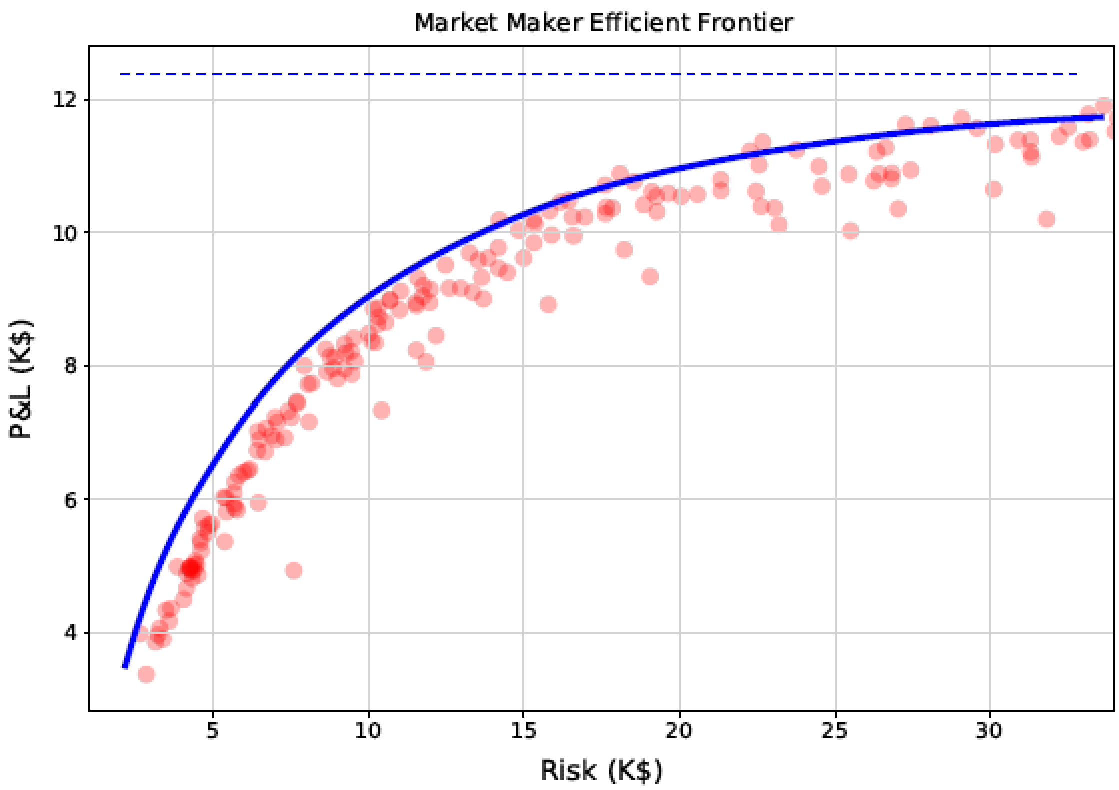
$$\delta_n^{b/a*} = \mp\tilde{\delta}_n \left(Az \pm (2Aq + B)\right)$$

Volume Share and Risk Neutralisation



Source: BBG, Risk, 2022

Efficient Frontier



Source: BBG, Risk, 2022

Challenges:

- Each currency pair provides a valuation of one currency in terms of the other
- Multi-dimensional HJB numerical solution is not scalable

Market Prices: $(S_t^1)_{t\in[0,T]}, \ldots, (S_t^d)_{t\in[0,T]}$ $\quad S_t^1 = 1 \text{ for all } t \in [0,T]$

Jump Processes: $J^{n,i,j}(dt,dz)$

$$\Lambda^{n,i,j}(z,\delta) = \lambda^{n,i,j}(z) f^{n,i,j}(z,\delta) \quad \text{with} \quad f^{n,i,j}(z,\delta) = \frac{1}{1+e^{\alpha^{n,i,j}(z)+\beta^{n,i,j}(z)\delta}}$$

Execution Processes: $\left(\xi_t^{i,j}\right)_{t\in[0,T]}$

Exchange Rate Dynamics:

$$\forall i \in \{1,\ldots,d\}, \quad dS_t^i = \underbrace{\mu_t^i S_t^i dt}_{drift} + \underbrace{\sigma^i S_t^i dW_t^i}_{volatility} + \underbrace{k^i \left(\sum_{j=i+1}^{d} \xi_t^{i,j} - \sum_{j=1}^{i-1} \xi_t^{j,i}\right) S_t^i dt}_{market\ impact}.$$

Inventory Dynamics:

$$dq_t^i = \sum_{n=1}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{d} \int_{z\in\mathbb{R}_+^*} \frac{z}{S_t^i} \left(J^{n,i,j}(dt,dz) - J^{n,j,i}(dt,dz)\right) + \left(\sum_{j=i+1}^{d} \frac{\xi_t^{i,j}}{S_t^i} - \sum_{j=1}^{i-1} \frac{\xi_t^{j,i}}{S_t^i}\right) dt.$$

Optimisation Problem

Cash Account Dynamics:

$$dX_t = \sum_{n=1}^{N} \sum_{1 \le i \ne j \le d} \int_{z \in \mathbb{R}_+^*} z\delta^{n,i,j}(t,z) J^{n,i,j}(dt,dz) - \sum_{1 \le i < j \le d} L^{i,j}\left(\xi_t^{i,j}\right)$$

spread capture

execution cost

Inventory Processes Measured in Reference Currency:

$$\begin{aligned} dY_t^i = {} & \mu_t^i Y_{t-}^i dt + \sigma^i Y_{t-}^i dW_t^i + k^i \left(\sum_{j=i+1}^{d} \xi_t^{i,j} - \sum_{j=1}^{i-1} \xi_t^{j,i} \right) Y_{t-}^i dt \\ & + \sum_{n=1}^{N} \sum_{\substack{j=1 \\ j \ne i}}^{d} \int_{z \in \mathbb{R}_+^*} z \left(J^{n,i,j}(dt,dz) - J^{n,j,i}(dt,dz) \right) + \left(\sum_{j=i+1}^{d} \xi_t^{i,j} - \sum_{j=1}^{i-1} \xi_t^{j,i} \right) dt. \end{aligned}$$

