Long-Term Care with Multi-State Models

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Outline

Demographic and insurance context

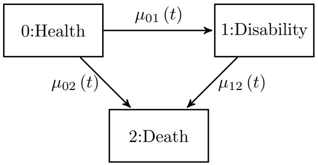
- Significant increase of health costs for elderly people in recent decades
- This trend will continue in future with a lot of uncertainty...
- Long-term care (LTC) insurance products in addition to social benefits
- Payment of benefits depends on the level of dependency (functional disability)
- No uniform definition and grid to measure the severity. Generally, contractual grids use criteria depending on the number of ADLs (wash, eat, dress, move, ...)
- A wide range of insurance products (short and long terms). LTC insurance may also be individual or collective.
- In France, contracts contain lots of policy clauses (whole life annuity vs. policy term, defered period, maximum age, deductible)

Regulatory context

- Solvency II offers a great role for actuaries
- Need for realistic (best estimate) assumptions. Actuaries are responsible for the data quality (accuracy, completeness) and the adequacy between data and models for reserving
- Pay close attention to bias (selection bias, information bias,...) and to the type of available data (e.g. continuous, discrete time, censorship) to select the best inference methods
- Need to regularly check biometric assumptions
- External data and expert opinion should be justified
- For LTC insurance, take account for the appropriately granular level and risk dynamics are great challenges

Current practices and available data

- Multi-state models are the most natural tools for pricing and reserving LTC guarantees, e.g. the illness-death model for only one heavy dependency state



- In the literature, large aggregated national dataset are usually used
- Introduce a Markov process X that describes the current state of a policyholder
- Quantities of interest:

$$p_{hj}(s,t) = \mathbb{P}(X_t = j \mid X_s = h) \text{ and } \mu_{hj}(t) = \lim_{\Delta t \to 0} \frac{p_{hj}(t, t+\Delta t)}{\Delta t}$$

Current practices and available data

- Researchers assume that the Markov assumption is satisfied and are interested in fitting the quantities of interest (e.g. Haberman and Pitacco, 1998; Pritchard, 2006; Levantesi and Menzietti, 2012; Fong *et al.*, 2015)
- Inference methodology → GLM Poisson models that depend on age x (CMIR12, 1991)

$$\eta\left(\mathbb{E}\left[\frac{d_{hj}\left(x\right)}{e_h\left(x\right)}\right]\right) = a_k x^k + a_{k-1}x^{k-1} + \ldots + a_0$$

- Lack of (detailed) national data. No covariate. Determining trends is quite complex (Gouriéroux and Lu, 2014)

Motivations

- Insurers should use their own data = longitudinal data in continuous time with censorship and truncation $\Longrightarrow$ we do not discuss the other cases
- It is time to develop statistical methods for multi-state models taking into account the data features. Non-parametric techniques $\rightarrow$ goodness-of-fit checks
- Practitioners often use methods developped for survival analysis (Guibert and Planchet, 2014)
- The Markov assumption is clearly not satistied

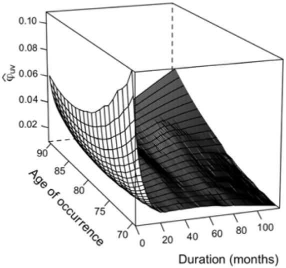


Figure: Fitted forces of mortality for LTC claimants (Tomas and Planchet, 2013)

Markov case: non-parametric inference for censored data

- Inference technique application to all Markov multi-state models
- C is an independent, non-informative, right censoring variable. We observe the censored process
- Based on counting process theory (Andersen *et al.*, 1993)

$$N_{hj}(t) = \#\{0 \leq \tau \leq t : X_\tau = j, X_{\tau-} = h, 0 \leq \tau \leq C\}$$

$$L_h(t) = \mathbb{1}_{\{X_{\tau-}=h, 0 \leq t \leq C\}} \text{ and } N(t) = \sum_{h,j} N_{hj}(t)$$

- Under $(\mathcal{F}_t)$ the canonical filtration generated by N and X_0 for all $h \to j$

$$N_{hj}(t) - \int_0^t L_h(\tau)\, dA_{hj}(\tau) \overbrace{=}^{\text{if abs. continuous}} N_{hj} - \int_0^t L_h(\tau)\, \mu_{hj}(\tau)\, d\tau$$

are martingale.

