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Modelling Credit Spread: A Fractional Integration Approach

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Abstract

We investigate the long memory properties of Treasury and corporate bond yields, spreads over Treasury and spreads between corporate yields, by considering a dynamic model of credit spreads consistent with recent econometric developments in modelling long range dependence in time series. We provide clear empirical evidence suggesting that applied analyses on credit spreads cannot be carried out within the classical I(0) vs I(1) framework, i.e. by testing for stationarity vs nonstationarity of spreads only. The empirical findings reported in this paper have strong implications for modelling credit spread dynamics and call for a reconsideration of the theoretical framework used to model the dynamics of credit spreads so far.

JEL Classification: C22, G10
Keywords: Credit spread, long range persistence, fractional cointegration.

1 Introduction

In this paper we provide direct evidence that the current modelling approaches to credit risk need to account for long memory properties of credit spread series. Specifically, we provide clear empirical evidence suggesting that applied analyses on credit spreads cannot be carried out within the classical I(0) vs I(1) framework, i.e. by testing for stationarity vs nonstationarity of spreads only. We conduct our analysis by considering a dynamic model of credit spreads consistent with recent econometric developments in modelling long range dependence in time series.

Two distinct approaches to modelling credit risk have been developed in the academic literature. The *structural* approach, following Merton (1974), views risky debt as a contingent claim written on the assets of a firm. A stochastic process is assumed for the evolution of the firm value, specifying the conditions generating default and the payoff in the event of default. The value of risky debt is derived from this framework. Papers that have built upon and extended Merton's model are, amongst others, Black and Cox (1976), Geske (1977), Kim, Ramaswamy and Sundaresan (1993), Shimko, Tejima and Van-Deventer (1993), Das (1995), Longstaff and Schwartz (1995a), Hull and White (1995), Anderson and Sundaresan (1996), Leland and Toft (1996). The *reduced-form* approach directly models the default process of risky debt. In combination with assumptions on the evolution of the risk-free rate and the recovery rate in the event of default, this is used to value risky debt. See, for example, Duffie and Singleton (1999) and Madan and Unal (1998). Models in this framework may also employ a ratings-based approach where default is attained through gradual changes in credit ratings driven by a Markov transition matrix. See, amongst others, Das and Tufano (1996) and Jarrow, Lando and Turnbull (1997). Other reduced-form models include Duffee (1999), Duffie and Huang (1996), Jarrow and Turnbull (1995), Kijima (1998), and Ramaswamy and Sundaresan (1986).

In both structural and reduced-form approaches credit spreads play a fundamental role in the pricing of risky debt and credit derivatives. However, several recent empirical studies have raised concerns on the ability of many of these models to explain the dynamics of credit spread. Pedrosa and Roll (1998) study daily credit spreads of USD-denominated bonds by industry, maturity and rating, and show, amongst others, that credit spread series are nonstationary and cointegrated, which suggests that common factors may explain the observed nonstationarity of spreads. Neal, Rolph and Morris (2000), using monthly data of Moody's and Treasury bond yields, show that Treasury and corporate yields are cointegrated and find that changes in interest rates have a different impact on credit spreads in the short and long term. Prigent, Renault and Scaillet (2001) use daily data of Moody's and Treasury bond yields to estimate the corporate spread dynamics nonparametrically. They determine that credit spreads are stationary and they find significant evidence of mean reversion, heteroskedasticity and jumps in spreads. Duellmann and Windfuhr (2000) test whether Ornstein-Uhlenbeck and square-root diffusion models are appropriate to capture the dynamics of sovereign spread series. They conclude that these models cannot capture all the shapes of the term structure of spreads and that none of the models outperforms the other. Kiesel, Perraudin and Taylor (2003) suggest that credit spreads are driven by combinations of stationary and random walk components and show that spread risk is the most important risk component for high credit quality portfolios.

Neither structural or reduced-form credit risk models nor their empirical counterparts have contemplated to go beyond considering spreads stationary or non stationary processes, while an important feature of this data is their degree of dependence. Thus it is important to distinguish between stationary long memory and nonstationary long memory behaviour of spreads.

No research has been carried out to date to investigate the long memory features of credit spreads. This paper aims to fill this important gap in the literature in two ways. First, we want to understand whether credit spreads are short or long memory processes, and whether, in the latter case, they are stationary or nonstationary. Such investigation is fundamental in assessing whether the dynamics of credit spread is consistent with the assumptions behind mainstream credit risk models, which are widely used by financial institutions. For example, the reduced-form models of Das and Tufano (1996), Jarrow, Lando, Turnbull (1997), and Duffie and Singleton (1999) assume stationarity of the risk-free rate process which implies stationarity of credit spreads. However authors do not specify whether credit spreads are short or long memory processes.

Second, we want to determine whether Treasury and corporate yields as well as corporate yields of different credit quality are cointegrated. For this purpose we use a fractional cointegration approach, which extends the classical framework to take into account non-integer orders of integration, retaining the idea that two or more variables are related and cannot evolve arbitrarily. The presence of (fractional) cointegration has strong implications for risky bond and credit spread option pricing. Spread option prices may be significantly biased if fractional cointegration is not taken into account. This, in turn, may generate bias in the prices of risky bonds: Duan and Pliska (2004) study the theory of option valuation in a classical cointegration framework and show that cointegration may significantly bias option prices when volatilities are stochastic. For example, (fractional) cointegrating relationships between rates are not accounted for in the models of Merton (1974), Kim, Ramaswamy and Sundaresan (1993), Das (1995), Longstaff and Schwartz (1995a,b), Duffee (1999), Duffie and Singleton (1999).

The paper is organised as follows. Section 2 describes the dataset. Section 3 reports the theoretical framework adopted to model the dynamics of the credit spread, consistent with econometric developments in modelling long range dependence in time series, that we briefly introduce in Section 4, where we also report the empirical results. Section 5 concludes.

2 Corporate Credit Spread: Data and Summary Statistics

Our dataset contains daily observations for the 30-year Historical US Treasury Constant Maturity Yields and Moody's *Aaa*, *Aa*, *A* and *Baa* Long-Term Corporate Bond Yield Averages. The data covers the period from December 1992 to November 2003, for 2703 observations. Spreads are calculated as the difference between corporate yields and Treasury yields, as well as between different corporate yields. In total, throughtout the paper we consider 30 series: Treasury yields (denoted by *Treas*), corporate yields (*Aaa, Aa, A,* and *Baa*), spreads over Tresasury (*sTreasAaa, sTreasAa, sTreasA,* and *sTreasBaa*), spreads between corporate yields (*sAaaAa, sAaaA, sAaaBaa, sAaA, sAaBaa,* and *sABaa*), and their first differences (denoted with a "*d*" in front of the relevant series name).

We use rating-specific indices rather than data on individual issues to analyse spreads for a number of reasons. First, the market for individual corporate bonds is often illiquid and the consistency of the credit spread component of corporate yields is strongly affected by liquidity constraints. Using indices can overcome this problem. Second, such indices, reflecting average spreads for well-diversified portfolios of corporate bonds with a relatively long maturity and for given credit classes, are useful market indicators. For example, investors can hedge corporate bond portfolios more efficiently against moves in the general level of spreads rather than hedging each individual bond. Speculative views on the direction of the market spreads can also be taken. Third, credit spread indices may be used as underlying to credit spread options. Currently, these credit derivatives are only traded on liquid individual issues.

The US Treasury constant maturity yield curve is a benchmark risk-free curve in empirical finance. The 30-year US Treasury Constant Maturity Yields are constructed by the US Treasury Department, based on the closing market bid yields of actively traded Treasury securities in the over-the-counter market. Yields are based on composite quotes reported by US Government securities dealers to the Federal Reserve Bank of New York. They are interpolated by the Treasury Department from a yield curve estimated on a daily basis using a cubic spline methodology. The constant maturity yields are the curve theoretical rates at the relevant maturities. A yield is therefore always available for a 30-year tenor even if no outstanding security has exactly this maturity.

Moody's Long-Term Corporate Bond Yield Averages are based on seasoned bonds with remaining maturities of at least 20 years. They are derived from yields on a population of almost 100 US corporate bonds, each with current outstanding of over $100 million. The bonds' maturities are as close as possible to 30 years; bonds are dropped from the list if their remaining life falls below 20 years, if they are redeemable or if their ratings change. Bonds with strong discounts or premiums to par are generally excluded. All rates are yields to maturity calculated on a semi-annual basis. Each observation is an unweighted average of the Average Industrial and Average Public Utility Yields.[1] Moody's Long-Term Corporate Bond Yield Averages are the only publicly available series of yields created per rating class. Other publicly available series average yields across rating categories.

According to Moody's debt rating definitions,[2] bond issues that are rated *Aaa* are judged to be of the best quality and carry the smallest degree of investment risk. Principal is secure and interest payments are protected by a large or exceptionally stable margin. Changes in the various protective elements are most unlikely to deteriorate the fundamentally strong position of such issues.

Bond issues that are rated *Aa* are judged to be of high quality by all standards. Together with the *Aaa* class are generally known as high-grade bonds. Margins of protection may not be as large as for *Aaa* bonds and changes in the protective elements may be of greater amplitude, thus making the long-term risk appear larger than the *Aaa* bonds.

[1] This has not been the case for the *Aaa* rating class since December 2001, when the Moody's utility index was suspended because only one issuer (TVA) was left in the index as a consequence of years of deregulation.

[2] See, 2003 Moody's investors service and http://www.moodys.com

Bond issues that are rated *A* have many favourable investment attributes and are to be considered as upper-medium-grade obligations. Factors giving security to principal and interest are judged adequate, however, elements may be present which suggest a susceptibility to impairment in the future.

Finally, bond issues that are rated *Baa* are considered as medium-grade obligation, being neither highly protected nor poorly secured. Interest payments and principal security appear adequate but some protective elements may be missing or unreliable in the future. Such bonds are subject to moderate credit risk, lacking outstanding investment characteristics and having speculative feature as well.

Data series of the US Treasury constant maturity yields were obtained from the Board of Governors of the Federal Reserve System, release H.15,[3] and data series of the Moody's corporate bond yields were obtained from Bloomberg. Relevant data issues related to the series used in this paper are discussed in full in Della Ratta (2005).

Table 1 shows summary statistics and normality test results for yields, spreads, and their first differences. For instance, over the period December 1992 to November 2003, *Treas* averaged 6.154%, *Aaa* 7.098%, *Aa* 7.346%, *A* 7.537% and *Baa* 7.895%. *sTreasAaa, sTreasAa, sTreasA,* and *sTreasBaa* averaged 0.944%, 1.193%, 1.383% and 1.741% respectively.

Figures 1-3 show the yields and spreads over the sample period. All spreads are positive and spreads over Treasury for lower rated bonds are consistently higher than spreads for higher rated bonds, as one would expect. Heteroskedasticity and volatility clustering are also two important features of the data.

[Insert Table 1 and Figures 1- 3 somewhere here]

Differences in credit spreads, with the exceptions of *dsTreasAaa*, *dsAaBaa* and *dsABaa* are positively skewed. This implies that the loss tail of the return distribution contains more probability than the normal distribution. Differences are also leptokurtic. This is a typical feature of credit risk, implying a small chance of very large returns combined with a large probability of small returns. Overall, the Jarque-Bera test for normality shows that distributions of yields and spreads, and their first differences, are non-normal.

