Dynamic Longevity Hedging in the Presence of Population Basis Risk: A Feasibility Analysis from Technical and Economic Perspectives

Kenneth Q. Zhou and Johnny S.-H. Li

University of Waterloo

September 3, 2014

Outline

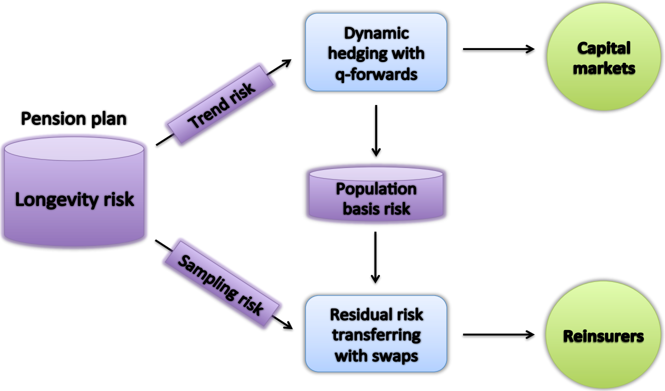

Figure 1: The outline of the proposed dynamic hedging strategy.

Overview

1. Dynamic Longevity Hedging
   - Constructing the dynamic hedging strategy
   - A two-population example
2. Residual Risk Transferring
   - Constructing the customized surplus swap
   - A multi-population example

Model and Approximation

The augmented common factor model (Li and Lee, 2005):

$$\ln(m_{x,t}^{(i)}) = a_x^{(i)} + B_x K_{t+1} + b_x^{(i)} k_{t+1}^{(i)} + e_{x,t}^{(i)}, \quad i = 1, 2, \ldots$$

where

- K_t follows a random walk:

$$K_t = c + K_{t-1} + \epsilon_t;$$

- $k_t^{(i)}$ follows an AR(1) process:

$$k_t^{(i)} = c^{(i)} + \phi^{(i)} k_{t-1}^{(i)} + \epsilon_t^{(i)}.$$

The approximation method - an extension of the work of Cairns (2011):

$$\begin{aligned} f_{x,t}^{(i)}(T, K_t, k_t^{(i)}) &= \Phi^{-1}(p_{x,t}^{(i)}(T, K_t, k_t^{(i)})) \\ &\approx D_{x,t,0}^{(i)}(T) + D_{x,t,1}^{(i)}(T) \cdot (K_t - \hat{K}_t) \\ &\quad + D_{x,t,2}^{(i)}(T) \cdot (k_t^{(i)} - \hat{k}_t^{(i)}) \end{aligned}$$

where

- Φ^{-1} is the probit function;
- $p_{x,t}^{(i)}(T, K_t, k_t^{(i)})$ is the time-t spot survival probability for T years;
- $\hat{K}_t = E(K_t|K_0)$;
- $\hat{k}_t^{(i)} = E(k_t^{(i)}|k_0^{(i)})$;
- $D_{x,t,0}^{(i)}(T) = f_{x,t}^{(i)}(T, \hat{K}_t, \hat{k}_t^{(i)})$;
- $D_{x,t,1}^{(i)}(T) = \left.\frac{\partial f_{x,t}^{(i)}(T, K_t, \hat{k}_t^{(i)})}{\partial K_t}\right|_{K_t = \hat{K}_t}$;
- $D_{x,t,2}^{(i)}(T) = \left.\frac{\partial f_{x,t}^{(i)}(T, \hat{K}_t, k_t^{(i)})}{\partial k_t^{(i)}}\right|_{k_t^{(i)} = \hat{k}_t^{(i)}}$.

Evaluation of the Required Quantities

To construct the dynamic hedging strategy, the following quantities are calculated using the approximation method:

- The time-t present value of the pension liabilities.
- The time-t present value of the hedging instruments.
- The first derivative with respect to K_t of the time-t present value of the pension liabilities, $\Delta_{K_t}^{liab}$.
- The first derivative with respect to K_t of the time-t present value of the hedging instruments, $\Delta_{K_t}^{hedge}$.