Markov case: non-parametric inference for censored data

- Transition intensities are estimated by the Nelson-Aalen estimators

$$\widehat{A}_{hj}(t) = \int_0^t \frac{dN_{hj}(\tau)}{L_h(\tau)} = \sum_{\{k:t_k \leq t\}} \frac{d_{hj}(t_k)}{L_h(t_k)}$$

- Heteregeneous population can be modeled with semi-parametric approaches, e.g. the Cox proportional hazard model

$$\mu_{hj}(t \,|\mathbf{Z}_{hj,i}, \boldsymbol{\theta}) = \mu_{0hj}(t) \exp\left(\boldsymbol{\theta}^\top \mathbf{Z}_{hj,i}\right)$$

- Transition probabilities matrices $\mathbf{p}$ are obtained with the Aalen-Johansen estimators

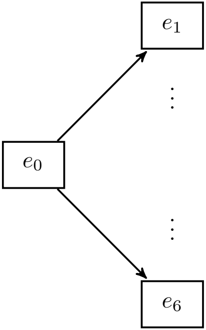
$$\widehat{\mathbf{p}}(s,t) = \mathop{\mathcal{P}}_{\tau \in]s,t]} \left(\mathbf{Id} + d\widehat{\mathbf{A}}(\tau)\right)$$

  with $\mathcal{P}$ the integral-product operator

- Generalization of the Kaplan-Meier (KM) estimator for survival data (Kaplan and Meier, 1958)

Example: LTC insurance data

- Database from a large French LTC insurer (Guibert and Planchet, 2014)
- Entry in dependency is distinguished by pathology (different waiting periods)
- $\simeq 210,000$ contracts observed during the period 1998-2010 after cleaning the database and almost 70% are censored



4 types of pathology and 2 direct exit causes.

	Exit causes	%
e_1	Neurologic pathologies	2.5%
e_2	Various pathologies	2.7%
e_3	Terminal cancers	2.4%
e_4	Dementia	5.4%
e_5	Death	52.2%
e_6	Cancel	34.8%

Example: LTC insurance data

Actuaries are interested in the inception rates $q_j(t) = p_{0j}(t, t+1)$

$$\widehat{q}_j(t) = \sum_{\{k: t < t_k \leq t+1\}} \frac{\widehat{S}(t_k)}{\widehat{S}(t)} \frac{d_{0j}(t_k)}{L_0(t_k)}$$

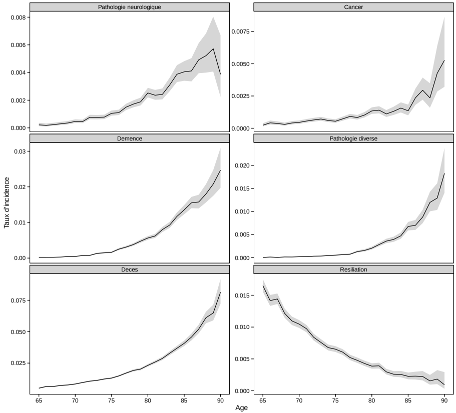


Figure: Inception rates estimates with approximate pointwise 95% confidence intervals

Semi-Markov model

- Let $S_1 < S_2 < \ldots < S_k < \ldots$ be the ordered jump times for the process X
- Let J_k be the discrete time process which gives the state occupied by X between times S_k and S_{k+1}
- Duration time in the current state $U_t = t - S_{N(t)}$

Definition

If the discrete time process (S_k, J_k) is a Markov process. It is called a Markov renewal process and is built up by an initial distribution and a semi-Markov kernel

$$Q_{hj}(s,t) = \mathbb{P}\left(\Delta S_{k+1} \leq t, J_{k+1} = j \mid S_k = s, J_k = h\right)$$

Then, (X_t, U_t) is Markov and the process (X_t) is called a semi-Markov process with its canonical filtration $(\mathcal{F}_t)$.