All yields and spreads series are highly persistent. The null hypothesis of the Ljung-Box Q-Statistic that yields and spreads are not serially correlated up to order 5 and up to order 20 is always rejected at 1% significance

[3] Data is available from the Federal Reserve at http://www.federalreserve.gov/releases/h15/data.htm

level. Yield and spread differences show different behaviours: whilst yield differences are not serially correlated, spread differences have a significant negative first order autocorrelation coefficient. Finally, volatility clustering is confirmed by the results of the autocorrelation tests for absolute and squared returns on yields and spreads.

3 The Dynamics of Credit Spread

In this paper we propose to model the dynamics of the credit spread according to the stochastic differential equation:

$$dS_t = \mu\left(S_t, t\right) dt + \sigma\left(S_t, t\right) dB_H\left(t\right), \tag{1}$$

where S_t is the relevant spread, μ_t and σ_t are the drift and diffusion term respectively, functions of S_t and possibly time-varying, and $B_H\left(t\right)$ is a fractional Brownian motion.

The fractional Brownian motion $B_H\left(t\right)$ is a Gaussian process with zero mean, stationary increments, variance $E\left(B_H^2\left(t\right)\right) = t^{2H}$ and covariance:

$$E\left(B_H\left(t\right) B_H\left(s\right)\right) = \frac{1}{2}\left(t^{2H} + s^{2H} - \left|t - s\right|^{2H}\right), \quad t, s \in R. \tag{2}$$

This framework is general enough to encompass a series of alternative models. Depending on the value assumed by the parameter H, the fractional Brownian motion has independent increments (for $H = 1/2$), positive covariance between two increments over non-overlapping time intervals (for $1/2 < H \leq 1$), or negative covariance between increments (for $0 < H < 1/2$). It is worth noting that mainstream models cited above only consider the case of $H = 1/2$, i.e. when the fractional Brownian motion reduces to a standard Brownian motion. H, the so-called Hurst exponent or coefficient (Hurst. 1951) , represents the classical parameter characterising long memory. Long memory typically occurs when $1/2 < H < 1$. For $0 < H < 1$, Mandelbrot and Van Ness (1968) define the fractional Brownian motion as a stochastic integral with respect to the standard Brownian motion:

$$B_H\left(t\right) = \int_{-\infty}^{t} k\left(t, s, H\right)\, dB_s, \tag{3}$$

where k is a deterministic kernel depending on H.

It is worth noticing that the fractional Brownian motion is part of a more general class of processes that exhibit long-range dependence under certain conditions. These processes, called self-similar processes, were introduced by Kolmogorov (1940) and are discussed in Mandelbrot and Van Ness (1968). (Sottinen and Valkeila, 2001, provide a useful overview on the topic). A centered stochastic process X_t, $0 \leq t \leq T$, is said to be statistically self-similar with Hurst coefficient H if it has the same distribution as $a^{-H} X_{at}$, for any $a > 0$. In addition, if X_t is square integrable with stationary increments we have

$$Cov\left(X_t, X_s\right) = \frac{Var\left(X_1\right)}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right). \tag{4}$$

If $H = 0$, the above equation implies that $X_t = 0$ identically. If $H = 1$, $Corr\left(X_t, X_1\right) = 1$, which is not an interesting case. If $H < 0$, $Var\left(X_0\right)$ is infinite. Finally, $H > 1$ is impossible as, for t big enough, one would obtain $Corr\left(X_t, X_1\right) > 1$. Therefore, in equation (4) it is usually assumed that $0 < H < 1$.

The autocorrelation function of a self-similar process is given by:

$$\rho_j = \frac{1}{2}\left[(j+1)^{2H} - 2j^{2H} + (j-1)^{2H}\right]. \tag{5}$$

We have:

$$\lim_{j \to \infty} \rho_j = H\left(2H-1\right) j^{2H-2} \tag{6}$$

or, equivalently:

$$\lim_{j \to \infty} \frac{\rho_j}{H\left(2H-1\right) j^{2H-2}} = 1. \tag{7}$$

Equation (7) is equivalent to the commonly used definition of long memory:

$$\lim_{j \to \infty} \frac{\rho_j}{c_\rho j^{-\alpha}} = 1,$$

with $\alpha = 2 - 2H$ and $c_\rho = H\left(2H-1\right)$ and constraint $0 < \alpha < 1$. Therefore self-similar processes exhibit long-range dependence when $1/2 < H < 1$. The process X_t admits a version with continuous sample paths when $1/2 < H < 1$.

Reduced-form models of credit risk typically assume a stochastic process for the instantaneous spread, as it is intimately related to the instantaneous probability of default. This representation is general and encompasses processes with different assumptions of the spread dynamics. For example, if we assume that spreads are generated by a random walk, the spread dynamics may be modelled with an arithmetic Brownian motion with drift, where $\mu(S_t, t) = \mu$, $\sigma(S_t, t) = \sigma$ and $H = 1/2$. Alternatively, if we assume that spreads are governed by a mean reverting process, we may adopt an Ornstein-Uhlenbeck process, where $\mu(S_t, t) = \alpha(\theta - S_t)$, $\sigma(S_t, t) = \sigma$ and $H = 1/2$. In the first case, the conditional variance of the process is equal to $VaR(S_t) = \sigma^2 t$, volatility increases with the square root of time, and the process is nonstationary. In the second case, the conditional variance of the process is equal to $VaR(S_t) = \frac{\left(1-e^{-2\alpha t}\right)\sigma^2}{2\alpha}$ which converges to a constant as t increases, and the process is stationary.

One may assume that spreads are neither pure random walk processes nor genuine stationary processes. This can be modelled by choosing $H \neq 1/2$, which introduces dependence between observations over time and therefore can describe a long memory process.

It is evident then that an estimate of H will allow to identify the underlying process characterising spreads. In the next section, we briefly describe the long-range models consistent with the dynamics of credit spread, and present the estimation methods used to estimate long range dependence parameter in time series. We also report and comment the empirical results.

4 Estimating Long-Range Dependence in Credit Spread

In this paper, we consider discrete time models, and two in particular. The simplest discrete-time long memory model is the fractional white noise (Adenstedt, 1974, Granger, 1980, Granger and Joyeux, 1980, and Hosking, 1981). It is defined as:

$$y_t = (1 - L)^{-d} \epsilon_t \tag{8}$$

where L is the lag operator, $E(\epsilon_t) = 0$, $E\left(\epsilon_t^2\right) = \sigma^2$, $E(\epsilon_t \epsilon_s) = 0$ for $s \neq t$, and

$$d = H - 1/2 \tag{9}$$

is the fractional difference parameter (see Adenstedt, 1974, and Taqqu, 1975). Let $y_t \sim I(d)$. If $d = 0$, $y_t = \epsilon_t$ and the process is serially uncorrelated. If $d > 0$ the process is said to have long memory and is mean square summable. It is also stationary for $d < 1/2$ and invertible for $d > -1/2$.

A more general class of processes that contains the fractional white noise as a particular case is the Autoregressive Fractionally Integrated Moving Average ($ARFIMA$) model introduced by Granger and Joyeux (1980), Granger (1980, 1981), and Hosking (1981). The $ARFIMA\,(p, d, q)$ process is defined as:

$$\phi\,(L)\;(1-L)^{d}\,(y_t-\mu)=\theta\,(L)\;\epsilon_t, \tag{10}$$

where $\phi\,(L)$ and $\theta\,(L)$ involve autoregressive and moving average coefficients of order p and q respectively and ϵ_t is a white noise process. The roots of $\phi\,(L)$ and $\theta\,(L)$ lie outside the unit circle. A fractional white noise process is equivalent to an $ARFIMA\,(0, d, 0)$ process. $ARFIMA$ processes are covariance stationary for $-1/2 < d < 1/2$, mean reverting for $d < 1$ and weakly correlated for $d = 0$. For $d \geq 1/2$, these processes have infinite variance but in the literature it is more usual to impose initial value conditions (extending that of autoregressive models with unit roots) so that y_t has changing, but finite, variance. See recent review by Robinson (2003) and Banerjee and Urga (2004).

Expression (9) links the credit spread model (1) to its empirical counterparts (8) or (10).

4.1 Semiparametric and Parametric Estimators of d

In this section, we report the estimates of d using two semiparametric methods, the Geweke and Porter-Hudak (1983, GPH henceforth) and Robinson (1995) estimators, and a parametric estimation method of an $ARFIMA\,(p, d, q)$ model, the approximate maximum likelihood (AML henceforth) of Haslett and Raftery (1989). The empirical findings, also commented in this section, are consistent across the two alternative sets of estimators of d.

4.1.1 Semiparametric methods

The GPH estimator. The GPH methodology provides an estimation of d in the frequency domain based on the fractionally integrated process representation of a long memory time series. For S_t^{Aaa}, this is:

$$(1-L)^{-d}\left(S_t^{Aaa}-\mu\right)=\epsilon_t, \tag{11}$$

where L is the lag operator and μ is the expectation of S_t^{Aaa}. The spectral density of S_t^{Aaa} is given by:

$$f(\lambda) = \left[4\sin^2\left(\frac{\lambda}{2}\right) \right]^{-d} f_{\epsilon}(\lambda), \tag{12}$$

where λ is the Fourier frequency and $f_{\epsilon}(\lambda)$is the spectral density corresponding to ϵ_t. In the GPH methodology, the fractional difference parameter is estimated by least squares using the following regression:

$$\ln f(\lambda_j) = \beta - d \ln \left[4\sin^2\left(\frac{\lambda_j}{2}\right) \right] + e_j \tag{13}$$

for $j = 1, 2, ..., m$, where m is the bandwidth parameter

In order to ensure that stationarity is achieved, we apply the GPH estimator to the first differences of the series. We test the null hypothesis that $d' = 0$, where d' is the long memory parameter of the first differences of the series and estimate $d = d' + 1$ as the long memory parameter of the series in levels. Test results, reported in Table 2A and Table 2B, show that the estimated value of d' for the first differences of yields and spreads is not statistically different from zero when $m = T^{0.5}$ or $m = T^{0.6}$ (where T is the number of observations) and the null $d' = 0$ is therefore not rejected. As a consequence, the estimated value of $d = d' + 1$ for yields and spreads is not statistically different from one and yields and spreads are therefore long memory and nonstationary processes. However, from the 95% confidence intervals we cannot conclude whether the series in first differences are short or long memory stationary processes. By looking at the point estimates, we notice that the d' estimate for the first differences of yields is generally negative, whilst the d' estimate for the first differences of spreads is generally positive. First differences of spreads are therefore more likely to be long memory than first differences of yields.

[**Insert Tables 2A-2B somewhere here**]

In order to test for robustness, estimations of d' are performed by using a number of alternative values of m. We find that the GPH estimator is not particularly sensitive to changes in the bandwidth, however, three exceptions arise. When $m = T^{0.4}$, *sAaaAa* and *sAaaA* seem to be nonstationary long memory series with $d = 0.67$ and $d = 0.65$ respectively. When $m = T^{0.7}$, *sTreasBaa* seems to be a nonstationary long memory process with $d = 1.13$.

Robinson (1995) estimator. Robinson (1995) refines the GPH log-periodogram regression. Specifically, the fractional difference parameter is estimated by least squares using the following regression:

$$\ln f(\lambda_j) = a + b \ln(\lambda_j) + e_j \tag{14}$$

for $j = 1, 2, ..., m$, and $\hat{d} = -\frac{1}{2}\hat{b}$ is the estimate of the fractional difference parameter. The least square estimator $\hat{b}$ is asymptotically normal and the corresponding theoretical standard error is given by $\pi(24m)^{-\frac{1}{2}}$. The Robinson estimator is more efficient than the GPH estimator and robust to nonnormality.

As for the GPH estimator, we apply the Robinson estimator to the first differences of the series. Results, summarised in Table 3A and Table 3B, are consistent with the GPH results and show that yields and spreads are long memory non stationary and their first differences are stationary. The Robinson estimator is robust to changes in the bandwidth.

