Robust Longevity Risk Management: Reinsurance v.s. Capital Solution

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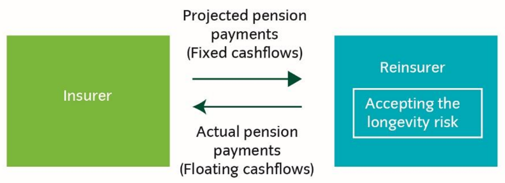
Longevity risk Transfer Market

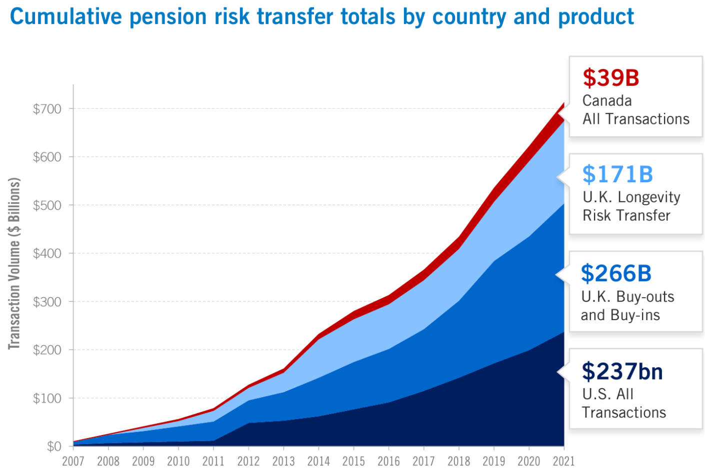
- Global population aging has called for effective means of longevity risk transfer from pension plans and life insurers to reinsurers.



- The longevity risk transfer market has been developing steadily over the past two decades. Brokerage and consultancy WTW forecasts over £40bn of bulk annuities and £20bn of longevity swaps transactions in 2023, making it the busiest year in history.[1]

[1] https://www.artemis.bm/news/20bn-of-longevity-swaps-forecast-for-2023-by-wtw/

Longevity risk Transfer Market Development



Data in USD billions. Cumulative totals. Sources: LIMRA, Hymans Robertson, LCP, and Prudential Financial, Inc. (PFI) analysis of EY 2020.

Longevity-Linked Capital Market

- Alternative longevity risk-transferring solution is the **longevity-linked capital market**, which is a way to share risks with capital market investors. (e.g. Reinsurance Sidecars)
- The benefit to **insurers** is that sidecars can provide protection against exposure to peak longevity risks. The benefit to **investors** is that they enjoy targeted non-correlated returns relating to specific short-horizon risks.
- However, capital market transactions have still been rather rare (Blake et al., 2019).
  - **Information Asymmetry**: Investors are concerned about the hedger having an informational advantage.
  - **Contract Duration Disparity**: There is the tension between the long-term nature of longevity risk and investor preference for a short-term investment horizon.

Static Hedging v.s. Dynamic Hedging

- **Static Hedging**: The risk exposure holder does not rebalance its hedging position overtime.
- **Dynamic Hedging**: The hedging position is adjusted over time.
- While static hedges have an initially defined and considerable cost, dynamic strategies have a cost, whose amount depends on the rebalancing technique, frequency, and the actual path of the underlying insurance contracts (De Rosa et al., 2017).
- Dynamic strategies are partial, leaving the insurer exposed to the longevity risk, but have potentially smaller costs than static hedges.

This paper

- This paper provides theoretical explains of the aforementioned observations in the longevity risk transfer market in the presence of ambiguity, i.e., the hedge provider (e.g., reinsurers or capital market investors) do not know the true mortality distribution of the hedger's portfolio.
- Under the presence of ambiguity, we show that:
  - the longevity risk transfer market could collapse;
  - the optimal hedging strategy (static vs. dynamic) depends on market sectors.

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Setup

- One hedger and one hedge provider, which could be a reinsurer or a general capital market investor.
- The hedger holds a whole-life annuity portfolio with $l_0^{(x)}$ annuitants aged x at time 0, each receiving \$1 per year for life.
- T: maximal remaining lifetime of the annuitants.
- $l_t^{(x)}$: the random number of remaining annuitants in year t.
- $l_{t+1}^{(x)} \sim B(l_t^{(x)}, \hat{p}_t^{(x)})$ under the reference measure $\mathbb{P}$.