- Transition probabilities: $p_{hj}(s,t,u,v) = \mathbb{P}\left(X_t = j, U_t \leq v \mid X_s = h, U_s = u\right)$
- Transition intensities: $\mu_{hj}(t,u) = \lim_{\Delta t \to 0} \dfrac{p_{hj}(t, t+\Delta t, u, \infty)}{\Delta t}$

Inference for homogeneous semi-Markov model

- Homogeneous semi-Markov process: $Q_{hj}(s,t) = Q_{hj}(t)$
- No problem to infer parametric model. We regard non-parametric model
- Model without loop: can be estimated similarly to a Markov model
  $\Rightarrow$ Many situations in actuarial science
- Model with loops: the semi-Markov kernel is estimated non-parametrically (Gill, 1980) by
  $$\widehat{Q}_{hj}(t) = \int_0^t \left(1 - \widehat{H}_h(\tau)\right) \frac{dN_{hj}(\tau)}{L_h(\tau)},$$
  where $H_h(u) = \mathbb{P}(\Delta S_{k+1} \leq u \mid J_k = h)$ and this function is estimated with Kaplan-Meier.
- The processes $N_{hj}(u)$ and $L_h(u)$ depend on the time u spends in state h
- **But** transition probabilities $\Psi_{hj}(t) = \mathbb{P}(X_t = j \mid X_0 = h)$ are tricky to compute (Spitoni *et al.*, 2012)

Non-homogeneous semi-Markov and non-Markov models

- Non-homogeneous semi-Markov without loop:
  - Most of the time, one of the time variable is considered as a covariate
  - The splitting of state approach (Haberman and Pitacco, 1998)
  - actuarial approach for the disability model: estimating the survival function in the disability state, for e.g. by Kaplan-Meier, splitting the sample by age (integer) $\simeq$ survival data with staggered entry
  - Cox semi-Markov model (Andersen and Perme, 2008)

$$\mu_{hj}\left(t \mid \boldsymbol{Z}_{hj,i}, U_t, \boldsymbol{\theta}\right) = \mu_{0hj}\left(t\right) \exp\left(\theta_0 f\left(U_t\right) + \boldsymbol{\theta}^\top \boldsymbol{Z}_{hj,i}\right)$$

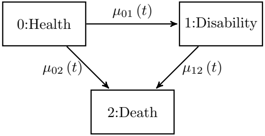
- Non-homogeneous semi-Markov with loops:
  - General framework not available for non-parametric inference
  - Cox specification is also applicable (Dabrowska, 1995)
  - Parametric approaches where intensities or kernels are written as a product of two uni-dimensional functions (Monteiro *et al.*, 2006; Mathieu *et al.*, 2007)

Approaches based on direct probabilities

- Meira-Machado *et al.* (2006): transition probabilities for an acyclic illness-death model without the Markov assumption
- Let S, the lifetime in healthy state and T the overall lifetime
- Let C a independent right-censored variable. We observe

$$\begin{cases} Y = \min(S, C) \text{ and } \gamma = \mathbb{1}_{\{S \leq C\}} \\ Z = \min(T, C) \text{ and } \delta = \mathbb{1}_{\{T \leq C\}} \end{cases}$$

- Transition probabilities are viewed as functional under the joint distribution of (S, T) and estimated using Kaplan-Meier integral



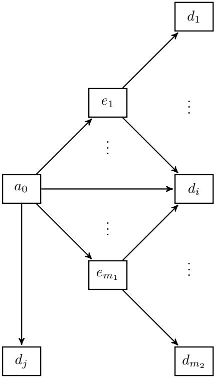
$$p_{00}(s,t) = \frac{\mathbb{P}(S > t)}{\mathbb{P}(S > s)}$$

$$p_{01}(s,t) = \frac{\mathbb{P}(s < S \leq t < T)}{\mathbb{P}(S > s)} = \frac{\mathbb{E}\left[\varphi_{st}^{(1)}(S,T)\right]}{\mathbb{P}(S > s)}$$

$$p_{11}(s,t) = \frac{\mathbb{P}(S \leq s, t < T)}{\mathbb{P}(S \leq s < T)} = \frac{\mathbb{E}\left[\varphi_{st}^{(2)}(S,T)\right]}{\mathbb{E}\left[\varphi_{ss}^{(2)}(S,T)\right]}$$

Acyclic multi-state model

Let an acyclic multi-state model which refers to a situation where both terminal and non-terminal events.