[Insert Tables 3A-3B somewhere here]

4.1.2 Parametric Methods: $ARFIMA(p, d, q)$ framework.

We now estimate d in an $ARFIMA(p, d, q)$ framework using the AML procedure of Haslett and Raftery (1989). For the *Aaa* spread series, the chosen *ARFIMA* model specification used to implement the AML procedure is based on Beran (1995) and is as follows:

$$\phi(L)\ (1-L)^{\delta}\left[(1-L)^{n} S_t^{Aaa} - \mu\right] = \theta(L)\ \epsilon_t, \tag{15}$$

where $-1/2 < \delta < 1/2$, $\phi(L)$ and $\theta(L)$ are polynomials with roots outside the unit circle. The integer n is the number of times that S_t^{Aaa} must be differenced to achieve stationarity and the difference parameter is given by $d = \delta + n$. This specification allows the estimation of $ARFIMA$ models for any $d > -1/2$.

For each yield and spread series and their first differences we estimate the long memory parameter d for the nine combinations of $ARFIMA(p, d, q)$ models where p and q are between 0 and 2 and we choose the model which minimises the Bayesian Information Criterion.

Results of the AML procedure[4] for the yields and spread series are summarised in Tables 4A-4C. The best model for the series of yields (*Treas*,

[4] We also implemented the exact maximum likelihood procedure (EML) of Sowell (1992) and the modified profile likelihood (MPL) procedure of Cox and Reid (1987). Both procedures provide similar results to the AML procedure in terms of the estimation of d, with the only exception of some of the series of spreads between corporate yields found to be long memory non stationary process. The full set of results is reported in Della Ratta (2005).

Aaa, *Aa*, and *Baa*) is an $ARFIMA\,(0,1,0)$, which confirms the long memory nonstationary property of Treasury and corporate yields. Consistently with this result, the best model for the first differences of yields is an $ARFIMA\,(0,0,0)$, indicating that these series are short memory stationary processes.

[Insert Tables 4A-4C somewhere here]

When we look at the series of spreads and their first differences the outcome is more variable. First, the best model for the series of spreads over Treasury (*sTreasAaa*, *sTreasAa*, *sTreasA*, and *sTreasBaa*) is an $ARFIMA\,(2,d,0)$, with d always statistically greater than 1. Point estimates range from 1.0732 for *sTreasAa* to 1.1367 for *sTreasBaa*. The series of spreads over Treasury are therefore long memory nonstationary processes. This result implies that the *differenced* series are long memory stationary processes. This is confirmed by the fact that the best model for the first differences of the spreads over Treasury is an $ARFIMA\,(2,d,0)$, where d is statistically greater than 0 and less than $\frac{1}{2}$.

Second, there is no unique result for the series of spreads between different corporate yields. In four cases (*sAaaAa*, *sAaaA*, *sAaBaa*, and *sABaa*) the best model is an $ARFIMA\,(2,0,0)$, in one case (*sAaaBaa*) an $ARFIMA\,(2,d,0)$ with $0 < d < \frac{1}{2}$, and in one case (*sAaA*) an $ARFIMA\,(1,d,1)$ with $-\frac{1}{2} < d < 0$. Therefore, while the series *sAaaBaa* is a long memory stationary process, the other series are short memory stationary processes. These results seem to contradict the previous outcome of the GPH and Robinson estimators which concluded that all spread series were long memory nonstationary processes. This discrepancy however can be explained by noticing that for each of the spread series the sum of the AR coefficients is close to unity, which indicates that the nonstationarity feature highlighted in the GPH and Robinson results has in fact been captured by the AR coefficients rather than by the d parameter in the $ARFIMA$ framework.

Third, the first differences of spreads between corporate yields are all short memory stationary processes, with some differences in the estimated long memory parameter. While in three cases (*dsAaaAa*, *dsAaaA* and *dsAaaBaa*) d is not statistically different from zero, in the other three cases (*dsAaA*, *dsAaBaa* and *dsABaa*) we have $-\frac{1}{2} < d < 0$.

Fourth, with regard to the $ARMA$ structure of the $ARFIMA(p,d,q)$ model specification, we notice that, unlike yields, which do not show any AR or MA component, 9 out of 10 spread series follow an $ARMA\,(2,0)$ process, while *sAaA* follows an $ARMA\,(1,1)$ process. This result, combined with the observations on the estimation of d, suggests that spreads are driven by a mixture of short memory and long memory processes.

To summarise, in contrast to what assumed by the existing relevant literature, yields and spreads are long memory nonstationary processes.

4.2 Fractional Cointegration

We turn now to assess whether Treasury and corporate yields follows a common sets of fundamentals via a long-run equilibrium relationship. We undertake this study by investigating the presence of cointegration or fractional cointegration for Treasury and corporate yields. For this purpose, we consider all 10 possible bivariate systems of yields, namely (*Treas*, *Aaa*), (*Treas*, *Aa*), (*Treas*, *A*), (*Treas*, *Baa*), (*Aaa*, *Aa*), (*Aaa*, *A*), (*Aaa*, *Baa*), (*Aa*, *A*), (*Aa*, *Baa*), and (*A*, *Baa*).

Two time series y_t and x_t, integrated of order d, are said to be fractionally cointegrated of order $(d,\, b)$ if the error correction term represented by the linear combination $z_t = y_t - \beta x_t$ is fractionally integrated of order $d - b$, where $0 < b \leq d$ and $d > 1/2$. It is also possible to have fractional cointegration with $d < 1/2$, as in Robinson (1994). In this case the error correction term is mean reverting and a shock to the system persists for some time but eventually dies out. Long run equilibrium exists amongst the variables even though adjustments to equilibrium may take a long time to realise. Granger (1986) has provided an error correction representation for fractionally cointegrated processes. Specifically, if y_t is an $I\,(d)$ vector of time series and z_t a set of cointegrating vectors such that the error correction term $z_t = \alpha' y_t$ is $I\,(d-b)$, the fractionally cointegrated system has an error representation of the form:

$$\Psi\,(L)\,(1-L)^d\, y_t = -\gamma\left(1-(1-L)^b\right)\cdot(1-L)^{d-b}\, z_t + c\,(L)\,\epsilon_t \qquad (16)$$

where $\Psi\,(L)$ is a polynomial matrix in the lag operator L, $\Psi\,(0)$ is the identity matrix, $c\,(L)$ is a finite order polynomial and ϵ_t is a white noise error term.

In what follows, we test this hypothesis via standard cointegration analysis (Section 4.2.1) and using GPH estimation method (Section 4.2.2). Dittmann (2004) proposes an alternative error correction model that can be employed to estimate fractionally cointegrated systems in three steps and in Section 4.2.3 we report the resuts from this application. Finally, in Section 4.2.4 we report results from applying AML procedure to the residuals of the cointegrating relationships. Once again, the empirical findings, also reported in this section, are consistent across the alternative methods to test for fractional cointegration.

4.2.1 Classical cointegration analysis

First, we focus on classical cointegration analysis. In order to test for cointegration, the cointegrating relationship

$$y_{it} = \alpha + \beta x_{it} + \epsilon_{it}$$

is estimated for each pair of yields x_{it} and y_{it}, and subsequently the Dickey and Fuller (DF), the Augmented Dickey and Fuller (ADF) and the Phillips and Perron (PP) tests are carried out on the estimated residuals. Phillips and Ouliaris (1990) show that the DF, ADF and PP unit root tests applied to the estimated cointegrating residuals do not have the usual Dickey-Fuller distributions under the null hypothesis of no cointegration. Consequently, we assess the significance of the tests against the critical values computed from the appropriate Phillips-Ouliaris distribution.

Test results, summarised in Table 5, are quite surprising. In contrast with previous indications, including Pedrosa and Roll (1998) and Neal, Rolph and Morris (2000), we do not find evidence of cointegration for any of the bivariate systems with the exception of the (Aaa, Aa) system. Specifically, we do not find cointegration between Treasury and corporate yields. This may seem counterintuitive, as corporate and Treasury yields are generally considered of being closely linked and do not certainly evolve in arbitrary ways. However, this result could be explained by the fact that the usual concept of cointegration may be too restrictive. Treasury and corporate yields may in fact be fractionally cointegrated. We evaluate this in the next sections.

[Insert Table 5 somewhere here]

4.2.2 GPH estimation

Testing for fractional cointegration requires testing for fractional integration in the error correction term. We use the GPH estimation method for this purpose. We apply the test to the first differences of residuals of the cointegrating relationship. Specifically, we test the null hypothesis that $d' = 0$, where d' is the fractional order of integration of the differenced residuals, against the alternative $d' < 0$. This is equivalent to a test of $d = 1$ (no cointegration) against $d < 1$ (fractional cointegration) on the residuals.

Since the test is applied to the estimated residuals, we cannot use the standard critical values of the GPH method. The error correction term obtained by minimising the residual variance of the cointegrating relationship will be biased towards stationarity, implying that the null hypothesis of no cointegration will be rejected more often than suggested by the nominal size of the test. We therefore use the critical values computed by Andersson and Lyhagen (1997) through Monte Carlo simulations. In addition, we adopted a conservative approach and used the bandwidth $m = T^{0.9}$ as determinant of the number of frequencies for the test given that the power of the test against fractional alternatives is maximum for $m = T^{0.9}$.

Table 6 reports the GPH results for fractional cointegration. Interestingly, we find evidence of fractional cointegration for 8 out of 10 bivariate systems, namely (*Treas*, *Aaa*), (*Treas*, *Aa*), (*Treas*, *A*), (*Aaa*, *Aa*), (*Aaa*, *A*), (*Aa*, *A*), (*Aa*, *Baa*), and (*A*, *Baa*). The only two systems which are not fractionally cointegrated are (*Treas*, *Baa*) and (*Aaa*, *Baa*). The fractional difference parameter takes values between 0.80 and 0.94 for the estimated residuals of the fractionally cointegrated systems, and is not statistically different from unity for the systems (*Treas*, *Baa*) and (*Aaa*, *Baa*).

[**Insert Table 6 somewhere here**]

Overall, we conclude that the error correction term of most bivariate systems follows a fractionally differenced process with long memory, indicating that there exists a binding long run equilibrium relationship between Treasury and corporate yields and between corporate yields of different rating categories. Therefore, even though the series may vary widely, deviations from the cointegrating relationship are mean reverting, so that a shock to the system will eventually die out. However, given that the fractional difference parameter assumes values close to unity, shocks may take several years to dissipate (see Della Ratta, 2005).

The absence of fractional cointegration for the system (*Treas*, *Baa*) may be explained by the fact that, as ratings worsen, components other than movements in the risk-free rate, such as spread risk and default risk, become more likely to be the primary drivers of changes in corporate yields. *Baa* is the lowest rating class amongst the ones considered. For example, whilst the average one year probability of default of a *Baa* exposure over the period 1981-1998 was approximately equal to 0.24%, the probability of default of an *A* exposure, which represents the next higher rating class, is six times lower, i.e. 0.04%. (See Standard and Poor's, 1999). Our results that the systems (*Treas*, *Aaa*), (*Treas*, *Aa*) and (*Treas*, *A*) are fractionally cointegrated is consistent with this explanation, and suggest that corporate rates for higher rating classes have a higher correlation with Treasury rates than corporate rates for lower rating classes. In this respect, our results are also consistent with the findings of Longstaff, Mithal and Neis (2004), who claim that the default risk component of the spread increases with the worsening of the rating.