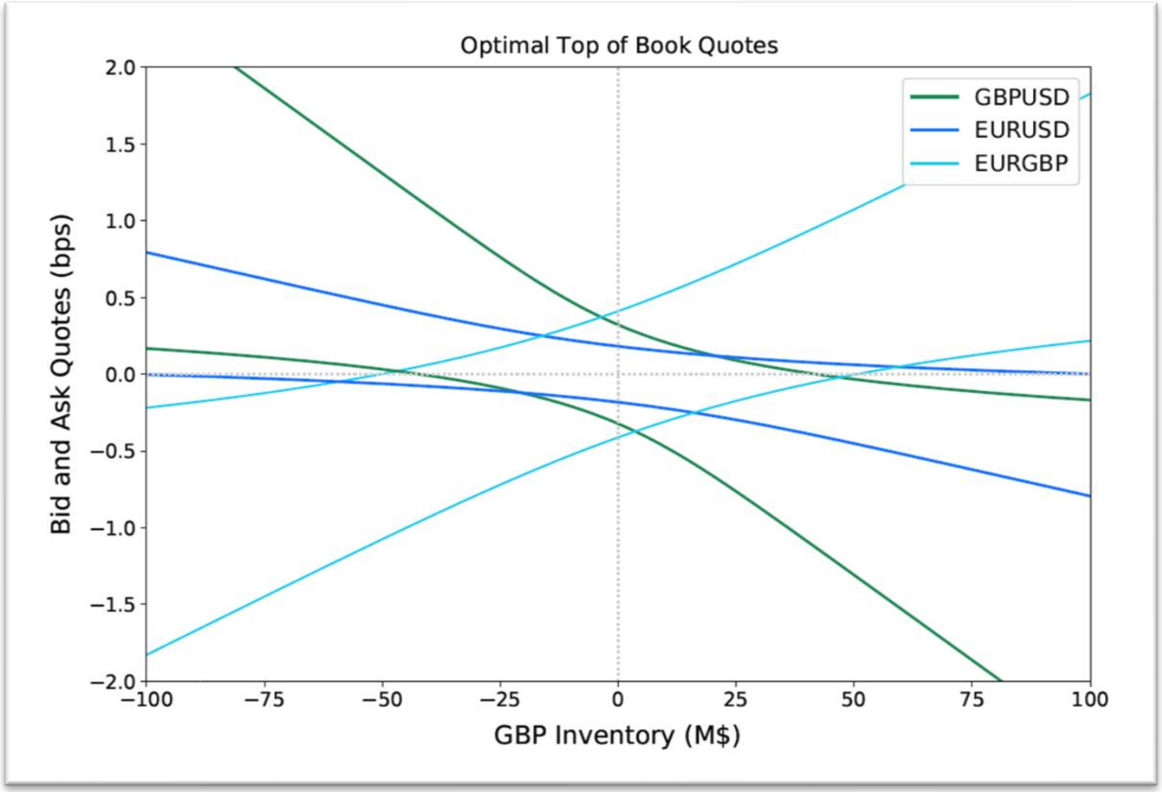
Objective:

$$\mathbb{E}\left[X_T + \sum_{i=1}^{d} Y_T^i - \frac{\gamma}{2} \int_0^T Y_t^{\intercal} \Sigma Y_t dt - \ell\left(Y_T\right) \right]$$