Dynamic Hedging

- To determine the optimal hedge ratio, h_t, the first derivatives of the pension liabilities and the hedging instruments with respect to K_t are matched:

$$\Delta_{K_t}^{liab} = h_t \cdot \Delta_{K_t}^{hedge}.$$

- At time t, h_t units of the hedging instruments are purchased to hedge the uncertainty of the pension liabilities from time t to $t+1$.
- At time $t+1$, the hedging instruments purchased at time t are closed out to compensate for the shortfall of the pension plan from the longevity risk.

Hedge Effectiveness

The hedge effectiveness at time t is calculated as:

$$HE_t = 1 - \frac{Var(H_t - L_t)}{Var(L_t)}$$

where

- L_t is the time-0 present value of the realized liabilities from time 0 to t and the future liabilities after time t;
- H_t is the time-0 present value of the payoffs of the hedging instruments up to time t with an initial reserve of $H_0 = L_0$.

A Two-Population Example

- Two populations: Continuous Mortality Investigation (CMI) and England and Wales (EW)
- Liability: 30 years of $1 pension liabilities subject to the mortality experience of a male individual aged 60 in year 2005 from the CMI population.
- Hedging instrument: 10-year age-75 q-forward contracts linked to the EW population, which are unlimitedly available and liquidly traded.

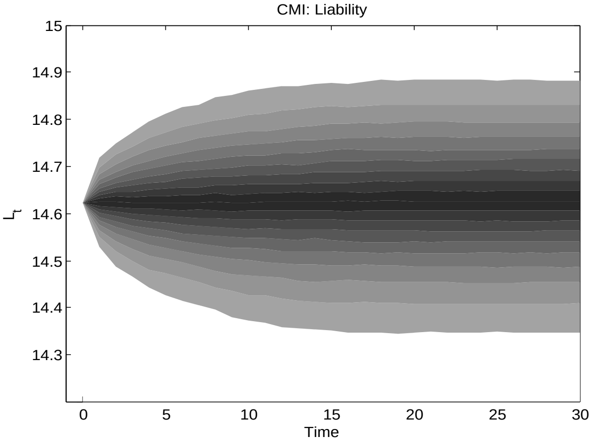

Figure 2: The time-0 present value of the liabilities over time.

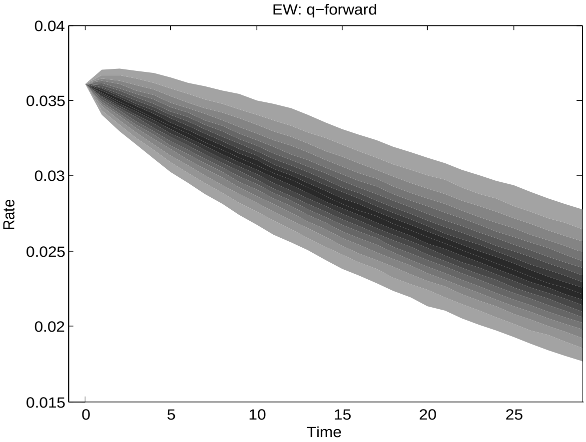

Figure 3: The 10-year age-75 q-forward rate over time.

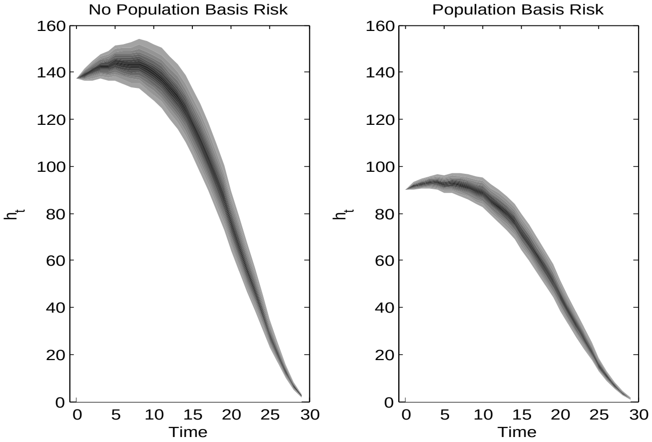

Figure 4: The optimal hedge ratio over time.