Setup

- The hedger's wealth process is:

$$X_{t+1} = X_t(1+r) - l_{t+1}^{(x)}, \qquad X_0 = W_0$$

- Consider a longevity reinsurance/swap contract which exchanges a predetermined premium with a random cash flow linked to $l_t^{(x)}$ at each t:

$$X_{t+1} = X_t(1+r) - l_{t+1}^{(x)} - \left[u_t(1+\eta)\hat{l}_{t+1}^{(x)} - u_t l_{t+1}^{(x)}\right],$$

u_t: hedge rate.

- The hedge provider's wealth process is:

$$Y_{t+1} = Y_t(1+r) + (1+\eta)u_t\hat{l}_{t+1}^{(x)} - u_t l_{t+1}^{(x)}.$$

Contract Design

- Long-Term Contract: (**Static contract**):

$$u_t = u, \qquad \hat{l}_{t+1}^{(x)} = \mathbb{E}_0^{\mathbb{P}}\left[l_{t+1}^{(x)}\right] = {}_{t+1}\hat{p}_0^{(x)} l_0^{(x)}$$

- All $\hat{l}_{t+1}^{(x)}$'s determined at time 0 and $u_t = u$ — static hedging, most commonly seen contracts in reality.

- Short-Term Contract: (**Dynamic contract**):

$$u_t = u_t, \qquad \hat{l}_{t+1}^{(x)} = \mathbb{E}_t^{\mathbb{P}}\left[l_{t+1}^{(x)}\right] = \hat{p}_t^{(x)} l_t^{(x)}$$

- $\hat{l}_{t+1}^{(x)}$ depends on information up to t and time-varying u_t — dynamic contracts/hedging, extensively studied in the literature yet rarely seen in reality.

Ambiguity

- Traditional finance models assume a decision maker (DM) has a single view $\mathbb{P}$ on the stochastic price dynamics but, in practice, the decision maker (DM) may be uncertain about the true probabilistic model $\mathbb{P}$.
- In the presence of ambiguity, the DM holds different views on the precise distributions of the price dynamics.
- This type of model uncertainty due to multiple probabilistic views is called **ambiguity** (e.g., two priors $\mathbb{Q}^{\lambda_1}$ and $\mathbb{Q}^{\lambda_2}$)
- Risk: we do not know the outcome, but know the probability distribution.
- Ambiguity: the true probability distribution is also unknown.

Ambiguity Modeling: Maxmin Expected Utility

- The most widely discussed ambiguity criterion is the **maxmin expected utility** (MEU) model, introduced by Gilboa and Schmeidler (1989):

$$\inf_{\mathbb{Q}\in\mathcal{Q}} \mathbb{E}^{\mathbb{Q}}\left[U(X)\right], \tag{1}$$

  where $\mathcal{Q}$ is a set of prior probability measures.

- The DM's objective is to find an optimal control to the random payoff under the worst-case perspective and thus it leads to a max-min form.

Ambiguity in Longevity Transfers

- The hedger knows the true probability distribution of mortality for its annuitants, $\mathbb{Q}^\lambda$.
- $\mathbb{Q}^\lambda$ could be equal to $\mathbb{P}$, or could be something else.
- The hedge provider does not know $\mathbb{Q}^\lambda$, but instead consider a set of probability distributions characterized by $\lambda \in R$.
- Under $\mathbb{Q}^\lambda$, ${}_t\mathrm{p}_0^{(x)}(\lambda)$ is defined by:

$$ {}_t\mathrm{p}_0^{(x)}(\lambda) = ({}_t\mathrm{p}_0^{(x)})^\lambda, $$

where ${}_t\mathrm{p}_0^{(x)}$ is the probability under $\mathbb{P}$.

How to measure ambiguity

- The range of λ is based on the hedge provider's degree of uncertainty in annuitants' life expectancy:

$$e_{(x)} = \sum_{k=1}^{\infty} {}_k p_0^{(x)}.$$

Table: Changes of life expectancy and ambiguity

Change of $e_{(65)}$	Interval of λ
$\pm 5\%$ $e_{(65)}$	[0.86, 1.15]
$\pm 10\%$ $e_{(65)}$	[0.67, 1.51]
$\pm 15\%$ $e_{(65)}$	[0.64, 1.53]
$\pm 20\%$ $e_{(65)}$	[0.54, 1.76]
$\pm 40\%$ $e_{(65)}$	[0.24, 3.18]

Optimal Contracting

- The optimal parameters of the contract are searched via a Stackelberg game, where
  - the hedge provider is the leader;
  - the hedger is the follower.
- The game proceeds as follows:
  1. Given any contract price (η), the hedger determines its optimal $(u_t)_t$ process.
  2. With the hedger's response in mind, the hedge provider chooses the optimal η.
- Separate games are played with the long-term and short-term contracts, respectively.