P&L

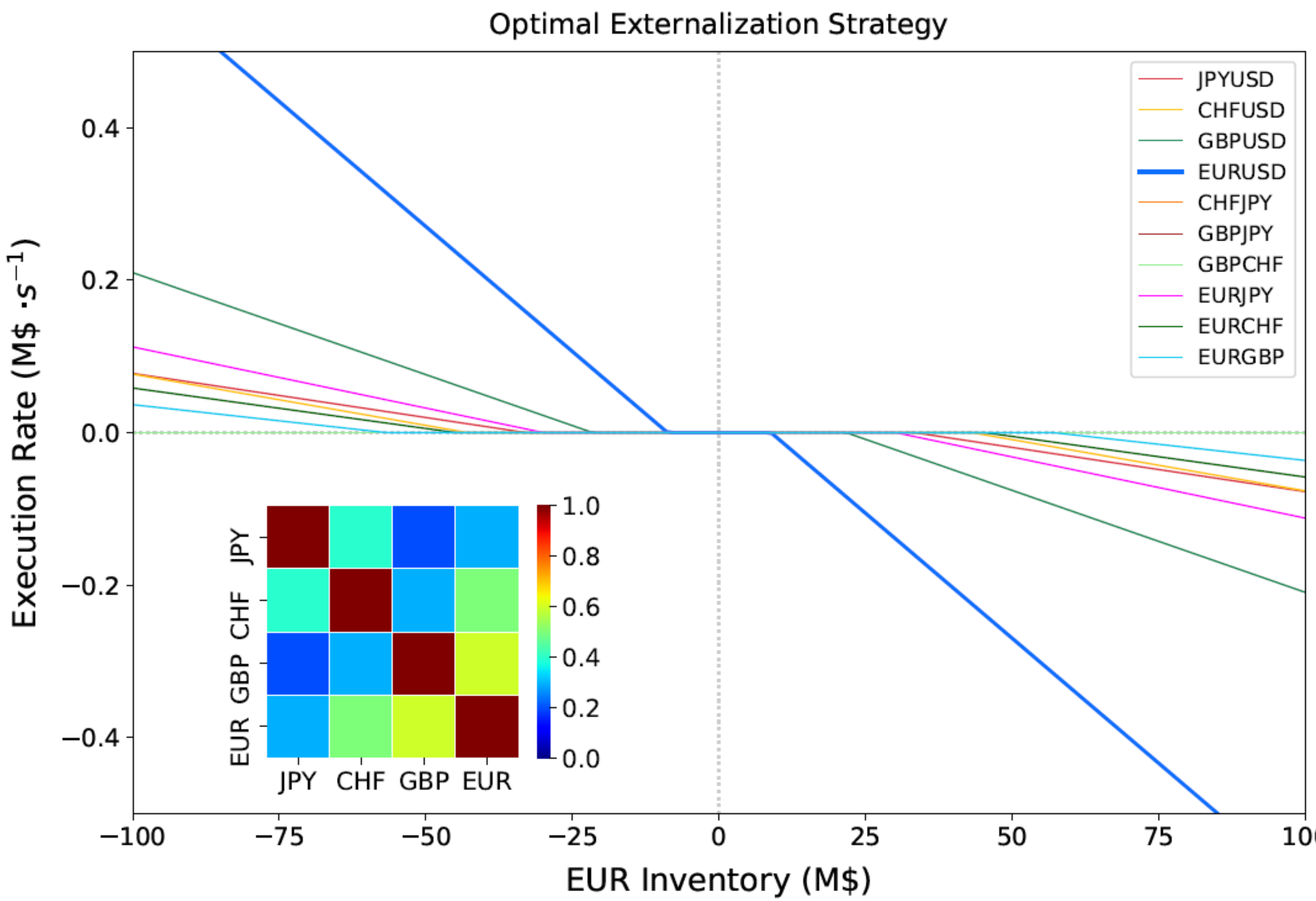
Risk

Optimal Pricing



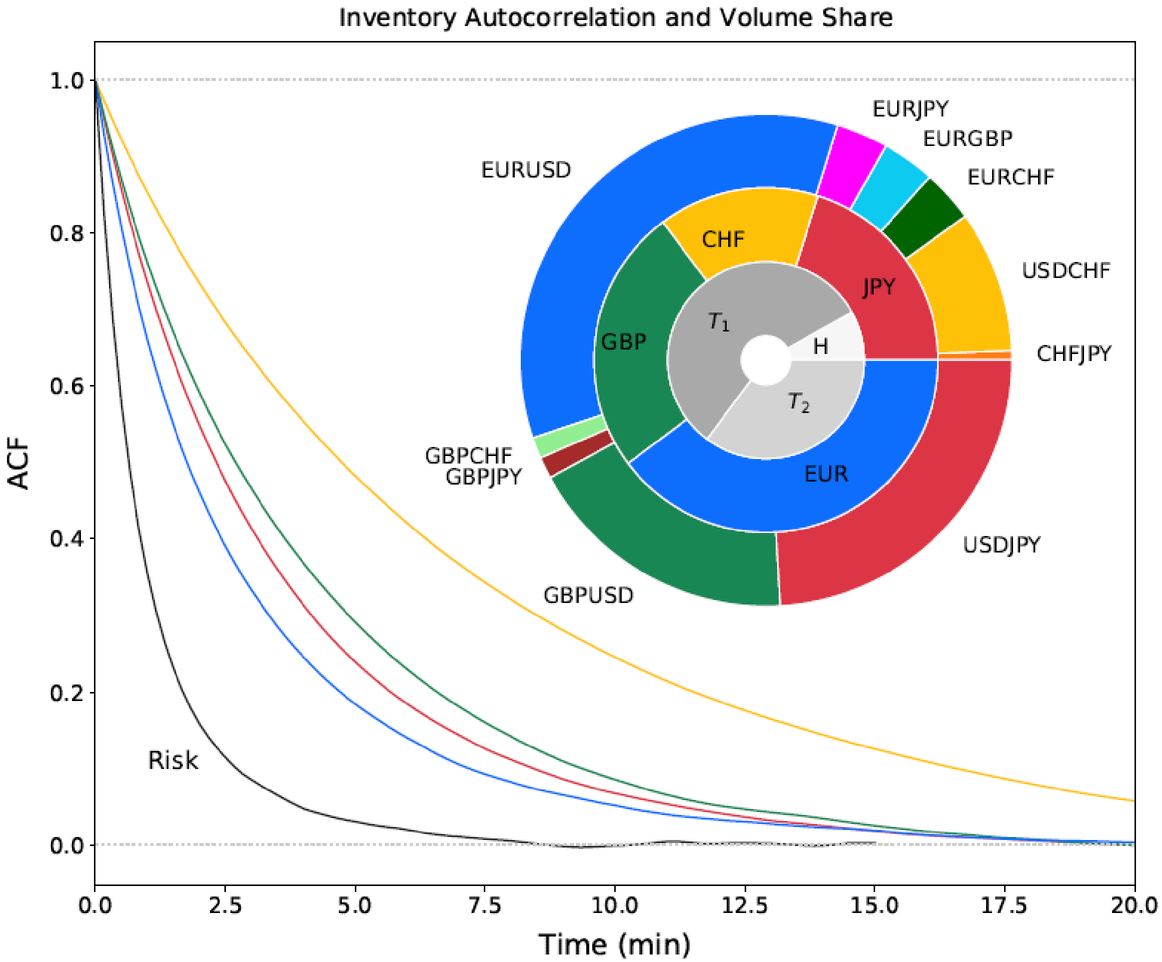
Source: BBG, Risk, 2023

Optimal Execution



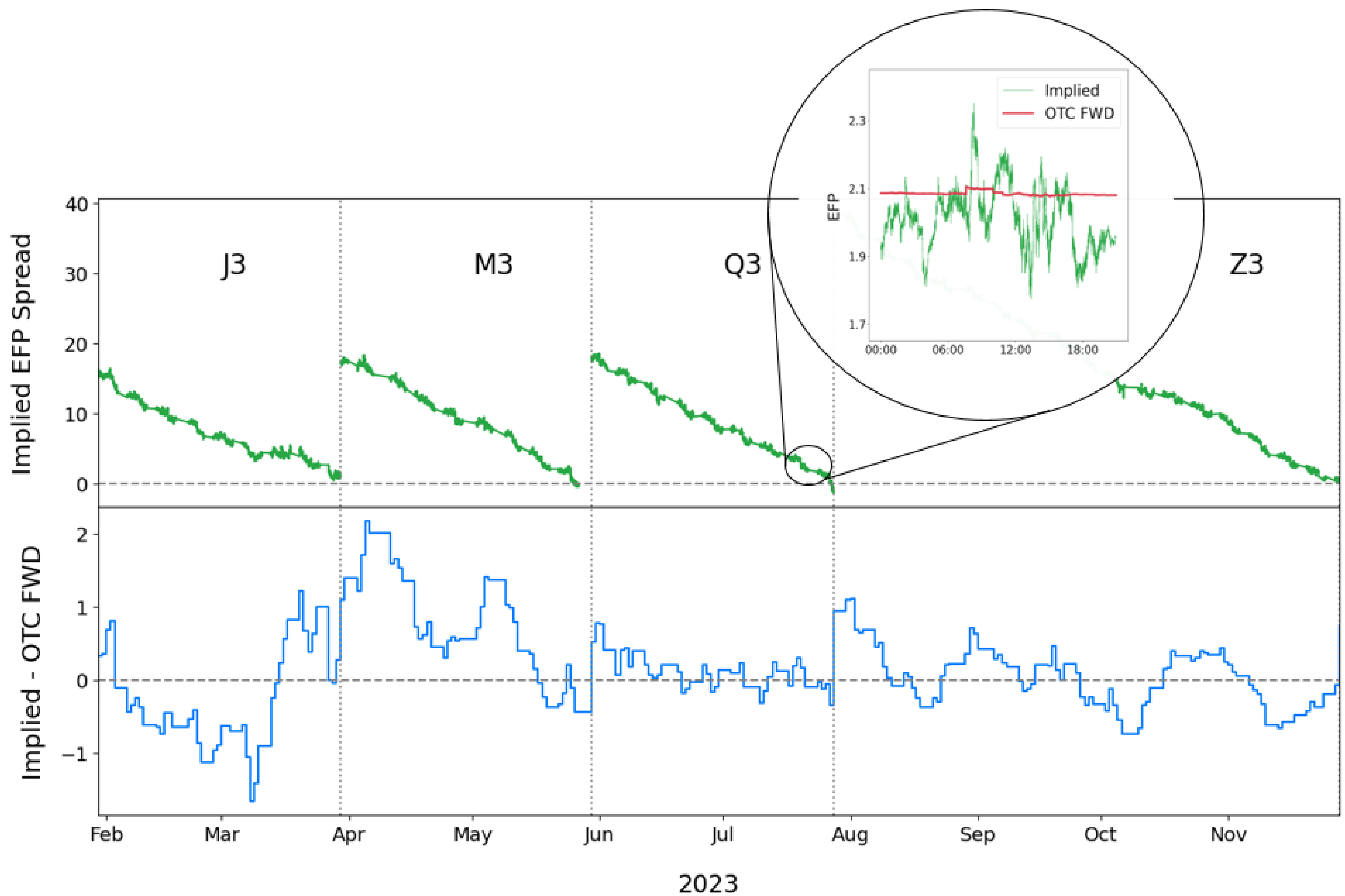
Source: BBG, Risk, 2023

Multi-Currency Risk Neutralisation



Source: BBG, Risk, 2023

Co-integrated Liquidity (Precious Metals, NDF)