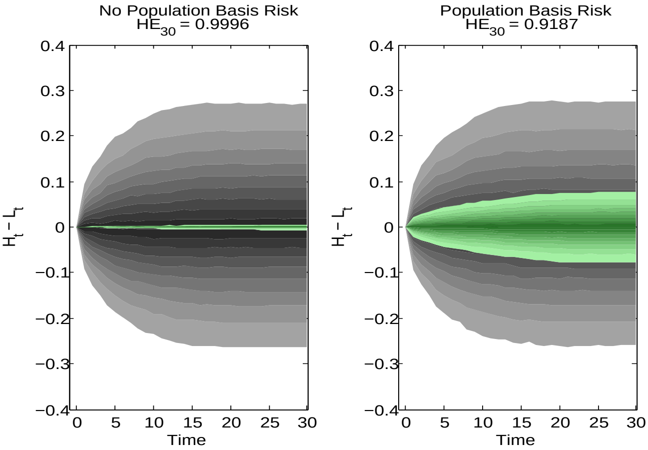

Figure 5: The hedge result over time.

Motivations

- Population basis risk exists if standardized index-linked hedging instruments are used.
- The degree of the population basis risk varies according to the populations involved in the hedge.
- Population basis risk cannot be diversified within a pension plan, but may be diversifiable across different pension plans.

The Customized Surplus Swap

- The swap is a cash exchange agreement between a pension plan and a reinsurer.
- It transfers population basis risk and sampling risk from one party to another.
- The swap exchanges the economic surplus of a pension plan after the implementation of a dynamic longevity hedging strategy.
- It eventually offloads the residual risk from the pension plan to the counterparty.

The Surplus of a Pension Plan

- The total liability of the pension plan at time t is

$$L_t = CL_t + FL_t,$$

where CL_t and FL_t are the time-t values of the current and future liabilities, respectively.

- The hedging portfolio for the pension plan at time t is

$$H_t = (1 + r)(H_{t-1} - CL_{t-1}) + P_t,$$

where P_t is the payoff of the hedging instruments at time t, and $H_0 = L_0$ is the initial reserve.

- Finally, the surplus of the pension plan at time t is defined as

$$SP_t = H_t - L_t.$$

The Cash Flow of the Swap

The goal is to have $SP_t = 0$. Hence, the cash flow of the swap at time t is defined as $CF_t = -SP_t$. It follows that

$$CF_t = L_t - (1 + r)(H_{t-1} - CL_{t-1}) - P_t.$$

If the swap is set up every year, then

$$CF_t = L_t - (1 + r)FL_{t-1} - P_t.$$

Both L_t and FL_{t-1} can be determined by whichever valuation methods agreed to by the two parties. The payoff of the hedging instruments will be determined by the market price at time t and the number of holdings at time $t - 1$.

A Multi-Population Example

- 20 national populations.
- Liability: 30 years of $1 pension liabilities subject to the mortality experience of a male individual aged 60 in year 2008 from each population.
- Hedging instrument: 10-year age-75 q-forward contracts linked to the EW population, which are unlimitedly available and liquidly traded.
- Sampling risk: a binomial frequency model with an assumed population size of 10,000 for each population.

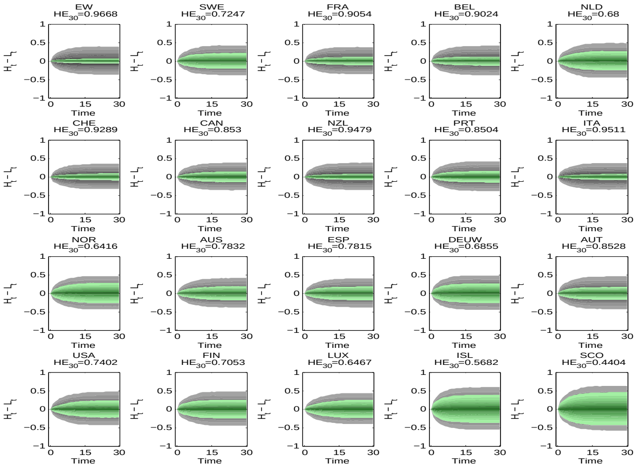

Figure 6: The hedge result over time for the 20 populations.