By turning our attention to the cointegration between corporate yields, we do not find fractional cointegration only for the system (*Aaa*, *Baa*). Rates in this system are at the opposite ends of the rating class spectrum of our sample. It is reasonable to assume that the dynamics of the *Aaa* and *Baa* yields, although may be explained by some common factors, are less correlated than the dynamics of, for example, the *Aaa* and *Aa* yields. Therefore, we expect that the system (*Aaa*, *Baa*) has a lower likelihood to be cointegrated than systems of corporate yields with adjacent or closer ratings. This is consistent with our empirical results that the other systems of corporate yields are fractionally cointegrated.

4.2.3 Dittman (2004) estimation of fractionally cointegrated systems

Dittman (2004) proposes an alternative to Granger (1986) error correction model for fractionally integrated systems, which can be employed to estimate a fractionally cointegrated system in three steps provided the integration order of all components of the system is the same. For example, for the bivariate system (*Treas, Aaa*), where the fractional difference parameter of both *Treas* and *Aaa* is not statistically different from one, the three steps are as follows. In the first step, the OLS regression:

$$Aaa = \alpha + \beta Treas + \epsilon_t$$

is performed to estimate α and β. Cheung and Lai (1993) show that this estimator is consistent. In the second step, the long memory parameter of the cointegrating equilibrium $\hat{\epsilon}_t = Aaa - \hat{\alpha} - \hat{\beta} Treas$ is estimated. As we use the GPH method for this estimation, these first two steps are equivalent to the GPH estimation procedure for fractional cointegration employed in (4.2.2), although we are now interested in the estimated value $\hat{d}$ of the long memory parameter and not only on the significance of the test statistic. In the third step, we use the estimates from the first two steps to compute the fractional difference $(1-L)^{\hat{d}}\left(Aaa - \hat{\alpha} - \hat{\beta} Treas\right)$ and verify that it is an $ARMA$ process.

Table 7 shows the test results. First, we notice that the slope coefficients estimated from the cointegrating relationships is less than one for all bivariate systems. This implies that an increase in the Treasury yield will produce a less than proportional increase in the corporate yield, with the effect of a reduction in the credit spread. Second, the fractional difference parameters of the cointegrating equilibria are statistically less than one, with the only exceptions of the systems (*Treas, Baa*) and (*Aaa,Baa*). In order to conclude, consistently with (4.2.2), that eight out of tem bivariate systems are fractionally cointegrated, we need to show that the fractionally differenced cointegrating equilibria are stationary processes. The KPSS test performed on the fractional differences of the cointegrating equilibria of the ten bivariate systems shows that they are stationary.

As suggested by Dittman (2004), we model these fractional differences as $ARMA$ processes. Specifically, in order to show stationarity, for each series we estimate by AML nine combinations of models where p and q are between 0 and 2. Given that the series have already been fractionally differenced using the appropriate estimated values of the fractional difference parameters, we expect the estimation of d to lie within the stationarity range. Table 8A and Table 8B show the results for the best models according to the Bayesian Information Criterion. We notice that, consistently with our expectations, in all cases the fractional differences are stationary process, as the estimates of d range between –0.27 and 0.12. Although d should always be statistically not different from zero, which is the case only for the systems (*Aaa, Aa*) and (*Aaa, Baa*), it is well known that $ARMA$ and $ARFIMA$ models can approximate the autocorrelation function of a series nearly equally well, and therefore can equally fit a fractionally integrated stationary process. (See, for example, Bos, Frances and Ooms (2002) and references within.)

[Insert Tables 7, 8A, 8B somewhere here]

4.2.4 AML Estimation of $ARFIMA(p, d, q)$ Models

The previous estimation methods relied on the application of the GPH procedure to the first differences of the residuals of the cointegrating relationship. In this paragraph, we employ $ARFIMA$ estimation to assess the presence of cointegration. Specifically, we apply the AML procedure to the residuals of the cointegrating relationship.

For each series, we estimate the nine combinations of models where p and q are between 0 and 2. Results are shown in Table 9. We find that, by modelling at the same time the fractional difference parameter and the AR and MA short memory components, eight out of ten systems are fractionally cointegrated. These are (*Treas, Aaa*), (*Treas, Baa*), (*Aaa, Aa*), (*Aaa, A*), (*Aaa, Baa*), (*Aa, A*), (*Aa, Baa*), and (*A, Baa*). The two systems which are not fractionally cointegrated are (*Treas, Aa*) and (*Treas, A*).

Differently from the results of the GPH method, the fractional difference parameter takes values between -0.07 and 0.11 for the estimated residuals of the cointegrating equilibria, while it is not statistically different from unity for the systems (*Treas, Aa*) and (*Treas, A*).

Most of the estimated values of the fractional difference parameter lie in the short memory or long memory stationary range rather than in the long memory nonstationary range as we found for the GPH method. Consistently with the comment given in the previous section, this can be explained by the fact that AR and MA components are able to explain, to a certain extent, fractionally integrated processes. For example, for the two nonstationary systems (*Treas, Aa*) and (*Treas, A*) the best fitting models according to the Akaike Information Criterion are $ARFIMA(0, 1, 1)$. However, the next best models are in both cases $ARFIMA(2, 0, 0)$. In this case, the AR components fully capture the nonstationarity feature.

[Insert Table 9 somewhere here]

4.3 Sensitivity to Heteroskedasticity and Volatility Clustering

In the estimation of the fractional difference parameter d we have so far assumed that the error terms of the relevant econometric representations of the time series are white noise processes. However, we have seen that the yield and spread series are heteroskedastic and show volatility clustering. In this paragraph, we want to assess the sensitivity to heteroskedasticity and volatility clustering by explicitly modelling the conditional variance of the error terms.

In order to do this, we re-estimate d in an $ARFIMA(p, d, q)$ framework by modelling at the same time the conditional volatility as a $GARCH(1, 1)$ process. We employ a Maximul Likelihood (ML) approach that uses the quasi-Newton method of the Broyden, Fletcher, Goldfarb and Shanno algorithm. We also assume that the innovation process is a Student-t distribution, which better captures the higher than observed kurtosis (see, for example, Baillie and Bollerslev, 1989, and Palm and Vlaar, 1997).

For each yield and spread series and their first differences, we estimate the fractional difference parameter d by fitting an $ARFIMA(p, d, q) - GARCH(1, 1) - t$ model, where the values of p and q have been chosen according to the best model from the AML estimation of an $ARFIMA(p, d, q)$

discussed in section 4.1.2.

Estimation results are shown in Tables 10A and 10B. Consistently with 4.1.2, the fractional difference parameter for yields lies in the long memory nonstationary range, with d not statistically different from 1 for all series with the exception of the Aaa series, where $d = 1.0320$. Also, the fractional difference parameter for the first differences of yields lies in the short memory stationary range, not being statistically different from 0 for all series.

For the series of spreads and their first differences, we also obtain consistent results between the $ARFIMA(p,d,q) - GARCH(1,1) - t$ and the $ARFIMA(p,d,q)$ models. However, in this case two important exceptions arise: the $sAaaAa$ series ($d = 0.9386$ vs $d = 0$ respectively) and the $sAaA$ series ($d = 0.9851$ vs $d = -0.0572$ respectively).

Overall, as far as the estimation of d is concerned, having found only two exceptions out of thirty estimations performed, we conclude that the $ARFIMA(p,d,q)$ model is robust to heteroskedasticity and volatility clustering.

[Insert Tables 10A and 10B somewhere here]

5 Conclusions

We have investigated the long memory properties of Treasury and corporate bond yields, spreads over Treasury and spreads between corporate yields. Using parametric and semiparametric estimators of the fractional difference parameter d, we have shown that yields and spreads are generally long memory nonstationary processes, whilst their first differences may be long memory or short memory stationary processes. We have shown that our results are robust to heteroskedasticity in an $ARFIMA(p,d,q)$ framework. Finally, there is evidence of fractional cointegration between yields.

The finding of (long memory) nonstationarity of spreads does not reconcile with the reduced-form model of Duffie and Singleton (1999), who assume a stationary risk-free rate process, which implies stationarity for the credit spread. This also applies, for instance, to the reduced-form models of Das and Tufano (1996) and Jarrow, Lando and Turnbull (1997). Thus, our empirical findings have strong implications for modelling credit spread dynamics and in particular it is equation (1) able to accommodate the long memory characteristics of credit spreads.

Our evidence of fractional cointegration in the bivariate systems of yields implies that there still exists a long run equilibrium relationship between yields, and deviations from the fractionally cointegrating relationship are mean reverting, so that a shock to the system will eventually die out. Given that for all systems the fractional difference parameter assumes values $d < 1$ but close to unity, shocks may take several years to dissipate. Fractional cointegration implies a link between yields, which is not captured, for instance, in the risky debt pricing models of Merton (1974), Kim, Ramaswamy and Sundaresan (1993) and Longstaff and Schwartz (1995a), as well as the reduced-form models of Duffee (1999) and Duffie and Singleton (1999) and the option pricing models of Das (1995) and Longstaff and Schwartz (1995b). Again, our findings have profound implications for modelling credit spreads and once again equation (1) seems the logical framework to adopt.

The empirical findings reported in this paper call for a reconsideration of the theoretical framework used to model the dynamics of credit spreads so far. We leave this to future research.

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Figure 1. Treasury, *Aaa*, *Aa*, *A* and *Baa* yields