Source: HSBC data

HSBC

Robustifying pure OU

$$dS_t = \sigma_S dW_t^S, \qquad \sigma_S > 0$$

Spot

$$dE_t = -k_E \left(E_t - D_t\right) dt + \sigma_E dW_t^E, \qquad k_E, \sigma_E > 0,$$

EFP

$$dD_t = -k_D \left(D_t - \bar{D}\right) dt + \sigma_D dW_t^D, \qquad k_D, \sigma_D \geq 0, \quad \bar{D} \in \mathbb{R},$$

$$dq_t^S = \int_{z=0}^{\infty} z J^b(dt, dz) - \int_{z=0}^{\infty} z J^a(dt, dz) + v_t^S dt,$$

Inventory

$$dq_t^F = v_t^F dt.$$

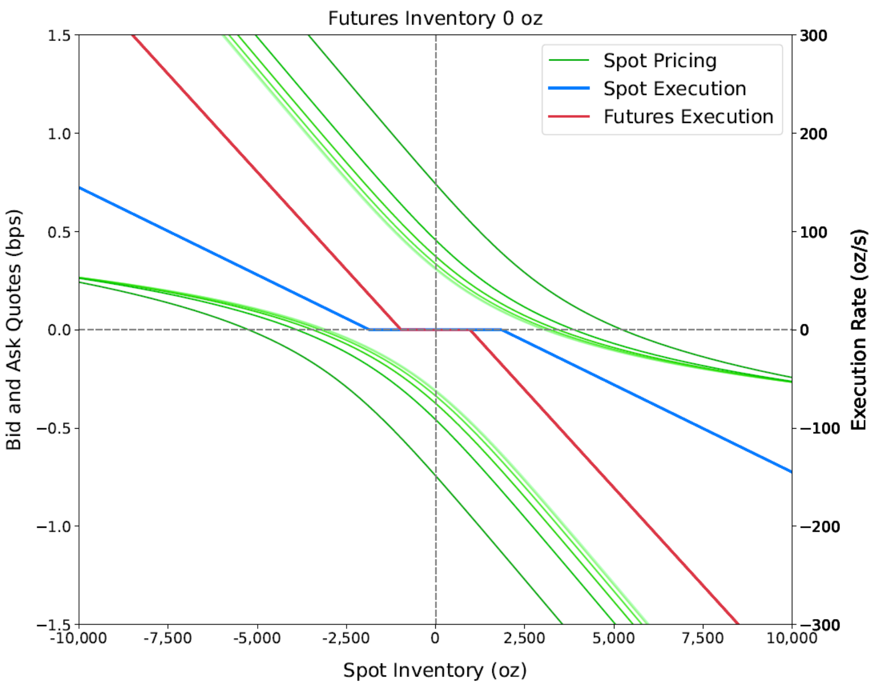
$$dX_t = \int_{z=0}^{\infty} S^a(t,z) z J^a(dt,dz) - \int_{z=0}^{\infty} S^b(t,z) z J^b(dt,dz) - v_t^S S_t dt - L^S(v_t^S) dt - v_t^F F_t dt - L^F(v_t^F) dt,$$

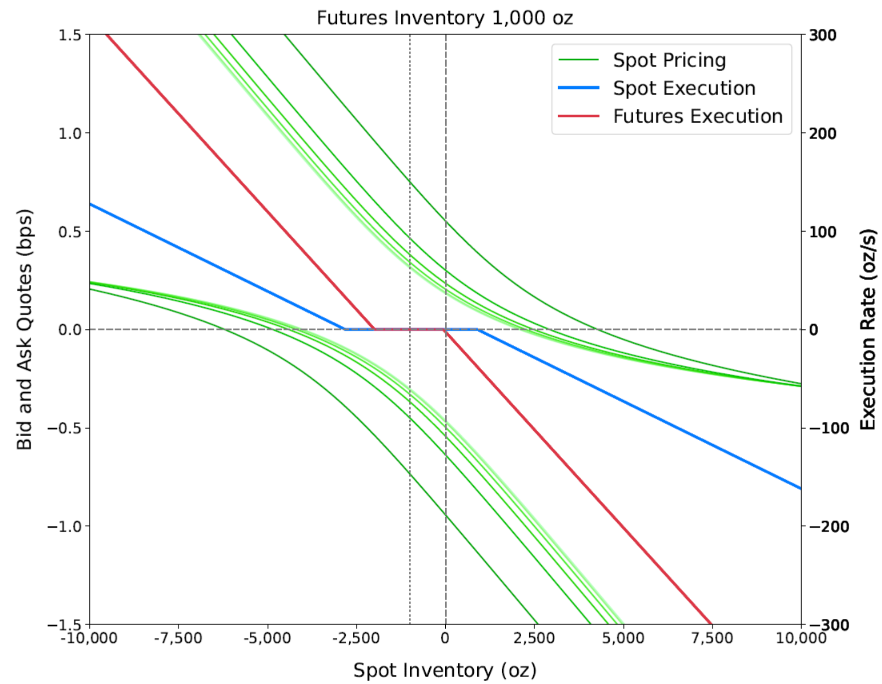
Wealth

$$\mathbb{E}\left[-\exp\left(-\gamma\left(X_T + q_T^S S_T + q_T^F F_T - K^S (q_T^S)^2 - K^F (q_T^F)^2\right)\right)\right]$$