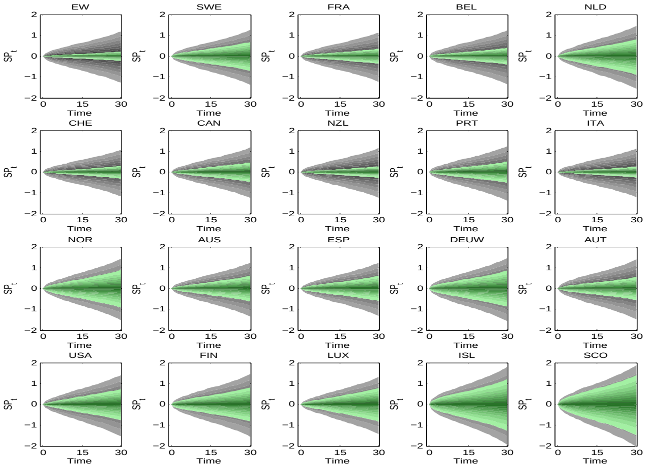

Figure 7: The surplus over time for the 20 populations.

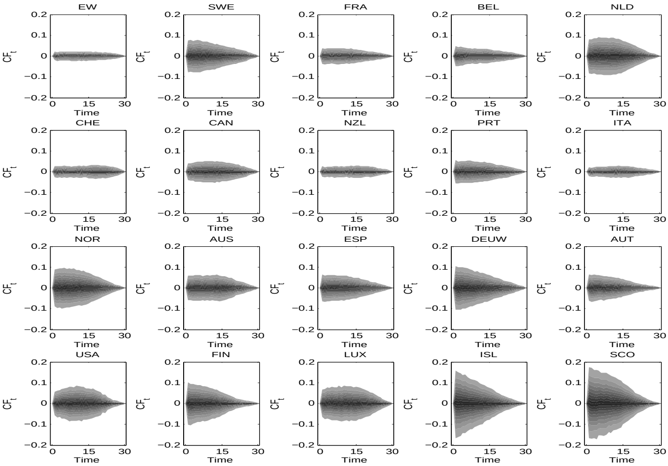

Figure 8: The cash flow over time for the 20 populations.

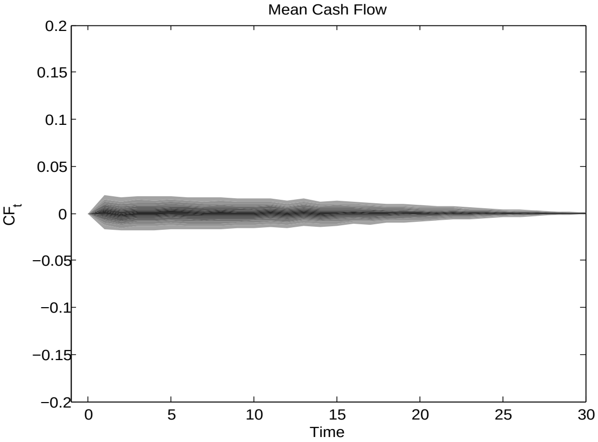

Figure 9: The mean cash flow of the 20 populations.

Conclusion

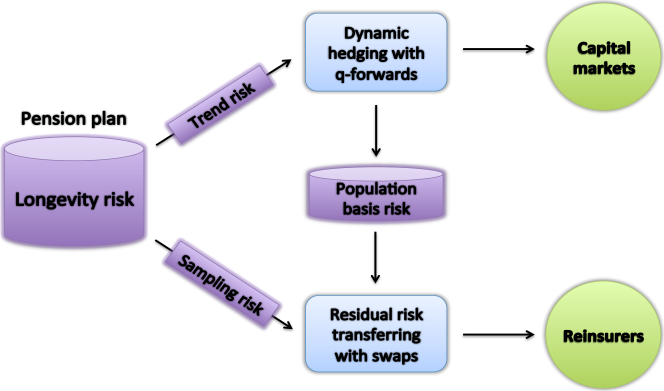

Figure 10: The outline of the proposed dynamic hedging strategy.