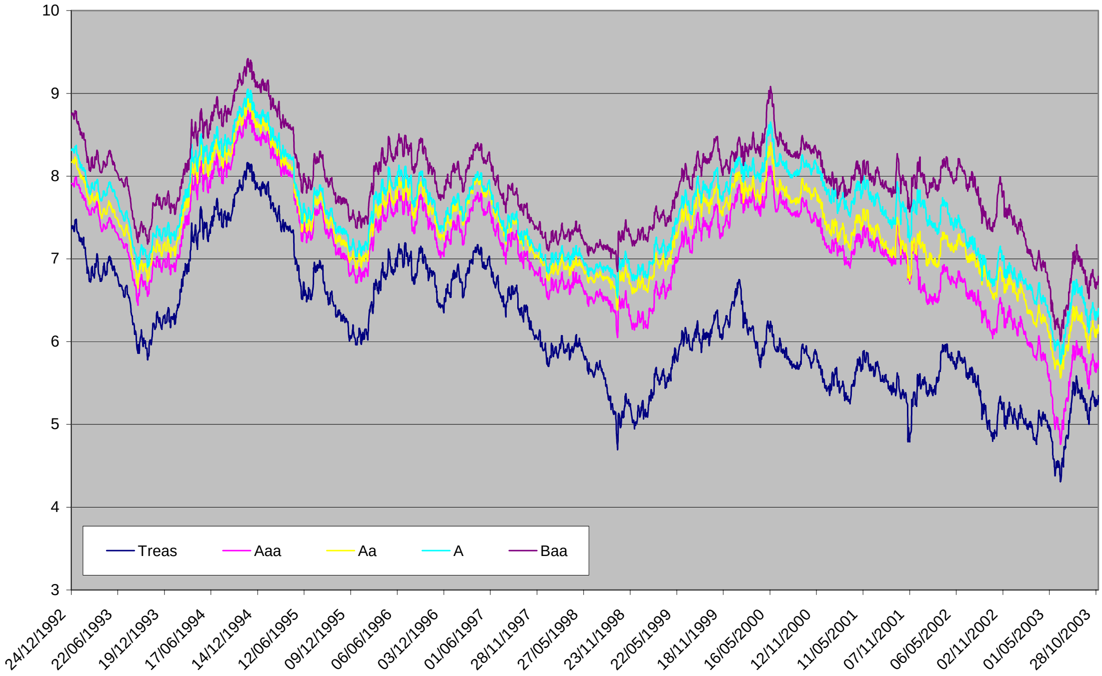

Figure 2. *Aaa*, *Aa*, *A* and *Baa* spreads over Treasury yields

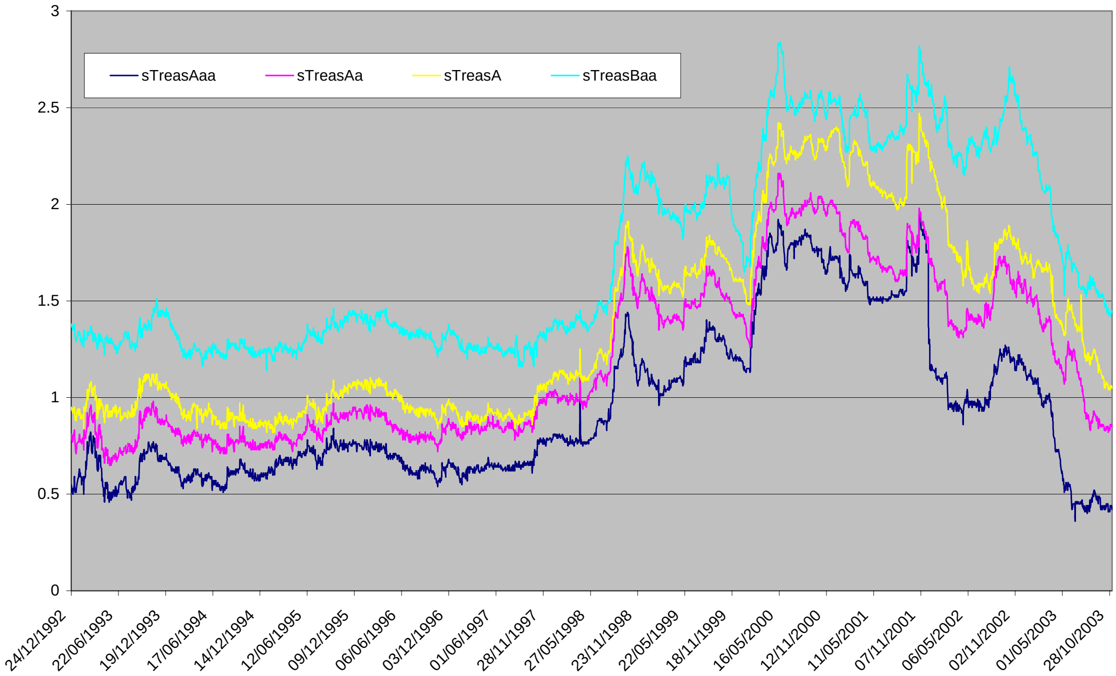

Figure 3. Spreads between corporate yields

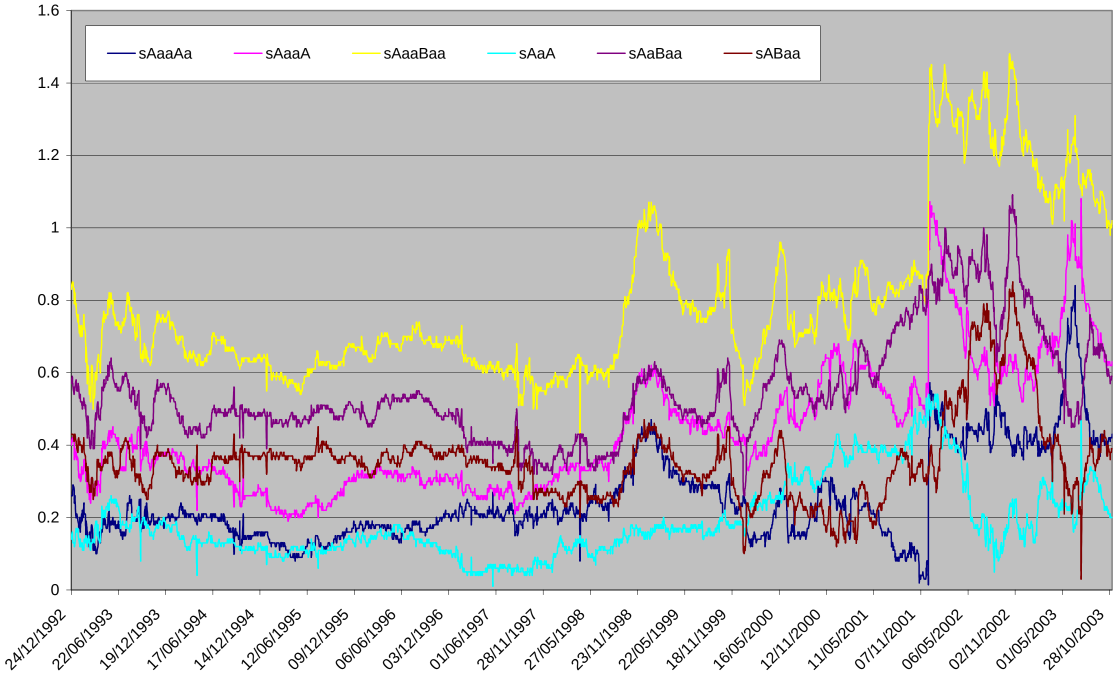

Table 1. Summary Statistics and Normality Tests

	Mean	Standard Deviation	Min	Max	Skewness	Excess Kurtosis	Jarque-Bera Test
Treas	6.1536	0.78947	4.31	8.16	0.30498	-0.57126	78.6550** (0.0000)
Aaa	7.0975	0.68853	4.76	8.79	-0.40296	0.37445	88.9433** (0.0000)
Aa	7.3463	0.60072	5.56	8.93	-0.12787	0.079963	8.0862* (0.0175)
A	7.5368	0.58879	5.73	9.05	-0.19240	-0.11221	18.0947** (0.0001)
Baa	7.8947	0.57677	6.01	9.42	-0.23461	0.29000	34.2692** (0.0000)
sTreasAaa	0.94396	0.40039	0.36	1.96	0.85386	-0.47124	353.4559** (0.0000)
sTreasAa	1.1927	0.41573	0.65	2.16	0.54433	-1.1281	276.8068** (0.0000)
sTreasA	1.3833	0.49232	0.82	2.47	0.61294	-1.0284	288.3570** (0.0000)
sTreasBaa	1.7412	0.49927	1.14	2.84	0.51375	-1.3408	321.3828** (0.0000)
sAaaAa	0.2487	0.12367	0.02	0.84	1.3572	2.0211	1289.9** (0.0000)
sAaaA	0.4393	0.17651	0.19	1.08	1.0234	0.68153	534.14** (0.0000)
sAaaBaa	0.7972	0.22790	0.43	1.48	1.2453	0.51050	727.98** (0.0000)
sAaA	0.1906	0.10543	0.01	0.54	1.0911	0.49992	564.49** (0.0000)
sAaBaa	0.5485	0.14479	0.26	1.09	1.1672	1.0723	743.29** (0.0000)
sABaa	0.3579	0.11427	0.03	0.85	1.5693	3.7498	2693.1** (0.0000)
dTreas	-7.44e-4	0.051915	-0.33	0.32	0.30619	2.0323	507.20** (0.0000)
dAaa	-7.92e-4	0.048762	-0.32	0.30	0.31655	2.6307	824.26** (0.0000)
dAa	-7.29e-4	0.046384	-0.20	0.30	0.43888	1.9246	503.78** (0.0000)
dA	-7.07e-4	0.047182	-0.20	0.31	0.43378	2.0412	553.84** (0.0000)
dBaa	-7.22e-4	0.047784	-0.20	0.32	0.42962	1.9843	526.41** (0.0000)
dsTreasAaa	-4.81e-5	0.024438	-0.45	0.24	-2.0015	55.643	350377.5** (0.0000)
dsTreasAa	1.48e-5	0.022002	-0.17	0.20	0.45617	8.3799	7999.658** (0.0000)
dsTreasA	3.70e-5	0.022995	-0.19	0.23	0.57711	14.692	24451.1** (0.0000)
dsTreasBaa	2.22e-5	0.022461	-0.17	0.22	0.59920	9.0243	9330.158** (0.0000)
dsAaaAa	6.29e-5	0.016227	-0.13	0.40	5.8072	144.87	2.37e+6** (0.0000)
dsAaaA	8.51e-5	0.017464	-0.21	0.40	4.9598	130.89	1.94e+6** (0.0000)
dsAaaBaa	7.03e-5	0.018205	-0.21	0.41	4.3241	112.28	1.43e+6** (0.0000)
dsAaA	2.22e-5	0.013231	-0.19	0.22	0.66513	52.071	3.05e+5** (0.0000)
dsAaBaa	7.40e-6	0.014956	-0.11	0.12	-0.19375	10.042	11371** (0.0000)
dsABaa	-1.48e-5	0.015527	-0.19	0.20	-0.10372	28.283	90065** (0.0000)

Notes to Table 1: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

The null hypothesis of the Jarque-Bera test is that the series are normally distributed and the test statistic has a χ^2 distribution with 2 degrees of freedom. P-values are shown in parentheses and the 5% and 1% levels of significance are indicated by one and two asterisks respectively.

Table 2A. Geweke and Porter-Hudak (GPH) Estimation of the Fractional Difference Parameter for Yields

		Treas	Aaa	Aa	A	Baa
α = 0.4	*d'* estimate	-0.1927	-0.0182	-0.1066	-0.0220	-0.0842
	test statistic	-1.1648	-0.1099	-0.6446	-0.1331	-0.5092
	std. error	0.165438	0.165438	0.165438	0.165438	0.165438
α = 0.5	*d'* estimate	-0.1629	-0.0552	-0.1244	-0.0913	-0.0636
	test statistic	-1.5990	-0.5421	-1.2216	-0.8959	-0.6243
	std. error	0.101861	0.101861	0.101861	0.101861	0.101861
α = 0.6	*d'* estimate	-0.0341	0.0359	-0.0870	-0.0358	-0.0302
	test statistic	-0.5278	0.5559	-1.3452	-0.5538	-0.4673
	std. error	0.064644	0.064644	0.064644	0.064644	0.064644
α = 0.7	*d'* estimate	-0.0461	-0.0117	-0.0574	-0.0221	0.0099
	test statistic	-1.0916	-0.2773	-1.3578	-0.5241	0.2342
	std. error	0.042259	0.042259	0.042259	0.042259	0.042259

Table 2B. Geweke and Porter-Hudak (GPH) Estimation of the Fractional Difference Parameter for Spreads

		sTreasAaa	sTreasAa	sTreasA	sTreasBaa	sAaaAa	sAaaA	sAaaBaa	sAaA	sAaBaa	sABaa
α = 0.4	*d'* estimate	-0.0803	0.0524	0.0237	-0.0050	-0.3259	-0.3470	-0.0949	-0.0051	-0.1126	0.0535
	test statistic	-0.4856	0.3166	0.1431	-0.0301	-1.9697*	-2.0975*	-0.5736	-0.0307	-0.6809	0.3233
	std. error	0.165438	0.165438	0.165438	0.165438	0.165438	0.165438	0.165438	0.165438	0.165438	0.165438
α = 0.5	*d'* estimate	0.0244	0.0324	0.0758	0.1062	-0.1917	-0.1790	-0.0424	-0.0046	-0.1817	-0.0786
	test statistic	0.2392	0.3180	0.7441	1.0422	-1.8817	-1.7571	-0.4162	-0.0451	-1.7839	-0.7714
	std. error	0.101861	0.101861	0.101861	0.101861	0.101861	0.101861	0.101861	0.101861	0.101861	0.101861
α = 0.6	*d'* estimate	0.0244	-0.0058	0.0311	0.0800	-0.1202	-0.0863	-0.0358	-0.0125	-0.0252	-0.1122
	test statistic	0.3776	-0.0893	0.4808	1.2373	-1.8601	-1.3354	-0.5543	-0.1941	-0.3906	-1.7360
	std. error	0.064644	0.064644	0.064644	0.064644	0.064644	0.064644	0.064644	0.064644	0.064644	0.064644
α = 0.7	*d'* estimate	0.0585	0.0726	0.0680	0.1256	-0.0208	0.0026	0.0412	-0.0384	0.0177	-0.0135
	test statistic	1.3837	1.7190	1.6102	2.9733**	-0.4923	0.0619	0.9758	-0.9075	0.4196	-0.3188
	std. error	0.042259	0.042259	0.042259	0.042259	0.042259	0.042259	0.042259	0.042259	0.042259	0.042259

Notes to Tables 2A and 2B: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

The GPH estimator is applied to the first differences of the series. The null hypothesis is $d' = 0$, where d' is the long memory parameter of the first differences of the series and $d = d' + 1$ is the long memory parameter of the levels of the series. Estimations of d' are performed by using a number of frequencies (bandwidth) equal to $m = T^{\alpha}$, where *T* is the number of observations and α is equal to 0.4, 0.5, 0.6 and 0.7. For each series, the table shows the estimated value of d', the GPH statistic for the null hypothesis $d' = 0$ with its level of significance (*=5%, **=1%), and the standard error of the estimate of d'.