CARA

Pricing

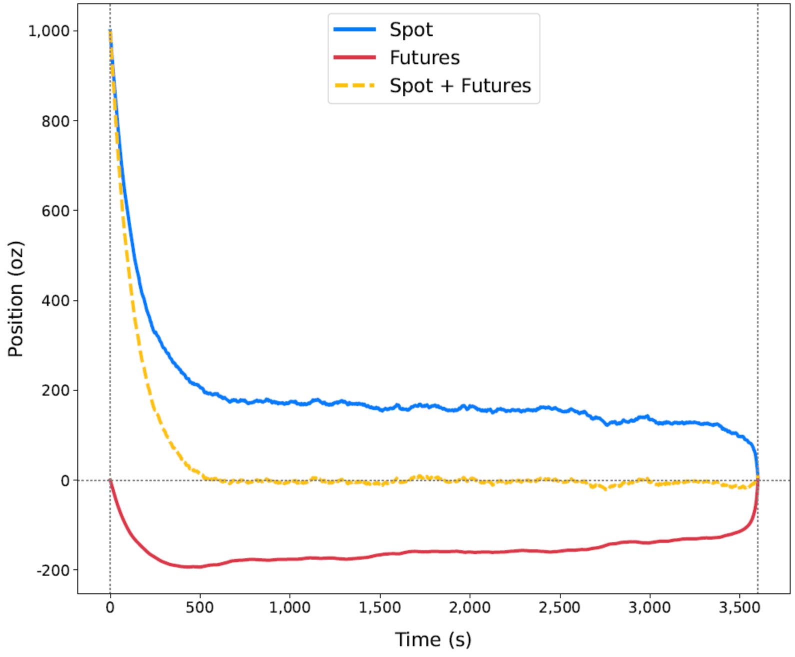




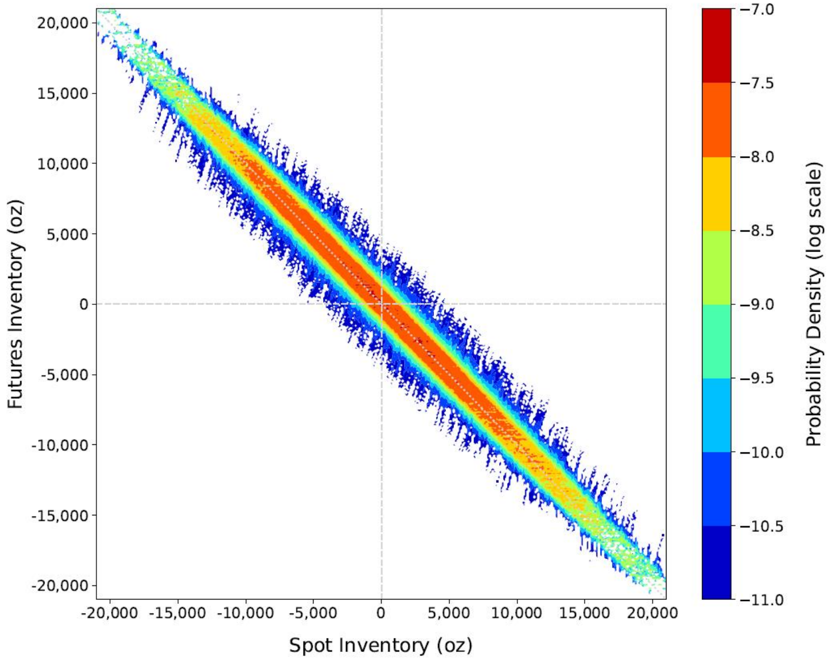
Source: BBG , Risk, 2024

Risk Relaxation Components

Relaxation

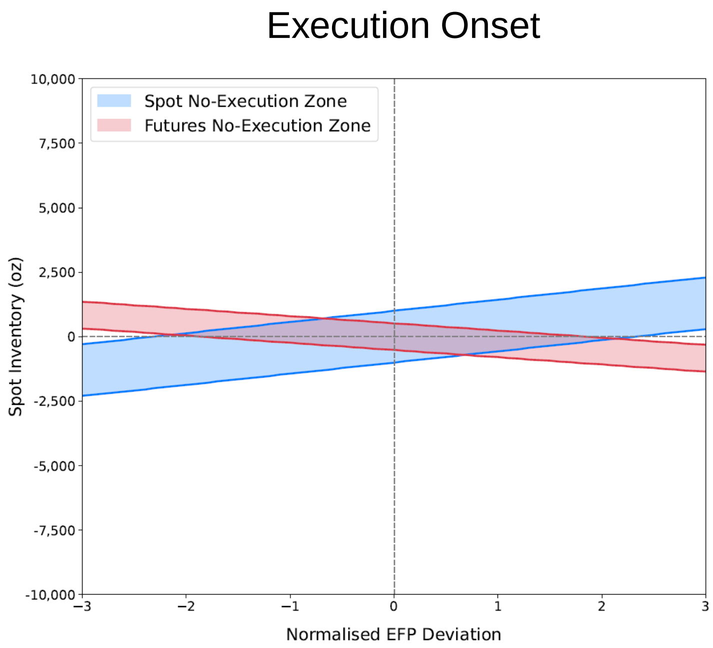


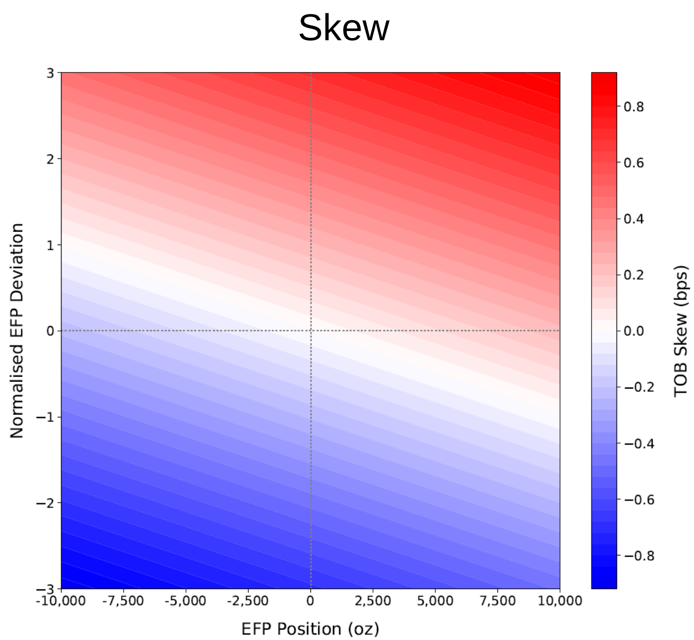
Inventory PDF

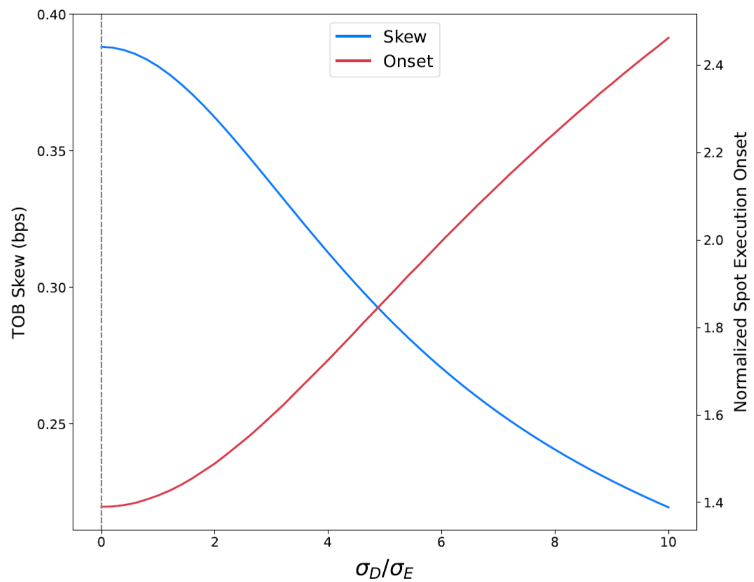


Source: BBG , Risk, 2024

Capitalizing on Mean Reversion

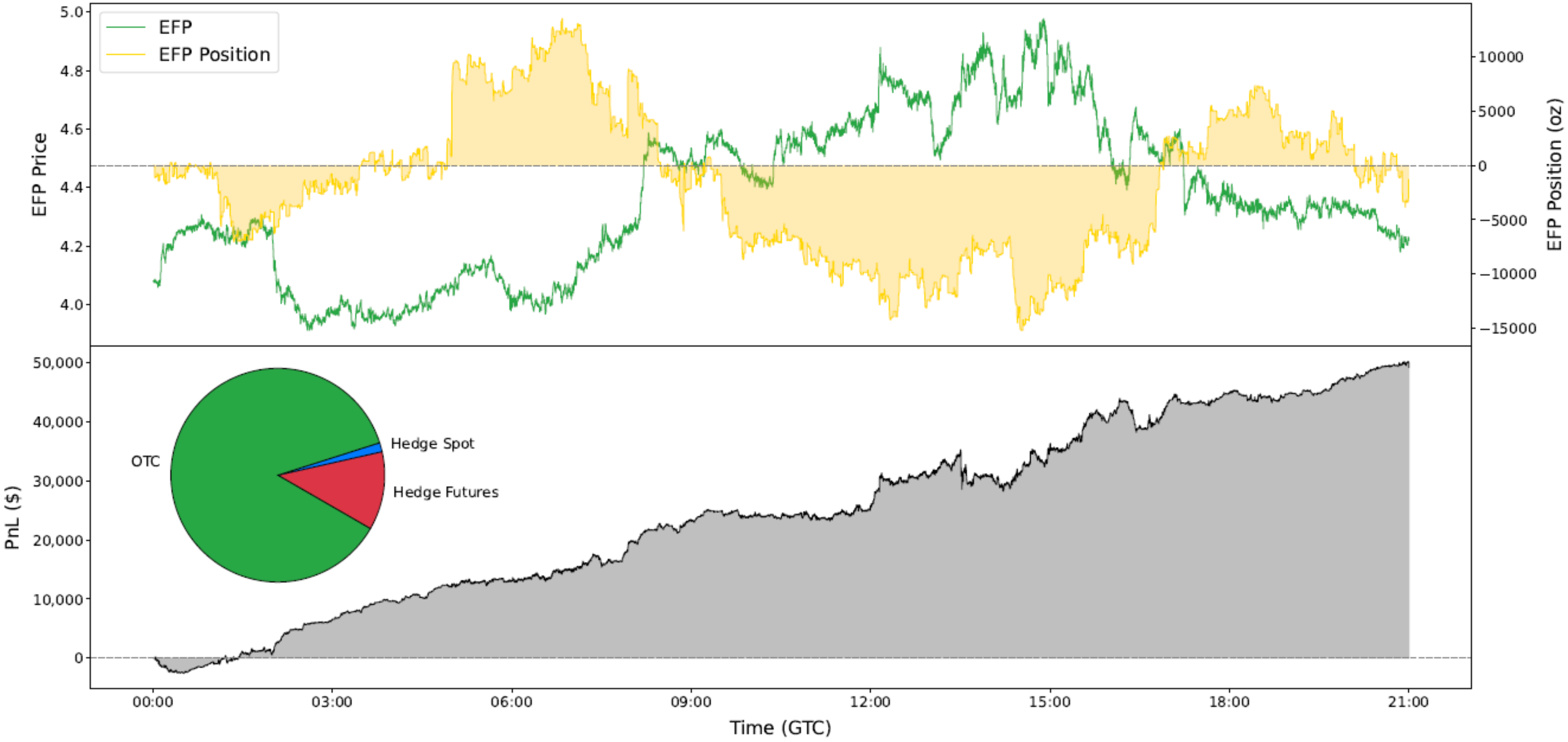






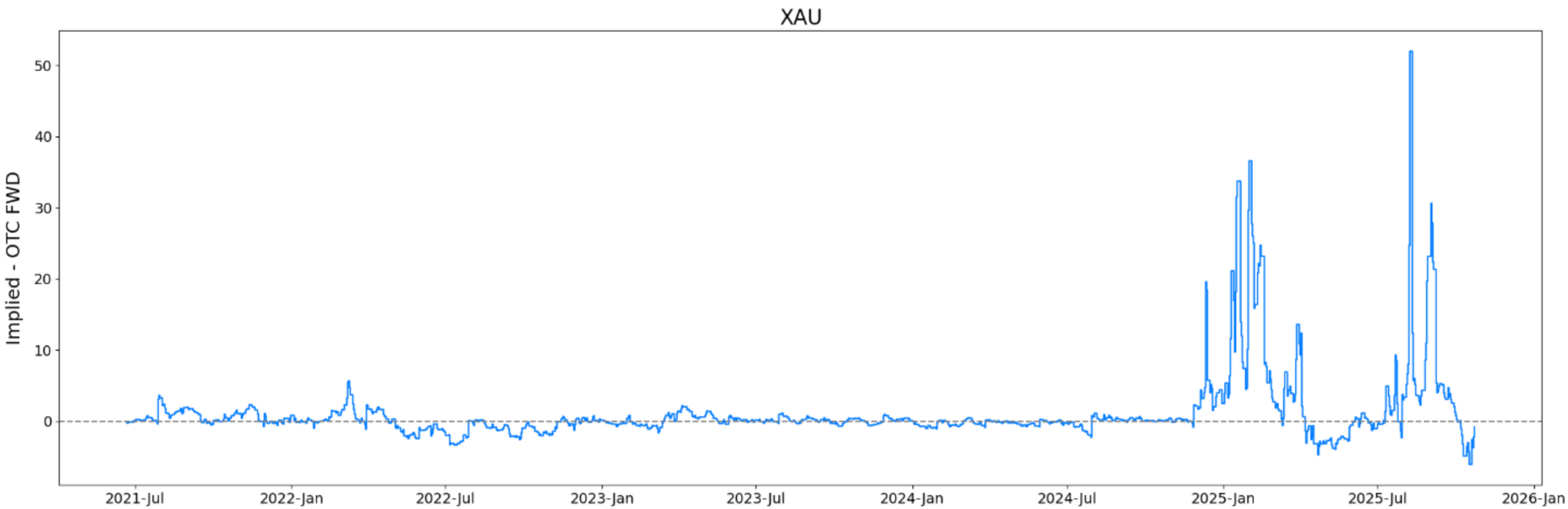
Source: BBG , Risk, 2024

Co-integrated Liquidity Demo



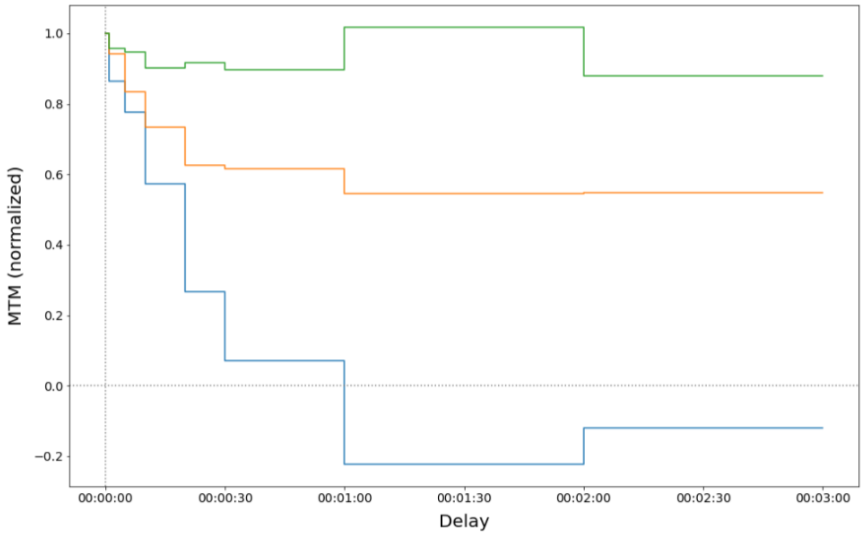
Source: BBG , Risk, 2024

Sobering Reality

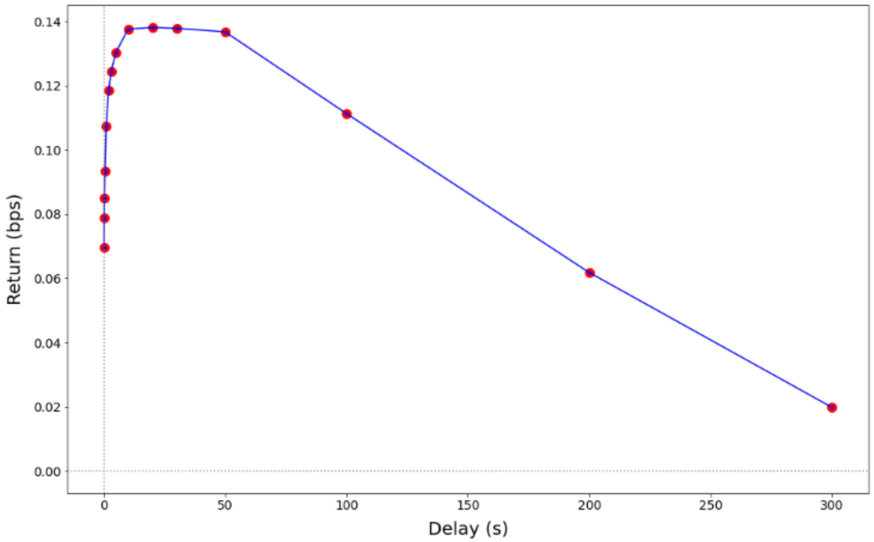


Source: HSBC Data