Formally, two lifetimes are identified:

- S, the lifetime in healthy state

$$S = \inf\{t : X_t \neq a_0\},$$

- T, the overall lifetime

$$T = \inf\{t : X_t \in \{d_1, \ldots, d_{m_2}\}\},$$

where $(X_t)_{t \geq 0}$ is the current state of the individual.

Main goals

- With independent right-censoring (non informative) variable
- No Markov assumption

Main goals

- Non-parametric estimation of transition probabilities for a such a right censoring acyclic multi-state model
- Define association measure between the failure time in healthy state and the overall lifetime

Existing Estimators for Competing Risks Data

- Let V be the indicator of the type of failure. The Aalen-Johansen (AJ) estimator for the cumulative incidence function (CIF) which is the joint distribution of (T, V) is

$$F^{(v)}(t) = \mathbb{P}(T \leq t, V = v)$$

Non-parametric estimator for CIF

- i.i.d. observations are composed of $(Z_i, \delta_i, \delta_i V_i,)_{1 \leq i \leq n}$
- Estimator can be expressed as a sum considering the ordered Z-values

$$\widehat{F}_n^{(v)}(z) = \sum_{i=1}^{n} \widetilde{W}_{in} J_{[i:n]}^{(v)} \mathbb{1}_{\{Z_{i:n} \leq z\}}, \quad \widetilde{W}_{in} = \frac{\delta_{[i:n]}}{n-i+1} \prod_{j=1}^{i-1} \left(\frac{n-j}{n-j+1} \right)^{\delta_{[j:n]}}$$

- $\widetilde{W}_{in}$ is the Kaplan-Meier (KM) weights and $J_i^{(v)} = \mathbb{1}_{\{V_i = v\}}$
- $\widehat{F}_n^{(v)}(\cdot)$ converges w.p.1 to $F^{(v)}(\cdot)$ and is asymptotically normal

Bivariate Competing Risks Data

- Idea: our model = a bivariate competing risks models with a unique right-censoring variable. Non-parametric inference is studied by Cheng *et al.* (2007) for a more general case
- Let (S, V_1) and (T, V) be 2 competing risks processes where:
  - V_1: indicator taking its values in the set of arrival states by direct transition from a_0
  - $V = (V_1, V_2)$ with is V_2 indicator taken its values in the set of arrival states from non-terminal events

Bivariate CIF estimator

$$\widehat{F}_{0n}^{(v)}(y, z) = \sum_{i=1}^{n} \widetilde{W}_{in} J_{[i:n]}^{(v)} \mathbb{1}_{\{Y_{[i:n]} \leq y, Z_{i:n} \leq z,\}}$$

- Simple form for the weights as (S, V_1) is observed whether T is observed
- $\widehat{F}_{0n}$ is weakly convergent under independent censoring

Aalen-Johansen Integrals Estimators

- Consider an integral of the form $S^{(v)}(\varphi) = \int \varphi \, dF_0^{(v)}$ with φ a generic function
- S can be considered as a covariate

AJ integrals

$$\widehat{S}_n^{(v)}(\varphi) = \int \varphi(s,t) \; \widehat{F}_{0n}^{(v)}(ds, dt) = \sum_{i=1}^{n} \widetilde{W}_{in}^{(v)} \varphi\left(Y_{[i:n]}, Z_{i:n}\right), \; 0 \leq s \leq t \leq \tau_Z.$$