Table 3A. Robinson Estimation of the Fractional Difference Parameter for Yields

		Treas	Aaa	Aa	A	Baa
α = 0.4	*d'* estimate	0.0081	-0.0037	-0.0013	-0.0063	0.0020
	test statistic	0.0621	-0.0280	-0.0099	-0.0478	0.0153
	std. error	0.130900	0.130900	0.130900	0.130900	0.130900
α = 0.5	*d'* estimate	0.0168	0.0035	0.0100	0.0092	0.0076
	test statistic	0.1885	0.0397	0.1123	0.1036	0.0850
	std. error	0.088929	0.088929	0.088929	0.088929	0.088929
α = 0.6	*d'* estimate	0.0025	-0.0033	0.0083	0.0039	0.0026
	test statistic	0.0424	-0.0555	0.1396	0.0644	0.0442
	std. error	0.059799	0.059799	0.059799	0.059799	0.059799
α = 0.7	*d'* estimate	0.0050	0.0018	0.0057	0.0030	-0.0015
	test statistic	0.1242	0.0436	0.1422	0.0740	-0.0375
	std. error	0.040317	0.040317	0.040317	0.040317	0.040317

Table 3B. Robinson Estimation of the Fractional Difference Parameter for Spreads

		sTreasAaa	sTreasAa	sTreasA	sTreasBaa	sAaaAa	sAaaA	sAaaBaa	sAaA	sAaBaa	sABaa
α = 0.4	*d'* estimate	0.0058	-0.0049	-0.0115	0.0023	0.0439	0.0376	0.0203	0.0025	0.0133	0.0105
	test statistic	0.0441	-0.0377	-0.0875	0.0177	0.3357	0.2875	0.1553	0.0193	0.1019	0.0806
	std. error	0.130900	0.130900	0.130900	0.130900	0.130900	0.130900	0.130900	0.130900	0.130900	0.130900
α = 0.5	*d'* estimate	-0.0070	-0.0060	-0.0116	-0.0124	0.0245	0.0223	0.0078	0.0010	0.0191	0.0141
	test statistic	-0.0783	-0.0677	-0.1308	-1.1393	0.2758	0.2506	0.0876	0.0117	0.2145	0.1580
	std. error	0.088929	0.088929	0.088929	0.088929	0.088929	0.088929	0.088929	0.088929	0.088929	0.088929
α = 0.6	*d'* estimate	-0.0057	-0.0017	-0.0074	-0.0079	0.0139	0.0101	0.0069	0.0020	0.0031	0.0146
	test statistic	-0.0959	-0.0293	-0.1232	-0.1325	0.2318	0.1688	0.1155	0.0333	0.0527	0.2441
	std. error	0.059799	0.059799	0.059799	0.059799	0.059799	0.059799	0.059799	0.059799	0.059799	0.059799
α = 0.7	*d'* estimate	-0.0080	-0.0096	-0.0083	-0.0136	0.0028	0.0005	-0.0041	0.0047	-0.0015	0.0022
	test statistic	-0.1973	-0.2388	-0.2055	-0.3385	0.0706	0.0130	-0.1023	0.1154	-0.0365	0.0539
	std. error	0.040317	0.040317	0.040317	0.040317	0.040317	0.040317	0.040317	0.040317	0.040317	0.040317

Notes to Tables 3A and 3B: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

The Robinson estimator is applied to the first differences of the series. The null hypothesis is $d' = 0$, where d' is the long memory parameter of the first differences of the series and $d = d' + 1$ is the long memory parameter of the levels of the series. Estimations of d' are performed by using a number of frequencies (bandwidth) equal to $m = T^{\alpha}$, where *T* is the number of observations and α is equal to 0.4, 0.5, 0.6 and 0.7. For each series, the table shows the estimated value of d', the Robinson statistic for the null hypothesis $d' = 0$ with its level of significance (*=5%, **=1%), and the asymptotic standard error of the estimate of d'.

Table 4A. Approximate Maximum Likelihood (AML) Estimation of ARFIMA(p, d, q) Models for Yields

	d estimate and standard error	95% confidence interval for d	t statistic for d and p-value	Best model	Log-Likelihood	Bayesian Information Criterion	Residuals
Treas	1.0076 (0.0151)	[0.9780362, 1.0371660]	66.7980 (0.0000)	**ARFIMA (0, 1, 0)**	4155.546	-8303.190	0.0519
Aaa	1.0148 (0.0151)	[0.9852615, 1.0443910]	67.2770 (0.0000)	**ARFIMA (0, 1, 0)**	4324.648	-8641.394	0.0488
Aa	1.0129 (0.0151)	[0.9833733, 1.0425030]	67.1518 (0.0000)	**ARFIMA (0, 1, 0)**	4459.479	-8911.057	0.0464
A	1.0071 (0.0151)	[0.9775593, 1.0366890]	66.7664 (0.0000)	**ARFIMA (0, 1, 0)**	4413.378	-8818.854	0.0472
Baa	1.0182 (0.0151)	[0.9886294, 1.0477590]	67.5003 (0.0000)	**ARFIMA (0, 1, 0)**	4379.725	-8751.549	0.0478
dTreas	0.0076 (0.0151)	[-0.0219275, 0.0372128]	0.5066 (0.6125)	**ARFIMA (0, 0, 0)**	4153.588	-8299.275	0.0519
dAaa	0.0148 (0.0151)	[-0.0147372, 0.0444031]	0.9832 (0.3256)	**ARFIMA (0, 0, 0)**	4322.564	-8637.228	0.0488
dAa	0.0129 (0.0151)	[-0.0166326, 0.0425077]	0.8575 (0.3912)	**ARFIMA (0, 0, 0)**	4457.327	-8906.754	0.0464
dA	0.0071 (0.0151)	[-0.0224473, 0.0366930]	0.4721 (0.6369)	**ARFIMA (0, 0, 0)**	4411.243	-8814.586	0.0472
dBaa	0.0182 (0.0151)	[-0.0113349, 0.0478054]	1.2087 (0.2269)	**ARFIMA (0, 0, 0)**	4377.603	-8747.305	0.0478

Notes to Table 4A: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

Nine $ARFIMA(p, d, q)$ models are estimated for each data series using p, q = 0, 1, 2. The table shows the best fitting model for each series that minimises the Bayesian Information Criterion.

Table 4B. Approximate Maximum Likelihood (AML) Estimation of ARFIMA(*p*, *d*, *q*) Models for Spreads

	d estimate and standard error	95% confidence interval for *d*	*t* statistic for *d* and p-value	Best model	Log-Likelihood	Bayesian Information Criterion	Residuals
sTreasAaa	1.0845 (0.0266)	[1.0323237, 1.1366890]	40.7337 (0.0000)	**ARFIMA (2, 1.0845, 0)**	6257.818	-12491.933	0.0239
sTreasAa	1.0732 (0.0265)	[1.0212250, 1.1252126]	40.4562 (0.0000)	**ARFIMA (2, 1.0732, 0)**	6534.745	-13045.787	0.0215
sTreasA	1.0831 (0.0260)	[1.0322230, 1.1340353]	41.7021 (0.0000)	**ARFIMA (2, 1.0831, 0)**	6429.76	-12835.82	0.0224
sTreasBaa	1.1367 (0.0261)	[1.0855605, 1.1877652]	43.5952 (0.0000)	**ARFIMA (2, 1.1367, 0)**	6464.95	-12906.20	0.0221
sAaaAa	0.0365 (0.0251)	[-0.0127532, 0.0856654]	1.4520 (0.1466)	**ARFIMA (2, 0, 0)**	7349.796	-14675.889	0.0159
sAaaA	0.0096 (0.0234)	[-0.0361509, 0.0553905]	0.4119 (0.6804)	**ARFIMA (2, 0, 0)**	7156.108	-14288.513	0.0171
sAaaBaa	0.1234 (0.0242)	[0.0759618, 0.1707982]	5.0997 (0.0000)	**ARFIMA (2, 0.0124, 0)**	7024.342	-14024.980	0.018
sAaA	-0.0572 (0.0218)	[-0.1000321, -0.0144464]	-2.6216 (0.0088)	**ARFIMA (1, -0.0572, 1)**	7983.754	-15943.805	0.0126
sAaBaa	0.0214 (0.0244)	[-0.0265061, 0.0692633]	0.8750 (0.3816)	**ARFIMA (2, 0, 0)**	7558.613	-15093.524	0.0148
sABaa	0.0141 (0.0245)	[-0.0339257, 0.0621690]	0.5761 (0.5646)	**ARFIMA (2, 0, 0)**	7486.915	-14950.127	0.0151
dsTreasAaa	0.0852 (0.0266)	[0.00632460, 0.1041857]	3.2040 (0.0014)	**ARFIMA (2, 0.0852, 0)**	6255.659	-12487.616	0.0239
dsTreasAa	0.0732 (0.0265)	[0.0211617, 0.1251519]	2.7577 (0.0059)	**ARFIMA (2, 0.0732, 0)**	6532.036	-13040.371	0.0216
dsTreasA	0.0831 (0.0260)	[0.0322271, 0.1340465]	3.2007 (0.0014)	**ARFIMA (2, 0.0831, 0)**	6427.213	-12830.723	0.0224
dsTreasBaa	0.1363 (0.0261)	[0.0851499, 0.1873926]	5.2246 (0.0000)	**ARFIMA (2, 0.1363, 0)**	6462.336	-12900.970	0.0221
dsAaaAa	-0.0405 (0.0315)	[-0.1022890, 0.0213316]	-1.2836 (0.1994)	**ARFIMA (2, 0, 0)**	7343.781	-14663.859	0.016
dsAaaA	-0.0022 (0.0212)	[-0.0437845, 0.0393147]	-0.1054 (0.9160)	**ARFIMA (1, 0, 0)**	7147.499	-14279.196	0.0171
dsAaaBaa	0.0231 (0.0322)	[-0.0400733, 0.0862651]	0.7166 (0.4737)	**ARFIMA (1, 0, 0)**	7020.195	-14016.687	0.018
dsAaA	-0.0917 (0.0274)	[-0.1454115, -0.0379831]	-3.3459 (0.0008)	**ARFIMA (2, -0.0917, 0)**	7979.459	-15935.215	0.0126
dsAaBaa	-0.3391 (0.0719)	[-0.4799292, -0.1981949]	-4.7176 (0.0000)	**ARFIMA (2, -0.3391, 1)**	7557.602	-15083.600	0.0147
dsABaa	-0.2761 (0.0729)	[-0.4188799, -0.1333124]	-3.7899 (0.0002)	**ARFIMA (2, -0.2761, 1)**	7483.496	-14935.388	0.0152