Adverse Selection (informed traders)



Price Reading (skew sniffers)



Source: HSBC Data

$$dS_t = \sigma\, dB_t - \sum_{\substack{n\in\mathcal{N}\\k\in\mathcal{K}}} \left(\bar{\zeta}^{n,k}\left(S_t - S_t^{n,k,b}\right) dN_t^{n,k,b} - \bar{\zeta}^{n,k}\left(S_t^{n,k,a} - S_t\right) dN_t^{n,k,a}\right)$$
$$+ \sum_{n\in\mathcal{N}} \bar{J}^n \left(\sum_{k\in\mathcal{K}} w^{n,k}\left(S_t^{n,k,a} - 2S_t + S_t^{n,k,b}\right)\right) dt, \qquad S_0 \text{ given},$$

Base quotes:

$$\widehat{\delta}^{n,k,b/a*}(q) = \frac{1}{\kappa^{n,k}} + \sigma\sqrt{\frac{\gamma e}{2\sum_{\substack{m\in\mathcal{N}\\j\in\mathcal{K}}} \Delta^j \Lambda_0^{m,j} \kappa^{m,j}}}\left(\pm q + \frac{\Delta^k}{2}\right).$$

First order price reading response:

$$\frac{\widehat{\eth}^{n,k,b/a*}(q) - \widehat{\delta}^{n,k,b/a*}(q)}{\varepsilon} = \frac{e\sum_{\substack{m\in\mathcal{N}\\j\in\mathcal{K}}} w^{m,j}}{\sum_{\substack{m\in\mathcal{N}\\j\in\mathcal{K}}} \Delta^j \Lambda_0^{m,j} \kappa^{m,j}}\left(\pm q + \frac{\Delta^k}{2}\right) \mp \frac{q w^{n,k}}{\Delta^k \kappa^{n,k}} \frac{1}{\Lambda^{n,k}\left(\widehat{\delta}^{n,k,b/a*}(q)\right)}$$

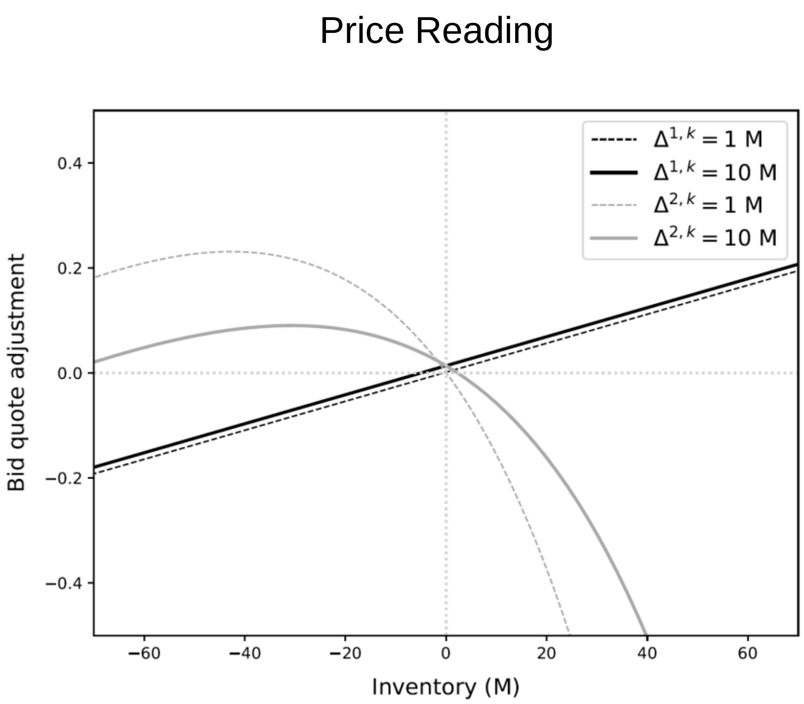
Risk widening *Reduce skew to sniffers*

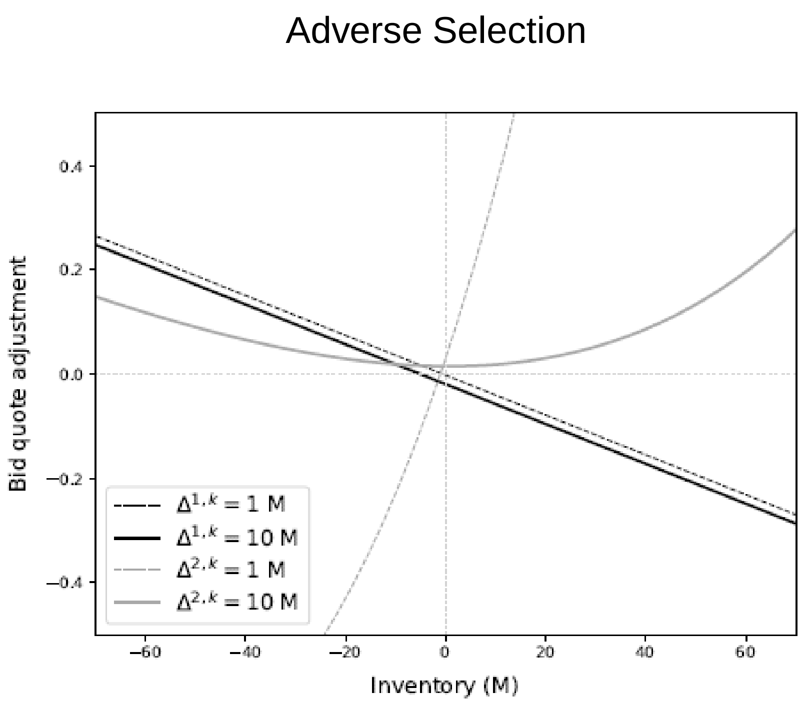
First order adverse selection response:

$$\frac{\widehat{\eth}^{n,k,b/a*}(q) - \widehat{\delta}^{n,k,b/a*}(q)}{\varepsilon} = \frac{\sum_{\substack{m\in\mathcal{N}\\j\in\mathcal{K}}} \alpha^{m,j}\Lambda_0^{m,j}(\beta^{m,j} - \kappa^{m,j}) e^{\frac{\beta^{m,j}}{\kappa^{m,j}}}}{\sum_{\substack{m\in\mathcal{N}\\j\in\mathcal{K}}} \Delta^j \Lambda_0^{m,j} \kappa^{m,j}}\left(\pm q + \frac{\Delta^k}{2}\right) \mp \frac{q \pm \Delta^k}{\Delta^k} \frac{\beta^{n,k} - \kappa^{n,k}}{\kappa^{n,k}} \zeta^{n,k}\left(\widehat{\delta}^{n,k,b/a*}(q)\right)$$

Global response *Targeted response*

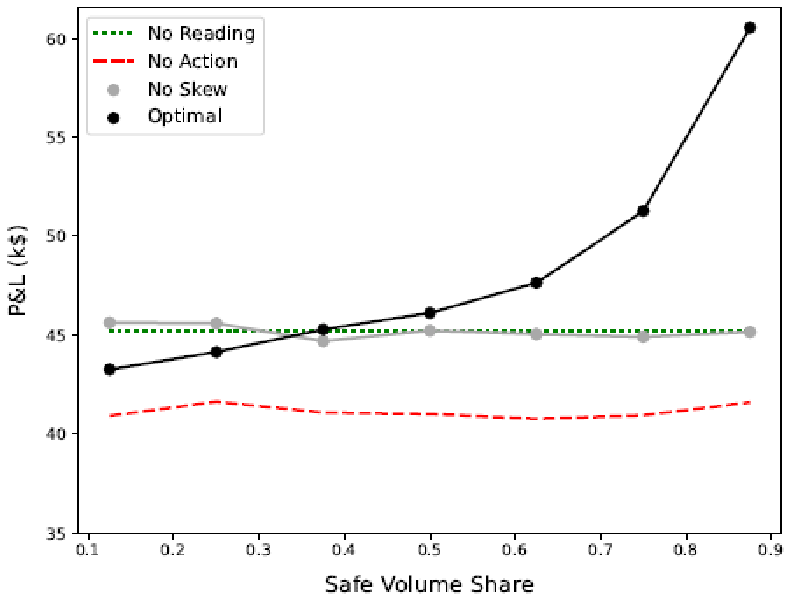
Pricing Adjustment

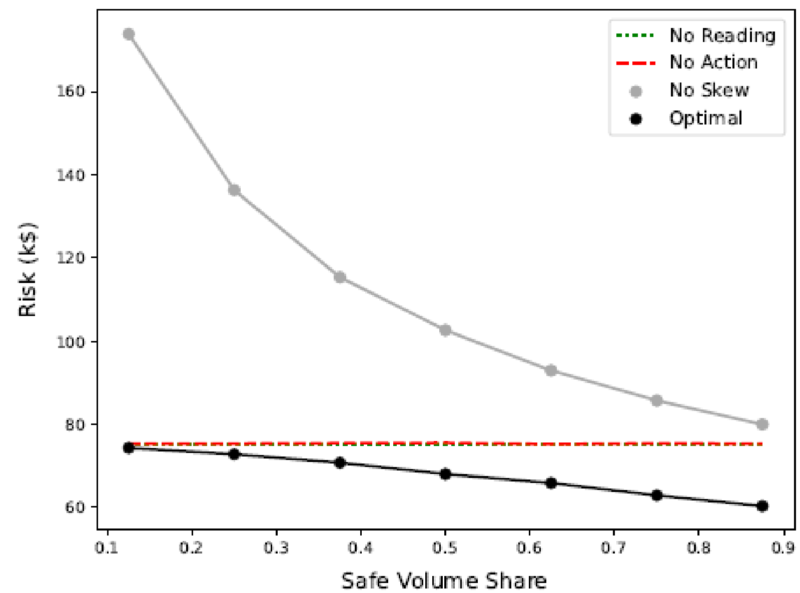




Source: BBG 2025

Optimal Risk Management with Price Reading





Source: BBG 2025

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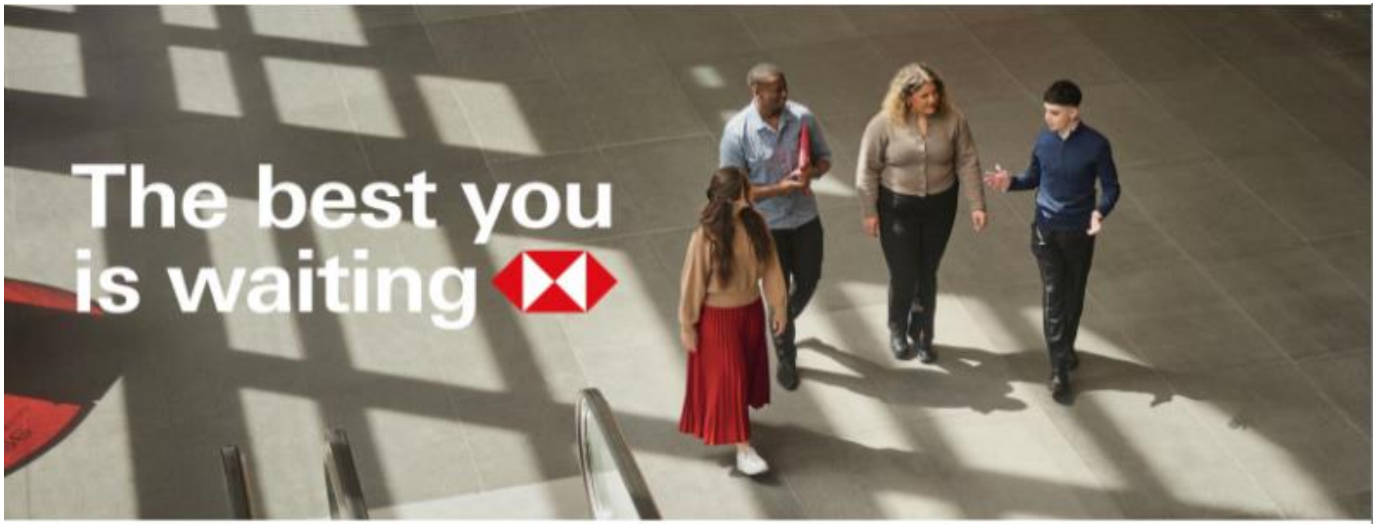
Disclaimer



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