- $W_{in}^{(v)} = W_{in} J_{[i:n]}^{(v)}$, AJ weights (Suzukawa, 2002) for competing risks data
- Possibility to take into account the left-truncation L considering

$$\widetilde{W}_{in}^{(v)} = \frac{\delta_{[i:n]} J_{[i:n]}^{(v)}}{n C_n(Z_{i:n})} \prod_{j=1}^{i-1} \left(1 - \frac{1}{n C_n(Z_{i:n})}\right)^{\delta_{[j:n]}},$$

where $C_n(x) = n^{-1} \sum_{i=1}^{n} \mathbb{1}_{L_i \leq x \leq Z_i}$

Transition Probabilities Estimators

Application for estimating key probabilities in actuarial science i.e.

$$p_{0e}(s,t,\eta) = \frac{\mathbb{P}(s < S \le \min(t, t-\eta), T > t, V_1 = e)}{\mathbb{P}(S > s)},$$

$$p_{ee}(s,t) = \frac{\mathbb{P}(S \le s, T > t, V_1 = e)}{\mathbb{P}(S \le s, T > s, V_1 = e)},$$

$$p_{ed}(s,t,\eta,\zeta) = \frac{\mathbb{P}(\eta < T - S \le \zeta, s < S \le t, V = (e,d))}{\mathbb{P}(T - S > \eta, s < S \le t, V_1 = e)}.$$

Remarking that $\{V_1 = e\} = \{V_1 = e, V_2 \in \mathcal{C}_e\}$ where $\mathcal{C}_e$ is the set of children (i.e transition states from e) related to the state e, we can refer to our AJ integrals estimators

Transition Probabilities Estimators

$$\widehat{p}_{0e}(s,t,\eta) = \frac{\widehat{S}_n^{(e,\mathcal{C}_e)}\left(\varphi^{(1)}_{s,t,\eta}\right)}{1-\widehat{H}_n(s)}, \text{ with } \varphi^{(1)}_{s,t,\eta}(x,y) = \mathbb{1}_{\{s<x\leq \min(t,t-\eta), y>t\}},$$

$$\widehat{p}_{ee}(s,t) = \frac{\widehat{S}_n^{(e,\mathcal{C}_e)}\left(\varphi^{(2)}_{s,t}\right)}{\widehat{S}_n^{(e,\mathcal{C}_e)}\left(\varphi^{(2)}_{s,s}\right)}, \text{ with } \varphi^{(2)}_{s,t}(x,y) = \mathbb{1}_{\{x\leq s, y>t\}},$$

$$\widehat{p}_{ed}(s,\eta,\zeta) = \frac{\widehat{S}_n^{(e,d)}\left(\varphi^{(3)}_{s,\zeta}\right)}{\widehat{S}_n^{(e,\mathcal{C}_e)}\left(\varphi^{(4)}_{s,\eta}\right)}, \text{ with } \varphi^{(3)}_{s,\zeta}(x,y) = \mathbb{1}_{\{s<x\leq t, \eta<y-x\leq\zeta\}},$$

$\varphi^{(4)}_{s,\eta}(x,y) = \mathbb{1}_{\{s<x\leq t, \eta<y-x\}}$ and $\widehat{H}_n$ is the KM estimator of the distribution function of S.

⇒ Our estimators generalize those of Meira-Machado *et al.* (2006)

Association measures

- Multivariate competing risks model (Scheike and Sun, 2012) $\rightarrow$ we introduce local association measures based on cross-odds ratio

$$\pi_0^{(e,d)}(s,t) = \frac{\text{odds}\left(T \leq t, V_2 = d \mid S \leq s, V_1 = e\right)}{\text{odds}\left(T \leq t, V_2 = d \mid V_1 = e\right)},$$

where $\text{odds}(A) = \dfrac{\mathbb{P}(A)}{1 - \mathbb{P}(A)}$.