Notes to Table 4B (see notes to Table 4A)

Table 4C. Approximate Maximum Likelihood (AML) Estimation of ARFIMA(*p*, *d*, *q*) Models: Statistics for the *AR* and *MA* Coefficients

	MA(1) coefficient estimate and standard error	*t* statistic for MA(1) coefficient and p-value	AR(1) coefficient estimate and standard error	*t* statistic for AR(1) coefficient and p-value	AR(2) coefficient estimate and standard error	*t* statistic for AR(2) coefficient and p-value
sTreasAaa	-	-	-0.2923 (0.0323)	-9.0580 (0.0000)	-0.0568 (0.0258)	-2.2029 (0.0277)
sTreasAa	-	-	-0.2709 (0.0321)	-8.4434 (0.0000)	-0.0671 (0.0255)	-2.6341 (0.0085)
sTreasA	-	-	-0.3082 (0.0315)	-9.7842 (0.0000)	-0.0807 (0.0254)	-3.1768 (0.0015)
sTreasBaa	-	-	-0.2927 (0.0316)	-9.2702 (0.0000)	-0.0806 (0.0253)	-3.1811 (0.0015)
sAaaAa	-	-	0.7820 (0.0290)	26.9226 (0.0000)	0.2088 (0.0275)	7.6007 (0.0000)
sAaaA	-	-	0.7954 (0.0279)	28.4880 (0.0000)	0.2006 (0.0272)	7.3834 (0.0000)
sAaaBaa	-	-	0.7542 (0.0280)	26.9704 (0.0000)	0.2376 (0.0266)	8.9341 (0.0000)
sAaA	0.2634 0.0254	10.2792 (0.0000)	0.9978 (0.0015)	654.7899 (0.0000)	-	-
sAaBaa	-	-	0.8214 (0.0291)	28.2584 (0.0000)	0.1732 (0.0281)	6.1711 (0.0000)
sABaa	-	-	0.7694 (0.0284)	27.0736 (0.0000)	0.2223 (0.0270)	8.2282 (0.0000)
dsTreasAaa	-	-	-0.2928 (0.0322)	-9.0815 (0.0000)	-0.0579 (0.0258)	-2.2458 (0.0248)
dsTreasAa	-	-	-0.2705 (0.0321)	-8.4298 (0.0000)	-0.0674 (0.0255)	-2.6466 (0.0082)
dsTreasA	-	-	-0.3080 (0.0315)	-9.7785 (0.0000)	-0.0809 (0.0254)	-3.1839 (0.0015)
dsTreasBaa	-	-	-0.2922 (0.0316)	-9.2501 (0.0000)	-0.0805 (0.0253)	-3.1783 (0.0015)
dsAaaAa	-	-	-0.1341 (0.0374)	-3.5838 (0.0003)	0.0607 (0.0271)	2.2423 (0.0250)
dsAaaA	-	-	-0.1916 (0.0266)	-7.2180 (0.0000)	-	-
dsAaaBaa	-	-	-0.1373 (0.0381)	-3.5984 (0.0003)	0.0809 (0.0276)	2.9361 (0.0034)
dsAaA	-	-	-0.2244 (0.0330)	-6.7956 (0.0000)	-0.0452 (0.0256)	-1.7673 (0.0773)
dsAaBaa	0.7252 (0.0204)	35.6042 (0.0000)	0.9019 (0.0842)	10.7136 (0.0000)	0.0113 (0.0580)	0.1951 (0.8454)
dsABaa	0.7062 (0.0240)	29.4292 (0.0000)	0.7619 (0.0872)	8.7352 (0.0000)	0.1066 (0.0516)	2.0679 (0.0387)

Notes to Table 4C: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve. Nine $ARFIMA(p, d, q)$ models are estimated for each data series using *p*, *q* = 0, 1, 2. The table summarises the statistics for the *AR* and *MA* coefficients for the best fitting models in Tables 4A and 4B..

Table 5. Cointegration Analysis: Unit Root and Stationarity Tests for Bivariate Systems

Test	Treas - Aaa	Treas - Aa	Treas - A	Treas - Baa	Aaa - Aa	Aaa - A	Aaa - Baa	Aa - A	Aa - Baa	A - Baa
DF	-1.112	-1.218	-1.228	-1.347	-5.103**	-3.378*	-2.793	-3.322*	-2.846	-3.641**
	(0.7132)	(0.6689)	(0.6644)	(0.6095)	(0.0000)	(0.0119)	(0.0594)	(0.0141)	(0.0521)	(0.0051)
ADF 1	-0.7606	-0.9838	-1.028	-1.265	-4.382**	-2.889*	-2.608	-2.495	-2.487	-2.981*
	(0.8293)	(0.7610)	(0.7455)	(0.6478)	(0.0003)	(0.0467)	(0.0915)	(0.1168)	(0.1188)	(0.0368)
ADF 2	-0.8167	-0.9864	-1.038	-1.323	-4.504**	-2.875*	-2.776	-2.274	-2.498	-2.950*
	(0.8138)	(0.7601)	(0.7417)	(0.6207)	(0.0002)	(0.0485)	(0.0619)	(0.1806)	(0.1159)	(0.0400)
ADF 3	-0.9028	-0.9910	-1.031	-1.373	-4.394**	-2.831	-2.830	-2.163	-2.503	-2.976*
	(0.7879)	(0.7585)	(0.7444)	(0.5971)	(0.0003)	(0.0541)	(0.0542)	(0.2202)	(0.1147)	(0.0373)
ADF 4	-0.9104	-0.9837	-1.003	-1.377	-4.481**	-2.797	-2.893*	-2.093	-2.552	-3.139*
	(0.7855)	(0.7611)	(0.7542)	(0.5951)	(0.0000)	(0.0588)	(0.0463)	(0.2477)	(0.1034)	(0.0239)
ADF 5	-0.9053	-0.9729	-0.9346	-1.347	-4.505**	-2.766	-2.843	-2.001	-2.538	-3.147*
	(0.7871)	(0.7648)	(0.7776)	(0.6096)	(0.0002)	(0.0634)	(0.0525)	(0.2866)	(0.1065)	(0.0234)
PP	-0.9226	-1.036	-1.042	-1.407	-4.681**	-2.926*	-2.869*	-2.304	-2.656	-3.249*
	(0.7815)	(0.7423)	(0.7403)	(0.5802)	(0.0000)	(0.0426)	(0.0492)	(0.1709)	(0.0820)	(0.0175)
KPSS	4.0090**	4.8108**	4.9315**	5.0898**	2.6821**	10.3348**	7.4391**	9.3078**	7.9490**	1.5756**

Notes to Table 5: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody’s and the 30-year constant maturity Treasury series is from the Federal Reserve.

Cointegration analysis is performed for all possible bivariate systems. For each pair of variables x_{it} and y_{it}, the Dickey-Fuller (DF), augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and KPSS tests are carried out on the estimated residuals ε_{it} of the regressions $y_{it} = \alpha + \beta x_{it} + \varepsilon_{it}$.

Table 6. Fractional Cointegration Analysis: GPH Test for Bivariate Systems

	Treas - Aaa	Treas - Aa	Treas - A	Treas - Baa	Aaa - Aa	Aaa - A	Aaa - Baa	Aa - A	Aa - Baa	A - Baa
d' estimate	-0.0732	-0.0649	-0.0564	0.0295	-0.0757	-0.0971	0.0067	-0.2037	-0.0698	-0.0997
test statistic	-3.6365**	-3.2230**	-2.8030**	1.4668	-3.7591**	-4.8226**	0.3322	-10.1197**	-3.4666**	-4.9513**
std. error	0.020127	0.020127	0.020127	0.020127	0.020127	0.020127	0.020127	0.020127	0.020127	0.020127

Notes to Table 6: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

Fractional cointegration analysis is performed for all the possible bivariate subsystems. The series x_{it} and y_{it} are fractionally cointegrated if there exists a cointegration relationship $x_{it} = \alpha + \beta y_{it} + \varepsilon_{it}$, where ε_{it} is a long-memory process. The fractional cointegration test is based on the null hypothesis of no cointegration, i.e. that ε_{it} is $I(1)$, against the alternative of cointegration, i.e. that ε_{it} is $I(d)$, with $d < 1$.

The GPH test is applied on the estimated residuals of the cointegration relationship. Specifically, for each bivariate subsystem, the GPH test is carried out on the series of the first difference of residuals of the cointegration relationship. The null hypothesis of the GPH test is the absence of cointegration, i.e. $H_0 : d' = 0$, where $d' = d - 1$ is the fractional difference parameter of the first differences of residuals, against the alternative $H_1 : d' < 0$. The model under the alternative is the $ARFIMA(0, d, 0)$ model.

Estimations of d' are performed by using a number of frequencies equal to 2703^{α}, with α equal to 0.9. For each series, the table shows the estimated value of d', the GPH test statistic for the null hypothesis $d' = 0$ with its level of significance (*=5%, **=1%), and the standard error of the estimate of d'. The following non-standard critical values derived by Andersson and Lyhagen (1997) are used:

Significance level		
1%	5%	10%
-3.20	-2.10	-1.59

The use of frequencies with α equal to 0.9 is justified by the fact that, as shown in Andersson and Lyhagen (1997), the power of the GPH test for fractional cointegration is highest when $\alpha = 0.9$.

Table 7. Fractional Cointegration Analysis: Dittman Estimation Procedure for Bivariate Systems

	Treas - Aaa	Treas - Aa	Treas - A	Treas - Baa	Aaa - Aa	Aaa - A	Aaa - Baa	Aa - A	Aa - Baa	A - Baa
Step 1										
α	2.4718	3.3413	3.9452	4.4063	1.2108	1.6262	2.2445	0.4482	1.0489	0.6521
β	0.7517	0.6508	0.5837	0.5669	0.8645	0.8328	0.7961	0.9649	0.9319	0.9610
Step 2										
d	0.9268**	0.9351**	0.9436**	1.0295	0.9243**	0.9029**	1.0067	0.7963**	0.9302**	0.9003**
Step 3										
KPSS test	0.4526	0.3764	0.3492	0.1719	0.0317	0.0766	0.0562	0.2230	0.0694	0.0531

Notes to Table 7: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

The Dittman (2004) procedure is performed for all the possible bivariate subsystems. The series x_{it} and y_{it} are fractionally cointegrated if there exists a cointegration relationship $x_{it} = \alpha + \beta y_{it} + \varepsilon_{it}$, where ε_{it} is a long-memory process. In step 1, the parameters α and β of the cointegration relationships are estimated via OLS. In step 2, the fractional difference parameter of the cointegrating equilibrium $\hat{\varepsilon}_{it} = x_{it} - \hat{\alpha} - \hat{\beta} y_{it}$ is estimated using the GPH test. Results are the same as in Table 13, although Table 14 reports the values of $d = d' + 1$ instead of d'. In step 3, we compute the fractional differences of the cointegrating equilibria and verify that they are stationary processes. For the systems (*Treas*, *Baa*) and (*Aaa*, *Baa*) the fractional difference is calculated as equal to the first difference as d is not statistically different from unity. The KPSS test is computed to test for stationarity in the fractionally differenced cointegrating equilibria.