Optimal Contracting

- The hedger's objective function:

$$\sup_{(u_t)_{(t=t_0,t_1,\ldots,T-1)}\in[0,1]} \mathbb{E}_t^{\mathbb{Q}^\lambda}\left[X_T\right] - \frac{\gamma_1}{2}\mathrm{Var}_t^{\mathbb{Q}^\lambda}\left[X_T\right]$$

- The hedge provider's objective function:

$$\sup_{\eta}\inf_{\mathbb{Q}\in\mathcal{Q}} \mathbb{E}^{\mathbb{Q}^\lambda}\left[Y_T\right] - \frac{\gamma_2}{2}\mathrm{Var}^{\mathbb{Q}^\lambda}\left[Y_T\right]. \tag{2}$$

- γ_1, γ_2: risk aversion parameters.

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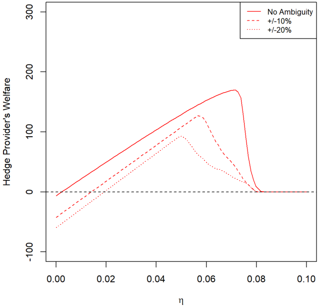
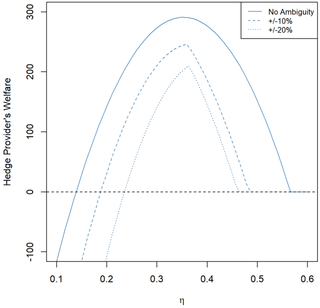
Market Settings

- **Risk Aversion Disparity**:
  - Insurance companies (hedgers) generally exhibit higher risk aversion due to their direct exposure to a myriad of individual policyholder risks — $\gamma_1 = 0.2$
  - Reinsurance companies (hedge providers) have lower risk aversion, stemming from their ability to spread risk across multiple insurers and maintain higher capitalization — $\gamma_2 = 0.1$

Hedge Provider's Welfare



(a) Short-Term Contract



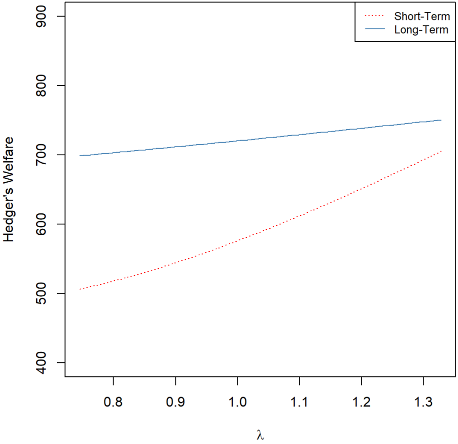
(b) Long-Term Contract

Figure: The hedge provider's welfare with the optimal contracts.

Observation

- **Ambiguity's Adverse Effects**: Uncertainty or ambiguity leads to a decline in welfare for both contract structures, underscoring the negative consequences of uncertainty on the hedge provider's welfare.
- **Constraining Effect of Ambiguity**: The market interval where positive welfare exists narrows with increasing ambiguity. This showcases how growth in ambiguity limits market existence.
- **Superiority of Long-term Contracts**: Despite the influence of ambiguity, the long-term contract continues to outperform the short-term contract concerning the hedge provider's welfare.

Hedger's welfare



(a) $\pm 10\% e_{(65)}$

Observation

- **Superiority of Long-term Contracts**: the long-term contract consistently outperforms the short-term contract across all prior probability distributions.
- **High Sensitivity of Short-term Contracts**: The welfare linked to the short-term contract is more sensitive to shifts in probability measures.
- The long-term contract transfers the longevity risk through the hedging horizon, whereas the short-term contract can be treated as a rolling strategy using 1-year contracts.