- Measure dependence between the lifetime in healthy state and the overall lifetime per cause
- Non-parametric estimator

$$\widehat{\pi}_{0n}^{(e,d)}(s,t) = \frac{\dfrac{\widehat{F}_{0n}^{(e,d)}(s,t)}{\widehat{H}_{0n}^{(e)}(s) - \widehat{F}_{0n}^{(e,d)}(s,t)}}{\dfrac{\widehat{F}_{n}^{(e,d)}(t)}{\widehat{H}_{0n}^{(e)}(\infty) - \widehat{F}_{n}^{(e,d)}(t)}},$$

where $\widehat{H}_{0n}^{(e)}$ is the estimator of the CIF of S for cause $V_1 = e$ and $\widehat{F}_n^{(e,d)}$ is that of T for cause $V = (e,d)$

AJ integrals estimators

Theorem (Consistency)

Assume that

- φ is an F_0-integrable function,
- F_0 and censoring distribution function G are continuous,
- C is independent from the vector (S, T, V).

Then, we have

$$\widehat{S}_n^{(v)}(\varphi) \longrightarrow S_\infty^{(v)}(\varphi) = \int \mathbb{1}_{\{t<\tau_Z\}} \varphi(s,t) \ F_0^{(v)}(ds, dt) \ , \ v \in \mathcal{V} \ \ w.p.1.$$

- **proof:** Apply a similar strategy than Stute (1993)

AJ integrals estimators

Theorem (Weak convergence)

Assume that:

- $$\int \frac{\varphi(S,T)^2\,\delta}{(1-G(T))^2}\,d\mathbb{P} < \infty,$$
- $$\int |\varphi(S,T)|\sqrt{C_0(T)}\mathbb{1}_{\{T<\tau_Z\}}\,d\mathbb{P} < \infty,$$

  where $C_0(x) = \int_0^{x-} \frac{G(dy)}{(1-M(y))(1-G(y))}$ and $M(z) = \mathbb{P}(Z \leq z)$.

With the previous assumptions and assuming $\operatorname{supp}(Z) \subseteq \operatorname{supp}(C)$, we have

$$\sqrt{n}\left\{\widehat{\boldsymbol{S}}_n(\varphi) - \boldsymbol{S}(\varphi)\right\} \xrightarrow{d} \mathcal{N}(0, \boldsymbol{\Sigma}(\varphi)).$$

- Extendable considering additional (discrete) covariates $\boldsymbol{U} = (U_1, \ldots, U_p)$ and assuming

$$\mathbb{P}(T \leq C \mid S, T, \boldsymbol{U}, V) = \mathbb{P}(T \leq C \mid T, \boldsymbol{U}, V).$$

- **proof:** Follows ideas used by Stute (1995)

Transition probabilities and association measures

Proposition (Asymptotic results for transition probabilities)

$\widehat{p}_{0e}(s,t,\eta)$, $\widehat{p}_{ee}(s,t)$ and $\widehat{p}_{ed}(s,t,\eta,\zeta)$ are consistent w.p.1 if the support of Z is included in that of C. These estimators admit a weak convergence result.

- Provide estimators when the Markov assumption is released.
- Application to goodness-of-fit testing. Practitioners often use simple multi-state Markov model or Cox semi-Markov model. Misspecification may lead to important errors.

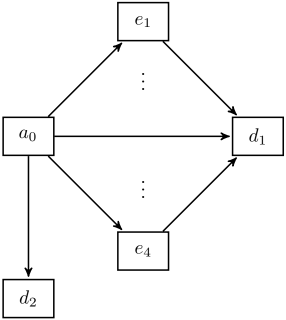
Proposition (Asymptotic results for assoaciation measures)

$\widehat{\pi}_{0n}^{(e,d)}(s,t)$ is consistent w.p.1 if the support of Z is included in that of C and admits a weak convergence result.

Possible applications to goodness-of-fit testing for models based on cross-odds ratios specification (see Scheike and Sun, 2012).