Table 8A. Approximate Maximum Likelihood (AML) Estimation of the *ARFIMA*(*p*, *d*, *q*) Model for the Fractionally Differenced Cointegrating Equilibria of Bivariate Systems

	d estimate and standard error	95% confidence interval for *d*	*t* statistic for *d* and p-value	Best ARFIMA (p, d, q) model	Log-Likelihood Information Criterion	Bayesian Information Criterion	Residuals
Treas - Aaa	0.1184 (0.0211)	[0.0769757, 0.1597539]	5.6051 (0.0000)	**ARFIMA (1, 0.1184, 0)**	6370.267	-12724.731	0.0229
Treas - Aa	0.0677 (0.0225)	[0.0236639, 0.1117668]	3.0113 (0.0026)	**ARFIMA (1, 0.0677, 0)**	6572.904	-13130.005	0.0212
Treas - A	0.0542 (0.0227)	[0.0097833, 0.0986271]	2.3916 (0.0168)	**ARFIMA (1, 0.0542, 0)**	6237.708	-12459.614	0.0240
Treas - Baa	0.0463 (0.0228)	[0.0016449, 0.0908750]	2.0322 (0.0422)	**ARFIMA (1, 0.0463, 0)**	6156.445	-12297.089	0.0247
Aaa - Aa	0.0091 (0.0321)	[-0.0537012, 0.0719581]	0.2848 (0.7758)	**ARFIMA (2, 0, 0)**	7456.623	-14889.543	0.0153
Aaa - A	0.0670 (0.0219)	[0.0240126, 0.1099781]	3.0549 (0.0023)	**ARFIMA (1, 0.067, 0)**	7178.899	-14341.996	0.0169
Aaa - Baa	0.0268 (0.0318)	[-0.0354163, 0.0890466]	0.8445 (0.3984)	**ARFIMA (2, 0, 0)**	6955.246	-13886.788	0.0184
Aa - A	0.1106 (0.0276)	[0.0564210, 0.1647090]	4.0024 (0.0001)	**ARFIMA (2, 0.1106, 0)**	7980.496	-15937.290	0.0126
Aa - Baa	-0.2746 (0.0317)	[-0.3367465, -0.2124033]	-8.6560 (0.0000)	**ARFIMA (1, -0.2746, 1)**	7528.017	-15032.331	0.0149
A - Baa	-0.1783 (0.0717)	[-0.3187871, -0.0378637]	-2.4883 (0.0129)	**ARFIMA (2, -0.1783, 1)**	7501.129	-14970.655	0.0151

Notes to Table 8A: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

Nine $ARFIMA(p,d,q)$ models are estimated for each cointegrating equilibrium using *p*, *q* = 0, 1, 2. The table shows the best fitting model for each series that minimises the Bayesian Information Criterion.

Table 8B. Approximate Maximum Likelihood (AML) Estimation of the *ARFIMA*(*p*, *d*, *q*) Model for the Fractionally Differenced Cointegrating Equilibria of Bivariate Systems: Statistics for the *AR* and *MA* Coefficients

	MA(1) coefficient estimate and standard error	*t* statistic for MA(1) coefficient and p-value	AR(1) coefficient estimate and standard error	*t* statistic for AR(1) coefficient and p-value	AR(2) coefficient estimate and standard error	*t* statistic for AR(2) coefficient and p-value
Treas - Aaa	-	-	-0.1980 (0.0264)	-7.4945 (0.0000)	-	-
Treas - Aa	-	-	-0.1012 (0.0285)	-3.5462 (0.0004)	-	-
Treas - A	-	-	-0.0896 (0.0288)	-3.1095 (0.0019)	-	-
Treas - Baa	-	-	-0.0834 (0.0289)	-2.8805 (0.0040)	-	-
Aaa - Aa	-	-	-0.0927 (0.0378)	-2.4501 (0.0143)	0.0597 (0.0267)	2.2384 (0.0253)
Aaa - A	-	-	-0.1382 (0.0277)	-4.9875 (0.0000)	-	-
Aaa - Baa	-	-	-0.1062 (0.0376)	-2.8271 (0.0047)	0.0565 (0.0267)	2.1122 (0.0348)
Aa - A	-	-	-0.2152 (0.0333)	-6.4699 (0.0000)	-0.0396 (0.0256)	-1.5454 (0.1224)
Aa - Baa	0.7265 (0.0155)	46.9529 (0.0000)	0.9213 (0.0146)	63.0087 (0.0000)	-	-
A - Baa	0.7080 (0.0237)	29.8258 (0.0000)	0.7736 (0.0860)	8.9950 (0.0000)	0.1024 (0.0522)	1.9596 (0.0501)

Notes to Table 8B: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

Nine $ARFIMA(p, d, q)$ models are estimated for each cointegrating equilibrium using *p*, *q* = 0, 1, 2. The table summarises the statistics for the *AR* and *MA* coefficients for the best fitting models in Table 8A.

Table 9. Approximate Maximum Likelihood (AML) Estimation of the *ARFIMA*(*p*, *d*, *q*) Model for the Cointegrating Equilibria of Bivariate Systems

	d estimate and standard error	*t* statistic for *d* and p-value	Best ARFIMA (p, d, q) model	Akaike Information Criterion
Treas - Aaa	0.0529 (0.0220)	2.41 (0.016)	**ARFIMA (2, 0.0529, 0)**	-12751.4252
Treas - Aa	1.0124 (0.0260)	38.9 (0.000)	**ARFIMA (0, 1, 1)**	-13156.5958
Treas - A	1.0035 (0.0359)	27.9 (0.000)	**ARFIMA (0, 1, 1)**	-12486.3046
Treas - Baa	0.0556 (0.0247)	2.25 (0.024)	**ARFIMA (2, 0.0556, 0)**	-12324.3922
Aaa - Aa	0.0272 (0.0293)	0.93 (0.352)	**ARFIMA (2, 0, 0)**	-14931.3722
Aaa - A	-0.0139 (0.0255)	-0.543 (0.587)	**ARFIMA (2, 0, 0)**	-14375.6265
Aaa - Baa	0.1088 (0.0288)	3.78 (0.000)	**ARFIMA (2, 0.1088, 0)**	-13924.4308
Aa - A	-0.0664 (0.0326)	-2.04 (0.042)	**ARFIMA (1, -0.0664, 1)**	-15971.7048
Aa - Baa	0.0393 (0.0245)	1.61 (0.108)	**ARFIMA (2, 0, 0)**	-15063.07
A - Baa	0.1050 (0.0504)	2.08 (0.037)	**ARFIMA (2, 0.1050, 1)**	-15010.9231

Notes to Table 9: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

Nine $ARFIMA(p, d, q)$ models are estimated for each cointegrating equilibrium using *p*, *q* = 0, 1, 2. The table shows the best fitting model for each series that minimises the Akaike Information Criterion.

Table 10A. Maximum Likelihood (ML) Estimation of the *ARFIMA*(*p, d, q*)-*GARCH*(*1, 1*)-*t* Model for Yields

	d estimate and standard error	*t* statistic for *d* and p-value	ARFIMA (p, d, q) model
Treas	1.0051 (0.0148)	68.10 (0.0000)	**ARFIMA (0, 1, 0)**
Aaa	1.0320 (0.0153)	67.59 (0.0000)	**ARFIMA (0, 1.0320, 0)**
Aa	1.0218 (0.0152)	67.17 (0.0000)	**ARFIMA (0, 1, 0)**
A	1.0257 (0.0155)	66.35 (0.0000)	**ARFIMA (0, 1, 0)**
Baa	1.0276 (0.0153)	67.10 (0.0000)	**ARFIMA (0, 1, 0)**
dTreas	-0.0003 (0.0149)	-0.0177 (0.9859)	**ARFIMA (0, 0, 0)**
dAaa	0.0241 (0.0154)	1.5650 (0.1178)	**ARFIMA (0, 0, 0)**
dAa	0.0146 (0.0154)	0.9479 (0.3433)	**ARFIMA (0, 0, 0)**
dA	0.0194 (0.0156)	1.2440 (0.2137)	**ARFIMA (0, 0, 0)**
dBaa	0.0225 (0.0155)	1.4540 (0.1461)	**ARFIMA (0, 0, 0)**

Notes to Table 10A: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve.

One $ARFIMA(p,d,q)-GARCH(1,1)-t$ model is estimated for each series in levels and first differences using the *p* and *q* values from the best fitting models of Table 4A.

Table 10B. Maximum Likelihood (ML) Estimation of the *ARFIMA*(*p, d, q*)-*GARCH*(*1, 1*)-*t* Model for Spreads

	d estimate and standard error	*t* statistic for *d* and p-value	ARFIMA (p, d, q) model
sTreasAaa	1.0112 (0.0170)	59.46 (0.0000)	**ARFIMA (2, 1, 0)**
sTreasAa	1.0426 (0.0213)	48.88 (0.0000)	**ARFIMA (2, 1.0426, 0)**
sTreasA	1.0641 (0.0207)	51.39 (0.0000)	**ARFIMA (2, 1.0641, 0)**
sTreasBaa	1.0936 (0.0233)	46.85 (0.0000)	**ARFIMA (2, 1.0936, 0)**
sAaaAa	0.9386 (0.0237)	39.55 (0.0000)	**ARFIMA (2, 0.9386, 0)**
sAaaA	-0.0196 (0.0280)	-0.6996 (0.4842)	**ARFIMA (2, 0, 0)**
sAaaBaa	0.0281 (0.0258)	1.089 (0.2761)	**ARFIMA (2, 0, 0)**
sAaA	0.9851 (0.0373)	26.41 (0.0000)	**ARFIMA (1, 0.9851, 1)**
sAaBaa	-0.0015 (0.0195)	-0.0771 (0.9385)	**ARFIMA (2, 0, 0)**
sABaa	-0.0037 (0.0197)	-0.1896 (0.8497)	**ARFIMA (2, 0, 0)**
dsTreasAaa	0.0101 (0.0169)	0.5958 (0.5514)	**ARFIMA (2, 0, 0)**
dsTreasAa	0.0402 (0.0211)	1.9070 (0.0566)	**ARFIMA (2, 0, 0)**
dsTreasA	0.0606 (0.0203)	2.9800 (0.0029)	**ARFIMA (2, 0.0606, 0)**
dsTreasBaa	0.0911 (0.0231)	3.9440 (0.0001)	**ARFIMA (2, 0.0911, 0)**
dsAaaAa	-0.0625 (0.0240)	-2.608 (0.0092)	**ARFIMA (2, -0.0625, 0)**
dsAaaA	-0.0324 (0.0175)	-1.856 (0.0635)	**ARFIMA (1, 0, 0)**
dsAaaBaa	0.0288 (0.0208)	1.3870 (0.1656)	**ARFIMA (1, 0, 0)**
dsAaA	-0.0611 (0.0237)	-2.5820 (0.0099)	**ARFIMA (2, -0.0611, 0)**
dsAaBaa	0.0396 (0.0422)	0.9397 (0.3475)	**ARFIMA (2, 0, 1)**
dsABaa	0.0434 (0.0405)	1.0700 (0.2848)	**ARFIMA (2, 0, 1)**

Notes to Table 10B: The statistics are based on 2703 daily observations from December 1992 to November 2003. The *Aaa*, *Aa*, *A* and *Baa* series are from Moody's and the 30-year constant maturity Treasury series is from the Federal Reserve. One $ARFIMA(p,d,q)-GARCH(1,1)-t$ model is estimated for each spread series in levels and first differences using the *p* and *q* values from the best fitting models of Table 4B.