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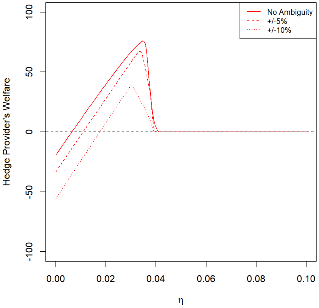
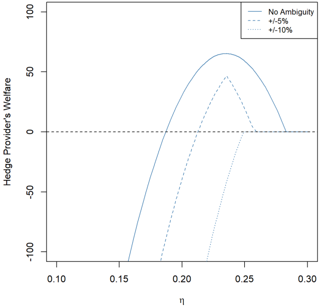
Market Settings

- The hedger could be a life (re)insurer, while the hedge provider is a general capital market investor (mutual fund, endowment, investment bank, etc).
- **Risk Aversion Disparity**:
  - Capital market investor's lesser familiarity with the nuances of life annuities and the underlying mortality risks can lead to a heightened perception of uncertainty.
  - $\gamma_1 = 0.1$ and $\gamma_2 = 0.3$.

Hedge Provider's Welfare



(a)



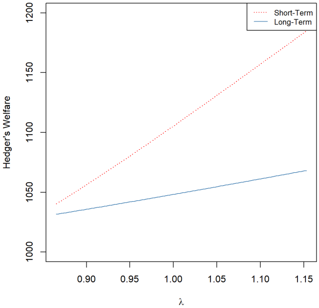
(b)

Figure: The hedge provider's welfare with the optimal contracts.

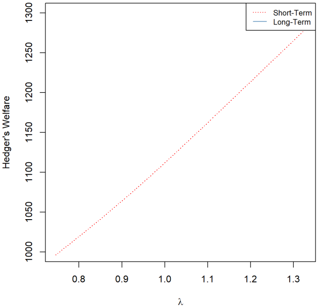
Observation

- **Adaptive nature of Short-Term Contracts**: The welfare associated with the short-term contract is slightly higher than that of the long-term contract within the capital market context.
- The short-term contract, due to its ability to update $\hat{l}_t$ over year based on realized outcomes, remains more robust against the presence of ambiguity.
- **Collapse of the Long-Term Contract Market**: As ambiguity levels escalate to $\pm 10\% \mathrm{e}_{(65)}$, the hedge provider's welfare becomes equal to or falls below zero, encompassing all potential risk loadings η.
- The short-term contract market remains viable even at the same level of ambiguity.
- **Implication:** The development of longevity-linked capital market could start with short-term transactions.

Hedger's Welfare



(a) $\pm 5\% \mathrm{e}_{(65)}$



(b) $\pm 10\% \mathrm{e}_{(65)}$

Figure: Hedger's welfare

Market Collapse Condition

Table: Market Collapse Condition (L & S)

	Long-Term	Short-Term
Reinsurance Market	$\pm 38\% e_{(65)}$	$\pm 35\% e_{(65)}$
Capital Market	$\pm 9\% e_{(65)}$	$\pm 22\% e_{(65)}$

1. The two markets prefer different contracts due to differences in risk preferences.
2. The reinsurance market generally displays a higher tolerance for ambiguity than the capital market.
3. The reinsurance market's established risk-sharing mechanisms and familiarity with life annuities may contribute to its greater tolerance against ambiguity.

Conclusion

- This paper examines the longevity risk transfers and optimal contract designs in the traditional reinsurance and capital market in the presence of ambiguity.
- We find that:
  - In the traditional market, both parties prefer the long-term contract due to its stable risk-sharing mechanism over an extended duration.
  - In the longevity-linked capital market, both parties prefer the short-term contract due to its inherent flexibility and adaptiveness.
  - Ambiguity could lead to market collapses.

Thank you for your attention!

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Mortality Model

- The Age-Period-Cohort-Improvement (APCI) model:

$$\ln(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)}(t - \bar{t}) + \kappa_t + \gamma_c,$$
$$\kappa_t = \kappa_{t-1} + \sigma_\kappa \varepsilon_t,$$

- Fitted to UK data, age 20 - 100, year 1956 - 2020.
- Insurance portfolio: $x = 65$.
- Reference measure: $\mathbb{P} = \mathbb{Q}^\lambda$ with $\lambda = 1$.

Parameters Setting

- the interest rate $r = 0.03$
- the length of contract $T = 35$
- the number of initial annuitants $l = 100$
- the age of annuitants $x = 65$

Blake, D., Cairns, A. J., Dowd, K., and Kessler, A. R. (2019). Still living with mortality: The longevity risk transfer market after one decade. *British Actuarial Journal*, 24:e1.

De Rosa, C., Luciano, E., and Regis, L. (2017). Basis risk in static versus dynamic longevity-risk hedging. *Scandinavian Actuarial Journal*, 2017(4):343–365.

Gilboa, I. and Schmeidler, D. (1989). Maxmin expected utility with a non-unique prior. *Journal of Mathematical Economics*, 18:141–153.