LTC insurance data

- Same dataset

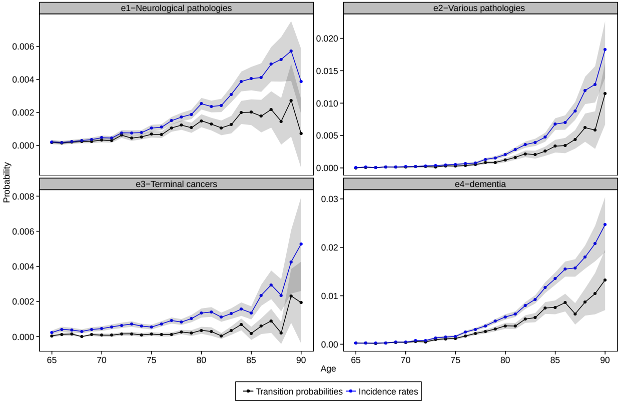


4 types of pathology and 2 direct exit causes.

	Exit causes	%
e_1	Neurologic pathologies	2.5%
e_2	Various pathologies	2.7%
e_3	Terminal cancers	2.4%
e_4	Dementia	5.4%
d_1	Death	52.2%
d_2	Cancel	34.8%

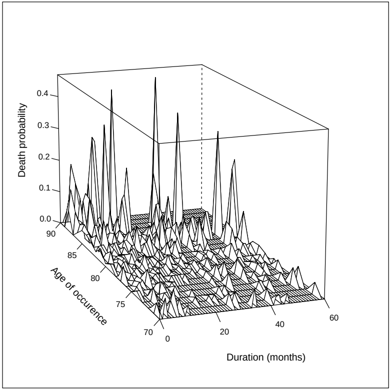
Transition probabilities

- Estimate annual transition probabilities to become dependent and stay at least one month in a disability state
- Compute pointwise 95% confidence interval from 500 bootstrap resamples

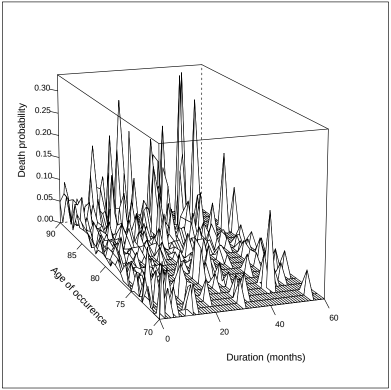


Transition probabilities

- Estimated surface of monthly death rates from each dependent state but quality is low due to missing data



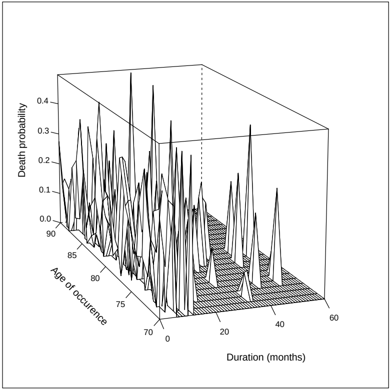
e_1-Neurologic pathologies.



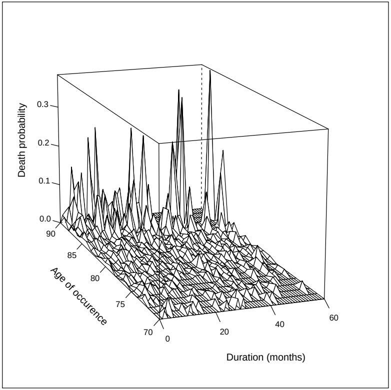
e_2-Various pathologies.

Transition probabilities

- Estimated surface of monthly death rates from each dependent state but quality is low due to missing data



e_3-Terminal cancers.



e_4-Dementia.

Summary

- Non-parametric estimation for AJ-integrals are applied to estimate this type of acyclic multi-state model under right-censoring
- These estimators and their properties stay valid if we consider covariates
- We provide new non-parametric estimators for transition probabilities
- We exhibit a non-parametric estimator for local association measures
- We apply them to LTC insurance data to estimate key probabilities

- Many outlooks
  - Consider framework for regression models
  - Develop more relevant bootstrap approach for AJ-integrals estimation
  - Develop semi-parametric approaches based on our local association measure
  - Consider general estimators for non-homogeneous semi-Markov models

Thank you for your kind attention.

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